Anderson localization on random regular graphs and Cayley trees (arXiv:1604.05353)

K. Tikhonov [1,2] A. Mirlin [2,3] M. Skvortsov [1,4]

[1] Landau Institute for Theoretical Physics

[2] Institut für Nanotechnologie, Karlsruhe Institute of Technology

[3] Institut für Theorie der Kondenserten Materie, Karlsruhe Institute of Technology

[4] Skolkovo Institute of Science and Technology

June 28, 2016

Motivation: Case of a quantum dot

- Quantum dot Hamiltonian:

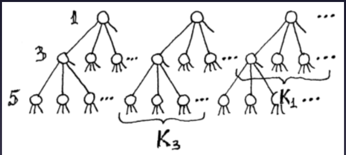
$$H = H_0 + H_1 = \sum_{\alpha} \epsilon_\alpha c^+_\alpha c_\alpha + \sum_{\alpha\beta\gamma\delta} V^{\alpha\beta}_{\gamma\delta} c^+_\gamma c^+_\delta c_\beta c_\alpha$$

Motivation: Case of a quantum dot

- Quantum dot Hamiltonian:

$$H = H_0 + H_1 = \sum_{\alpha} \epsilon_\alpha c_\alpha^+ c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\gamma\delta}^{\alpha\beta} c_\gamma^+ c_\delta^+ c_\beta c_\alpha$$

- Structure of perturbation theory reminds a Cayley tree (B Altshuler et al 1996):



$\Upsilon^\alpha = c_\alpha^+ \psi_{GS}$: one particle in the state α - first generation
$\Upsilon_\gamma^{\alpha\beta} = c_\alpha^+ c_\beta^+ c_\gamma \psi_{GS}$: two particles and one hole - second generation etc

Anderson localization on a Cayley tree

- Metal-insulator transition is well established for Anderson model (R Abou-Chacra, P Anderson, D Thouless 1973)

Anderson localization on a Cayley tree

- Metal-insulator transition is well established for Anderson model (R Abou-Chacra, P Anderson, D Thouless 1973)
- Various sigma-models ($N \gg 1$-orbital Anderson model) provide symmetry-breaking picture of AL (M Zirnbauer 1986, K Efetov 1987)

Anderson localization on a Cayley tree

- Metal-insulator transition is well established for Anderson model (R Abou-Chacra, P Anderson, D Thouless 1973)
- Various sigma-models ($N \gg 1$-orbital Anderson model) provide symmetry-breaking picture of AL (M Zirnbauer 1986, K Efetov 1987)
- Original Anderson model in supersymmetric treatment (A Mirlin, Y Fyodorov 1991)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)
- IPR in the localized phase (M Zirnbauer 1985)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)
- IPR in the localized phase (M Zirnbauer 1985)
- Density-density correlations (K Efetov 1987, A Mirlin Y Fyodorov 1991)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)
- IPR in the localized phase (M Zirnbauer 1985)
- Density-density correlations (K Efetov 1987, A Mirlin Y Fyodorov 1991)
- Distribution function of LDOS (A Mirlin, Y Fyodorov 1994)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)
- IPR in the localized phase (M Zirnbauer 1985)
- Density-density correlations (K Efetov 1987, A Mirlin Y Fyodorov 1991)
- Distribution function of LDOS (A Mirlin, Y Fyodorov 1994)
- Statistics of root-leaf transmission coefficients: numerics (C Monthus, T Garel 2009)

Anderson Localization on a Cayley Tree: physical quantities

- Position of the mobility edge (R Abou-Chacra et al 1973)
- IPR in the localized phase (M Zirnbauer 1985)
- Density-density correlations (K Efetov 1987, A Mirlin Y Fyodorov 1991)
- Distribution function of LDOS (A Mirlin, Y Fyodorov 1994)
- Statistics of root-leaf transmission coefficients: numerics (C Monthus, T Garel 2009)
- Statistics of wave function at root: this work

Sparse Random Matrix and Random Regular Graph ensembles

- Sparse random matrix ensemble (Y Fyodorov, A Mirlin 1991): $N \to \infty$ and finite mean number of elements per row p:

$$\mathcal{P}(H_{ij}) = (1 - \frac{p}{N})\delta(H_{ij}) + \frac{p}{N}h(H_{ij})$$

Sparse Random Matrix and Random Regular Graph ensembles

- Sparse random matrix ensemble (Y Fyodorov, A Mirlin 1991): $N \to \infty$ and finite mean number of elements per row p:

$$\mathcal{P}(H_{ij}) = (1 - \frac{p}{N})\delta(H_{ij}) + \frac{p}{N}h(H_{ij})$$

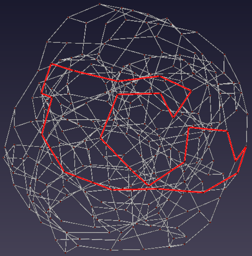
- Random Regular Graph ensemble
  1. Generate random graph G with given number of nodes N and constant connectivity K.
  2. Build adjacency matrix A of G - sparse matrix of 0 and 1 with KN non-zero elements (out of N^2).
  3. Add diagonal disorder: $H = A +$ diag$(w_1, w_2, ..., w_n)$ with $w_i \sim W$unif$(-0.5, 0.5)$

Random Regular Graphs: almost loop-less

- RRG with N vertices has small diameter $d \propto \log N$ and with high probability, the length of the shortest loop passing through a given vertex is $l \propto \log N$. As a result, RRG is locally tree-like

Random Regular Graphs: almost loop-less

- RRG with N vertices has small diameter $d \propto \log N$ and with high probability, the length of the shortest loop passing through a given vertex is $l \propto \log N$. As a result, RRG is locally tree-like
- Distance-preserving (approx.) embedding of RRG in 2D:



Anderson localization on a Random Regular Graph: physical quantities

- Spectral statistics: the joint distribution of $\{E_n\}$ (Y Fyodorov, A Mirlin 1991)

Anderson localization on a Random Regular Graph: physical quantities

- Spectral statistics: the joint distribution of $\{E_n\}$ (Y Fyodorov, A Mirlin 1991)
- Statistics of normalized eigenvectors ψ_n (Y Fyodorov, A Mirlin 1991)

Anderson localization on a Random Regular Graph: physical quantities

- Spectral statistics: the joint distribution of $\{E_n\}$ (Y Fyodorov, A Mirlin 1991)
- Statistics of normalized eigenvectors ψ_n (Y Fyodorov, A Mirlin 1991)
- Recent numerical studies of spectral and WF statistics (G Biroli et al 2012, A De Luca et al 2014, K Tikhonov et al 2016 etc)

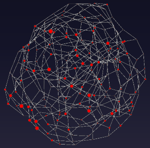
Wavefunctions on a Random Regular Graph: Weak and strong disorder limits

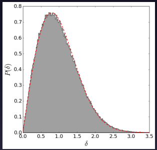
Typical wavefunction and PDF of the nearest level spacings:

Wavefunctions on a Random Regular Graph: Weak and strong disorder limits

Typical wavefunction and PDF of the nearest level spacings:

- Weak disorder $W = 5 \ll W_c$:

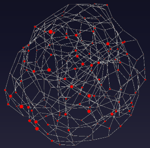


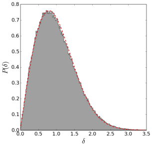


Wavefunctions on a Random Regular Graph: Weak and strong disorder limits

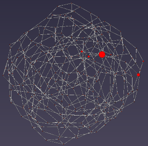
Typical wavefunction and PDF of the nearest level spacings:

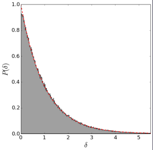
- Weak disorder $W = 5 \ll W_c$:





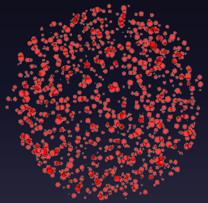
- Strong disorder $W = 25 \gg W_c$:



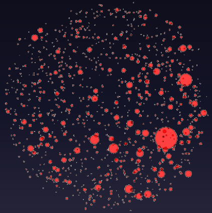


IPR and ergodicity. Inside the delocalized phase: $W = 5$, $W = 10$

- Typical WF in the delocalized phase: evolution with disorder:



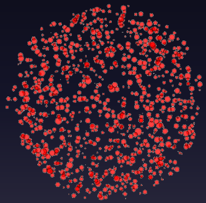
$W = 5$



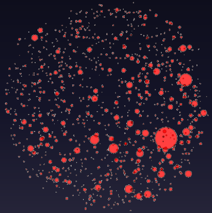
$W = 10$

IPR and ergodicity. Inside the delocalized phase: $W = 5$, $W = 10$

- Typical WF in the delocalized phase: evolution with disorder:



$W = 5$



$W = 10$

- Inverse Participation Ratio (IPR): $I_2 = N\left\langle |\psi|^4 \right\rangle$. For uniform spreading of the WF: $I_2(N \to \infty) = C/N$ with C independent on the system size.

Anderson localization on a Random Regular Graphs: summary of theoretical results

Theory predicts only one phase transition, at which, simultaneously:

- Delocalization transition happens: ψ becomes delocalized over macroscopic number of sites.

Anderson localization on a Random Regular Graphs: summary of theoretical results

Theory predicts only one phase transition, at which, simultaneously:

- Delocalization transition happens: ψ becomes delocalized over macroscopic number of sites.
- Spectral statistics turns to Wigner-Dyson: level repulsion establishes.

Anderson localization on a Random Regular Graphs: summary of theoretical results

Theory predicts only one phase transition, at which, simultaneously:

- Delocalization transition happens: ψ becomes delocalized over macroscopic number of sites.
- Spectral statistics turns to Wigner-Dyson: level repulsion establishes.
- Eigenfunctions become ergodic, being spread uniformly all over the system.

Anderson localization on a Random Regular Graphs: Recent numerical studies

- Exact diagonalization on RRG ($m = 2$) with diagonal disorder: G Biroli et al (2012, unpublished), A. De Luca et al (2014)

Anderson localization on a Random Regular Graphs: Recent numerical studies

- Exact diagonalization on RRG ($m = 2$) with diagonal disorder: G Biroli et al (2012, unpublished), A. De Luca et al (2014)
- Main claim: existence of delocalized non-ergodic phase on the RRG, characterized by fractal behaviour of wavefunctions at the delocalized side, i. e.:

$$I_2(N \to \infty) = CN^{-\alpha}$$

with $\alpha < 1$ in the thermodynamic limit of $N \to \infty$.

Anderson localization on a Random Regular Graphs: Recent numerical studies

- Exact diagonalization on RRG ($m = 2$) with diagonal disorder: G Biroli et al (2012, unpublished), A. De Luca et al (2014)
- Main claim: existence of delocalized non-ergodic phase on the RRG, characterized by fractal behaviour of wavefunctions at the delocalized side, i. e.:

$$I_2(N \to \infty) = CN^{-\alpha}$$

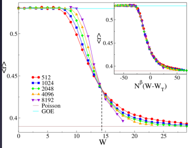
  with $\alpha < 1$ in the thermodynamic limit of $N \to \infty$.
- Contradicts existing analytical results and of (potentially) high impact.

Anderson localization on a Random Regular Graphs: Recent numerical studies

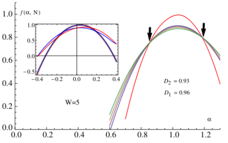
- Interpretation of numerics is a very delicate issue in the absence of analytical results for finite-N corrections.

Anderson localization on a Random Regular Graphs: Recent numerical studies

- Interpretation of numerics is a very delicate issue in the absence of analytical results for finite-N corrections.
- Crossing points put forward to advocate for existence of non-ergodic phases.



G. Biroli et al



A. De Luca et al

Interpretation of finite-size behaviour of IPR: running critical exponents $I_2 \sim N^{-\alpha}$

- Define pseudo-fractal exponent α as:

$$\alpha = -\frac{\partial \log I_2}{\partial \log N}.$$

Interpretation of finite-size behaviour of IPR: running critical exponents $I_2 \sim N^{-\alpha}$

- Define pseudo-fractal exponent α as:

$$\alpha = -\frac{\partial \log I_2}{\partial \log N}.$$

- 1. Localized phase: $\alpha(N \to \infty) = 0$
  2. Ergodic phase: $\alpha(N \to \infty) = 1$

Interpretation of finite-size behaviour of IPR: running critical exponents $I_2 \sim N^{-\alpha}$

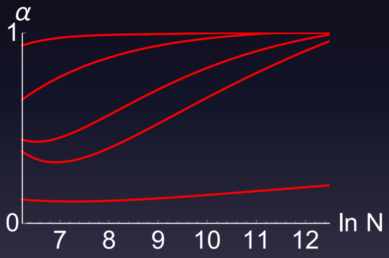
- Define pseudo-fractal exponent α as:

$$\alpha = -\frac{\partial \log I_2}{\partial \log N}.$$

- 1. Localized phase: $\alpha(N \to \infty) = 0$
  2. Ergodic phase: $\alpha(N \to \infty) = 1$
- Expectation on RRG: Delocalized $\equiv$ Ergodic

Interpretation of finite-size behaviour of IPR: running critical exponents $I_2 \sim N^{-\alpha}$

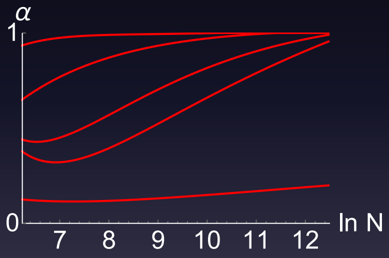
- Exact diagonalization with size up to $N = 262K$:



$\alpha(\ln N)$ for $W = 5, 8, 10, 11, 14$.

Interpretation of finite-size behaviour of IPR: running critical exponents $I_2 \sim N^{-\alpha}$

- Exact diagonalization with size up to $N = 262K$:

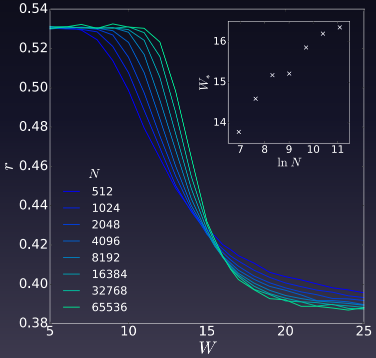


$\alpha(\ln N)$ for $W = 5, 8, 10, 11, 14$.

- Non-monotonous behaviour of $\alpha(\ln N)$ with minimum at $N_{min}(W)$.

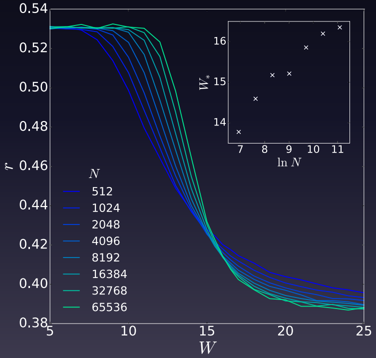
Interpretation of finite-size behaviour of IPR: spectral statistics

- Mean adjacent gap ratio: $r = \langle \delta_{i+1}/\delta_i \rangle$ as a function W for various N:



Interpretation of finite-size behaviour of IPR: spectral statistics

- Mean adjacent gap ratio: $r = \langle \delta_{i+1}/\delta_i \rangle$ as a function W for various N:



- Apparent crossing point $W_*(N)$ drifts logarithmically (alternatively: $N_*(W)$).

Critical length on the disordered RRG

- The crossover should happen at

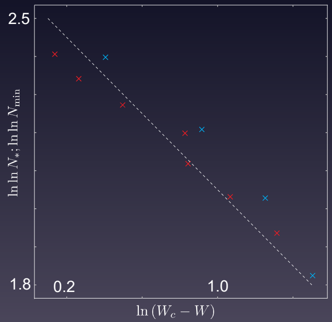
$$N \sim N_c \propto \exp\left[(W - W_c)^{-1/2}\right]$$

Critical length on the disordered RRG

- The crossover should happen at

$$N \sim N_c \propto \exp\left[(W - W_c)^{-1/2}\right]$$

- As extracted from $r(\ln N)$ and $\alpha(\ln N)$:

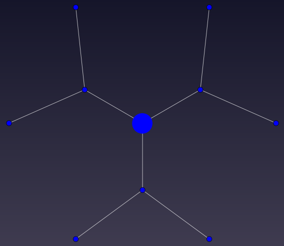


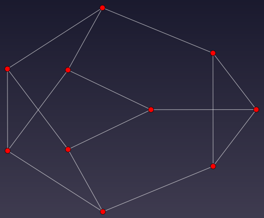
Random Regular Graph vs Cayley Tree

- Critical point: $N \to \infty$: $W^{(c)}_{RRG} \equiv W^{(c)}_{CT}$

Random Regular Graph vs Cayley Tree

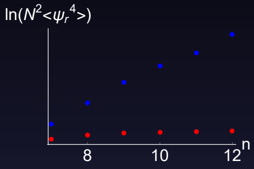
- Critical point: $N \to \infty$: $W^{(c)}_{RRG} \equiv W^{(c)}_{CT}$
- What can we say about finite-size behaviour? Compare statistics of ψ^4 at root of CT (n generations) and $\langle \psi^4 \rangle$ at RRG of the same size.



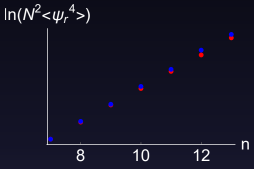


Random Regular Graph vs Cayley Tree

- Statistics of wavefunctions



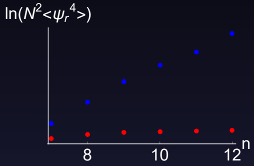
$W = 5$



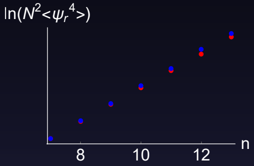
$W = 14$

Random Regular Graph vs Cayley Tree

- Statistics of wavefunctions

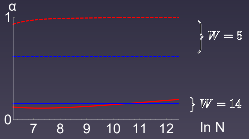


$W = 5$



$W = 14$

- In terms of (pseudo)-fractal exponent α:



Wavefunction statistics at finite-size CT

- For disordered electronic systems:

$$|\psi(r)|^2 = \frac{1}{4\pi\nu V} \lim_{\eta\to 0} \eta^{-1} \langle G_R(r,r) G_A(r,r) \rangle$$

Wavefunction statistics at finite-size CT

- For disordered electronic systems:

$$|\psi(r)|^2 = \frac{1}{4\pi\nu V} \lim_{\eta\to 0} \eta^{-1} \langle G_R(r,r) G_A(r,r) \rangle$$

- $$|\psi(r)|^2 = -\frac{1}{2\pi\nu\eta V} \lim_{\eta\to 0} \int Q_{11}(r) Q_{22}(r) e^{-S(Q)} dQ$$

  In the limit of $\eta \to 0$ only the dependence on $1 \lesssim \lambda_1 < \infty$ persists:

$$P(Q) \to P(\eta\lambda_1).$$

Wavefunction statistics at finite-size CT

- For disordered electronic systems:

$$|\psi(r)|^2 = \frac{1}{4\pi\nu V} \lim_{\eta\to 0} \eta^{-1} \langle G_R(r,r) G_A(r,r) \rangle$$

- $$|\psi(r)|^2 = -\frac{1}{2\pi\nu\eta V} \lim_{\eta\to 0} \int Q_{11}(r) Q_{22}(r) e^{-S(Q)} dQ$$

  In the limit of $\eta \to 0$ only the dependence on $1 \lesssim \lambda_1 < \infty$ persists:

$$P(Q) \to P(\eta\lambda_1).$$

- Explicit expressions for the integral kernels a cumbersome for realistic model, let us start with a toy example.

Simple example: Vector O(n,1) sigma-model on a CT

- Minimal model to study localization transition. Introduced as a toy model by Zirnbauer 1990. Later studied by I Gruzberg and A Mirlin 1996:

$$\mathcal{H} = J \sum_{ij} \vec{n}_i \cdot \vec{n}_j + \eta \sum_i \sigma_i$$

with $\vec{n} = (\sigma, \vec{\pi})$ for $n+1$ - component vector constrained by $\vec{n}^2 = \sigma^2 - \vec{\pi}^2 = 1$ and η for symmetry-breaking field.

Simple example: Vector O(n,1) sigma-model on a CT

- Minimal model to study localization transition. Introduced as a toy model by Zirnbauer 1990. Later studied by I Gruzberg and A Mirlin 1996:

$$\mathcal{H} = J \sum_{ij} \vec{n}_i \cdot \vec{n}_j + \eta \sum_i \sigma_i$$

  with $\vec{n} = (\sigma, \vec{\pi})$ for $n+1$ - component vector constrained by $\vec{n}^2 = \sigma^2 - \vec{\pi}^2 = 1$ and η for symmetry-breaking field.
- Shows phase transition at $J = J_c$ for $0 \le n < 1$. We will be interested in $n = 0$. For $m = 2$: $J_c \approx 0.026$.

Simple example: Vector O(n,1) sigma-model on a CT

- Minimal model to study localization transition. Introduced as a toy model by Zirnbauer 1990. Later studied by I Gruzberg and A Mirlin 1996:

$$\mathcal{H} = J \sum_{ij} \vec{n}_i \cdot \vec{n}_j + \eta \sum_i \sigma_i$$

  with $\vec{n} = (\sigma, \vec{\pi})$ for $n+1$ - component vector constrained by $\vec{n}^2 = \sigma^2 - \vec{\pi}^2 = 1$ and η for symmetry-breaking field.
- Shows phase transition at $J = J_c$ for $0 \leq n < 1$. We will be interested in $n = 0$. For $m = 2$: $J_c \approx 0.026$.
- Statistical interpretation: vertex-reinforced jump process (VRJP) on a tree.

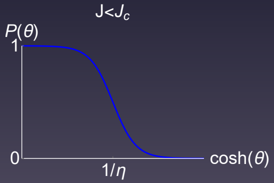
Vector $O(n,1)$ sigma-model: $N \to \infty$ phase transition

- At $N \to \infty$ distribution function of an order parameter $P(\vec{n})$ satisfies the integral equation:

$$P(\vec{n}) = \int d\vec{n}' L(\vec{n}, \vec{n}') D(\vec{n}') P^m(\vec{n}')$$

with $L(\vec{n}, \vec{n}') = e^{-J\vec{n}\cdot\vec{n}'}$ and $D(\vec{n}) = e^{-\eta\sigma}$.

- Does the order parameter $P(\theta)$ depend on η in the limit of $\eta \to 0$? The answer depends on J:



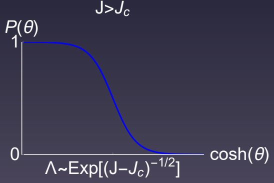
Vector $O(n,1)$ sigma-model: $N \to \infty$ phase transition

- At $N \to \infty$ distribution function of an order parameter $P(\vec{n})$ satisfies the integral equation:

$$P(\vec{n}) = \int d\vec{n}' L(\vec{n},\vec{n}') D(\vec{n}') P^m(\vec{n}')$$

with $L(\vec{n},\vec{n}') = e^{-J\vec{n}\cdot\vec{n}'}$ and $D(\vec{n}) = e^{-\eta\sigma}$.

- Does the order parameter $P(\theta)$ depend on η in the limit of $\eta \to 0$? The answer depends on J:



Vector $O(n,1)$ sigma-model: finite N, $\eta \to 0$.

- The symmetry breaking factor $D(\vec{n})$ is significant for $\theta \sim \ln 1/\eta$. Introduce $t = \ln(\eta \cosh\theta)$ and consider $\eta \to 0$.

Vector $O(n,1)$ sigma-model: finite N, $\eta \to 0$.

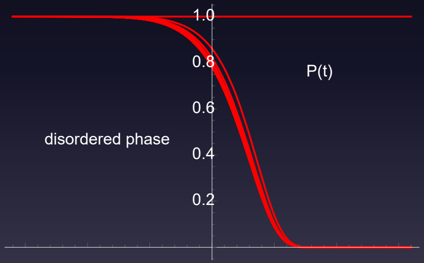
- The symmetry breaking factor $D(\vec{n})$ is significant for $\theta \sim \ln 1/\eta$. Introduce $t = \ln(\eta \cosh \theta)$ and consider $\eta \to 0$.
- Integration out of the nodes layer by layer ($n = 1, 2, ..., \ln N / \ln m$):

$$P_{n+1}(t) = \int L(t-t')e^{-e^{t'}} P_n^m(t')dt'$$

with $L(t) = \frac{1}{2K_{1/2}(J)} e^{t/2 - J\cosh t}$.

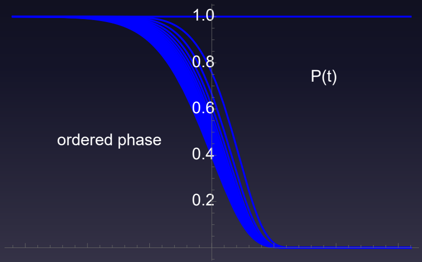
Tail analysis

- Iterations produce a drifting kink:



Tail analysis

- Iterations produce a drifting kink:



Tail analysis: cont.

- For $t \to -\infty$, where $P = 1 + \delta P(t)$:

$$\delta P_{n+1}(t) = m \int L(t - t')\delta P_n(t')dt'$$

Tail analysis: cont.

- For $t \to -\infty$, where $P = 1 + \delta P(t)$:

$$\delta P_{n+1}(t) = m \int L(t - t')\delta P_n(t')dt'$$

- $\delta P(t) = e^{\lambda t}$ are eigenfunctions of $\hat{L}$ with spectrum ϵ_λ

Tail analysis: cont.

- For $t \to -\infty$, where $P = 1 + \delta P(t)$:

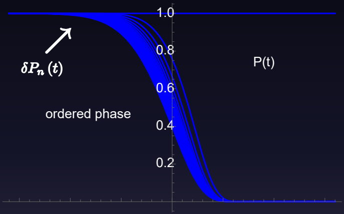
$$\delta P_{n+1}(t) = m \int L(t-t')\delta P_n(t')dt'$$

- $\delta P(t) = e^{\lambda t}$ are eigenfunctions of $\hat{L}$ with spectrum ϵ_λ

- In the ordered phase: $m\epsilon_\lambda > 1$, there is no solution for asymptotic self-consistent equation.

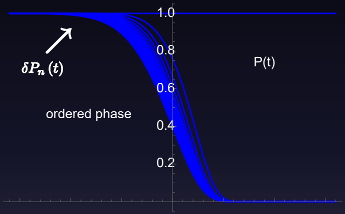
Tail analysis: cont.

- The drifting kink is formed instead



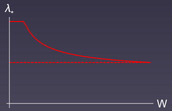
Tail analysis: cont.

- The drifting kink is formed instead



- The tail: $\delta P_n(t) = -\# e^{\lambda(t+\frac{n}{\lambda}\ln m\epsilon_\lambda)}$. On the linear level: $\frac{1}{2} \leq \lambda \leq 1$ are possible. Non-linearities dynamically select λ according to:

$$\frac{\ln m\epsilon_\lambda}{\lambda} \to \min$$



Wavefunction statistics at finite-size CT

- Analysis is exactly the same with more complicated kernels.

Wavefunction statistics at finite-size CT

- Analysis is exactly the same with more complicated kernels.
- Distribution function of $u = \psi_r^2$ is expressed in terms of $P(t)$:

$$\mathcal{P}(u) = N^{-1} \partial_u^2 P_{\ln N / \ln m}(t)$$

Wavefunction statistics at finite-size CT

- Analysis is exactly the same with more complicated kernels.
- Distribution function of $u = \psi_r^2$ is expressed in terms of $P(t)$:

$$\mathcal{P}(u) = N^{-1} \partial_u^2 P_{\ln N / \ln m}(t)$$

- Recall the drifting kink equation:

$$P_n(t) = \begin{cases} 1 - \# e^{\lambda(t + \alpha \ln N)}, \ t + \alpha \ln N < 0 \\ 0, \ \text{otherwise} \end{cases}$$

with $\alpha = \frac{\min_\lambda(\epsilon_\lambda/\lambda)}{\ln m}$.

Distribution function of wavefunction $u = \psi_r^2$ at the root

- As a result, WF at root is distributed according to:

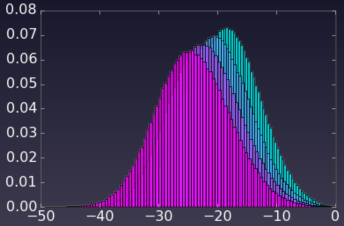
$$\mathcal{P}(u) = \begin{cases} N^{-1+\alpha\lambda_*} u^{\lambda_* - 2}, \ uN^\alpha \lesssim 1 \\ 0, \ \text{otherwise} \end{cases}$$

Distribution function of wavefunction $u = \psi_r^2$ at the root

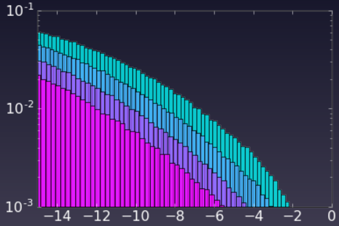
- As a result, WF at root is distributed according to:

$$\mathcal{P}(u) = \begin{cases} N^{-1+\alpha\lambda_*} u^{\lambda_*-2}, & uN^\alpha \lesssim 1 \\ 0, & \text{otherwise} \end{cases}$$

- Statistics of the WF from the numerics (W=14):



$\mathcal{P}(\ln \psi^2)$



$\ln \mathcal{P}(\ln \psi^2)$

Distribution function of wavefunction $u = \psi_r^2$ at the root

- For the 2nd moment at root:

$$N \left\langle \psi_r^4 \right\rangle = N^{-\alpha}.$$

Compare with $\alpha = 1$ for RRG at $N \to \infty$.

Distribution function of wavefunction $u = \psi_r^2$ at the root

- For the 2nd moment at root:

$$N \left\langle \psi_r^4 \right\rangle = N^{-\alpha}.$$

  Compare with $\alpha = 1$ for RRG at $N \to \infty$.
- More generally, for $q \geq 1/2$:

$$\left\langle \psi_r^{2q} \right\rangle = N^{-q+(q-1)(1-\alpha)}$$

Specific examples: $\alpha(J)$ for some concrete models.

- Exact ϵ_λ for SM are provided in the original papers.

Specific examples: $\alpha(J)$ for some concrete models.

- Exact ϵ_λ for SM are provided in the original papers.
- For Anderson model one may use quasi-ACAT approximation:

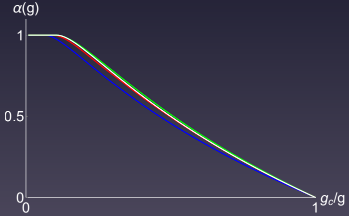
$$\epsilon_\lambda \approx \frac{1}{\lambda - 1/2} \frac{1}{W - 4/W} \left[(W/2)^{2\lambda - 1} - (W/2)^{-2\lambda + 1} \right]$$

Specific examples: $\alpha(J)$ for some concrete models.

- Exact ϵ_λ for SM are provided in the original papers.
- For Anderson model one may use quasi-ACAT approximation:

$$\epsilon_\lambda \approx \frac{1}{\lambda - 1/2} \frac{1}{W - 4/W} \left[(W/2)^{2\lambda - 1} - (W/2)^{-2\lambda + 1} \right]$$

- As a result, for $m = 2$ we obtain (introducing $g = W^{-2}$ for the Anderson model):

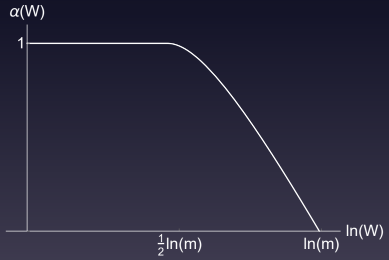


Large-m behaviour of Anderson model

- For $m \gg 1$ for all considered models $\alpha(g)$ collapse to a single curve.

Large-m behaviour of Anderson model

- For $m \gg 1$ for all considered models $\alpha(g)$ collapse to a single curve.
- Consider Anderson model as an example:



On recent preprint by B Altshuler at al arXive: 1605.02295

On recent preprint by B Altshuler at al arXive: 1605.02295

- It is admitted that the phase at $W < 10$ on RRG is ergodic.

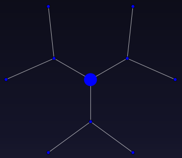
On recent preprint by B Altshuler at al arXive: 1605.02295

- It is admitted that the phase at $W < 10$ on RRG is ergodic.
- First-order phase transition point ($N \to \infty$) at $W = 10$ is proposed instead.
  Not consistent with exact diagonalization up to 262K for $W = 11$.

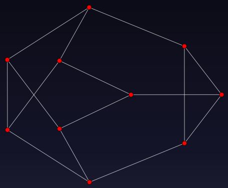
On recent preprint by B Altshuler at al arXive: 1605.02295

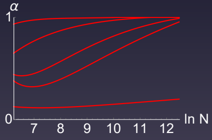
- It is admitted that the phase at $W < 10$ on RRG is ergodic.
- First-order phase transition point ($N \to \infty$) at $W = 10$ is proposed instead.
  Not consistent with exact diagonalization up to 262K for $W = 11$.
- Population dynamics: simulates a loop-less graph.
  Confirms the fractal statistics of the wavefunction at the root of a CT.

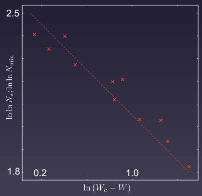
Conclusion

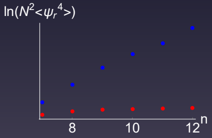


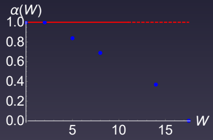
$\neq$











Outlook

- Deviations of the RRG ensemble from RMT at finite N

  For spectral properties: F Metz, G Parisi and L Leuzzi 2014

Outlook

- Deviations of the RRG ensemble from RMT at finite N

  For spectral properties: F Metz, G Parisi and L Leuzzi 2014

- Implications for Many-Body Localization

  For a nice recent discussion of connections of a quantum dot problem with hopping Hamiltonian on a Bethe lattice see I Gornyi, A Mirlin and D Polyakov 2016

Acknowledgements

- V. Kravtsov and M. Feigel’man for useful discussions

Acknowledgements

- V. Kravtsov and M. Feigel'man for useful discussions
- A. Prokofiev, A. Poenicke, L. Shchur , L. Barash, I. Oseledets for help with (free) computer power

Acknowledgements

- V. Kravtsov and M. Feigel'man for useful discussions
- A. Prokofiev, A. Poenicke, L. Shchur , L. Barash, I. Oseledets for help with (free) computer power

Thank You!