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ICTP The Abdus Salam International Centre for Theoretical Physics

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Markus Müller

Magnetoresistance in insulators close to the superconductor-insulator transition

Localization, Interactions and Superconductivity,
Landau Institute, Chernogolovka, June 27-July 1, 2016

Outline

- Huge magnetoresistance peak near certain SI transitions:
  Origin of **strongly non-monotonous** R(B)?

  - Insulator: Inhom. mixture of localized pairs and single electrons;
  - Understanding features of the peak?

- Nature of transport in the insulator ?

  - Strongly disordered superconductors:
    systems undergoing manybody localization??
  - Activated resistance in the Bose glass
  - GMR Peak: Interplay of interference and density of states effects!

Insulator: Giant magnetoresistance

Most studied material:

4 decades of research...

Buckley prize 2015...

Ever more puzzles and delights to come?

InOx

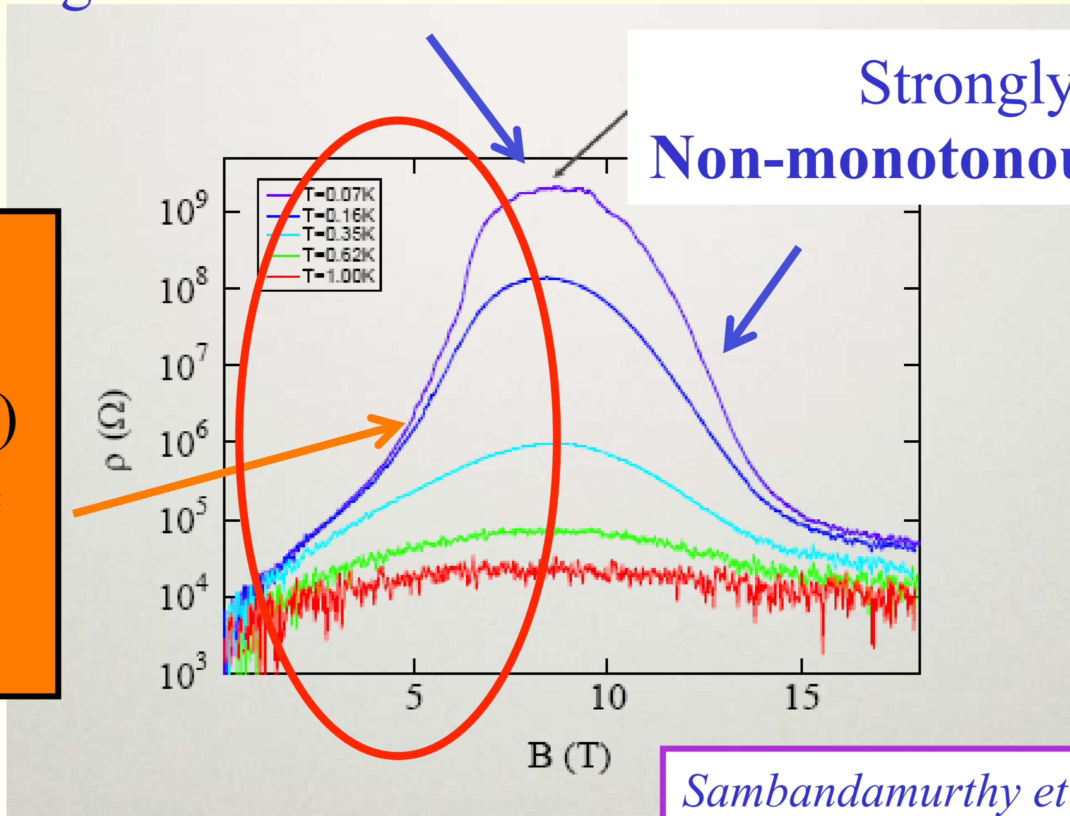
Hebard-Palaanen
Gantmakher
Kapitulnik, Mason
Goldman
Ovadyahu
Shahar
Sacepe, Chapelier
(many more)
...

Insulator: Giant magnetoresistance

Giant magnetoresistance

Strongly
Non-monotonous R(B)!

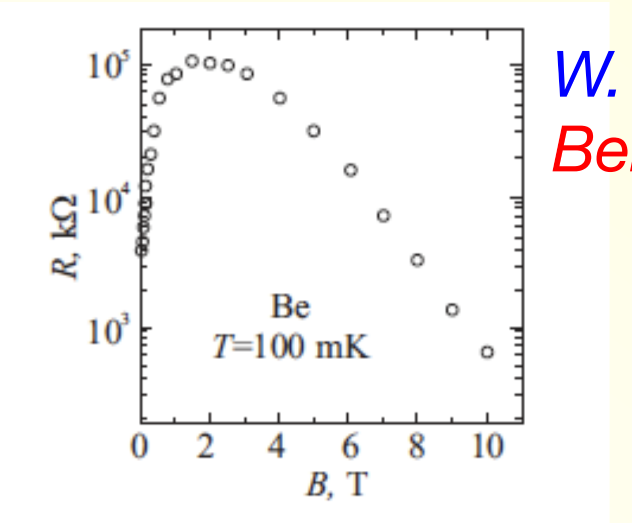
Common belief:
Pairs (bosons) survive in the insulator:
Bose glass



Sambandamurthy et al. (PRL 2005)

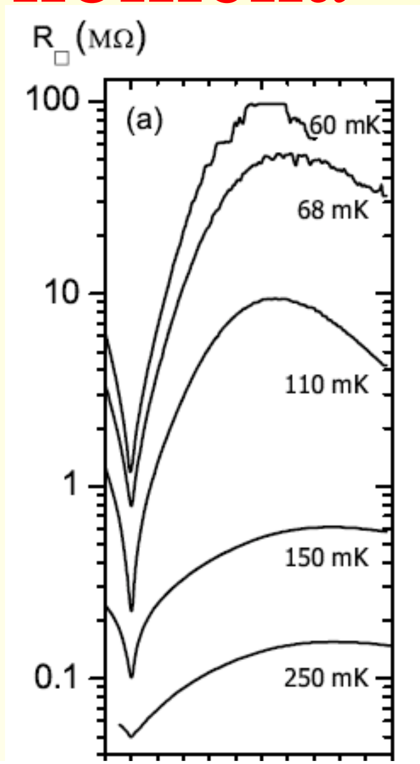
Insulating behavior **enhanced** by local superconductivity!

Other systems - similar phenomena

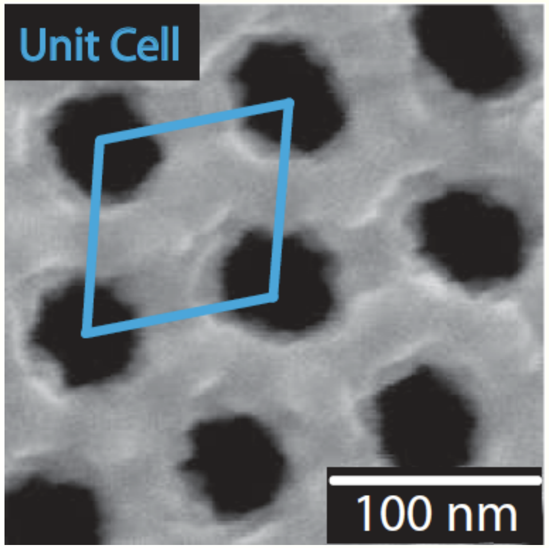


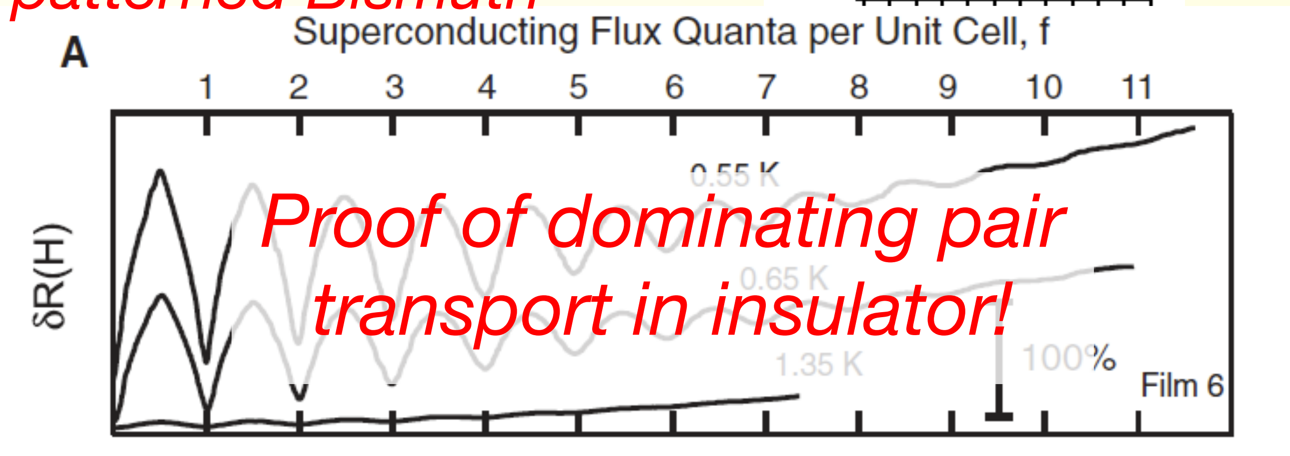
W. Wu - Berryllium

T. Baturina et al. - TiN



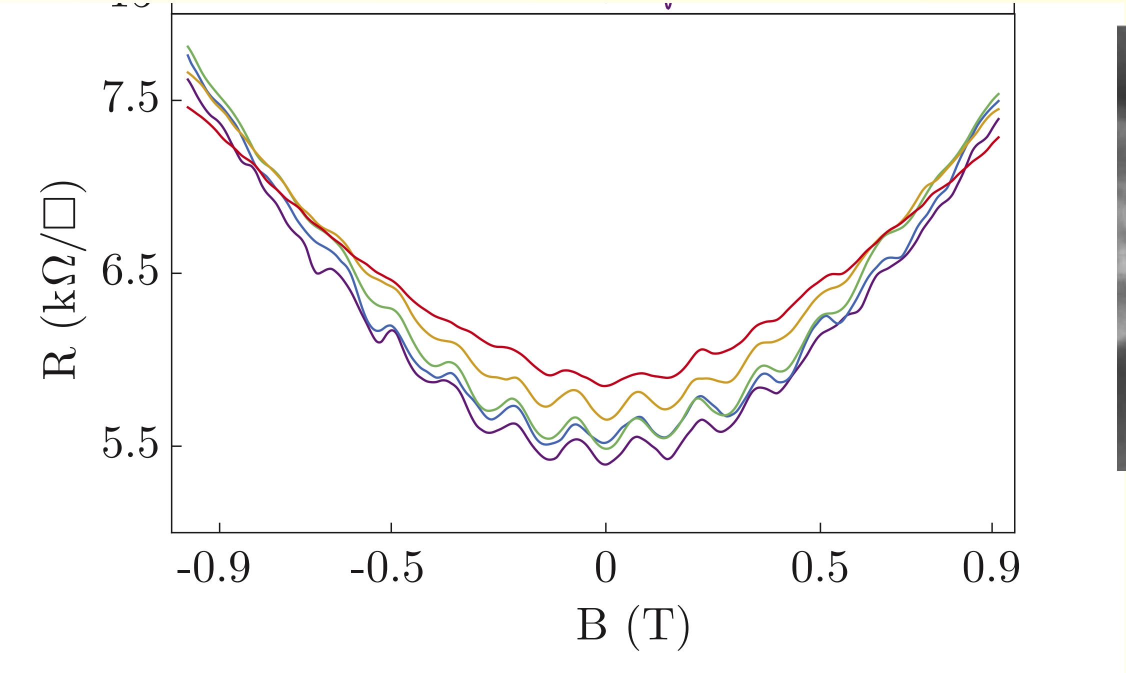
J. Valles et al. – patterned Bismuth

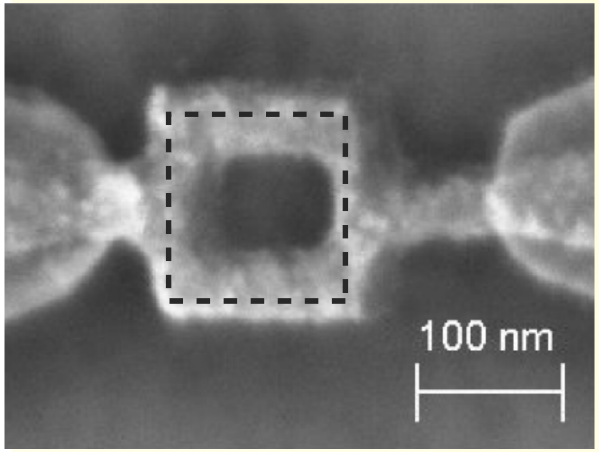




Proof of dominating pair transport in insulator!

Further evidence: Little Parks oscillations in weakly insulating ring of InO_x





Gurovich, Tikhonov, Mahalu, and Shahar (2015)

Oscillation period corresponds to charge 2e!

Road map of this talk

I. Orbital magnetoresistance of bosons (& contrast with fermions)

II. Activated magneto-transport in Bose insulator with long range Coulomb

III. Pair-to-electron crossover & MR peak

Magnetoresistance in Bose and Fermi insulators?

How are hard core bosons different from free fermions?

Disordered insulators
Simplest model: Hopping+disorder

Model

$$H = \sum_i \varepsilon_i n_i - \sum_{\langle i,j \rangle} t_{ij} (b_j^\dagger b_i + b_i^\dagger b_j), \quad n_i = b_i^\dagger b_i.$$

Fermions

$$\{b_i, b_j\} = 0, , \quad \{b_i^\dagger, b_j\} = \delta_{ij}$$

P. W. Anderson (1958)
.....

Hard core bosons
(↔ spin ½)

$$[b_i, b_j] = 0, \quad [b_i^\dagger, b_j] = \delta_{ij}(2n_i - 1)$$

Krauth, Trivedi, Randeria;
Feigelman, Ioffe, Kravtsov;
Ioffe, Mézard, Feigelman;
Syzranov, Moor, Efetov;
Yu, MM

Disordered insulators
Simplest model: Hopping+disorder

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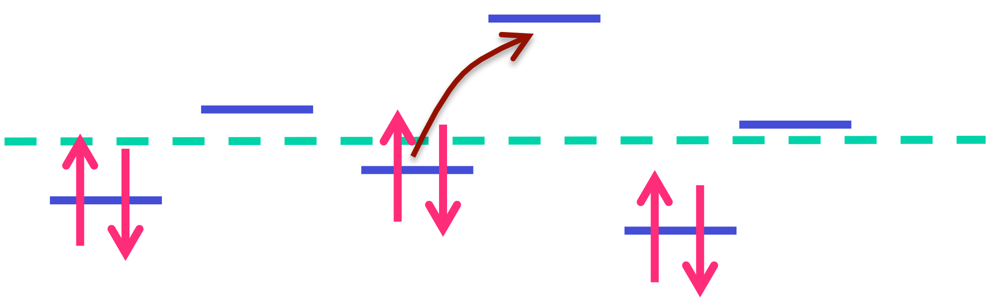
P. W. Anderson (1958)
.....

Hard core bosons
($\leftrightarrow$ spin ½)

Example: Localized Anderson pseudospins
= doubly occupied or empty orbitals

M. Ma and P. A. Lee (1985),
Kapitulnik and Kotliar (1985)

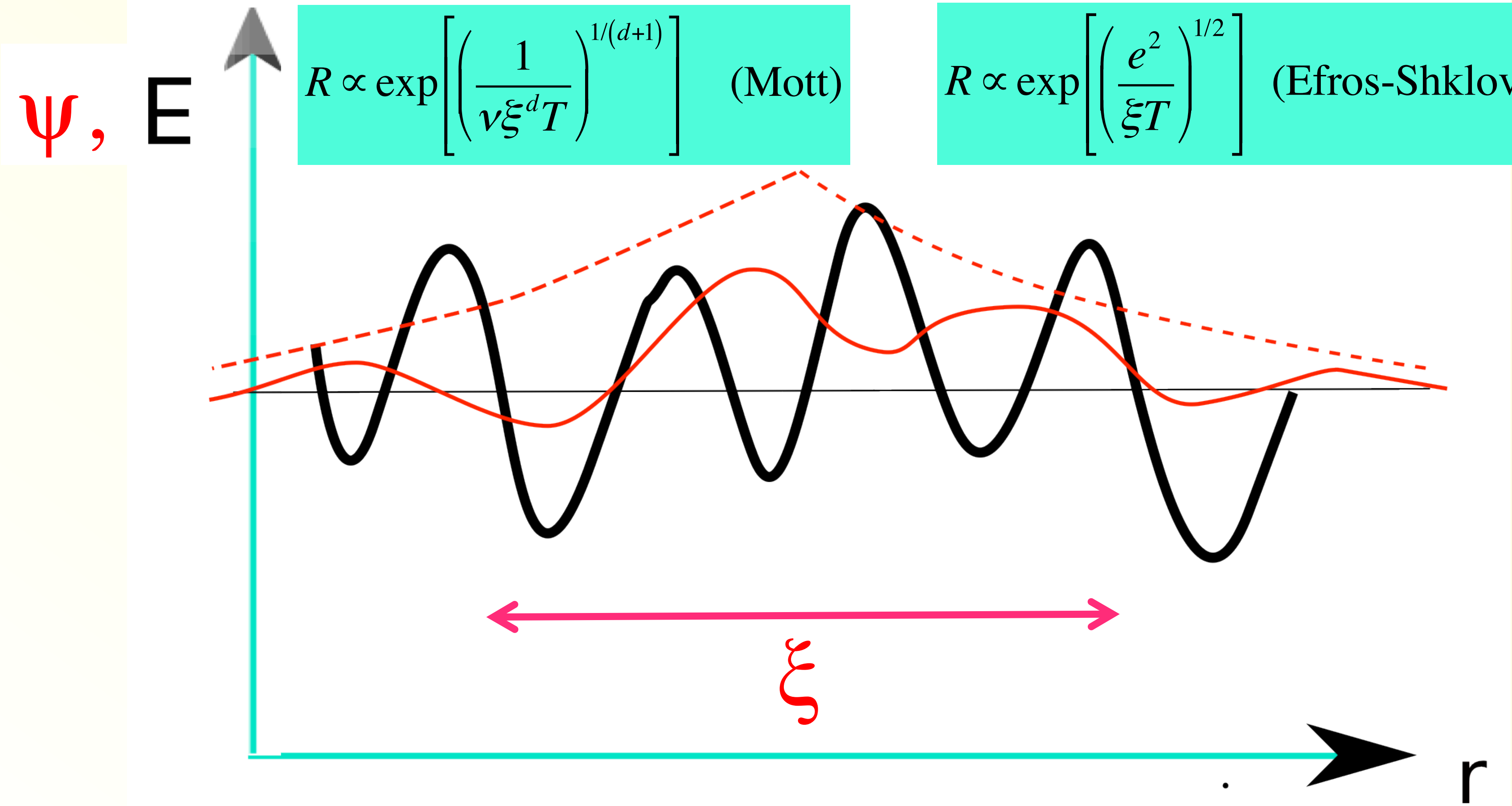


Localization length

Strong insulators: Hopping transport! - Localization length ξ?

$$R \propto \exp\left[\left(\frac{1}{\nu\xi^d T}\right)^{1/(d+1)}\right] \quad \text{(Mott)}$$

$$R \propto \exp\left[\left(\frac{e^2}{\xi T}\right)^{1/2}\right] \quad \text{(Efros-Shklovskii)}$$



Mobility edge ω_c $\leftrightarrow$ $\xi(\omega \to \omega_c) \to \infty$

Localization length

Fermions $$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle\{b_i(t), b^\dagger_0(t')\}\rangle$$

Bosons $$G^R_{i,0}(t-t') = -i\Theta(t-t')\langle[b_i(t), b^\dagger_0(t')]\rangle$$

Generalized localization length (also interacting!)

$$\xi(\omega)^{-1} = -\lim_{\vec{r}_i\to\infty} \overline{\ln[|G^R_{i,0}(\omega)/G^R_{0,0}(\omega)|]/|\vec{r}_i - \vec{r}_0|}.$$

Free fermions: no features near E_F: $\xi(\omega) \sim$ const.

What about bosons and/or interactions?

Locator expansion and forward scattering

Fermions

J. Hubbard (1963): Equation of motion for Green's function!

$$\left(i\frac{d}{dt}-\varepsilon_i\right)G^R_{i,0}(t)$$

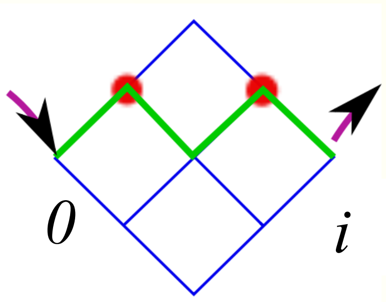
$$= \delta(t)\delta_{i,0} + i\Theta(t-t')\left\langle\left\{\overbrace{\sum_{j\in\partial i} t_{ij} b_j(t)}^{i\frac{d}{dt}b_i(t)}, b_0^\dagger(t')\right\}\right\rangle$$

$$= \delta(t)\delta_{i,0} - \sum_{j\in\partial i} t_{ij}G^R_{j,0}(t)$$

Fourier transform → Anderson-Feynman sum over paths *Anderson (1958)*

Forward scattering approximation: Sum over shortest paths!

Spivak, Shklovskii, Nguyen (1983)



$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p}\frac{1}{\varepsilon_{j_p}-\omega}$$

Locator expansion and forward scattering

Fermions

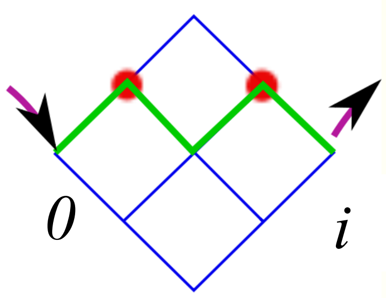
Magnetoresistance: **negative** *(Nguyen, Spivak, Shklovskii)*

Path amplitudes: real with random signs!

B-field: $t_{ij} \longrightarrow t e^{-i\phi_{ij}}$ makes destructive interference less likely $\rightarrow$ ξ and 1/R increase.

Forward scattering approximation: Sum over shortest paths!

Spivak, Shklovskii, Nguyen (1983)



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Locator expansion and forward scattering

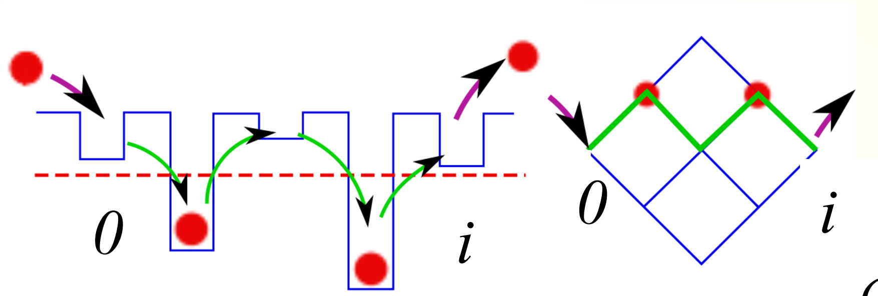
Bosons
(hard core)

MM (EPL '13)
X. Yu, MM, Ann. Phys '13

Equation of motion for Green's function!

$$\left(i\frac{d}{dt}-\varepsilon_i\right)G^R_{i,0}(t)=\delta(t)\delta_{i,0}(1-2\langle n_0\rangle)$$
$$+i\Theta(t-t')\left\langle\left[(-1)^{n_i(t)}\sum_{j\in\partial i}t_{ij}b_j(t),b_0^\dagger(t')\right]\right\rangle$$
$$\approx\delta(t)\delta_{i,0}(1-2\langle n_0\rangle)-\mathrm{sgn}(\varepsilon_i)\sum_{j\in\partial i}t_{ij}G^R_{j,0}(t)$$

Forward scattering: Sum over shortest paths, lowest order in t!



Sign difference Bosons/Fermions:
Loop of two paths:
Ring exchange of particles

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Locator expansion and forward scattering

Bosons (hard core)

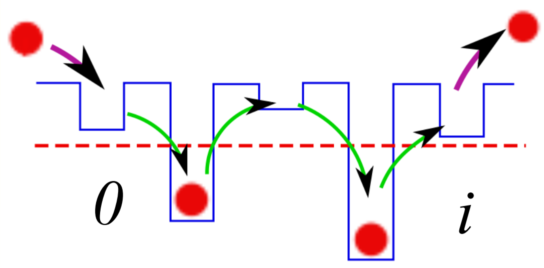
Magnetoresistance: **positive**

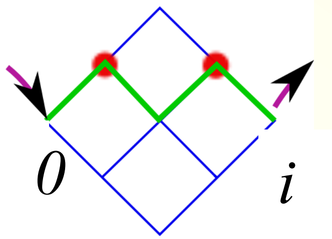
cf also Zhou, Spivak (1991)
Syzranov et al (2012)

Path amplitudes: **all** positive at ($\omega \to 0$) !

B-field: $t_{ij} \longrightarrow t e^{-i\phi_{ij}}$ destroys constructive interference, ξ and 1/R decrease.

Forward scattering: Sum over shortest paths, lowest order in t!





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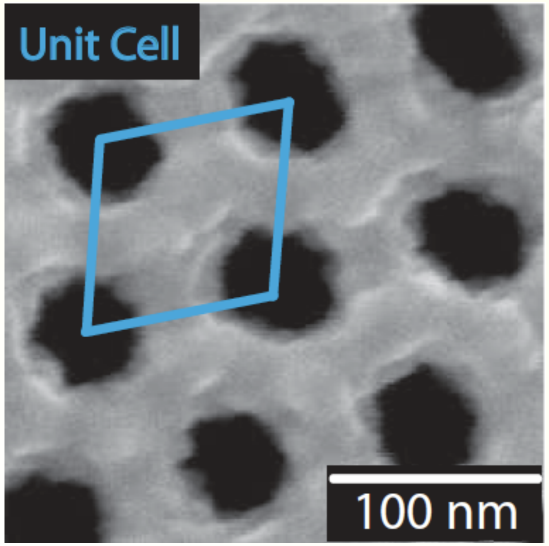
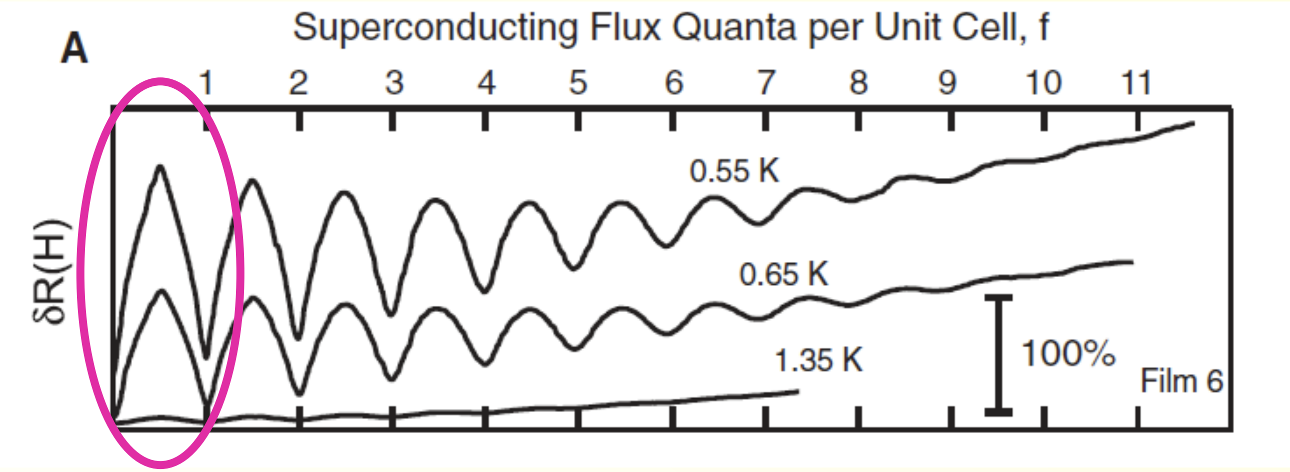




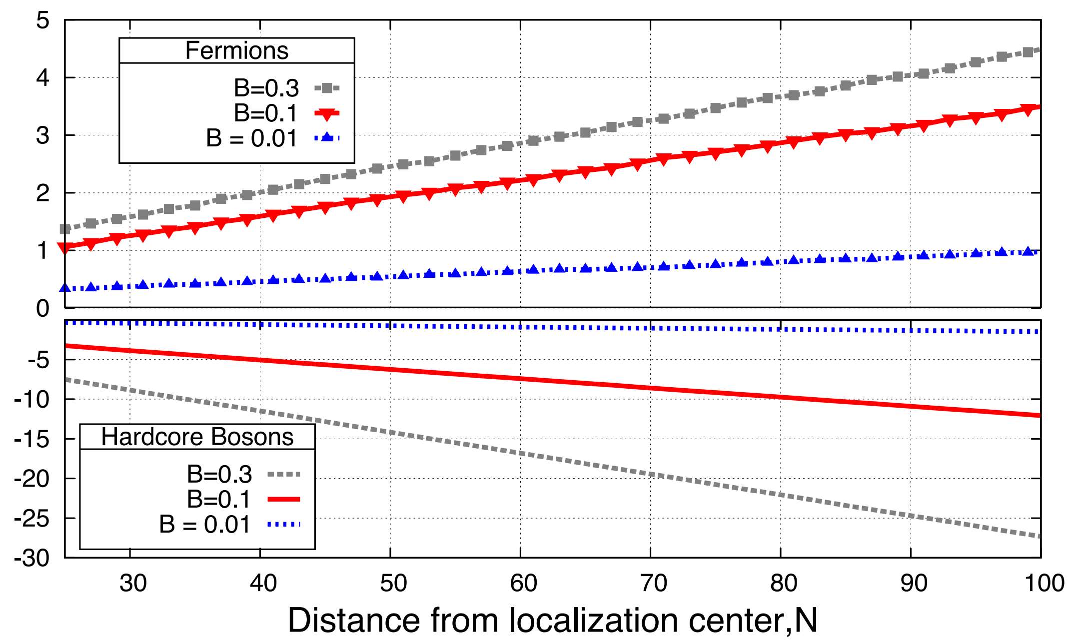
J. Valles et al. (2007-11): Patterned Bi films - Oscillations start with pos. MR: smoking gun for bosons!

Bosons vs fermions?

Bosons (hard core) | Mag. Resistance | Fermions
Strongly positive ← Mag. Resistance → Weakly negative
Shrinks ← Wavefunction → Expands

$$N\left[\frac{1}{\xi(B)}-\frac{1}{\xi(0)}\right]$$

$$\left(=\Delta\left[\log\left|G_N\right|\right]\right)$$



F

B

Bosons: Change in localization length is ~7 times bigger than fermions!
Exponentially strong effect on resistance!

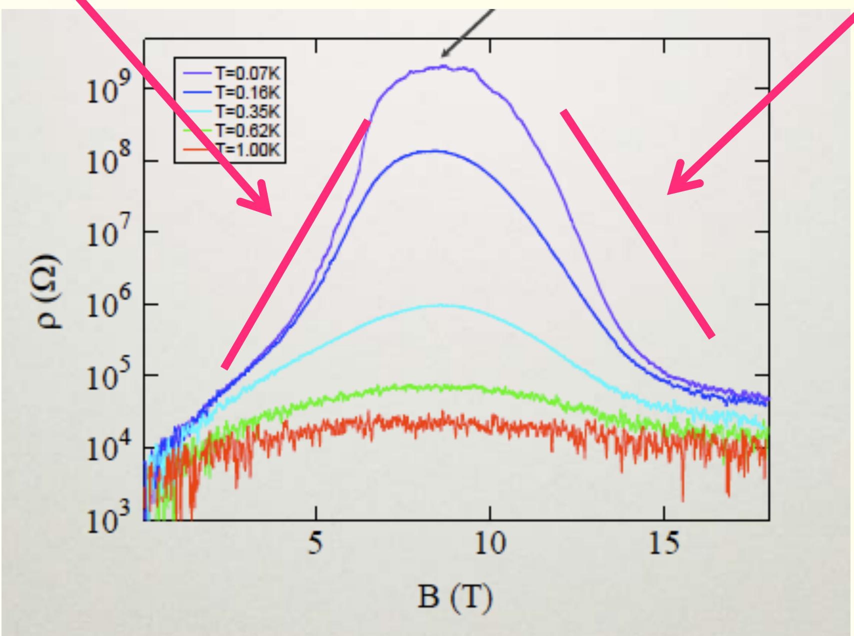
Magnetoresistance peak

Hebard+Palaanen, Gantmakher et al., Shahar et al, Baturina et al, W. Wu, Valles et al., Goldman et al.

One ingredient to MR peak in superconducting films:

Local pairs = bosons
→ exponentially positive MR

Unpaired fermions
→ exponentially negative MR

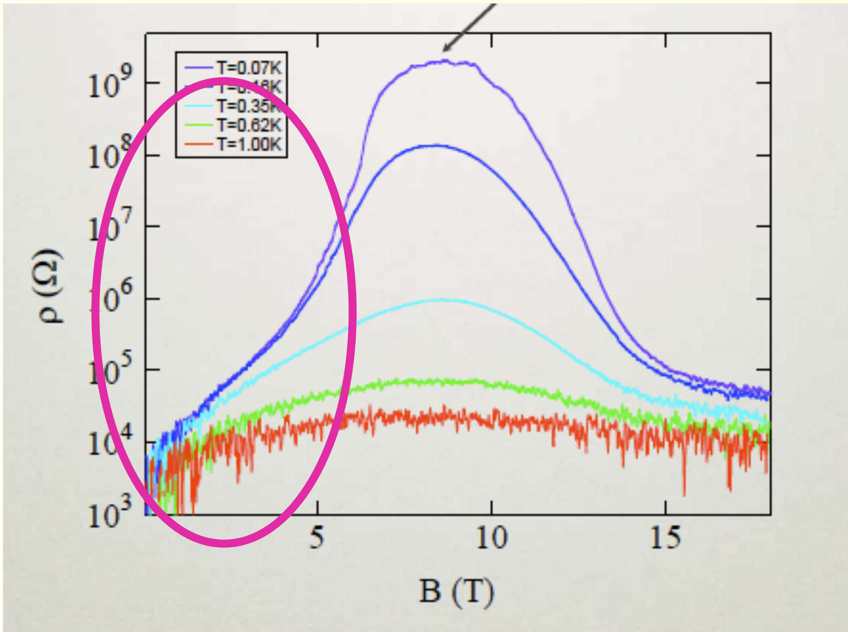


Sambandamurthy, Shahar et al. (2005) - InO_x

Is that really the main ingredient? NO! See part III

Before turning to the Pair-electron crossover:

II. Transport puzzles in the Bose glass



Puzzles

1. Experiment: Simple activation in R(T);
   [and possibly precursor traces of MBL??]

2. Evidence for purely electronic transport mechanism

$$R(T) = R(T_{electron}) \quad \textbf{not } R(T_{phonon}) \;!$$

Demonstrated via overheating instability of electrons

$\rightarrow$ Phonon-less transport!

Shahar et al.; Kravtsov et al.

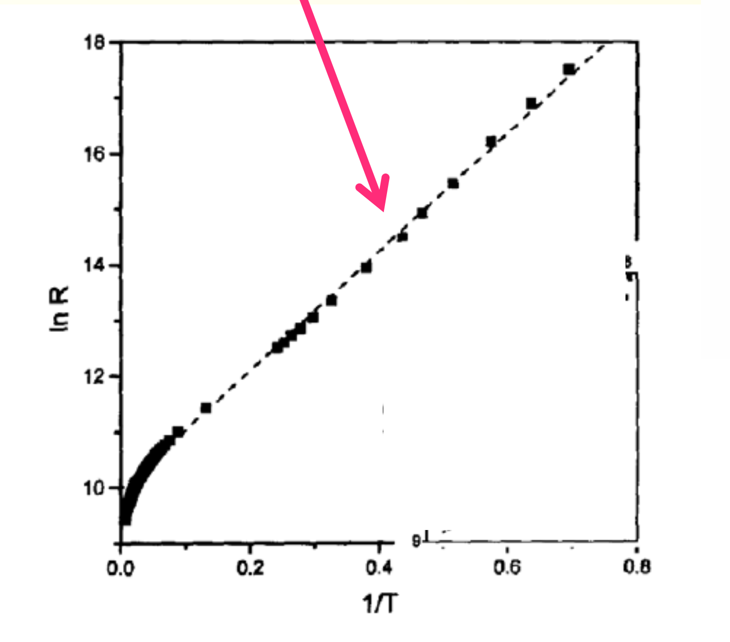
Mechanism?

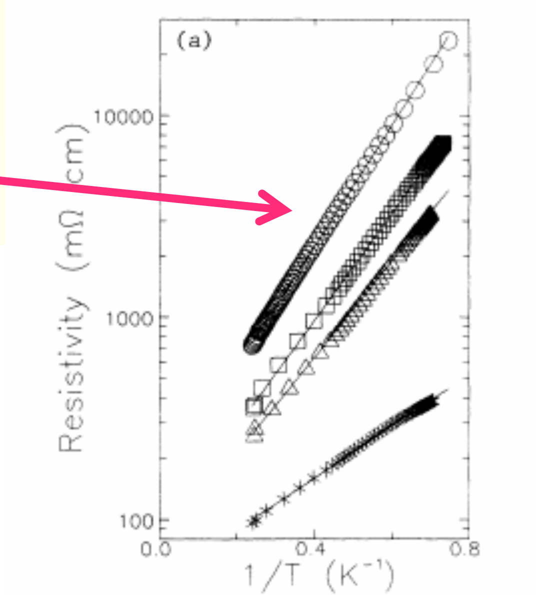
Activated transport near the SIT

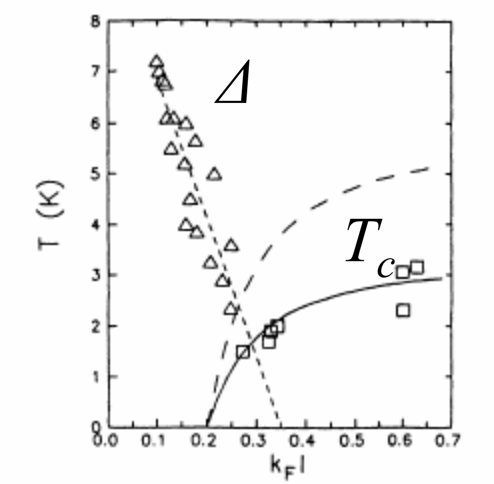
*D. Shahar, Z. Ovadyahu, PRB **46**, 10971 (1992).*

Insulating InO_x

Simple activation!







$$R(T) = R_0 \exp\left[-\frac{\Delta}{T}\right]$$

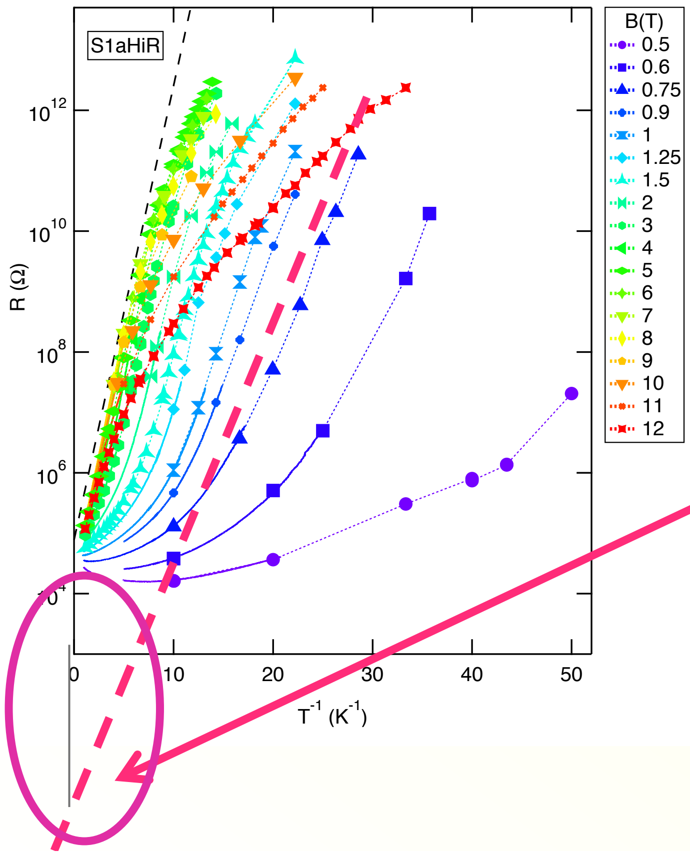
Activation energy increases with distance to SIT

*D. Kowal and Z. Ovadyahu, Sol. St. Comm. **90**, 783 (1994).*

MBL in the pair insulator?

Evidence for a Finite-Temperature Insulator

M. Ovadia,[1,2] D. Kalok,[1] I. Tamir,[1] S. Mitra,[1] B. Sacépé,[1,3,4] and D. Shahar[1,*]



1. Interpretation:

Maybe just simple activation with anomalously small prefactor

$$R_0 \sim \frac{R_Q}{10^4} \quad ?$$

[Expected from asymptotic "MBL" with "bubbles"]

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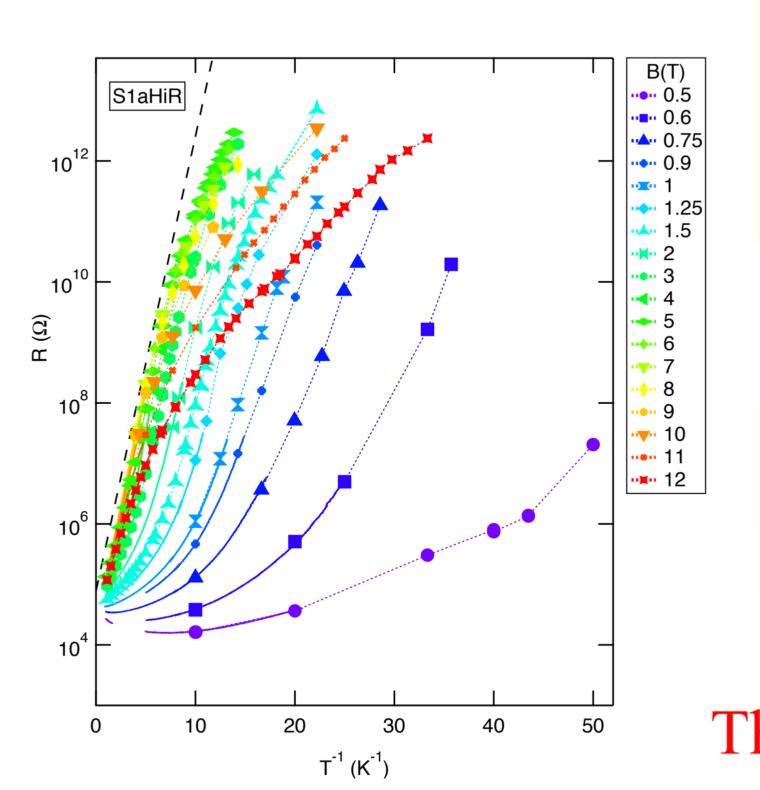


2. Empiric fit to super-insulation

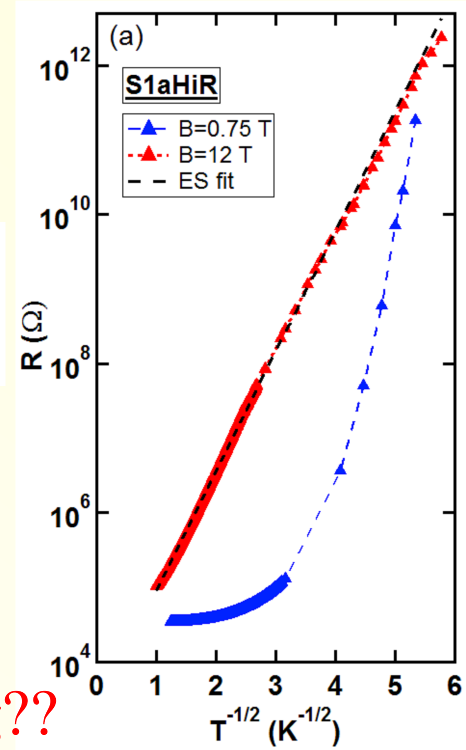
$$\sigma(T) = \sigma_0 exp[-\frac{T_0}{T - T^*}]$$

T^*=0.031 K

T_0=0.138

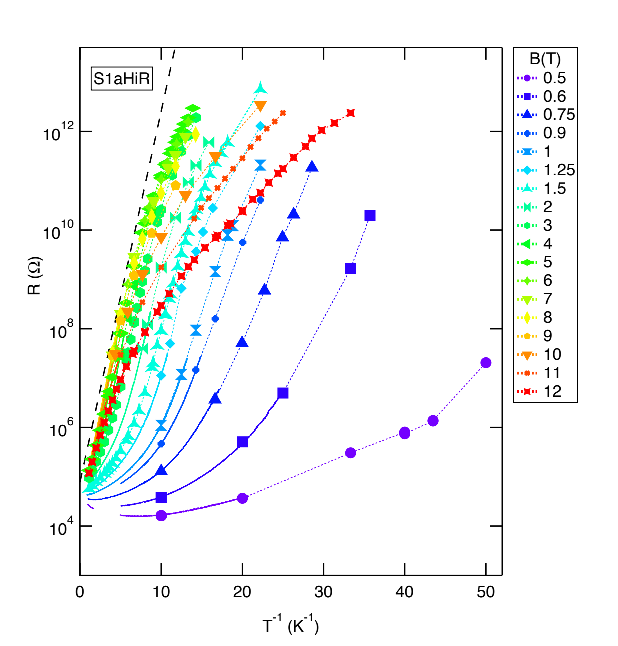
for T > 1.4 T*

Theoretical underpinning??



Activated transport near the SIT

Insulating InO_x



$$R(T) = R_0 \exp\left[-\frac{\Delta}{T}\right]$$

Origin of simple activation?

• Gap in the density of states?
A: NO! Too disordered systems!
No Mott gap!

• Why no variable range hopping?
A: Phonons are inefficient at low T.
Also: Would give too large prefactor R_0.

• Nearest neighbor hopping?
A: NO! Inconsistent with the experimental prefactor of Arrhenius

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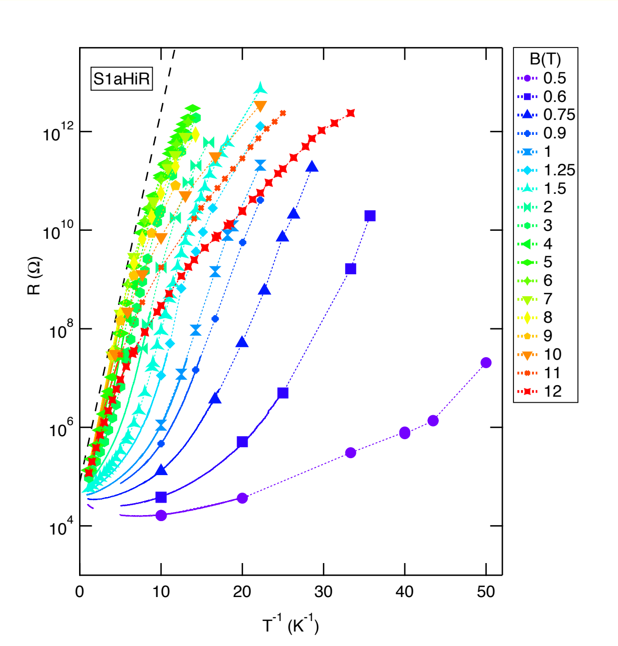

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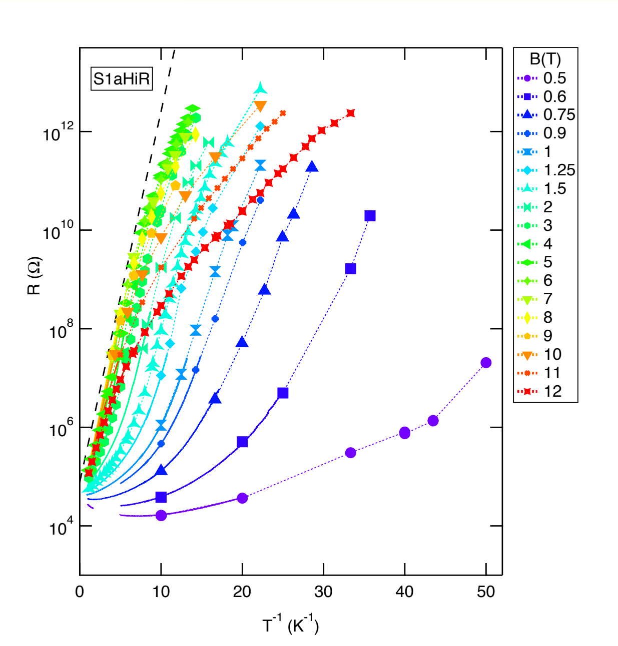

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A: NO! Inconsistent with the experimental prefactor of Arrhenius

- Δ is not a depairing gap (pos. MR!)
- But: Boson mobility edge !?
  (suggested by Feigelman-Ioffe-Mezard)

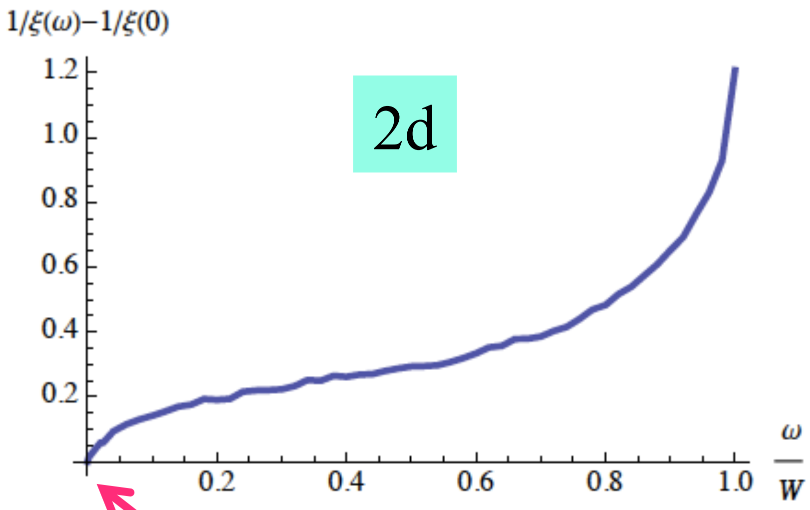
Boson localization as fct of E?

(F)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{1}{\varepsilon_{j_p} - \omega}$$

(B - XY)

$$\frac{G^R_{i,0}(\omega)}{G^R_{0,0}(\omega)} = \sum_{\mathcal{P}=\{j_0=0,\ldots,j_\ell=i\}} \prod_{p=1}^{\ell} t_{j_{p-1},j_p} \frac{\mathrm{sgn}(\varepsilon_{j_p})}{\varepsilon_{j_p} - \omega}$$



Localization **weakest** at lowest energies: **ξ(0) > ξ(ω)!**

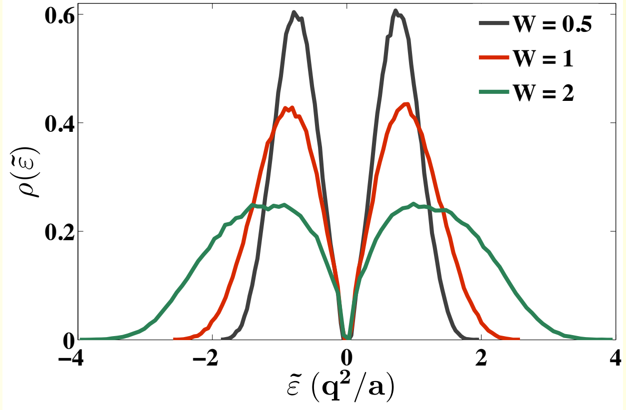
→ Hardcore bosons delocalize best at low energy!

→ **No mobility edge??**

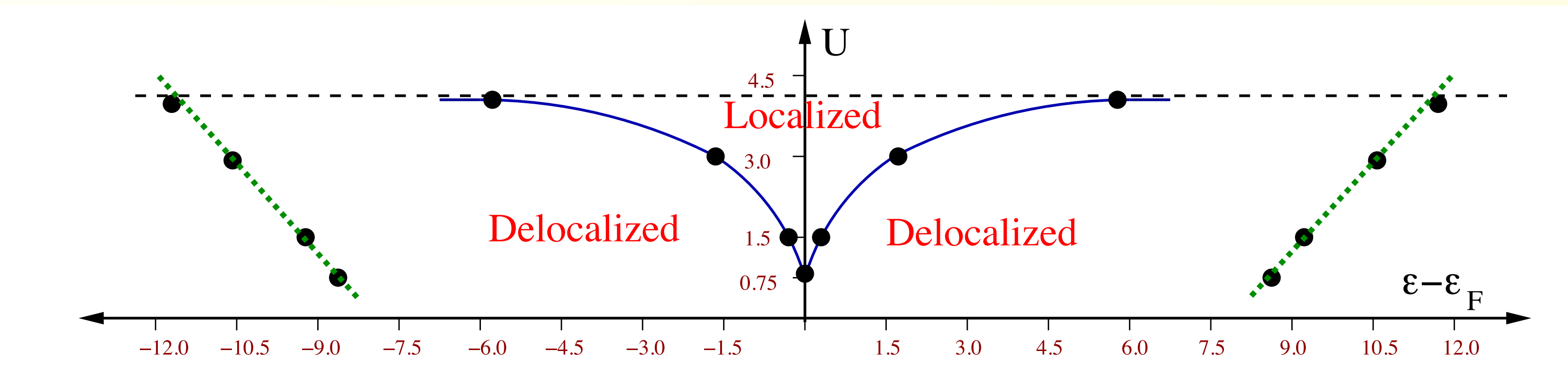
Boson localization as fct of E?

Crucial element missing: Repulsive interactions!

E.g. Coulomb:
→ Coulomb gap;
suppression of low energies



Consequence studied in detail for Mott-Anderson transition:
Mobility edge in the insulator; mobility gap closes at transition



Amini, Kravtsov, MM, NJP 2014; see also Burmistrov et al. 2014

Boson localization as fct of E?

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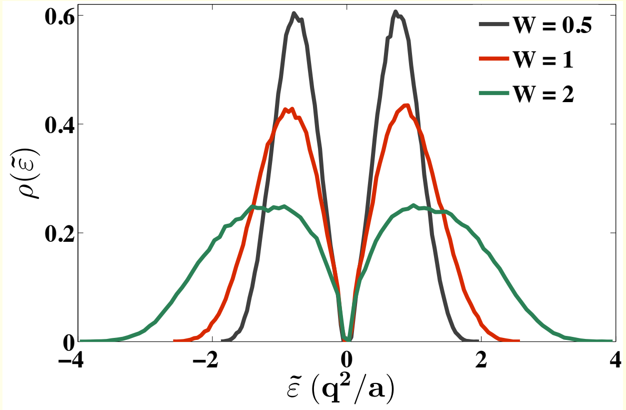

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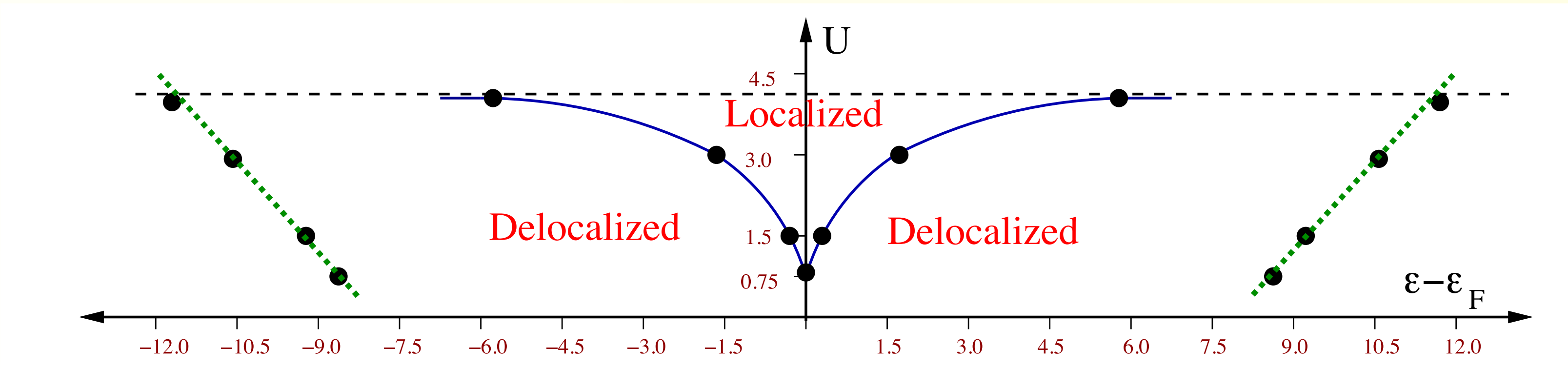


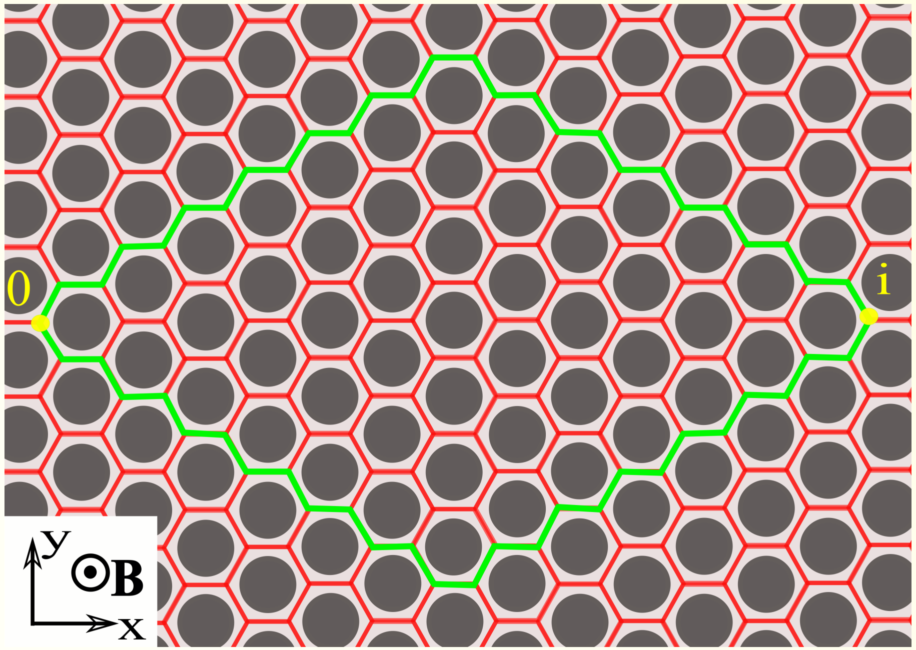
Essentially the same expected for SI transition of bosons with Coulomb

Mobility edge of bosons
with
Coulomb
+
magnetic field?

Magneto-oscillations of mobility edge

T. Nguyen and MM (2016)

$$H = -t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j + \mathrm{h.c.}\right) + \sum_i \varepsilon_i n_i + \frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}} n_i n_j$$



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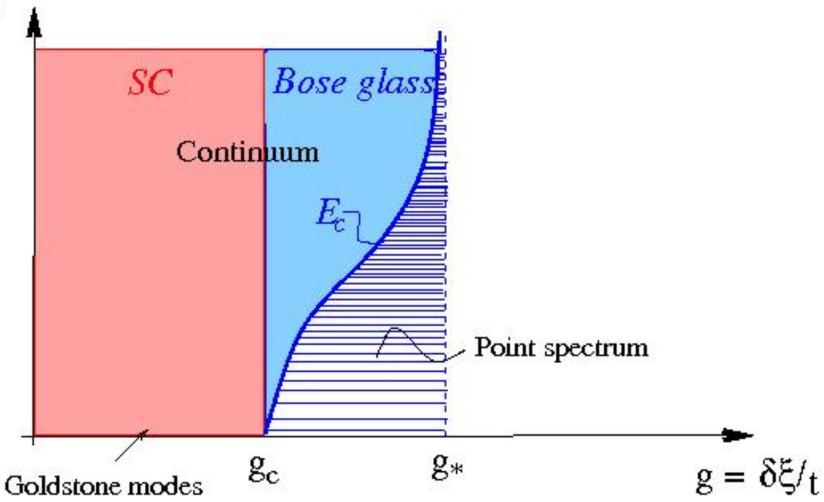
Defining a mobility edge in interacting systems

T = 0

$$\xi^{-1}(\varepsilon_0, B) = -\lim_{r_{0i}\to\infty}\frac{1}{r_{0i}}\overline{\ln\left|\frac{G_{0,i}(\omega,B)}{G_{0,0}(\omega,B)}\right|}_{\omega\to\varepsilon_0}$$

$$\epsilon_c = \inf\{E|\xi(E) = \infty\}$$

- Well-defined *finite energy* mobility edge ε_c in insulators in d>2 at T = 0



Magneto-oscillations of mobility edge

T. Nguyen and MM (2016)

$$H=-t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j+\mathrm{h.c.}\right)+\sum_i \varepsilon_i n_i+\frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j$$

Defining a mobility edge in interacting systems

- In d=2: *finite* energy excitations are localized!

BUT: crossover from weak to strong localization:

Define:

Effective ε_c: Forward approx. diverges $\epsilon_c = \min\{E|\xi^{\mathrm{FSA}}(E)=\infty\}$

[weak inelastic processes at T>0 suffice to really delocalize]

Expect: $R(T)\approx R_0\exp\left(\frac{\varepsilon_c}{T}\right)$

In d=2, at very low T: $\varepsilon_c(T\to 0)\to\infty$

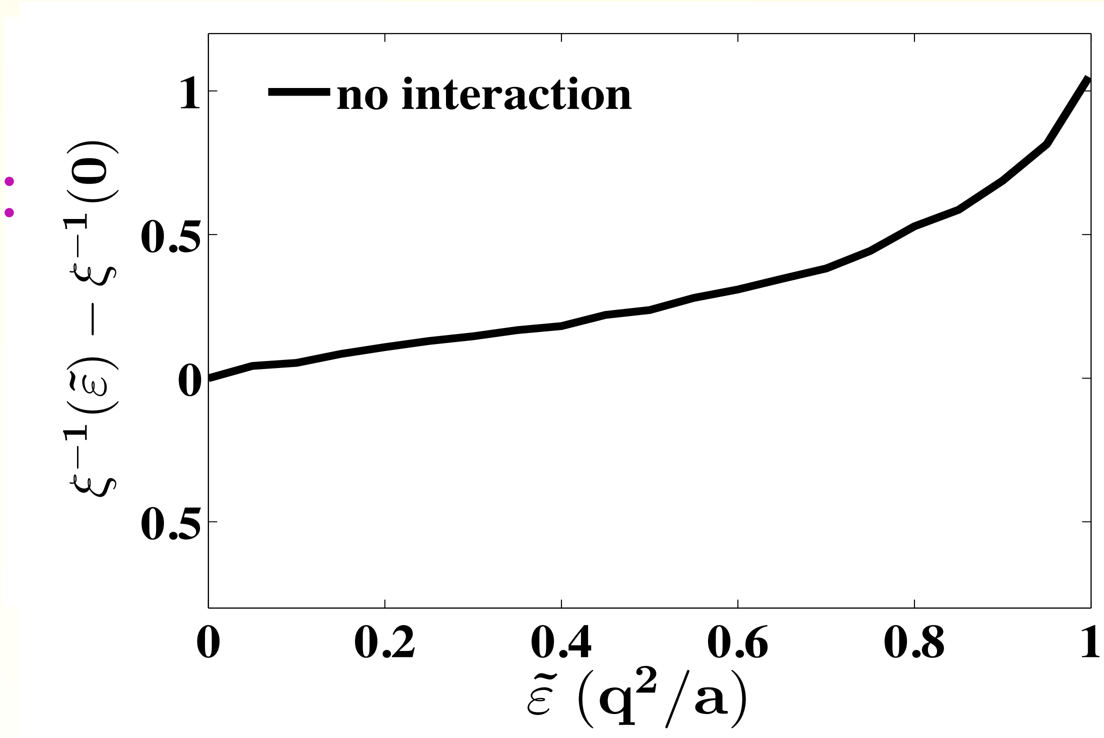
How? Interesting open problem!

Magneto-oscillations of mobility edge

T. Nguyen and MM (2016)

$$H = -t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j + \mathrm{h.c.}\right) + \sum_i \varepsilon_i n_i + \cancel{\frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j}$$

Reminder:

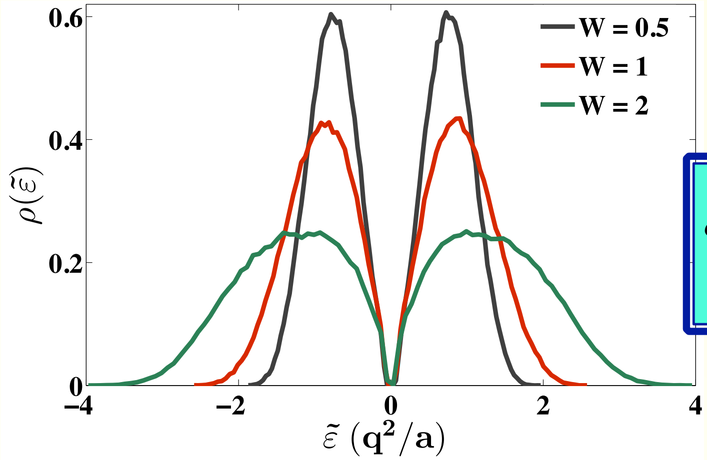


Magneto-oscillations of mobility edge

T. Nguyen and MM (2016)

$$H = -t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j + \text{h.c.}\right) + \sum_i \varepsilon_i n_i + \frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j$$

Coulomb gap

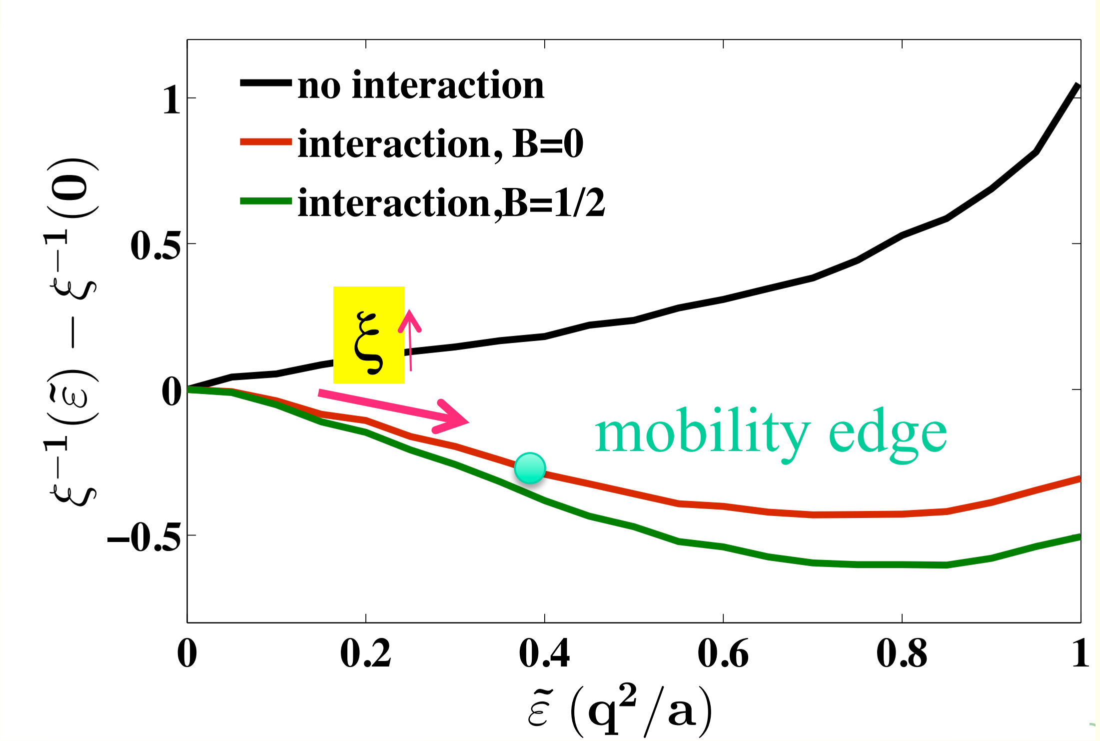


$$\tilde{\varepsilon}_i = \varepsilon_i + \sum_{j\in\partial i} J_{ij} n_j$$

Magneto-oscillations of mobility edge

T. Nguyen and MM (2016)

$$H = -t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j + \text{h.c.}\right) + \sum_i \varepsilon_i n_i + \frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}} n_i n_j$$



Apply magnetic flux: oscillation of mobility edge!

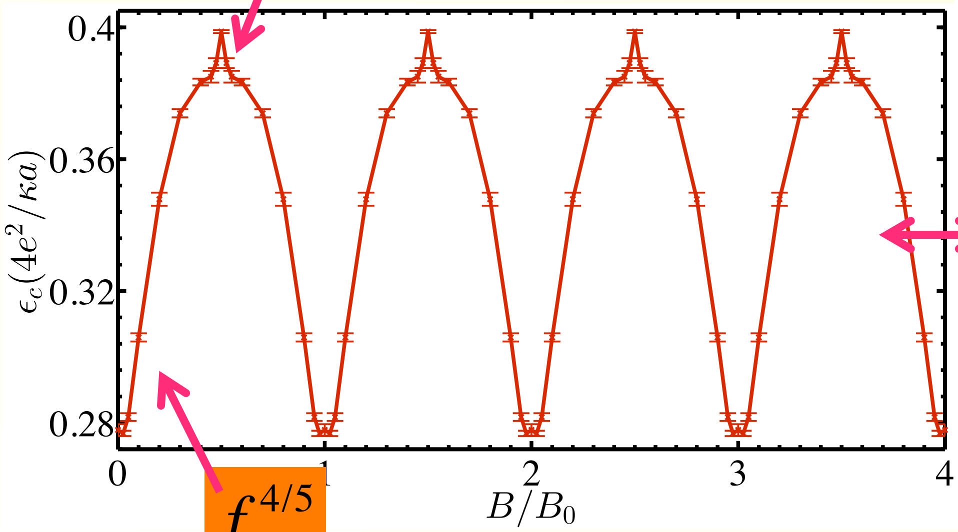
Magneto-oscillations of boson mobility edge

T. Nguyen and MM (2016)

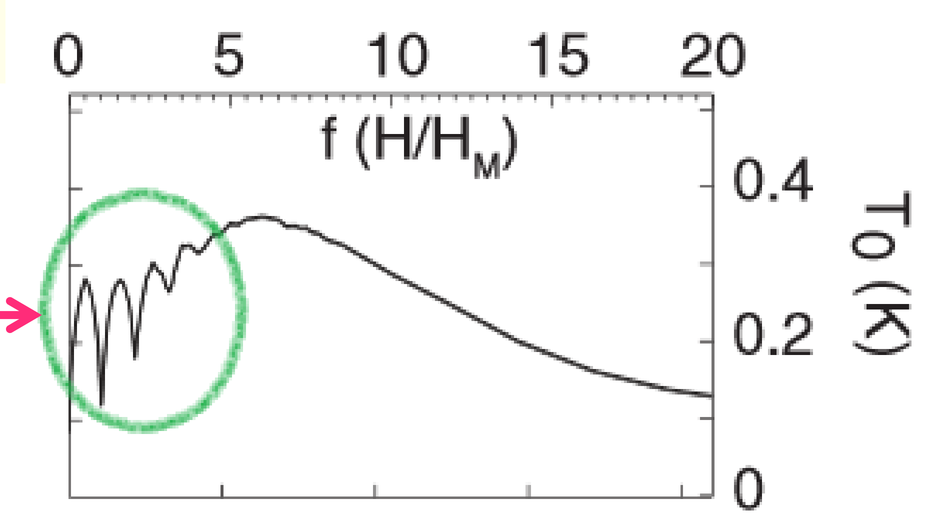
Substantial oscillations with cusps

J. Valles group,
*PRL **103**, 157001 (2009)*

$|f - 1/2|^{4/5}$



$f^{4/5}$



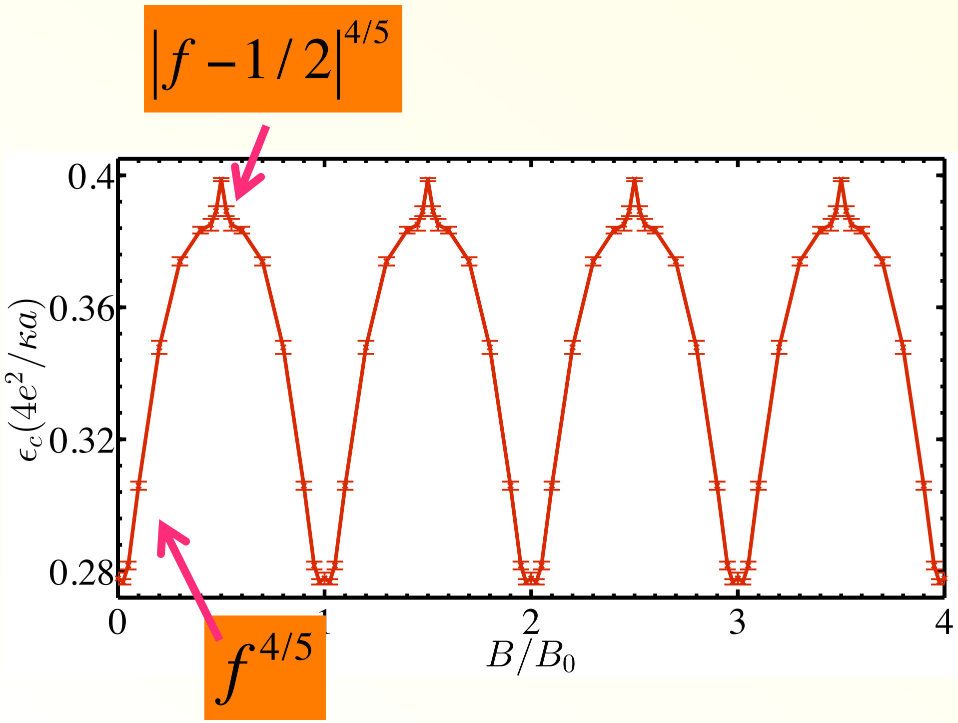
$$R_\square = R_0 \exp\left(\frac{T_0}{T}\right)$$

If bath is bad: expect transport by activation to mobility edge!

Magneto-oscillations of boson mobility edge

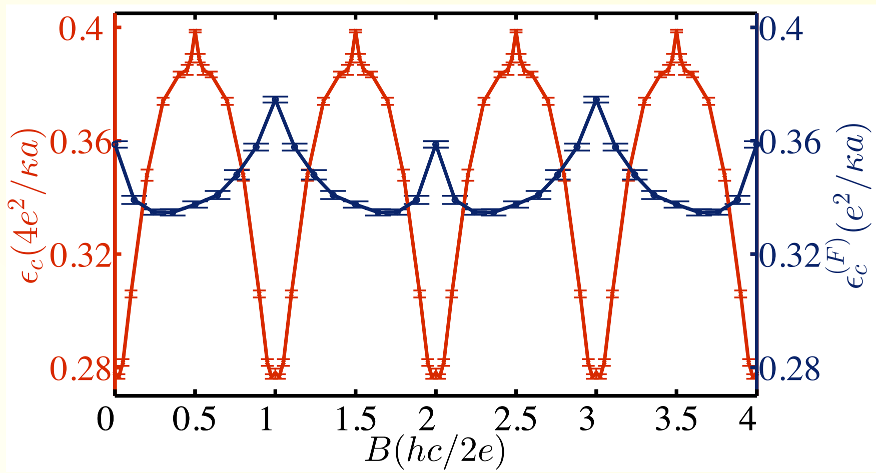
T. Nguyen and MM (2016)

Substantial oscillations with cusps



Comparison with fermions:

- Opposite, downward lobes
- Smaller amplitude



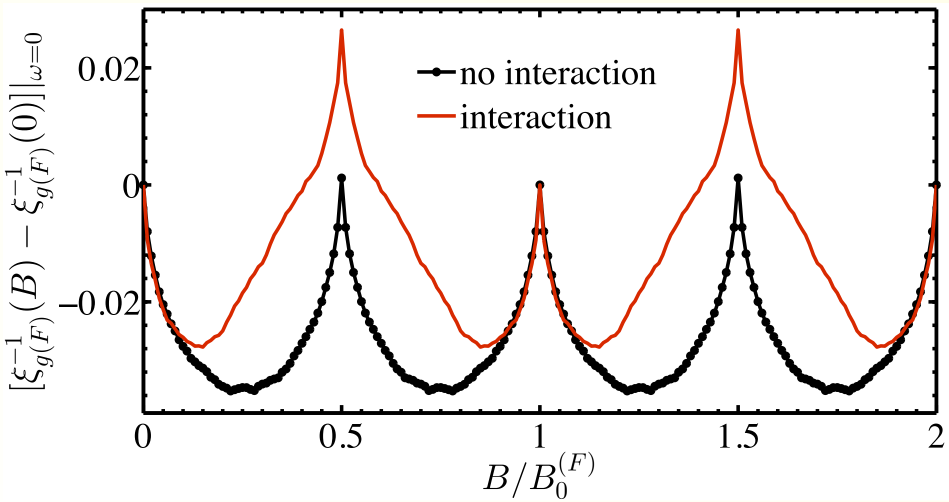
If phonon bath is bad: expect transport by activation to mobility edge!

Magneto-oscillations of boson mobility edge

T. Nguyen and MM (2016)

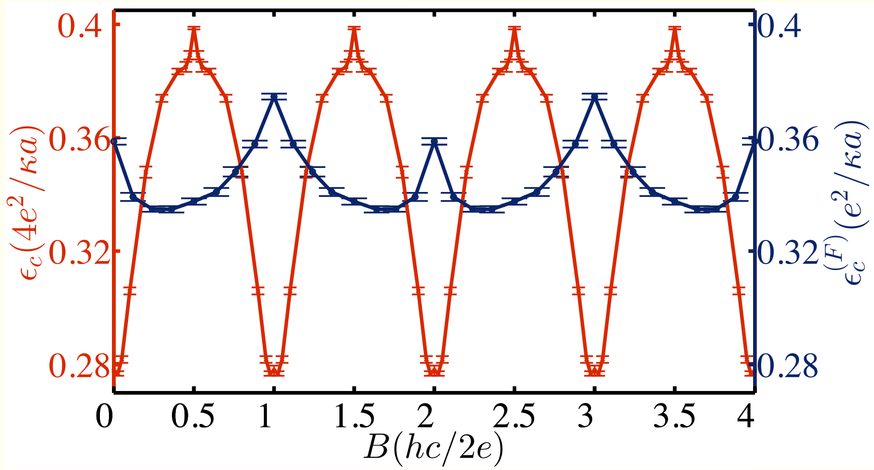
Fermions:

Non-interacting electrons
↔ Coulomb glass



Comparison with fermions:

- Opposite, downward lobes
- Smaller amplitude
- **2 unequal** maxima per period: reflect Coulomb correlations!

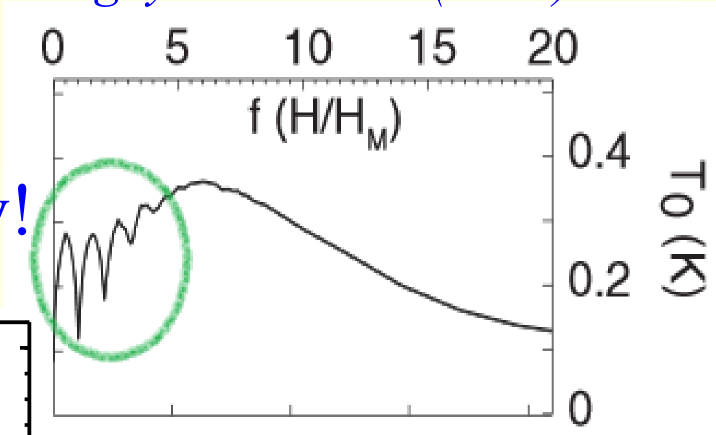


If phonon bath is bad: expect transport by activation to mobility edge!

Magneto-oscillations of boson mobility edge

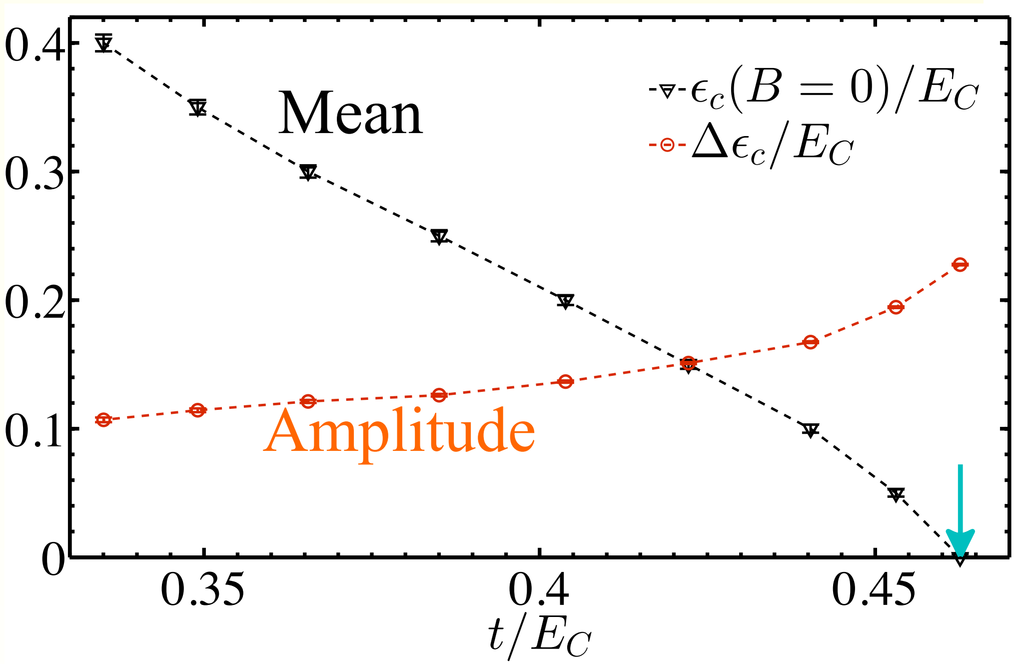
T. Nguyen and MM (2016)

Amplitude of oscillations:
Increase upon approach to criticality!



more insulating

INS



SC

more insulating

Magneto-oscillations of mobility edge

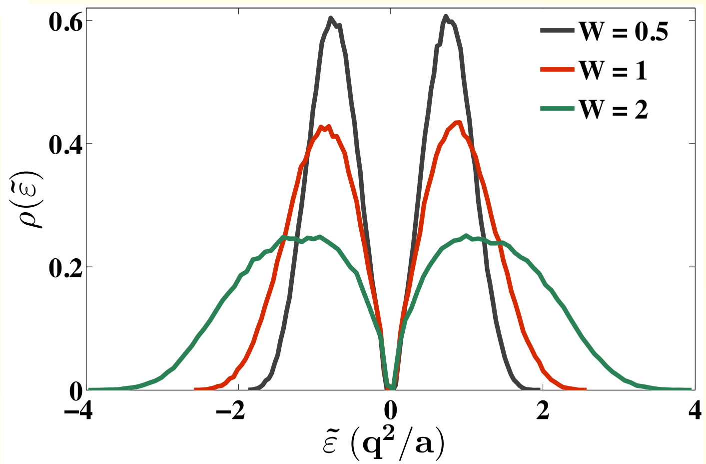
T. Nguyen and MM (2016)

$$H=-t\sum_{\langle i,j\rangle}\left(b_i^{+}b_j+\mathrm{h.c.}\right)+\sum_i \varepsilon_i n_i+\frac{1}{2}\sum_{\langle i,j\rangle}\frac{e^2}{r_{ij}}n_i n_j$$

Defining a mobility edge in interacting systems

T = 0

$$\epsilon_c=\min\{E|\xi^{\mathrm{FSA}}(E)=\infty\}$$



- What if insulation is so strong that forward approximation converges at all E? → no mobility edge → MBL??

- NOT quite (but it may look like it!):

MBL spoilers: 1. Coulomb (1/r) in d>3/2 [Burin]
2. hot bubbles; 3. d>1: rare ergodic spots might act as baths

Beyond the pure Bose glass:

III. Boson-Fermion crossover?

Include depairing!

Pairs and electrons:
Antagonists increase the insulation!

Boson-Fermion crossover

Simple model for mixed pair / electron glass

- Each site can host 0, 1 or 2 electrons (spin up/down)

$$H = \sum_{i,\sigma}(\varepsilon_i - \mu)n_{i,\sigma} - \sum_i (\lambda_i n_{i\uparrow}n_{i\downarrow} - BE_Z(n_{i\uparrow} - n_{i\downarrow}))$$

Disorder Local attraction Zeeman depairing

Random site properties: $P(\varepsilon) = \frac{1}{2W}\Theta(W-\varepsilon)\Theta(\varepsilon+W).$ $P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right)$

$$-\sum_{\langle i,j\rangle,\sigma}\left(t_1^{(ij)}c^\dagger_{i,\sigma}c_{j,\sigma} + \text{h.c.}\right) - \sum_{\langle i,j\rangle}\left(t_2^{(ij)}c^\dagger_{i,\uparrow}c^\dagger_{i,\downarrow}c_{j,\downarrow}c_{j,\uparrow} + \text{h.c.}\right)$$

Single electron and pair hopping

(At resulting phenomenological level:
Some similarity with resistor model by Dubi, Avishai, Meir)

Boson-Fermion crossover

Simple model for mixed pair / electron glass

- Each site can host 0, 1 or 2 electrons (spin up/down)

$$H = \sum_{i,\sigma}(\varepsilon_i - \mu)n_{i,\sigma} - \sum_i (\lambda_i n_{i\uparrow} n_{i\downarrow} - BE_Z(n_{i\uparrow} - n_{i\downarrow}))$$

Disorder Local attraction Zeeman depairing

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Single electron and pair hopping

- Simplification: no long range Coulomb; purely local interaction

[With Coulomb, but σ=0: **non-universal** Coulomb gap! - But: MR peak requires σ>0

Mitchell, Gangopadhyay, Galitski, MM, 2012]

Boson-Fermion crossover

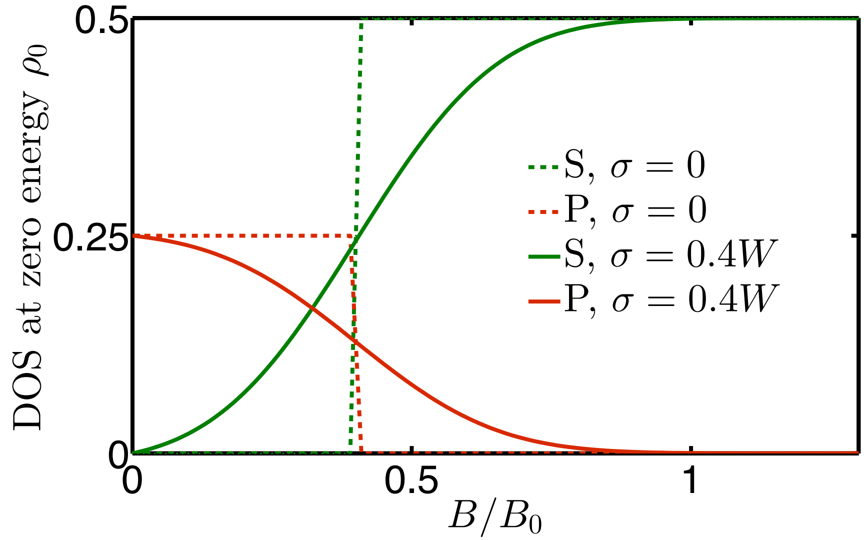
Our Approach

1. Solve classical part (trivial) $\longrightarrow$ density of states: $\rho_{pair}(E;B)$, $\rho_{single}(E;B)$

$$H = \sum_{i,\sigma} (\varepsilon_i - \mu) n_{i,\sigma} - \sum_i (\lambda_i n_{i\uparrow} n_{i\downarrow} - B E_Z (n_{i\uparrow} - n_{i\downarrow}))$$

$$P(\varepsilon) = \frac{1}{2W} \Theta(W - \varepsilon) \Theta(\varepsilon + W).$$

$$P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\lambda - \lambda_0)^2}{2\sigma^2}\right)$$



No dispersion in attraction ($\sigma = 0$):

$\longrightarrow$

One species hard gapped; jump in R(B) at $B_c = \lambda_0$

Boson-Fermion crossover

Our Approach

1. Solve classical part (trivial) $\longrightarrow$ density of states: $\rho_{pair}(E;B)$, $\rho_{single}(E;B)$

$$H = \sum_{i,\sigma}(\varepsilon_i - \mu)n_{i,\sigma} - \sum_i (\lambda_i n_{i\uparrow}n_{i\downarrow} - BE_Z(n_{i\uparrow} - n_{i\downarrow}))$$

$$P(\varepsilon) = \frac{1}{2W}\Theta(W-\varepsilon)\Theta(\varepsilon+W).$$

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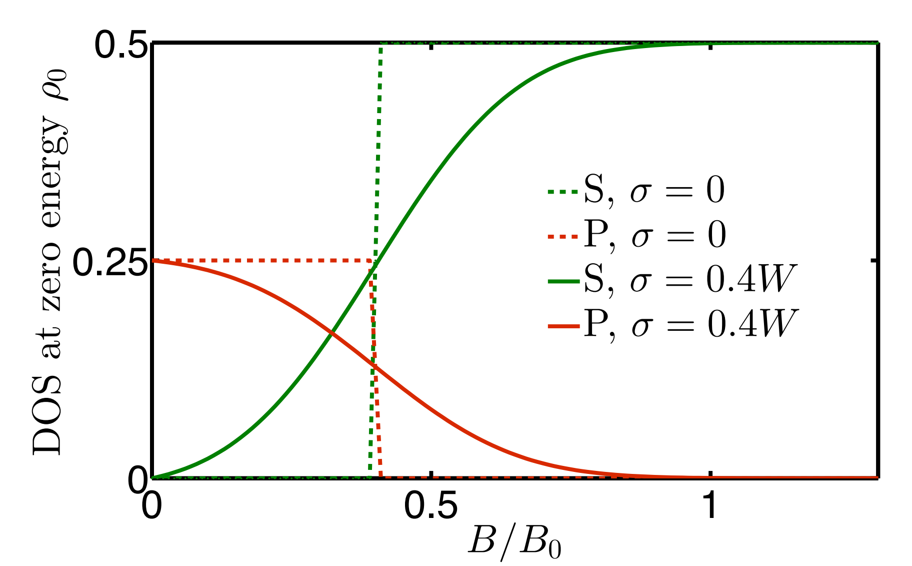

Widely distributed attraction ($\sigma > 0$):

$\rightarrow$

As B: $\uparrow$ $\rho_{pair}(0)\downarrow$

$\rho_{single}(0)\uparrow$

$\rightarrow$ main drive for MR peak

Boson-Fermion crossover

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$$P(\varepsilon) = \frac{1}{2W}\Theta(W-\varepsilon)\Theta(\varepsilon+W).$$

$$P(\lambda) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right)$$

2. Treat hopping perturbatively on the classical background $\longrightarrow$ ξ_{pair}, ξ_{single}

$$-\sum_{\langle i,j\rangle,\sigma}\left(t_1^{(ij)} c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.}\right) - \sum_{\langle i,j\rangle}\left(t_2^{(ij)} c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} c_{j,\downarrow} c_{j,\uparrow} + \text{h.c.}\right)$$

As B$\uparrow$: DOS effect: $\xi_{pair}\downarrow$ $\xi_{single}\uparrow$

Forward approximation, neglecting virtual break-up of pairs

NB: can even drive an MIT at strong B!

Boson-Fermion crossover

Our Approach

1. Solve classical part (trivial) $\longrightarrow$ density of states: $\rho_{pair}(E;B)$, $\rho_{single}(E;B)$

$$H = \sum_{i,\sigma} (\varepsilon_i - \mu) n_{i,\sigma} - \sum_i (\lambda_i n_{i\uparrow} n_{i\downarrow} - BE_Z(n_{i\uparrow} - n_{i\downarrow}))$$

$$P(\varepsilon) = \frac{1}{2W} \Theta(W - \varepsilon) \Theta(\varepsilon + W).$$

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As $B_{perp} = B \sin\theta \uparrow$: $\xi_{pair} \downarrow$, $\xi_{single} \uparrow$

$\longrightarrow$ angle(θ)-dependence!

Forward approximation, neglecting virtual break-up of pairs

Boson-Fermion crossover

Our Approach

1. Solve classical part (trivial) $\longrightarrow$ density of states: $\rho_{pair}(E;B)$, $\rho_{single}(E;B)$

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$$P(\varepsilon) = \frac{1}{2W}\Theta(W-\varepsilon)\Theta(\varepsilon+W).$$

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2. Treat hopping perturbatively on the classical background $\longrightarrow$ ξ_{pair}, ξ_{single}

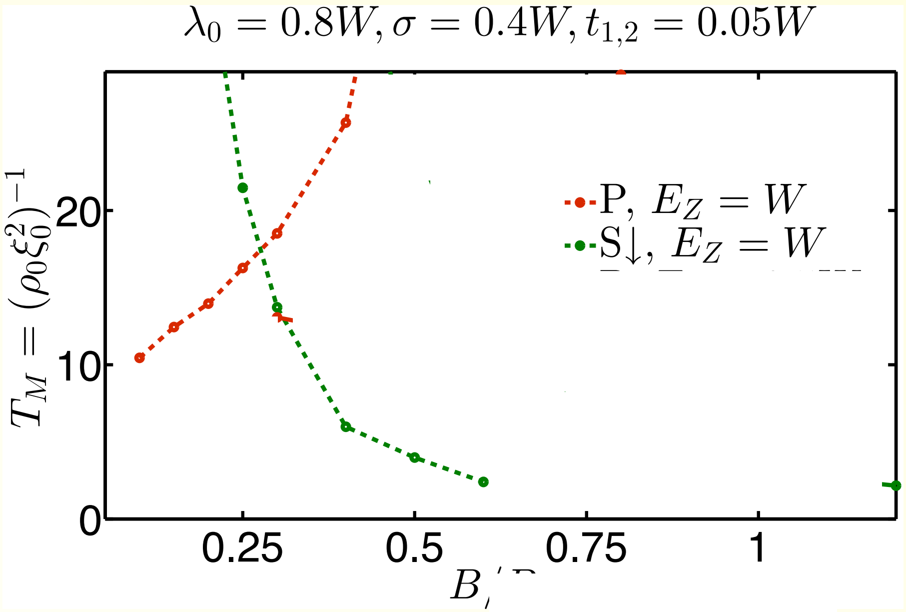
$$-\sum_{\langle i,j\rangle,\sigma}\left(t_1^{(ij)} c_{i,\sigma}^\dagger c_{j,\sigma} + \text{h.c.}\right) - \sum_{\langle i,j\rangle}\left(t_2^{(ij)} c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger c_{j,\downarrow} c_{j,\uparrow} + \text{h.c.}\right)$$

3. **Assume a bath** (always there!) $\longrightarrow$ Mott variable range hopping at low T:

$$R(T) = \exp\left[\left(\frac{T_M}{T}\right)^{1/(1+d)}\right] \quad ; \quad T_M = \min\left[T_M^{\text{pair}}, T_M^{\text{single}}\right] \quad ; \quad T_M^{\alpha} = \frac{Cst.}{\rho_\alpha(E=0)\xi_\alpha^2}$$

Boson-Fermion crossover

Crossing of Mott temperature

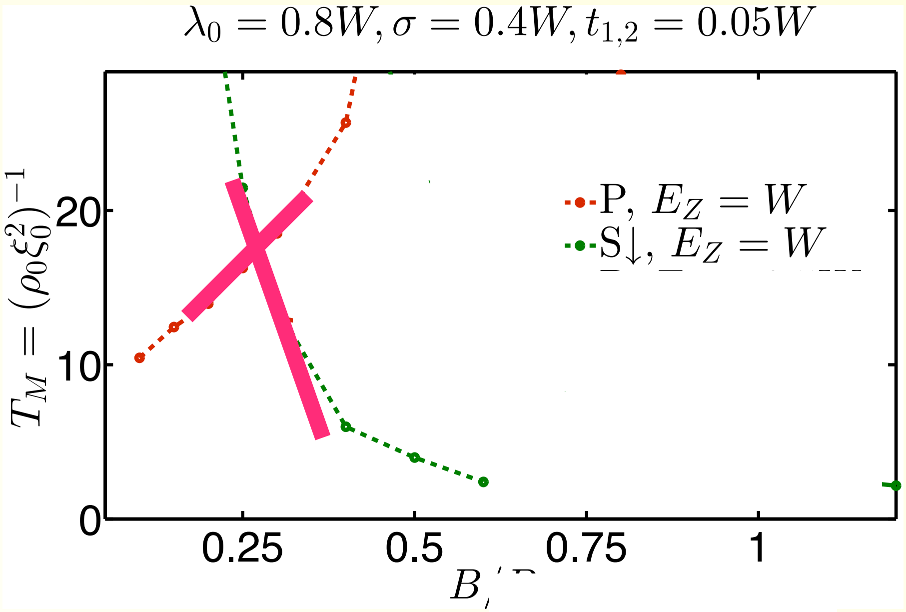


Analytical result for peak position ($u = \xi_{pair}/\xi_{single}$)

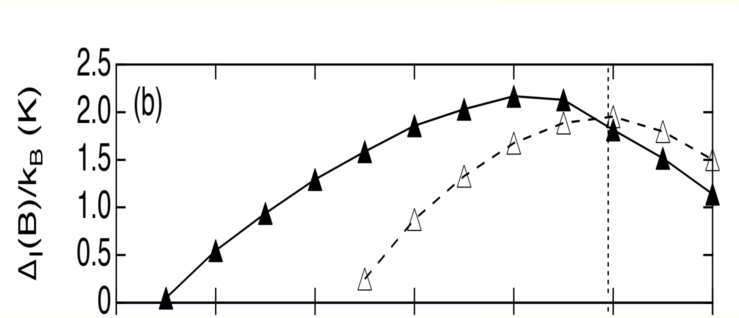
$$B_c E_Z = \frac{\lambda_0}{2} + \frac{\sigma}{\sqrt{2}} \mathrm{erf}^{-1} \left[\frac{\frac{u^2}{2} \mathrm{erf}\left(\frac{2(W-\mu)-\lambda_0}{\sigma\sqrt{2}} \right) - \mathrm{erf}\left(\frac{\lambda_0}{\sigma\sqrt{2}} \right)}{1 + \frac{u^2}{2}} \right]$$

Boson-Fermion crossover

Crossing of Mott temperature



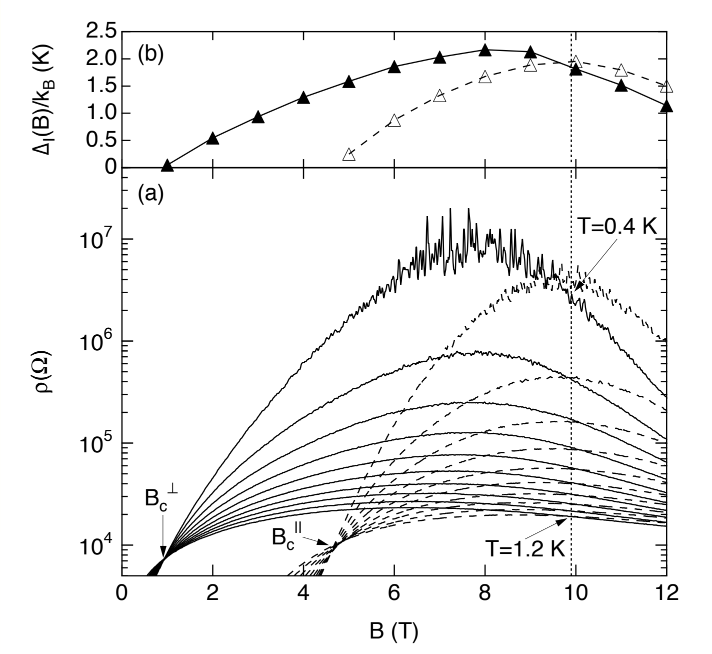
Asymmetric shape of peak: fermionic side steeper

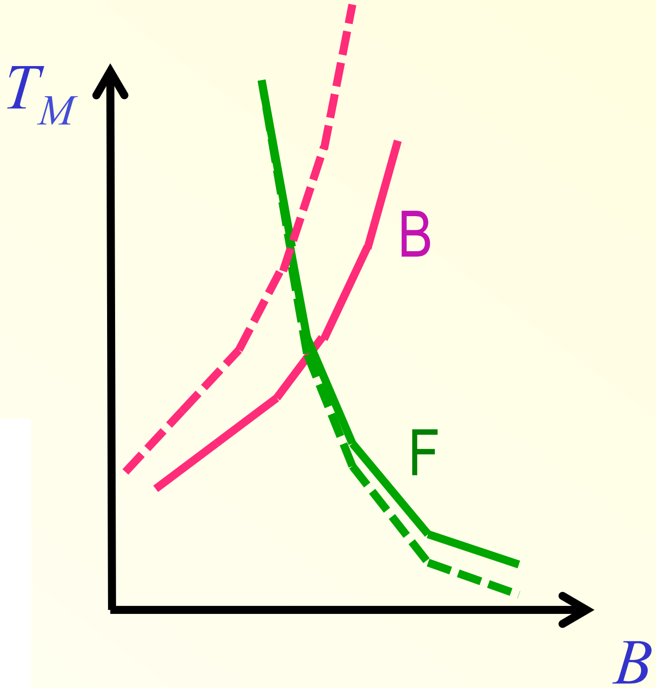


$$\left.\frac{d\ln\rho_{\text{pair}}}{d\ln B}\right|_{\text{peak}} \approx -\frac{1}{2}\left.\frac{d\ln\rho_{\text{single}}}{d\ln B}\right|_{\text{peak}}$$

A. Johannson, D. Shahar et al., (2006)

Dependence on angle of B-field





Parallel field

Perp. field:

B: stronger localized

$T_M \uparrow$

F: less localized

$T_M \downarrow$

$\longrightarrow$

B_{Peak} lower &

$T_{M,Peak}$ higher

More asymmetric

A. Johannson, D. Shahar et al., (2006)

Conclusions

- ξ of bosons shrinks in B-field, fermions inflate
- Effect of Coulomb gap:
  $\longrightarrow$ Bosonic mobility edge ("effective edge" in d=2)
  $\longrightarrow$ Magneto-oscillations of mobility edge: like exp. features
- MR peak results from antagonizing fermions and pairs:
  Hampering each other's transport
  & On top of that: opposite orbital MR

  Angle dependence, asymmetry, peak position qualitatively reproduced by very simple, minimal model
- Open Q: Why is there activated transport ONLY on pair side??