Probing many-body localized phase and delocalization transition with matrix elements

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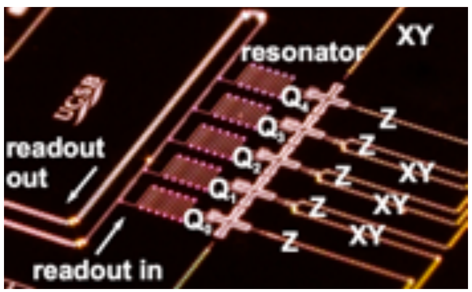
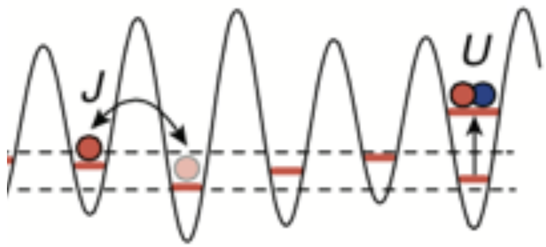
Chernogolovka, 2016

Artificial quantum systems

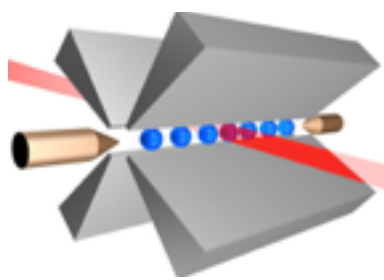
mutli-qubit systems



cold atoms



trapped ions

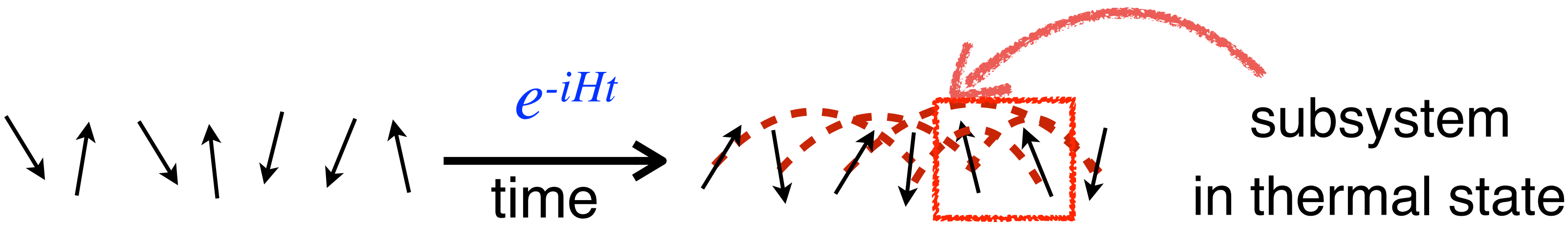


NV centers in diamond,
polar molecules,
....

Universality in **isolated quantum systems** out-of-equilibrium?

Outcomes of unitary dynamics

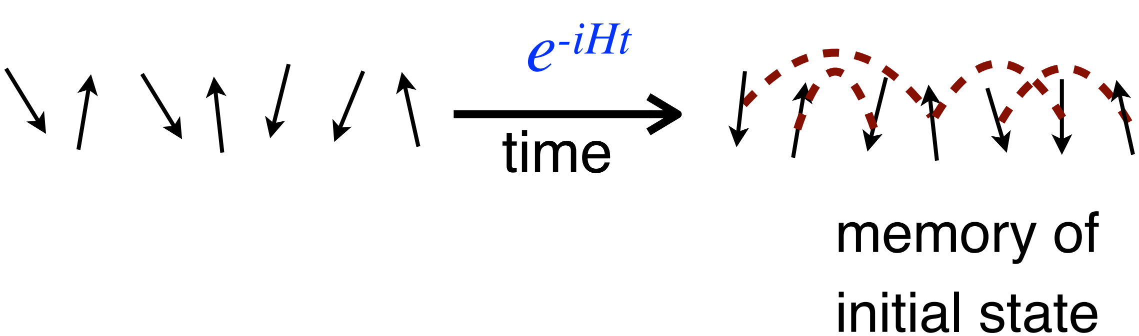
- Thermalization:



[Deutsch'91] [Srednicki'94]
[Rigol,Dunjko,Olshanii'08]

- MBL phase: breakdown of thermalization

[Anderson, Fleishman'80]
[Basko, Aleiner,Altshuler'05]
[Gorniy, Polyakov, Mirlin'05]
[Oganesyan,Huse'08]
[Znidaric,Prosen'08]
[Pal,Huse'10]



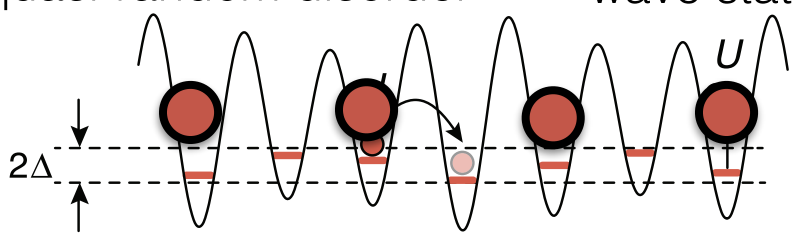
Experiments on MBL

- Signatures of localized phase with interactions:

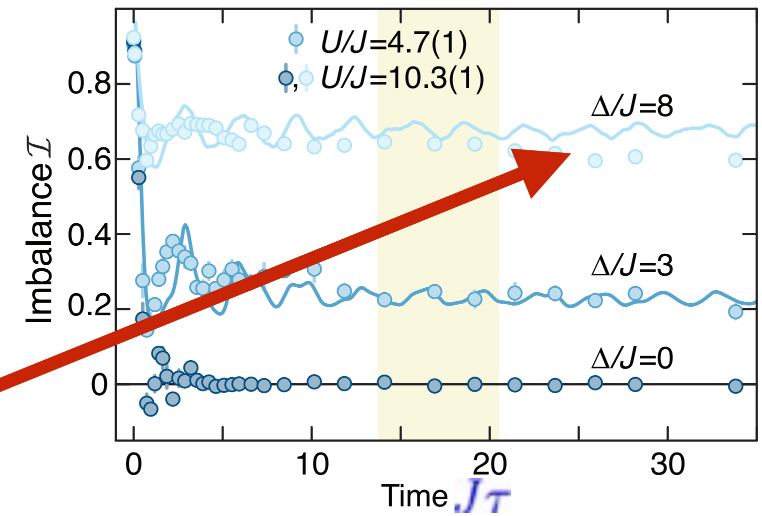
[M. Schreiber, et. al. Science'15] [P. Bordia, et. al. arXiv: 1509.00478]
also B. DeMarco, C. Monroe and others

interacting fermions& quasi-random disorder

initial density wave state



Imbalance $\mathcal{I} = \dfrac{N_e - N_o}{N_e + N_o}$ **saturates!**

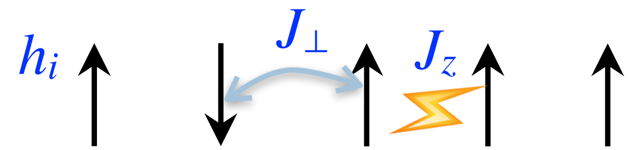


- MBL signatures in dynamics? Local observables?

Quasi-local conserved quantities

- If model is in MBL phase, rotate basis

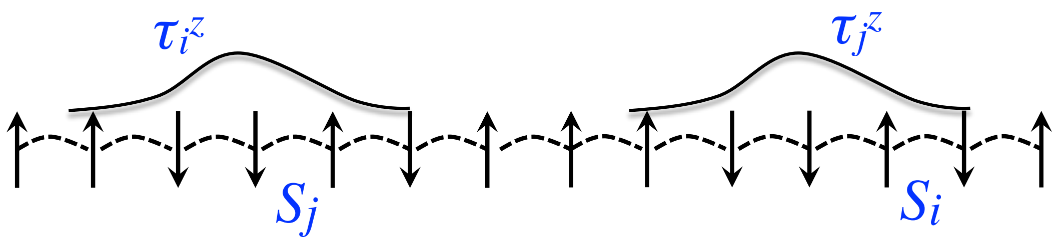
$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + h_i S_i^z$$



- Conserved spins: $\tau_i = U^\dagger \, S_i \, U$ quasi-local; complete basis

$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \sum_{ijk} H_{ijk} \tau_i^z \tau_j^z \tau_k^z + \dots$$

$$H_{ij} \propto \exp(-|i-j|/\xi)$$



- Consequences: no transport, ETH breakdown, power-law relaxation

[MS, Papic, Abanin '13]
[Huse, Oganesyan '13]
[Imbrie'14]

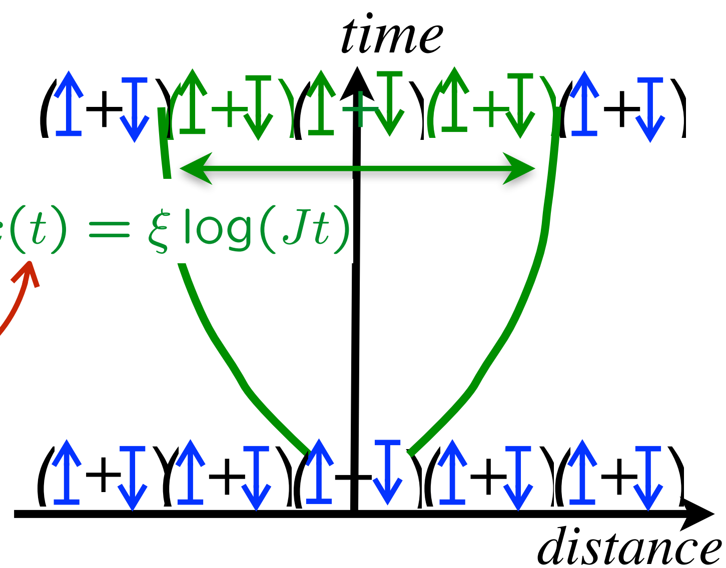
Dynamics in MBL phase

$$\hat{H} = \sum_i H_i \tau_i^z + \sum_{ij} H_{ij} \tau_i^z \tau_j^z + \sum_{ijk} H_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots \qquad H_{ij} \propto J e^{-|i-j|/\xi}$$

- Phases randomize

  on distance $x(t)$:

$$t H_{ij} = t J \exp(-x/\xi) \sim 1$$



- Logarithmic growth of entanglement [MS, Papic, Abanin, PRL'13]

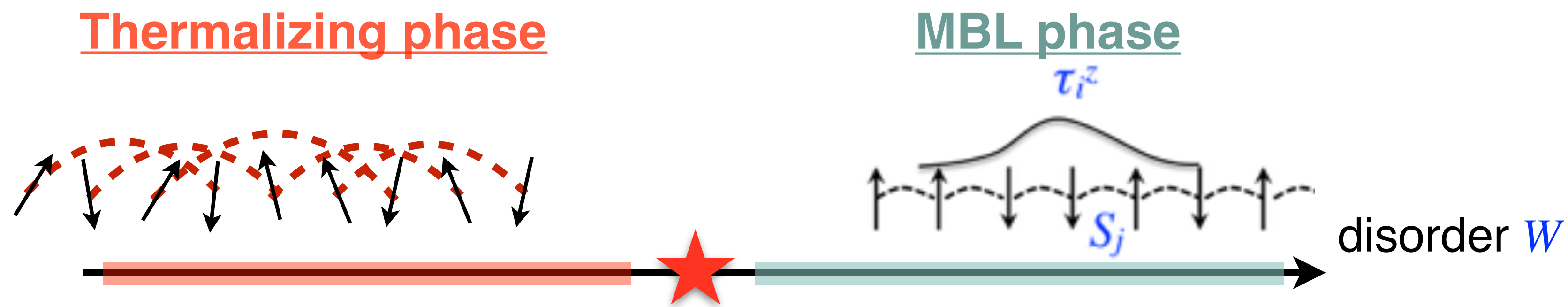
- Power-law relaxation of local observables:

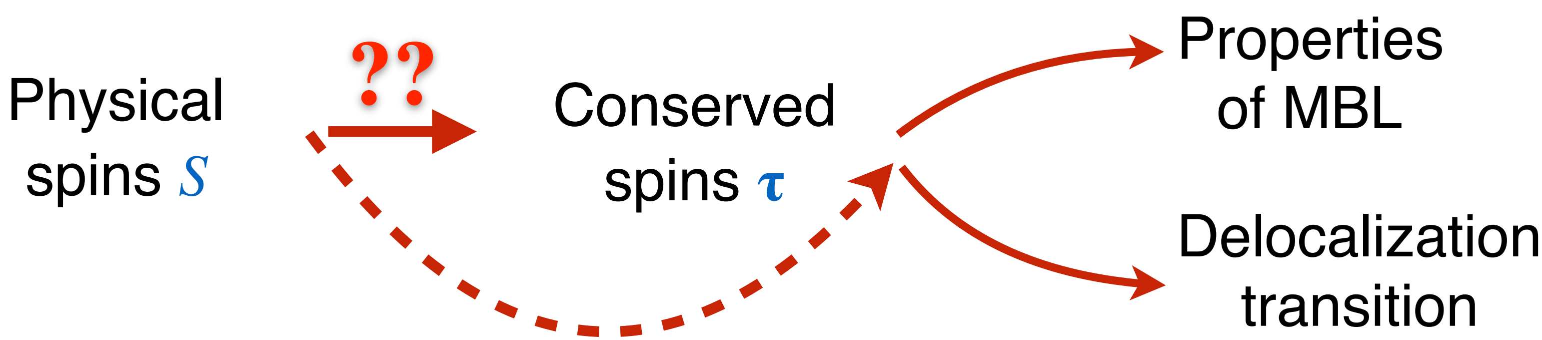
$$\left| \langle \hat{\mathcal{O}}(t) \rangle - \langle \mathcal{O}(\infty) \rangle \right| \sim \frac{1}{t^a}$$

[MS, Papic, Abanin, PRB'14]

memory of initial state

Outline





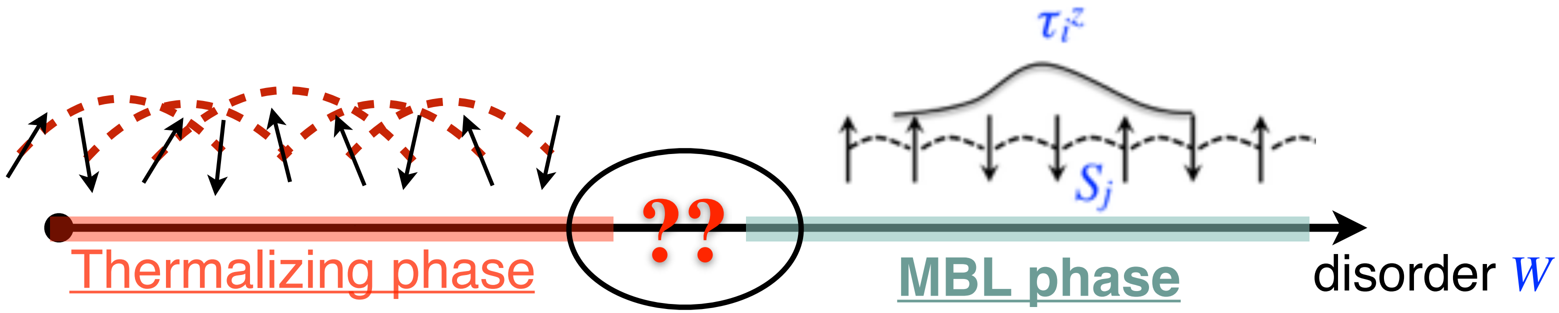
- **This talk:** matrix elements

1. Delocalization transition [MS, Papic, Abanin,PRX'15]
2. Entanglement spectrum in MBL phase

[MS, Michailidis, Abanin, Papic, arXiv:1605.05737]

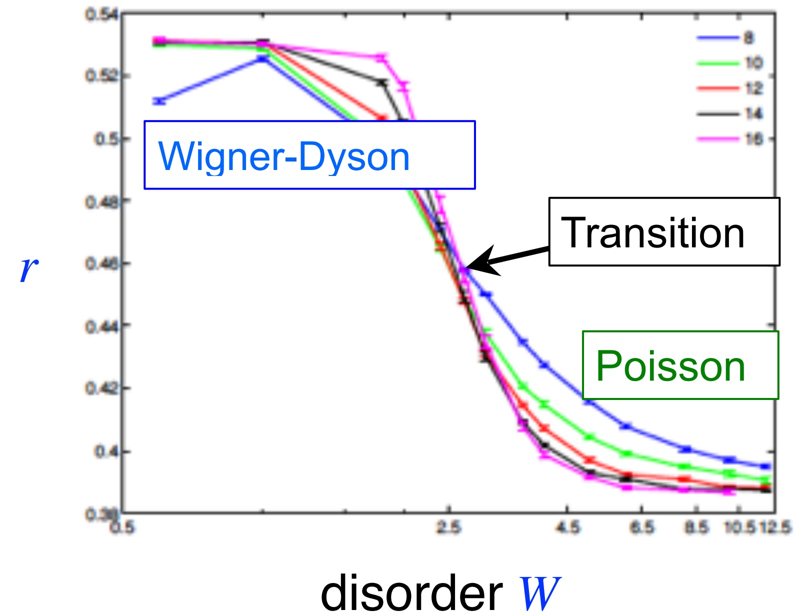
1. Delocalization transition

How to detect delocalization transition?



- Level statistics:

  [Oganesyan,Huse,PRB'07]
  [MS,Moore,PRB'16]



- Entanglement fluctuations in eigenstates, other probes

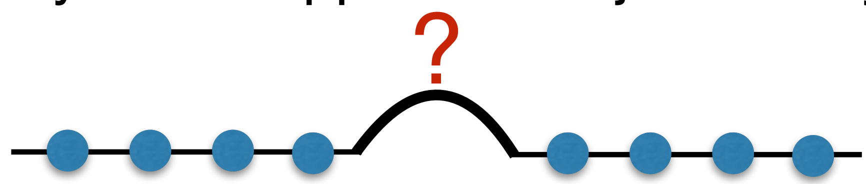
  [Kjall et al, PRL'14]
  [Luitz et al, PRB'15]

- Alternatives? Scaling parameter for transition?

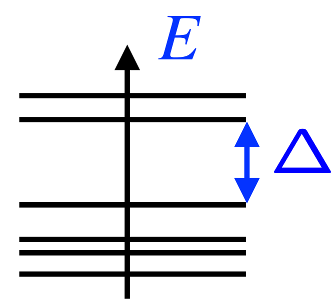
  [MS, Papic, Abanin,PRX'15]

Thouless conductance in Anderson localization

- How to quantify what happens if we join two systems?



- Coupling **vs** Level spacing $g = \frac{E_T}{\Delta}$

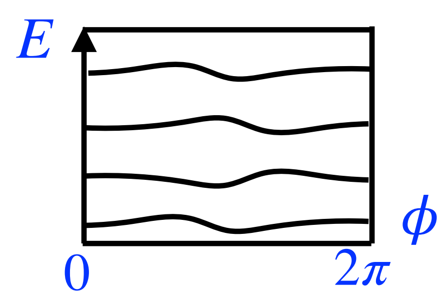


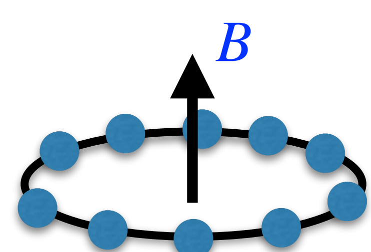
  - g>>1: states become extended
  - g<<1: localization!

- Scaling theory of localization: $g = \frac{\sigma L^{d-2}}{e^2/h}$ [Abrahams,Anderson, Licciardello,Ramakrishnan]

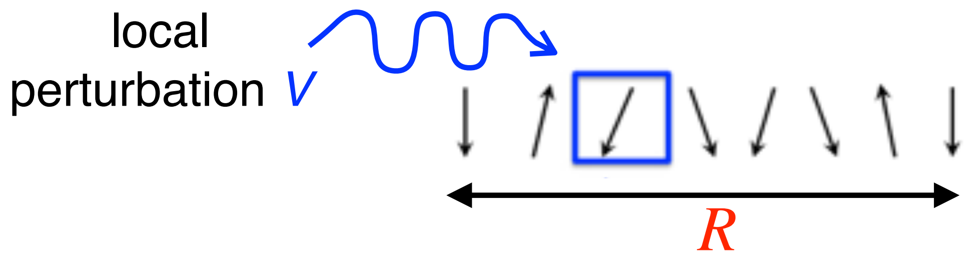
- Practical definition of *g*:

$$g = \frac{1}{\Delta}\frac{\partial^2 E}{\partial \phi^2}$$





Matrix elements of local operators



- Matrix elements from ETH

$$\langle i|S^z|j\rangle = e^{-S(E,R)/2} f(E_i, E_j) R_{ij}$$

[Srednicki'99]

narrow distribution:

$$\langle i|S^z|j\rangle \sim 1/\sqrt{2^R}$$

- Matrix elements from τ_i

$$S^z = \sum_{\{\alpha\}} \hat{\tau}^{\{\alpha\}} \hat{B}^{\{\alpha\}}[\tau^z]$$

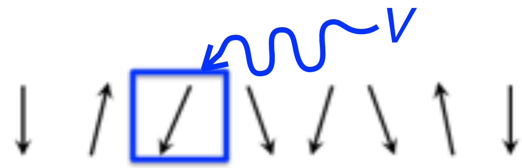
$\hat{B}^{\{\alpha\}}[\tau^z] \rightarrow \langle i|S^z|j\rangle$

broad distribution:

$$\langle i|S^z|j\rangle \sim \exp(-\kappa' R)$$

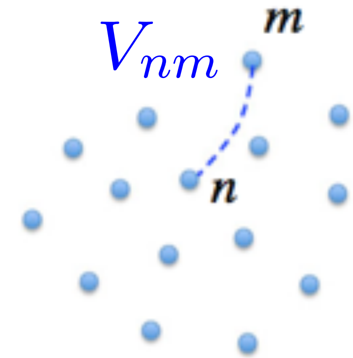
Many-body analogue of Thouless conductance

- Effect of local perturbation on eigenstates:



$$H \rightarrow H + V$$

$$H|n\rangle = E_n|n\rangle \qquad (H+V)|\alpha\rangle = E_\alpha|\alpha\rangle$$



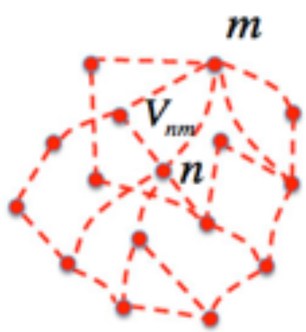
- New eigenstates are localized/delocalized?

- Parameter: $\mathcal{G} = \log \frac{V_{i,i+1}}{E_i - E_{i+1}}$

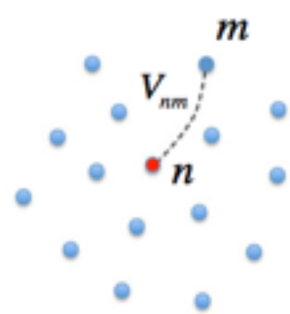
[MS, Papic, Abanin,PRX'15]

$\mathcal{G} \gg 1$

strong mixing
all spins perturbed



Delocalized



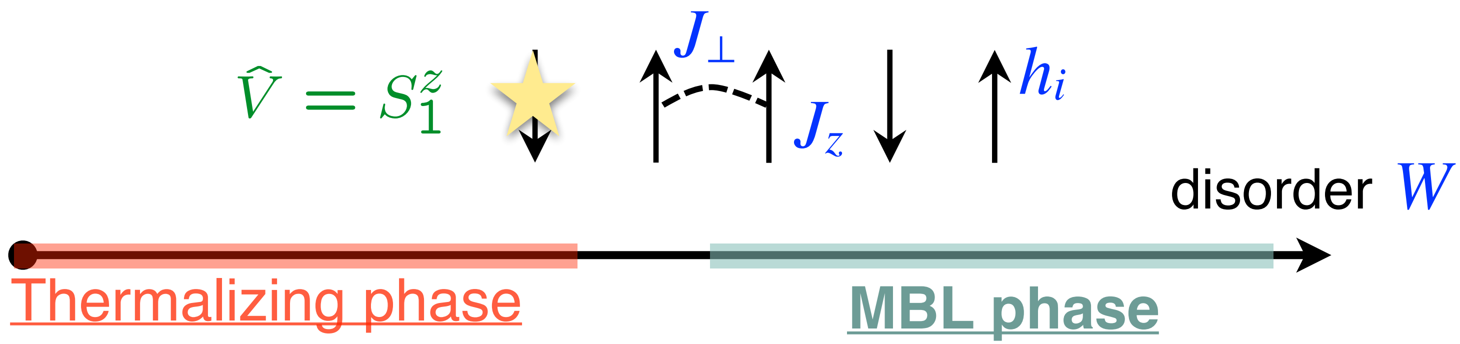
MBL phase

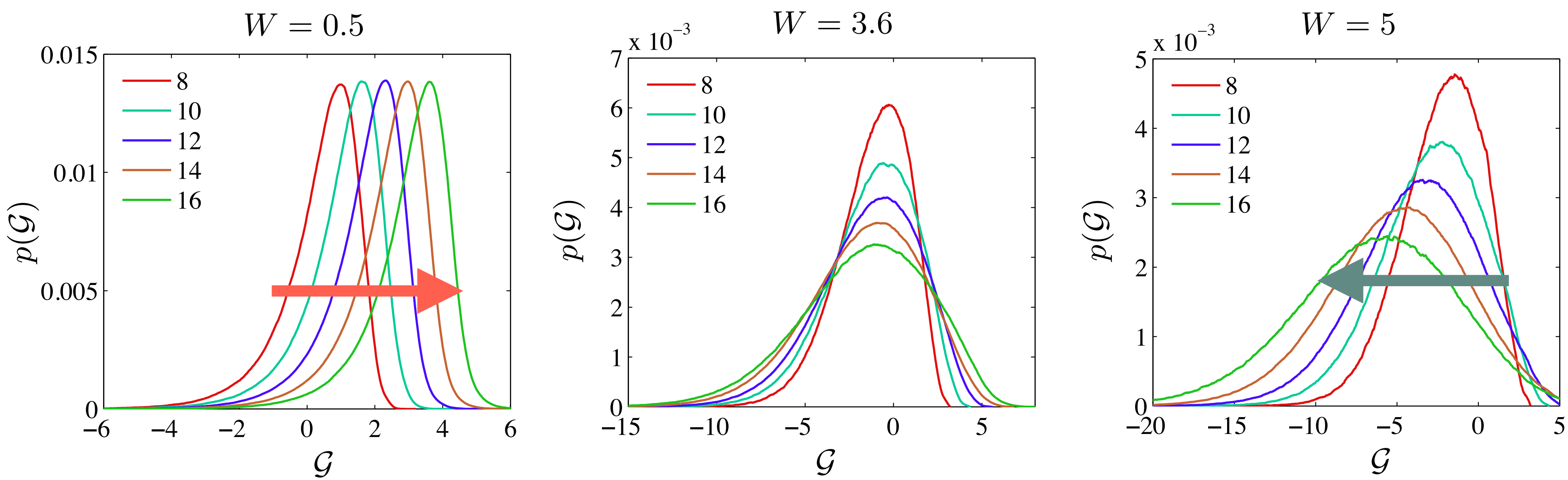
$\mathcal{G} \ll 1$

no resonances
τ are local

Distribution of Thouless conductance

- Scaling parameter: $\mathcal{G}(L) = \log \frac{V_{n,n+1}}{|E_n - E_{n+1}|}$
- Numerical results for XXZ spin chain: [MS, Papic, Abanin,PRX'15]



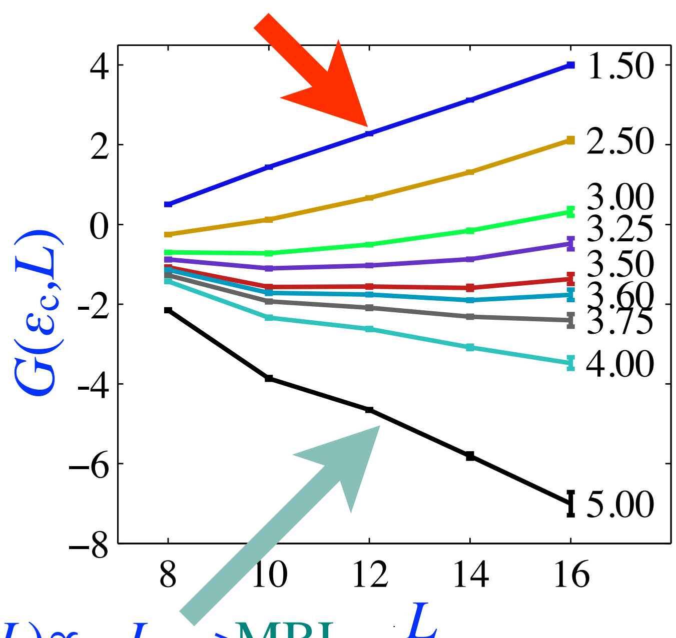


Scaling of G and many-body mobility edge

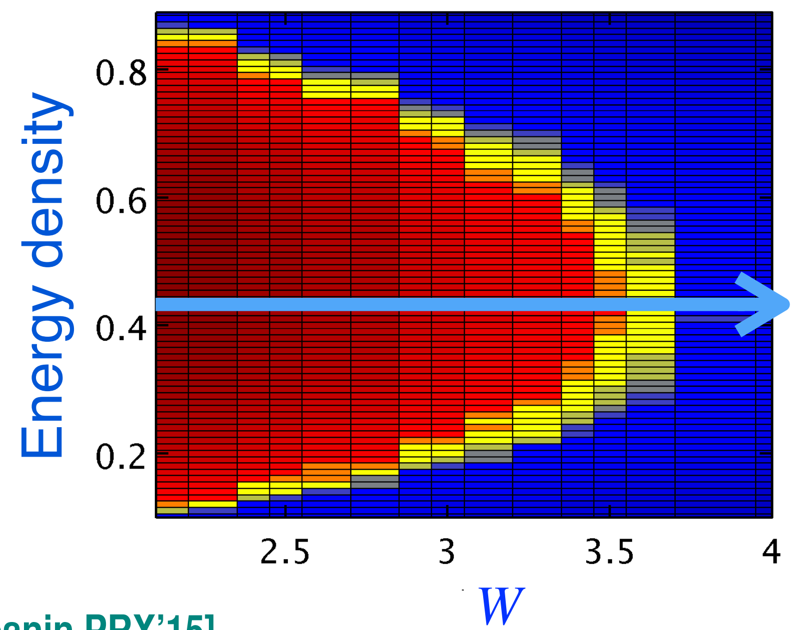
- Energy resolved $\mathcal{G}$: $$\mathcal{G}(L,\varepsilon) = \left\langle \log \frac{\langle i|S^z|j\rangle}{|E_i - E_j|} \right\rangle$$
- Numerical results for XXZ spin chain
- Exponent $\nu=0.7\pm0.1$ agrees with numerics for $L=22$ spins

[Luitz, Laflorencie, Alet PRB'15]

$G(\varepsilon,L)\propto +L$ →delocalized



$G(\varepsilon,L)\propto -L$ →MBL

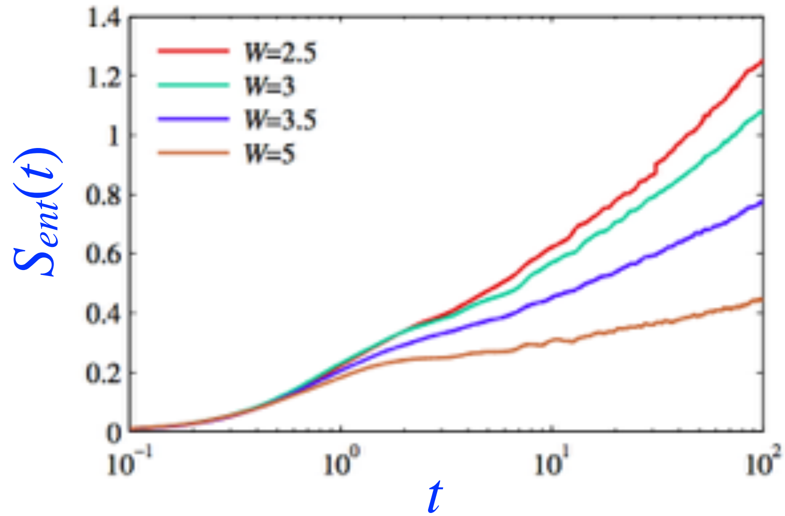


[MS, Papic, Abanin,PRX'15]

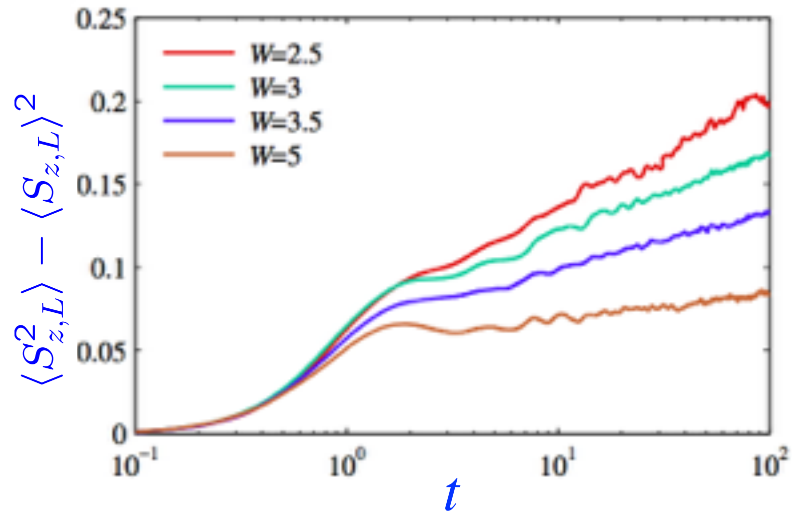
Dynamics at the transition

- Qualitative argument: $\mathcal{G}$(L) = const+O(log L) at transition→ logarithmically slow transport **[MS, Papic, Abanin,PRX'15]**
- tEBD simulations for L=24 spins:

$S_{ent}(t) \propto \log t$



$\langle S_{z,L}^2 \rangle - \langle S_{z,L} \rangle^2 \propto \log t$

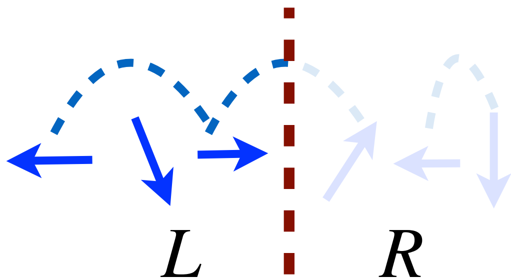


- Consistent with RG studies **[Vosk,Huse,Altman,PRX'15]** **[Potter,Vasseur,Parameswaran,PRX'15]**

2. MBL phase: entanglement spectrum

From entanglement entropy to spectrum

- "Quantumness" of the pure state:



trace out $R \to$

$$\rho_L = \mathrm{Tr}_R \, |\psi\rangle\langle\psi|$$

- Entanglement entropy: $S_{\mathrm{ent}} = -\sum_i \lambda_i \log \lambda_i$
  - ✳ ground states: probes topological order

    [Levin&Wen], [Kitaev&Preskill]

  - ✳ excited states: probes ergodicity

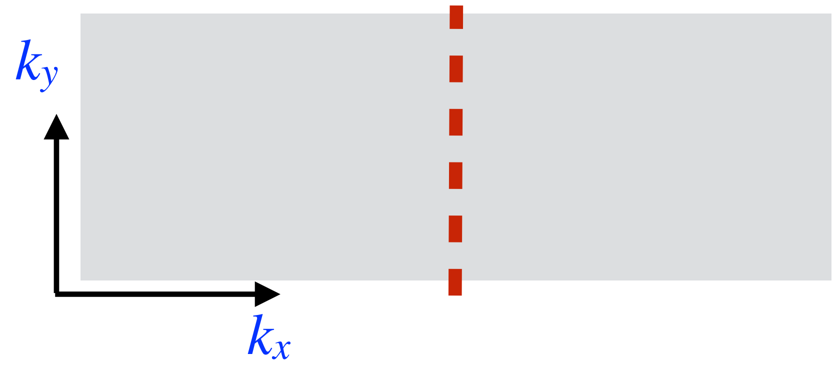
- Beyond entanglement? More information in $\{\lambda_\mathrm{i}\}$

  [Li & Haldane]

Organization of entanglement spectrum

- Quantum Hall wave function:

  k_y to organize ES
  [Li & Haldane]



- MBL phase: conserved quantities label ES

$$|\uparrow\uparrow\uparrow\uparrow\rangle = c_0 \; |\uparrow\uparrow\rangle|\uparrow\uparrow\rangle + e^{-\kappa} \underbrace{|\uparrow\downarrow\rangle}_{r=1}|\uparrow\uparrow\rangle + e^{-2\kappa} |\uparrow\downarrow\rangle|\downarrow\uparrow\rangle_{r=2} + \ldots$$

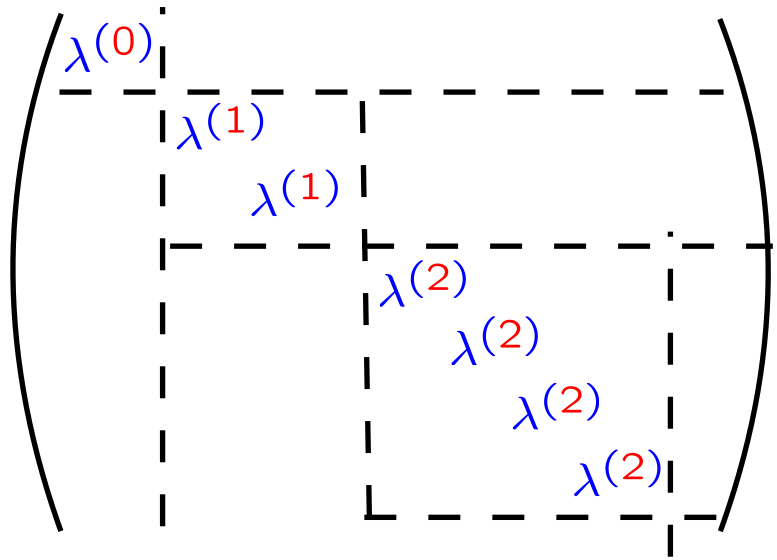
$$+ \; e^{-4\kappa} \underbrace{|\downarrow\downarrow\rangle|\downarrow\downarrow\rangle}_{r=4} + \; \ldots..$$

- Coefficients decay as $\left|C_{\uparrow\ldots\uparrow\underbrace{\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow}_{r}\uparrow\ldots\uparrow}\right| \propto e^{-\kappa r}$

Power-law entanglement spectrum

- Hierarchical structure of $\rho_L = \sum_{r=0}^{L} |\psi^{(r)}\rangle\langle\psi^{(r)}|$

$$\langle\psi^{(r)}|\psi^{(r)}\rangle \propto e^{-2\kappa r}$$ but non-orthogonal



- Orthogonalize perturbatively

$$\lambda^{(r)} \propto e^{-4\kappa r}$$

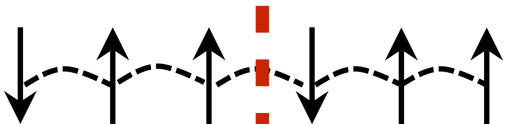
multiplicity is 2^r

- Power-law entanglement spectrum

$$\lambda_k \propto \frac{1}{k^\gamma} \qquad \gamma \approx \frac{4\kappa}{\ln 2}$$

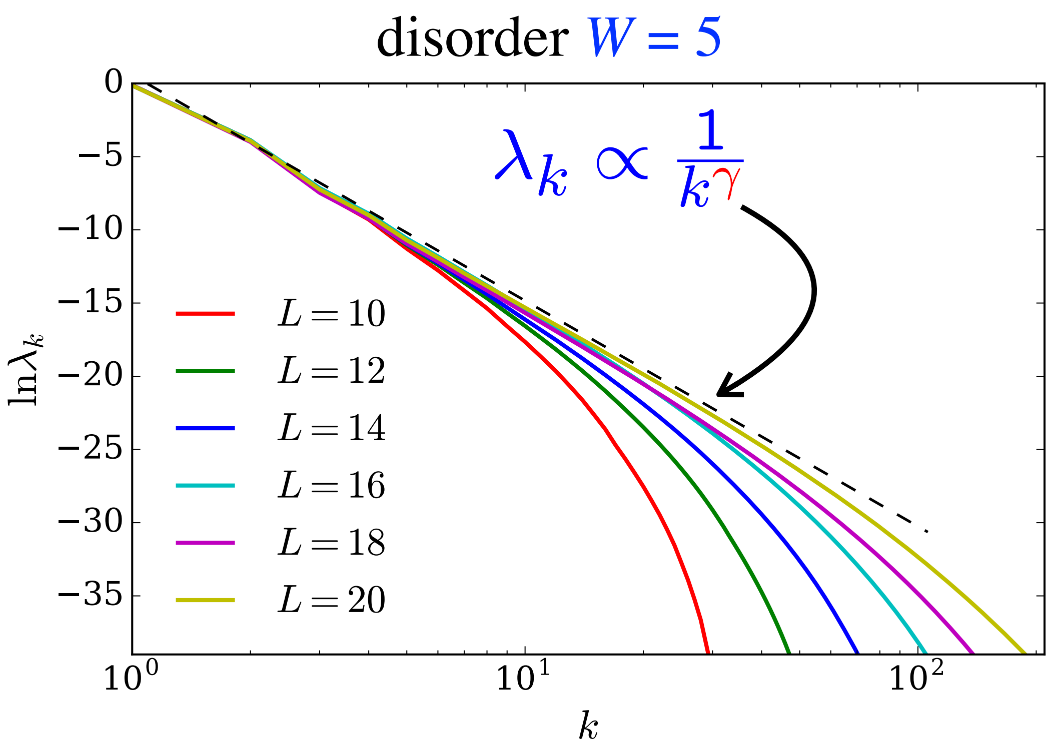
Numerics for XXZ spin chain

- Numerical studies for XXZ spin chain, $J_\perp = J_z = 1$



$$H = \sum_i (h_i S_i^z + J_\perp S_i^+ S_{i+1}^- + h.c.) + \sum_i J_z S_i^z S_{i+1}^z$$

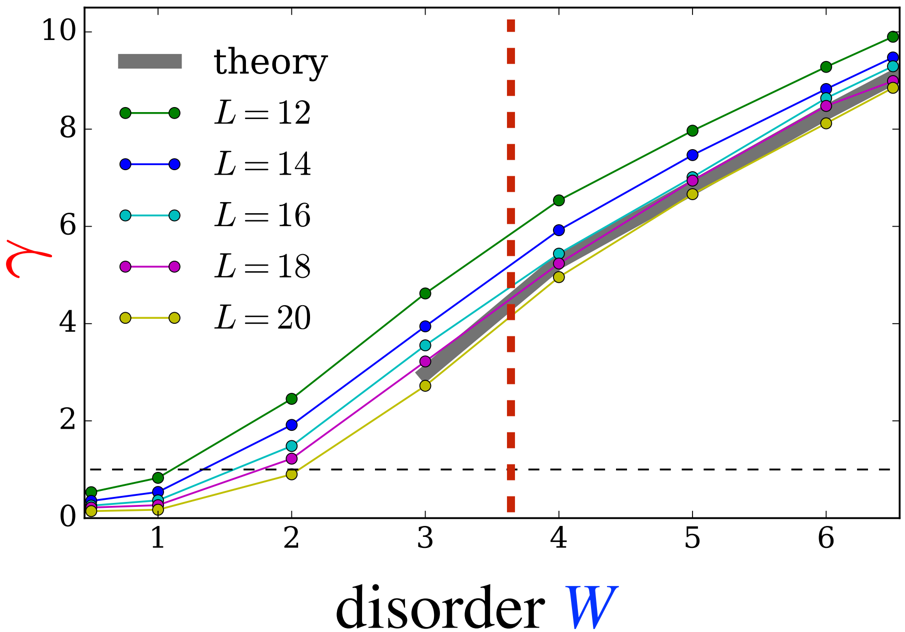
- Power law entanglement spectrum:



more details in:
[arXiv:1605.05737]

Decay of entanglement spectrum

- γ controls decay of entanglement spectrum $\lambda_k \propto \frac{1}{k^{\gamma}}$



$$\gamma \approx \frac{4\kappa}{\ln 2}$$

perturbation theory

$$\kappa = 2\kappa' + \ln 2$$

$$\mathcal{G}(L) \propto e^{-\kappa' L}$$

- Large value of γ → MPS description!

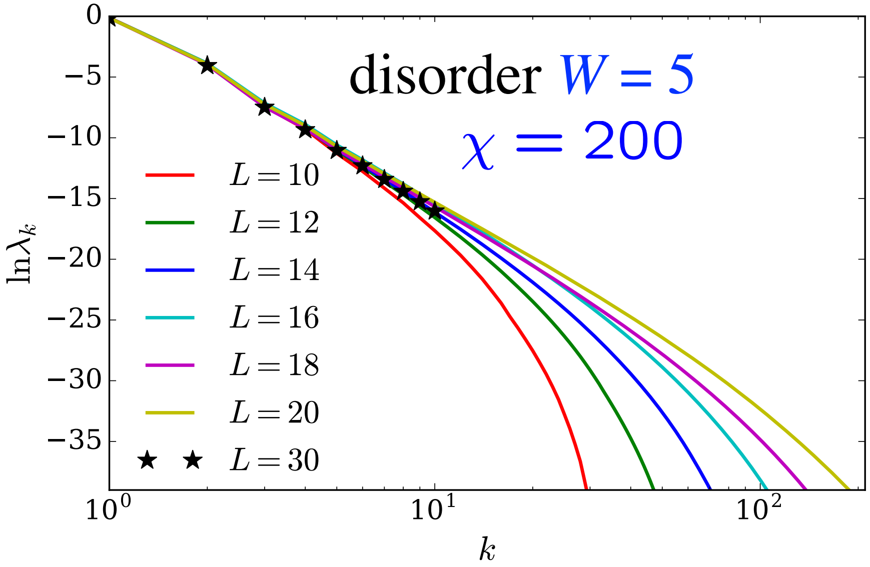
MPS algorithm close to MBL transition

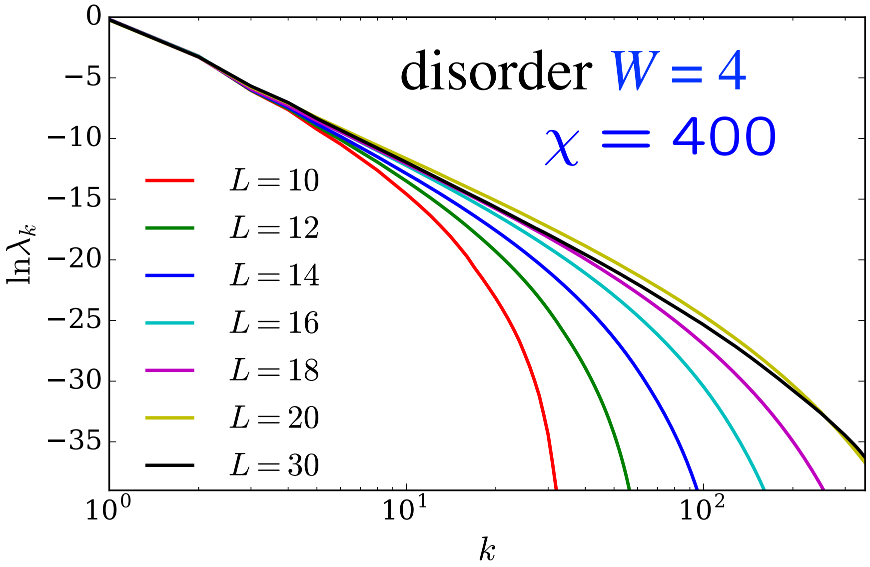
- Theoretically: γ controls truncation error $\propto 1/\chi^{\gamma-1}$

  large γ → MPS is ok close to MBL transition

- Practically: MPS-based algorithm in MBL phase

  use ES as a very stringent test for DMRG:





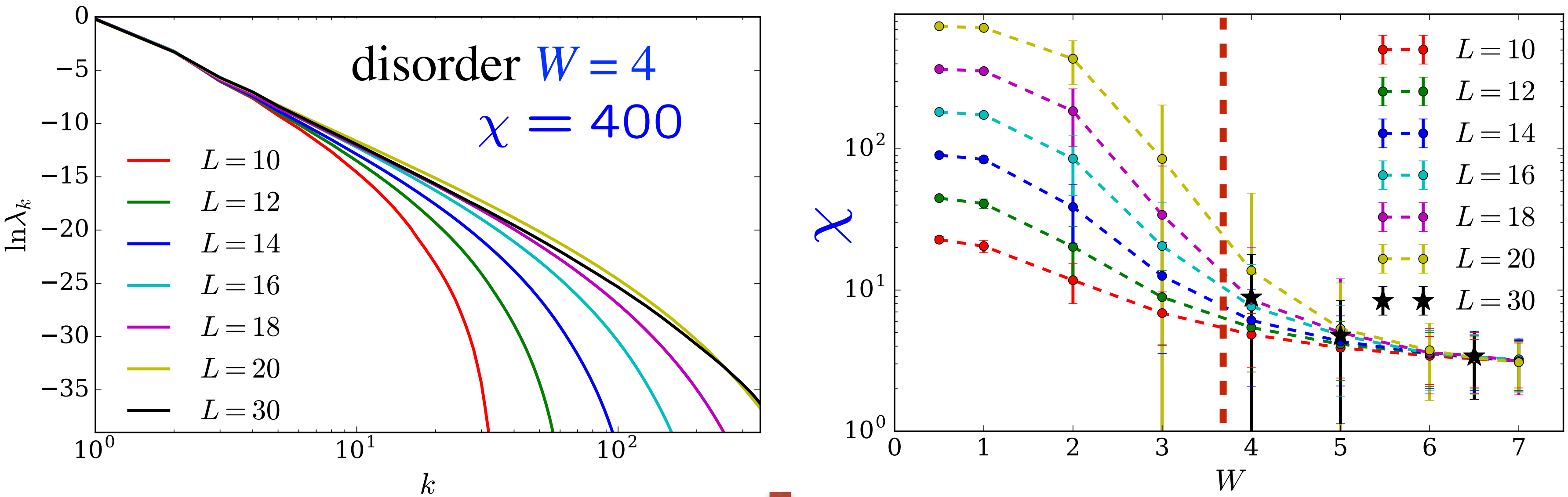
more details:
[arXiv:1605.05737]

also: [Yu et al arXiv:1509.01244] [Lim&Sheng arXiv:1510.08145] [Pollmann et al arXiv:1509.00483] [Kennes&Karrasch arXiv:1511.02205]

Estimates for the bond dimension

- Entanglement spectrum deviates at small λ

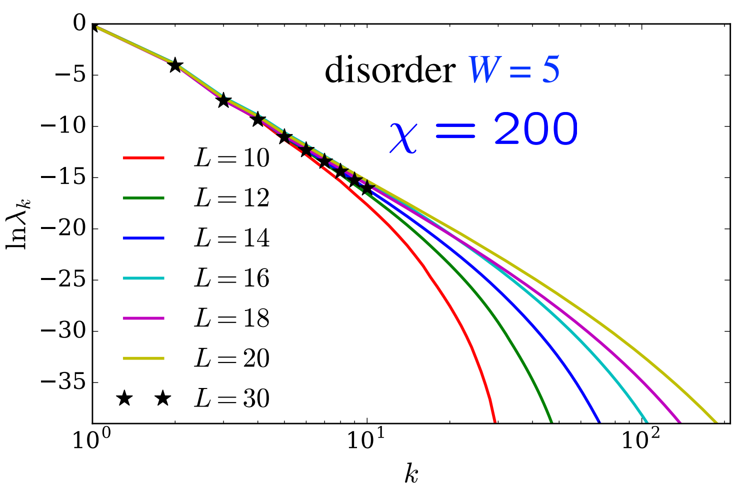
- Average bond dimension to converge S_{ent} up to 1%:

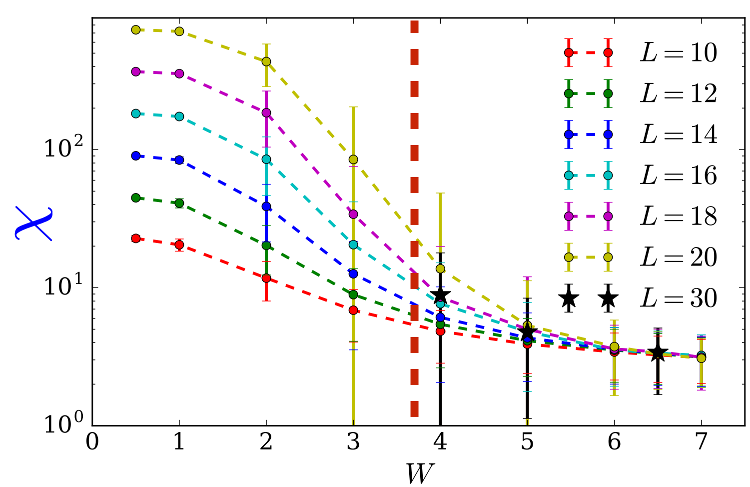


- Uses: DMRG close to MBL transition, probe MBL phase

Summary

- Scaling parameter for transition $\mathcal{G} = \log \frac{V_{i,i+1}}{E_i - E_{i+1}}$ [PRX 5, 041047 (2015)]
  →logarithmic transport at transition
- Power-law entanglement spectrum in MBL $\lambda_k \propto \frac{1}{k^{\gamma}}$ [arXiv:1605.05737]
  →power γ related to scaling of $\mathcal{G}$
  →implementation of MPS algorithm close to transition
- Global goal: delocalization transition & thermalization





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UC Berkeley

GORDON AND BETTY MOORE FOUNDATION

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