Anderson localization in QCD

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Strong interaction exhibits Anderson transition

- Quantum chromodynamics — strongly interacting quarks
- Energy scale 200 MeV – 1 GeV
- Computer “experiments” — lattice QCD simulation

QCD — the model

- Basic fields
  - SU(3) gauge field $A_\mu(x)$
  - quarks — $\psi(x)$ Dirac spinor, SU(3) fundamental representation
- Dynamics
  - Action: $S = \int d^4x \left[\mathscr{L}_g[A(x)] + \bar{\psi}(x)\{D[A(x)] + M\}\psi(x) \right]$
  - D[A(x)] — covariant Dirac operator
  - Quantization — path integral $Z = \int \mathscr{D}\psi \mathscr{D}A \; e^{iS}$

How to make sense of this? The lattice

- Regularization: 4d continuum $\longrightarrow$ 4d hypercubic lattice
- How to get rid of the lattice? Continuum limit
  - lattice spacing $a \to 0$
  - mass^{-1} = ξ (correlation length)
  - $\xi_{\text{lattice}} a = (\text{physical mass})^{-1}$ $\Rightarrow$ $\xi_{\text{lattice}} \to \infty$
  - tune system to critical point
- Wick rotation: $t \to -it$
  - $e^{iLt} \longrightarrow e^{-Ht}$
  - $Z = \int e^{iLt} \longrightarrow Z = \int e^{-Ht}$ stat. phys. partition sum
  - temporally finite box of size L_t $\longrightarrow$ temperature $T = 1/L_t$

Lattice QCD

- Partition function (integrating out quarks):

$$Z = \int \mathscr{D}\psi \mathscr{D}\bar{\psi} \mathscr{D}U \; e^{-S_g[U] \; - \; \bar{\psi}\{D[U]+M\}\psi}$$
$$= \int \mathscr{D}U \; det\{D[U]+M\} \cdot e^{-S_g[U]}$$

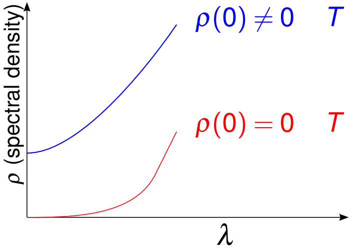
- Statistical physics system (4-dimensional, Euclidean)
- Dynamical variables: $U_i \in SU(3)$ on lattice links
- Temperature: $T = \frac{1}{L_t}$ (L_t: extension in Euclidean time)

Dirac operator: $D[U]$

- discretized differential op.
- nearest neighbor hopping — random $SU(3)$ phase
- random sparse matrix localization?

Dirac spectrum for $T < T_c$ and $T > T_c$

- cross-over at T_c where $1/T_c = L_{tc} \approx$ correlation length
- transition at $T_c \approx 170\text{MeV}$:



$\rho(0) \neq 0$ $T < T_c$ hadronic (χ-broken)

$\rho(0) = 0$ $T > T_c$ QG plasma (χ-symmetric)

Eigenvalue statistics $T > T_c$ (quark-gluon plasma)

in different regions of the spectrum

unfolded level spacing distribution $s = \frac{\lambda_{n+1} - \lambda_n}{\langle \lambda_{n+1} - \lambda_n \rangle}$

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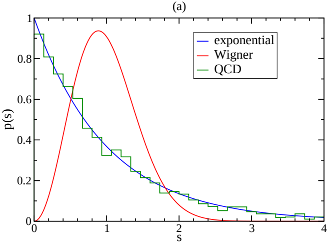

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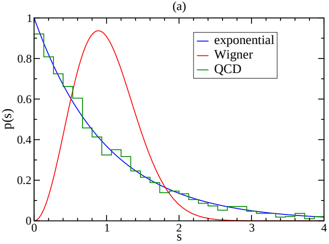

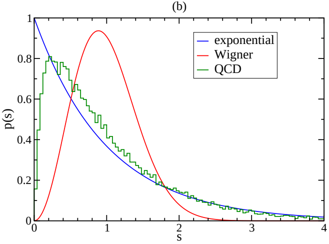

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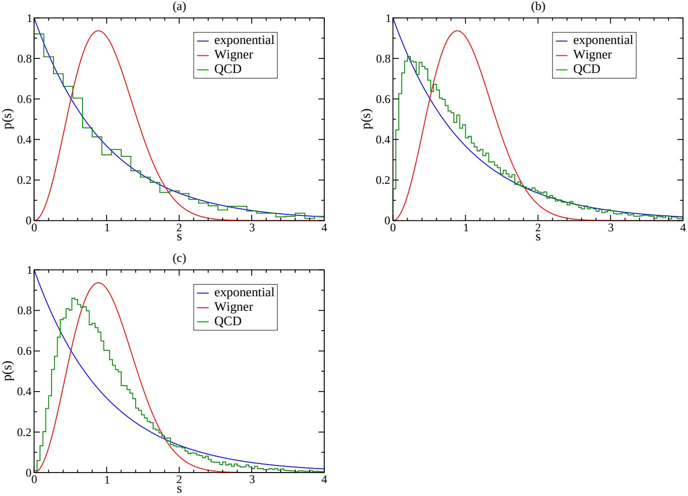

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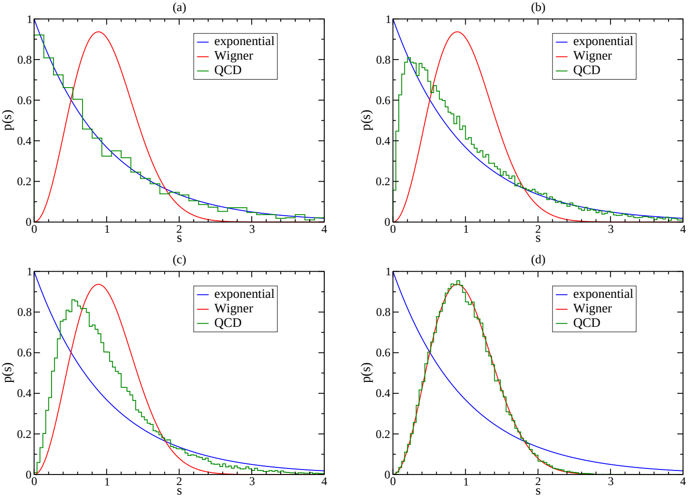
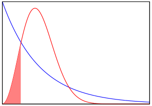
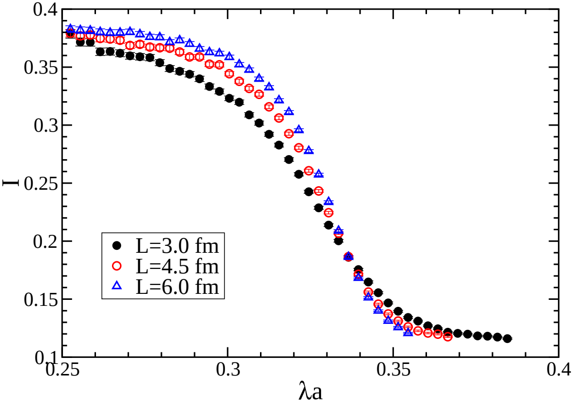


Integrated level spacing distribution

$$I = \int_0^{0.5} p(s)ds$$

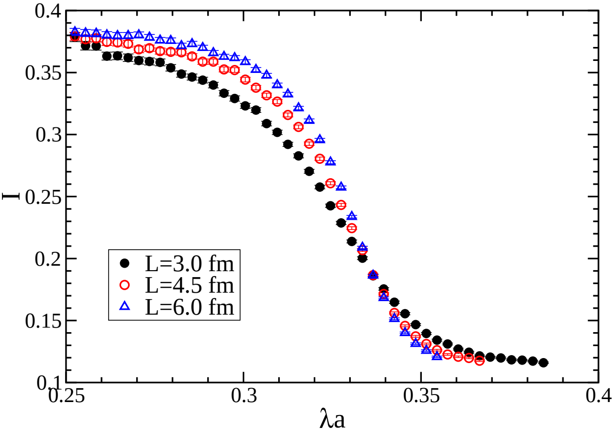




Finite size scaling

$$I(\lambda, \mu, L) = f\left(L^{1/\nu}(\lambda - \lambda_c)\right)$$

Is it possible to choose ν and λ_c to have data collapse?



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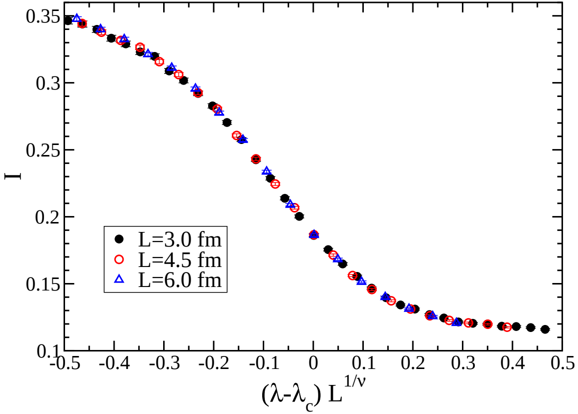
Yes! $\longrightarrow$ λ_c, ν



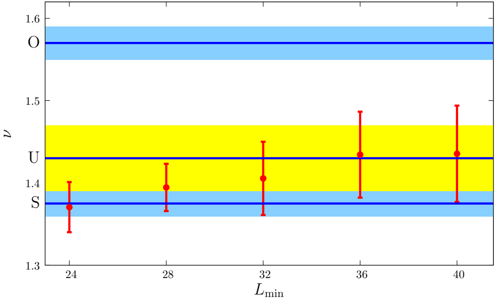
The exponent ν

- Use only systems larger than L_{min} for the fit
  - system sizes: $L^3 = 24^3, 28^3, 32^3, 36^3, 40^3, 48^3, 56^3$

K. Slevin & T. Ohtsuki, PRL 82 (1999)

K. Slevin & T. Ohtsuki, PRL 78 (1997)

Y. Asada, K. Slevin & T. Ohtsuki, J.Phys.Soc.Jpn. 74 (2005)

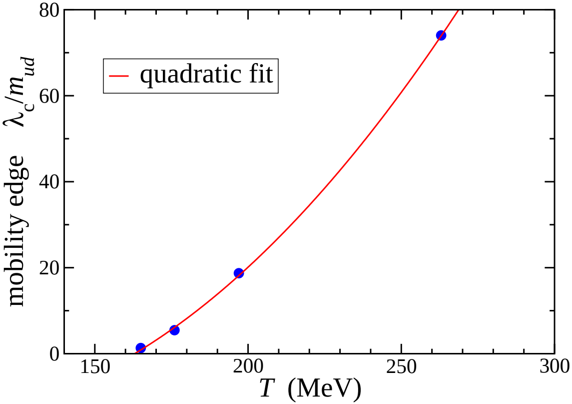


- ν compatible with 3d Anderson model in unitary class

M. Giordano, TGK, F. Pittler, PRL 112 (2014)

The temperature controls the mobility edge

Localized modes appear around T_c



Fit: $\lambda_c \to 0$ at $T = 163(2)$ MeV $\approx T_c$ cross-over to quark-gluon plasma

Why is there a mobility edge?

Free quarks at temperature T

- Antiperiodic temporal boundary condition

$\rightarrow$ Gap in the Dirac spectrum:

πT — lowest Matsubara mode

Interactions switched on; $T < T_c$

- $1/T > \xi$ (correlation length)
- Quarks "do not feel" the temporal boundary condition
- Matsubara picture does not apply $\rightarrow$ no gap, χSB

Temporal twist determines the lowest eigenvalues

Interactions switched on; $T > T_c$

- $1/T < \xi$ (correlation length) $\rightarrow$ quarks "feel" the b.c.
- Lowest Matsubara mode $\propto$ phase of temporal "twist"
- Temporal "twist":
  - π boundary condition
  - ϕ phase of the gauge parallel transporter around temporal direction
    Polyakov loop
- Polyakov loop can only decrease the twist

  $\rightarrow$ eigenvalues move down

  $\rightarrow$ Matsubara bound becomes fuzzy

  $\rightarrow$ gap disappears, gets sparsely populated

$$\pi T \longrightarrow \lambda_c$$

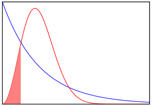
Analogy with the Anderson model

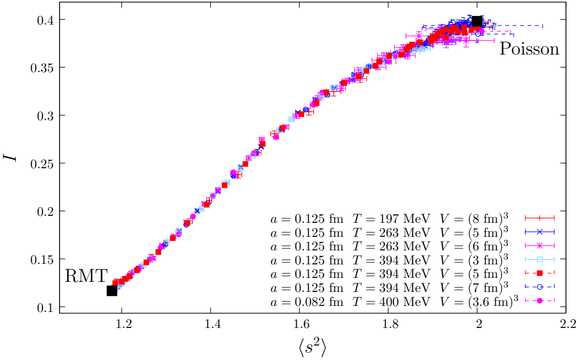
- High T $\rightarrow$ dimensional reduction to 3d
- Local Polyakov loop $\rightarrow$ on site random potential
  - disorder correlated
  - but correlation length finite
- Spatial gauge links $\rightarrow$ hopping terms
- 3d effective model:
  - P-loop $\rightarrow$ spin model
  - spins $\rightarrow$ disorder for Anderson model
  - exhibits same type of transition   Giordano, TGK, Pittler, JHEP 2015, JHEP 2016

Wigner-Dyson → Poisson

Unfolded level spacing distribution

$$I = \int_0^{0.5} p(s)\,ds$$

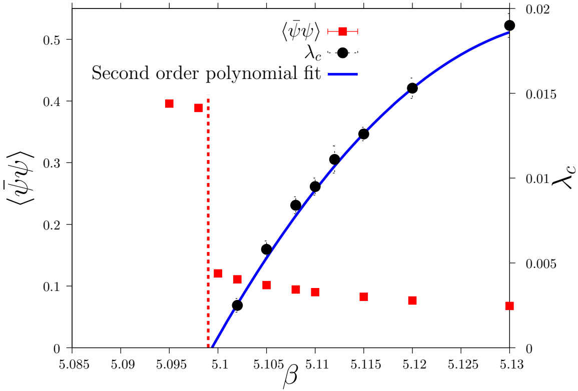




Conclusions

- Anderson transition in QCD at high T
  - high T: dimensionally reduced $4d \to 3d$
  - mobility edge controlled by temperature
  - role of Polyakov loop fluctuations
  - ν compatible with Anderson model
  - multifractal analysis of eigenvectors confirms this
    Giordano, TGK, Ujfalusi, Pittler, Varga, PRD 2015
- Phase transition or not?
  - The transition to QGP is only a cross-over
  - Have we found a genuine phase transition?
  - No! In QCD no thermodynamic quantity is singular

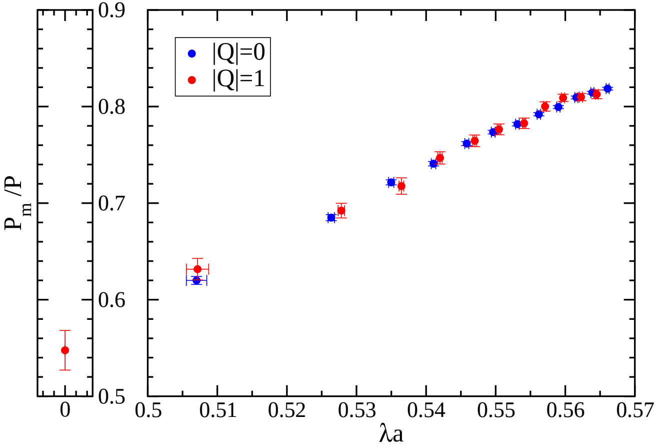
QCD with staggered quarks on a coarse lattice



Localized modes correlate with P-loop fluctuations

$P_m = \sum_x P(x)|\psi(x)|^2$

Bruckmann, Schierenberg, TGK, PRD 2011



How to describe the transition?

- Random matrix models?
  - Power-law random banded matrices
  - Moshe-Neuberger-Shapiro model
  - Invariant ensemble with log potential
- Renormalization group description?
- Three “fixed points”?
  - Wigner-Dyson
  - Poisson
  - Critical
- Universality?