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# Local density of states and its mesoscopic fluctuations near the transition to a superconducting state

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# Outline:

- Experimental motivation
- Formalism: Nonlinear sigma-model
- Results for LDOS (averaged + spatial fluctuations):
  - near SC transition in 2D with short-range interactions
  - near SC transition in 2D with Coulomb interaction
  - SIT near Anderson transition with short-range interactions
- Conclusions

# Experimental motivation

**Tunneling spectroscopy** of 2D superconducting films:

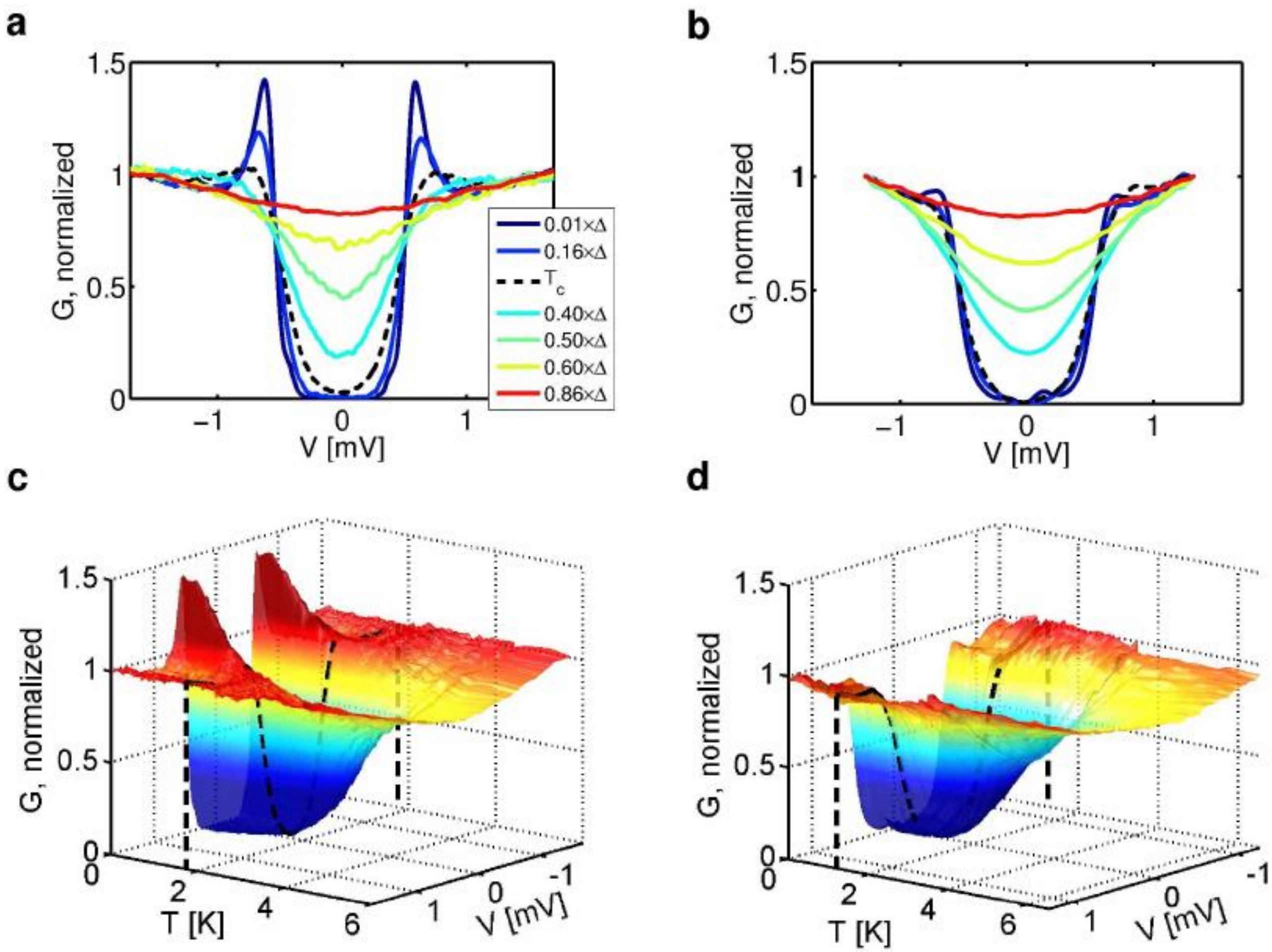
- pronounced **soft gap** in the tunneling spectrum survives across the superconductor-metal transition (at $T > T_c$)
- … and across the superconductor-insulator transition
- strong point-to-point **fluctuations of LDOS**

**Popular interpretations** ("bosonic mechanism"):

- preformed Cooper pairs ➔ "pseudogap" in non-SC states
- localization of Cooper pairs on the SC side of transitions
- emergent "self-granularity" of homogeneous films

# Experimental motivation

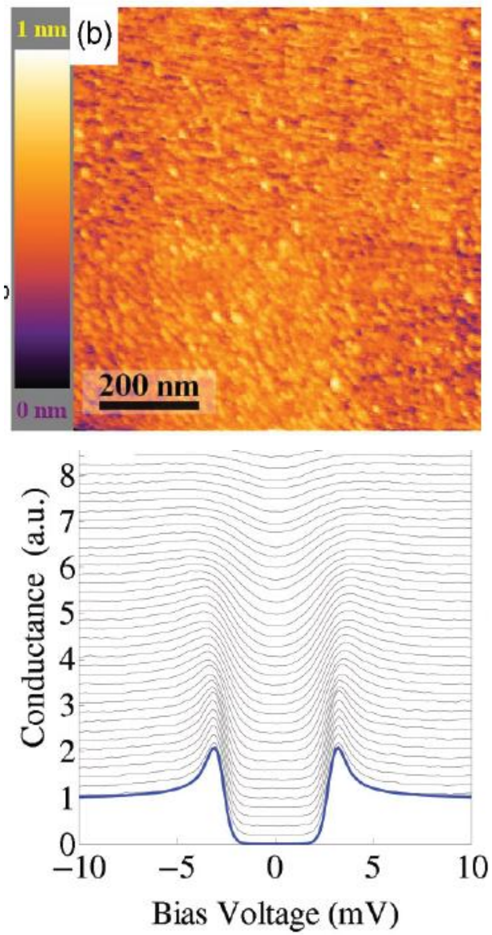
InO thin films: Differential tunneling conductance, $T_c = 1.2 \div 1.7$ K

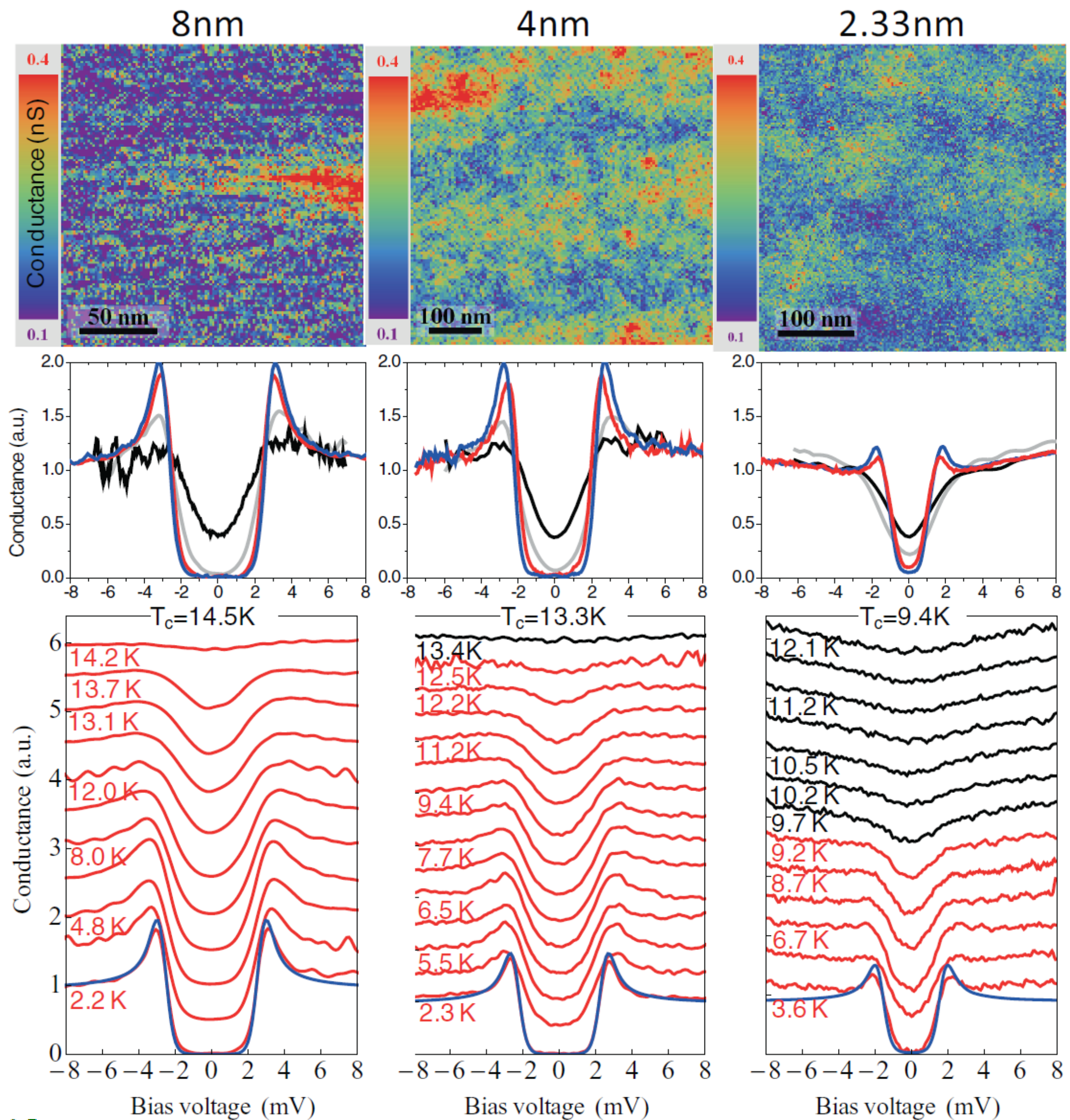



[adapted from Sacépé et al. (2011)]

# Experimental motivation

homogeneously disordered NbN thin films:

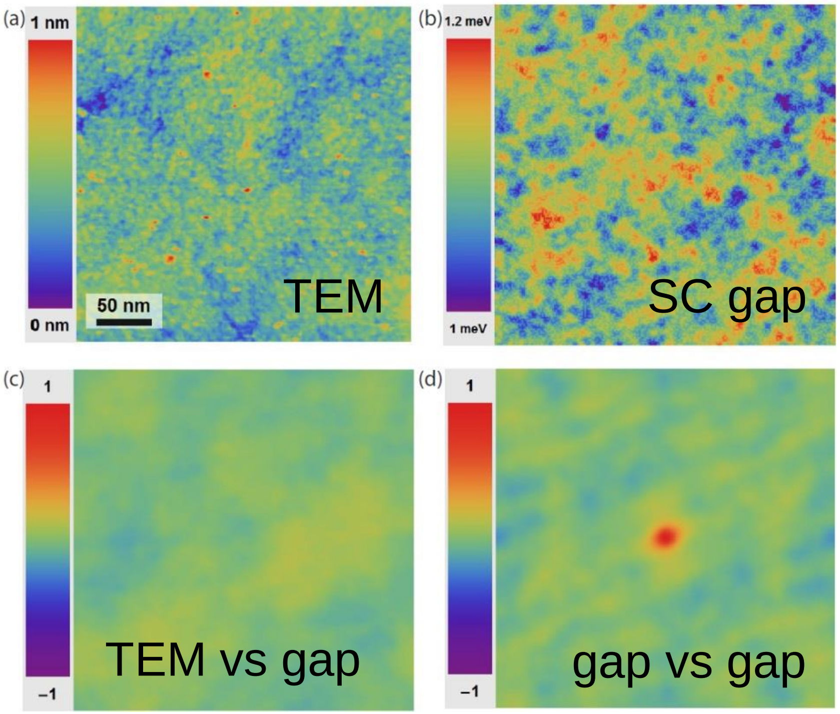


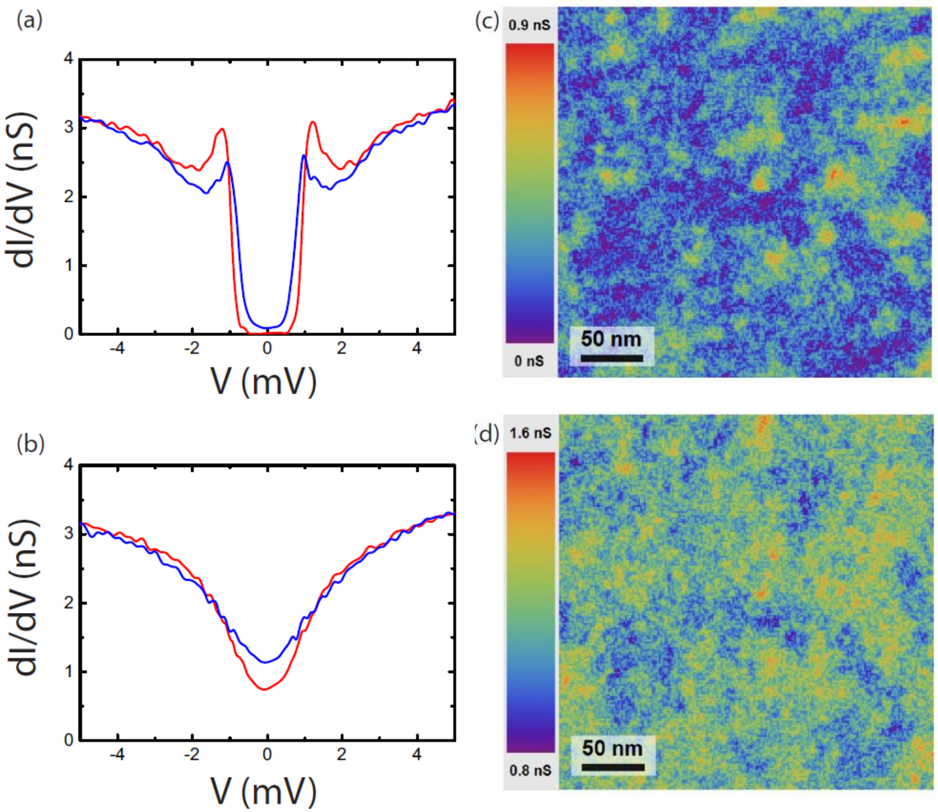




[adapted from Noat et al. (2013)]

# Experimental motivation

homogeneously disordered NbN thin films:




[adapted from Carbillet et al. (2016)]

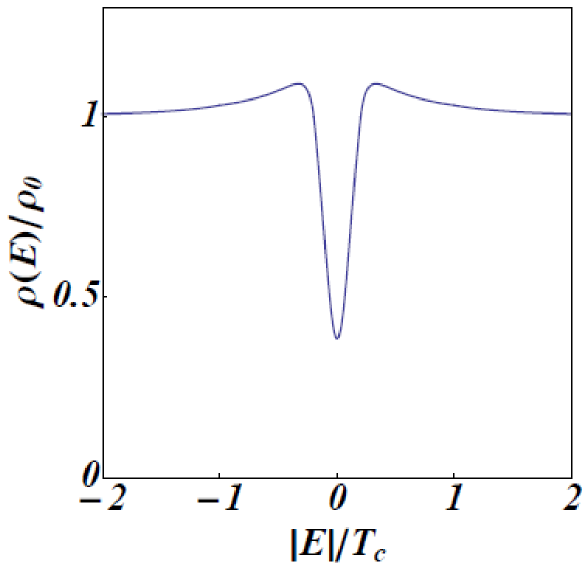
# Tunneling DOS in the presence of SC correlations in 2D

[Abrahams, Redi, Woo (1970); Huralt, Maki (1970)]

- suppression of LDOS near superconducting transition temperature, $T - T_c \ll T_c$

$$\frac{\langle \rho(E) \rangle}{\rho_0} = \begin{cases} 1 - 8(1-\ln 2) t_0 (T_c \tau_{GL})^2, & |E| \ll \tau_{GL}^{-1}, \\ 1 + 2t_0 (T_c/E)^2 \ln(|E|\tau_{GL}), & \tau_{GL}^{-1} \ll |E| \ll T_c, \\ 1 + 2t_0 (T_c/E)^2 \ln(T_c \tau_{GL}), & T_c \ll |E| \end{cases}$$

where $\tau_{GL}^{-1} = 8(T - T_c)/\pi$ and $t_0 = 2/(\pi g_0)$ denotes bare resistance.



# Tunneling DOS:  Zero-bias anomaly in diffusive 2D metals

Altshuler, Aronov '79,  Altshuler, Aronov, Lee '80,  Finkelstein '83,
Nazarov '89,  Levitov & Shytov '97,  Kamenev & Andreev '99

- zero-bias anomaly in $d = 2$ with Coulomb repulsion

$$\langle \rho(E, r) \rangle_{\mathrm{dis}} \sim \exp\left( -\frac{1}{4\pi g} \ln(|E|\tau) \ln \frac{|E|}{D^2\kappa^4\tau} \right)$$

where

$g$ - conductance in units $e^2/h$,
$D$ - diffusion coefficient,
$\kappa = e^2\rho_0/\varepsilon$ - inverse static screening length

Physics:  Diffusive/hydrodynamic charge spreading affected by gauge-type phase fluctuations (Maxwell relaxation + “Debye-Waller factor”)

# Fluctuations of LDOS (no interaction)

review: Evers & Mirlin RMP´08

- local density of states (LDOS)

$$\rho(E, \boldsymbol{r}) = \sum_{\alpha} \left|\psi_{\alpha}(\boldsymbol{r})\right|^2 \delta(E - \epsilon_{\alpha})$$

- multifractality in the moments of LDOS

$$\left\langle [\rho(E, \boldsymbol{r})]^q \right\rangle_{\text{dis}} \sim L^{-\Delta_q}$$

- spatial correlations of LDOS

$$\langle \rho(E, \boldsymbol{r}) \rho(E, \boldsymbol{r} + \boldsymbol{R}) \rangle_{\text{dis}} \sim (R/L)^{\Delta_2} \qquad R \ll L$$

in the absence of interaction average LDOS is non-critical for Wigner-Dyson classes

# Fluctuations of LDOS with interactions (normal state)

Burmistrov, IG, Mirlin, PRL´13, PRB´14, PRB´15

- **Multifractality** of LDOS does exist in disordered electron systems with **Coulomb interaction**
- Coulomb interaction: **multifractal exponents** are **different** from those in noninteracting case
- **Metal-insulator transition**: From Altshuler-Aronov-Finkelstein **zero-bias anomaly** to Efros-Shklovskii **Coulomb gap**:
- This work: LDOS near superconducting transition ($T>T_c$)

# Field-theory approach: Interacting nonlinear sigma-model

Finkelstein (1983)

- nonlinear sigma-model action

$$\mathcal{S}[Q] = -\frac{g}{32}\int d\boldsymbol{r}\,\mathrm{tr}(\nabla Q)^2 + 4\pi T Z_\omega \int d\boldsymbol{r}\,\mathrm{tr}\,\eta Q$$
$$-\frac{\pi T}{4}\sum_{r=0,3}\sum_{j=0,\dots,3}\sum_{\alpha,n}\int d\boldsymbol{r}\,\Gamma_j\,\mathrm{tr}\,I_n^\alpha t_{rj} Q\,\mathrm{tr}\,I_{-n}^\alpha t_{rj} Q$$
$$-\frac{\pi T}{4}\sum_{r=1,2}\sum_{\alpha,n}\int d\boldsymbol{r}\,\Gamma_c\,\mathrm{tr}\,L_n^\alpha t_{r0} Q\,\mathrm{tr}\,L_n^\alpha t_{r0} Q$$

matrix field $Q$ (in Matsubara, particle-hole, spin and replica spaces):

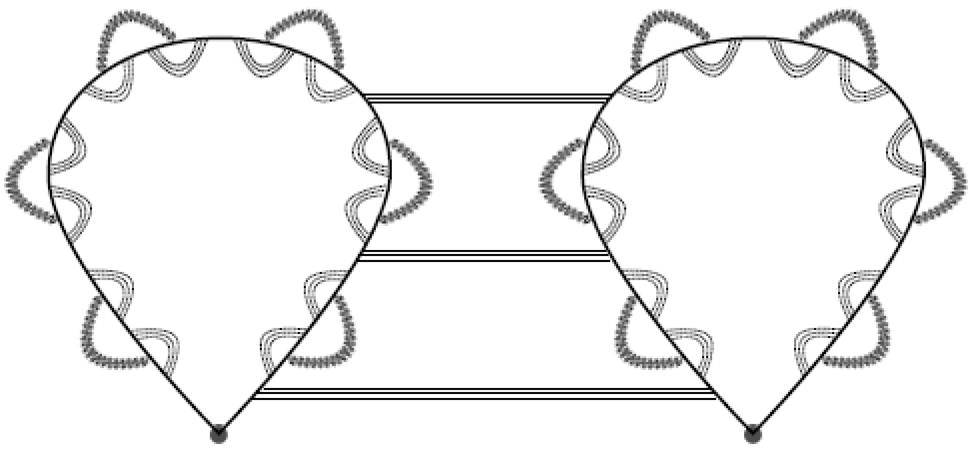
$$Q^2(\boldsymbol{r}) = 1, \qquad \mathrm{tr}\,Q(\boldsymbol{r}) = 0, \qquad Q^\dagger(\boldsymbol{r}) = C^T Q(\boldsymbol{r}) C, \qquad C = it_{12}$$

$g$ – conductivity in units $e^2/h$, $\Gamma_0 \equiv \Gamma_s$ – interaction amplitude in the singlet channel,
$\Gamma_1, \Gamma_2, \Gamma_3 \equiv \Gamma_t$ – interaction amplitude in the triplet channel, $\Gamma_c$ – Cooper channel
$t_{rj} = \tau_r \otimes s_j$, $\boldsymbol{\tau}$, $\boldsymbol{s}$ are Pauli matrices in particle-hole and spin spaces

$$\Lambda_{nm}^{\alpha\beta} = \mathrm{sgn}\,n\,\delta_{nm}\delta^{\alpha\beta}, \qquad \eta_{nm}^{\alpha\beta} = n\,\delta_{nm}\delta^{\alpha\beta}, \qquad (I_k^\gamma)_{nm}^{\alpha\beta} = \delta_{n-m,k}\delta^{\alpha\beta}\delta^{\alpha\gamma}, \qquad (L_k^\gamma)_{nm}^{\alpha\beta} = \delta_{n+m,k}\delta^{\alpha\beta}\delta^{\alpha\gamma}$$

## Strategy

- Construct and renormalize operators describing the moments of LDOS
- Renormalize NLSM down to energy scales ~max[E,T]
- Include superconducting fluctuations from real processes



# Mesoscopic fluctuations of LDOS: Field theory

- disorder-averaged LDOS

$$\left\langle \rho(E,\boldsymbol{r}) \right\rangle_{\text{dis}} = \rho_0 \,\mathrm{Re}\, \langle P_1^R(E) \rangle_{\mathcal{S}}, \qquad P_1(i\varepsilon_n) = Q_{nn}^{\alpha\alpha}(\boldsymbol{r})$$

where $\varepsilon_n = \pi T(2n+1)$ is fermionic Matsubara frequencies

- disorder-averaged 2nd moment of LDOS

$$\left\langle \rho(E,\boldsymbol{r})\rho(E',\boldsymbol{r}) \right\rangle_{\text{dis}} = (\rho_0^2/2)\,\mathrm{Re} \left\langle P_2^{RR}(E,E') - P_2^{RA}(E,E') \right\rangle_{\mathcal{S}}$$

$$P_2(i\varepsilon_n, i\varepsilon_m) = Q_{nn}^{\alpha_1\alpha_1}(\boldsymbol{r}) Q_{mm}^{\alpha_2\alpha_2}(\boldsymbol{r}) - Q_{nm}^{\alpha_1\alpha_2}(\boldsymbol{r}) Q_{mn}^{\alpha_2\alpha_1}(\boldsymbol{r})$$

$$\cdots$$

- $P_q$ are eigenoperators of renormalization group transformations

  renormalization of $P_q$ by means of perturbation theory around $Q = \Lambda$

# Mesoscopic fluctuations of LDOS: RG

Disorder-averaged DOS: $\langle \rho(E) \rangle = \rho_0 [Z(E)]^{1/2},$

$$-\frac{d \ln Z}{dy} = -[\ln(1+\gamma_s) + 3\ln(1+\gamma_t) + 2\gamma_c]t + O(t^2),$$

$q$th moment of the DOS: $\langle \rho^q \rangle = \langle \rho \rangle^q m_q,$

$$-\frac{d \ln m_q}{dy} = \frac{q(1-q)}{2}\{2t + [c(\gamma_s) + 3c(\gamma_t) - 2\gamma_c]t^2\}.$$

$$c(\gamma) = 2 + \frac{2+\gamma}{\gamma}\mathrm{li}_2(-\gamma) + \frac{1+\gamma}{2\gamma}\ln^2(1+\gamma),$$

$\gamma_{s,t,c} = \Gamma_{s,t,c}/Z_\omega$; $\gamma_s = -1$ for the Coloumb interaction

# RG of 2D sigma-model

Finkelstein (1983)

Full set of one-loop RG equations $[t = 2/(\pi g)]$ :

Burmistrov, IG, Mirlin, PRB´15

$$\frac{dt}{dy} = t^2[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c],$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + 3\gamma_t + 2\gamma_c + 4\gamma_c^2\big),$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}(1+\gamma_t)[\gamma_s - \gamma_t - 2\gamma_c(1 + 2\gamma_t - 2\gamma_c)],$$

$$\frac{d\gamma_c}{dy} = -2\gamma_c^2 - \frac{t}{2}\big[(1+\gamma_c)(\gamma_s - 3\gamma_t) - 2\gamma_c^2 + 4\gamma_c^3 + 6\gamma_c(\gamma_t - \ln(1+\gamma_t))\big],$$

$$\frac{d\ln Z_\omega}{dy} = \frac{t}{2}\big(\gamma_s + 3\gamma_t + 2\gamma_c + 4\gamma_c^2\big)$$

# Short-range interactions in 2D: Enhancement of $T_c$

Burmistrov, IG, Mirlin, PRL´12, PRB´15

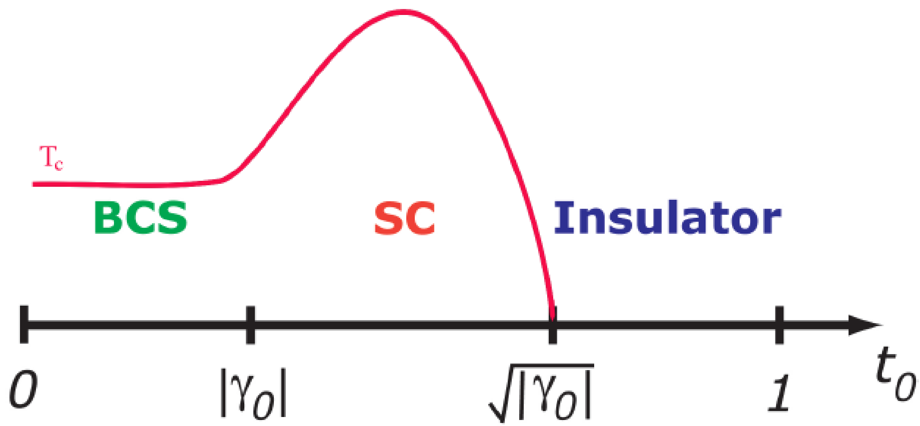
- RG after approaching BCS line $\gamma_s = -\gamma_t = -\gamma_c = -\gamma$:

$$dt/dy = t^2, \qquad d\gamma/dy = 2t\gamma - 2\gamma^2/3\,.$$

- superconducting transition temperature is nonmonotonous function of $t_0$:

$$T_c \sim \frac{1}{\tau}\exp\left(-\frac{2}{t_0}+\frac{2}{t_c}\right) \qquad t_c = 3t_0^2/(2|\gamma_0|) \ll 1$$

- $T_c \gg T_c^{BCS} \sim \tau^{-1}\exp(-1/|\gamma_{c,0}|)$ in the interval $|\gamma_0| \ll t_0 \ll \sqrt{|\gamma_0|} \ll 1$




3D near Anderson transition:
Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007); Feigelman, Ioffe, Kravtsov, Cuevas (2010)

# Short-range interactions in 2D: Average LDOS

$$\frac{\langle\rho(E)\rangle}{\rho_0} = Z_s^{1/2}(T)[1 - 8(1-\ln 2)t_c(T_c\tau_{\mathrm{GL}})^2] \qquad \text{for } |E| \ll \tau_{GL}^{-1},$$

$$\frac{\langle\rho(E)\rangle}{\rho_0} = Z_s^{1/2}(T)\left[1 + 2t_c\frac{T_c^2}{E^2}\ln(|E|\tau_{\mathrm{GL}})\right] \qquad \text{for } \tau_{GL}^{-1} \ll |E| \ll T_c,$$

$$\frac{\langle\rho(E)\rangle}{\rho_0} = Z_s^{1/2}(|E|)\left[1 + 2t(L_E)\frac{T_c^2}{E^2}\ln(T_c\tau_{\mathrm{GL}})\right]$$

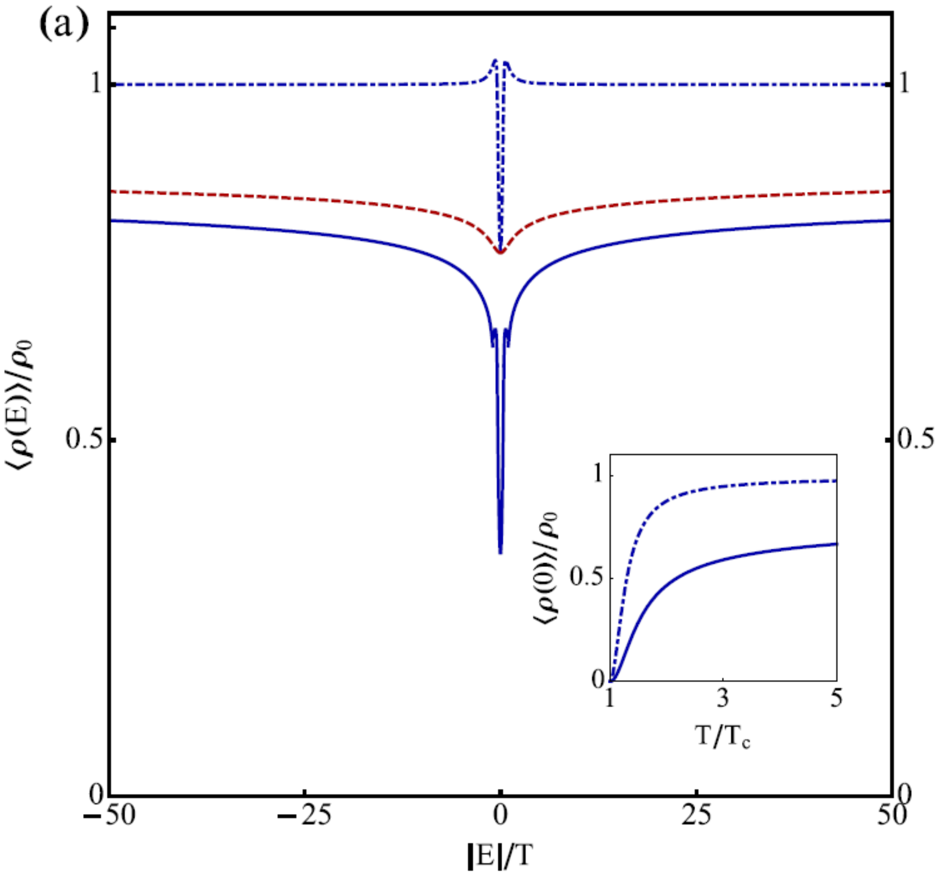
$$\simeq 1 + 2t(L_E)\frac{T_c^2}{E^2}\ln(T_c\tau_{\mathrm{GL}}) - \frac{3t^2(L_E)}{2t_c} \qquad \text{for } T_c \ll |E| \ll 1/\tau$$

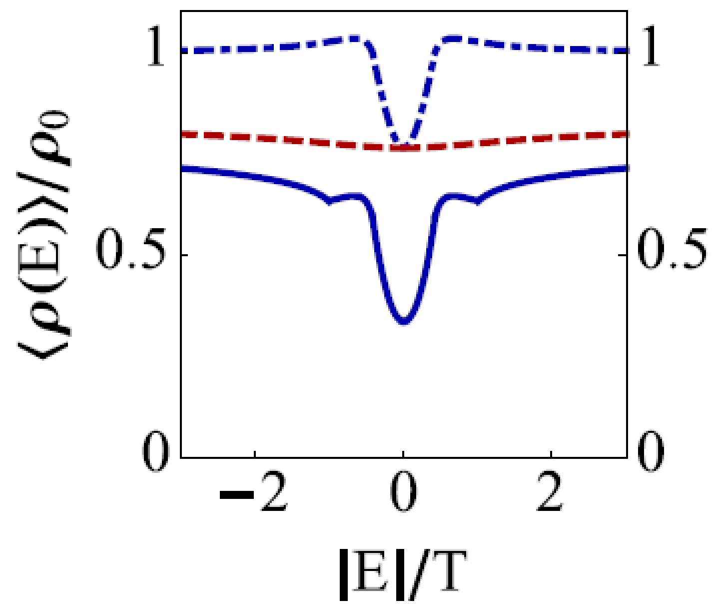
$$Z_s^{1/2}(T) = \left(\frac{16}{3\pi e}\frac{T_c\tau_{\mathrm{GL}}}{t_c}\right)^{-3t_c} \qquad \qquad \tau_{\mathrm{GL}}^{-1} = 8T_c|\gamma^{-1}(L_T)|/\pi$$

not a superposition of AA and SC corrections close to the SC transition

# Short-range interactions in 2D: Average LDOS

$$T = 1.6T_c \qquad t_c = 0.05,\ t_0 = 0.02,\ \text{and}\ T_c\tau_{\mathrm{GL}} = 2.5$$



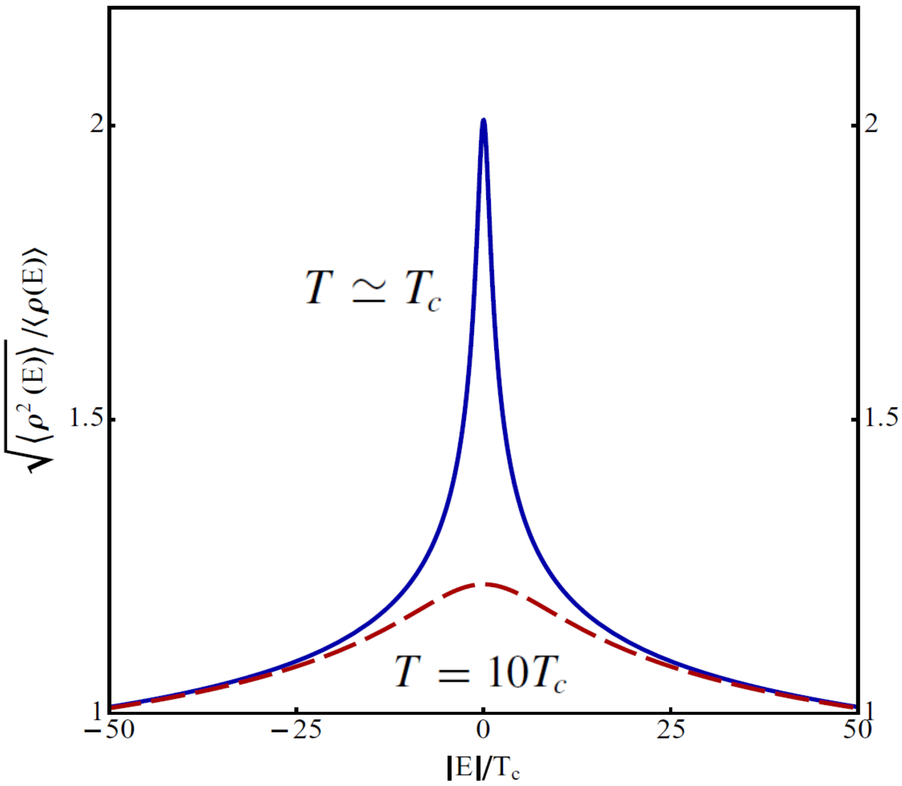

Dotted-dashed blue curve ignores renormalization; dashed red curve: $T$=10$T_c$

# Short-range interactions in 2D: LDOS fluctuations

$$\frac{\langle \rho^q(E)\rangle}{\langle \rho(E)\rangle^q} = \left(\frac{t_c}{t_0}\right)^{q(q-1)} \left(1 + \frac{t_c}{2}\ln\frac{E}{T_c}\right)^{q(1-q)} \qquad \text{for } T_c \ll |E|$$

$$\frac{\langle \rho^q(E)\rangle}{\langle \rho(E)\rangle^q} = \left(\frac{t_c}{t_0}\right)^{q(q-1)} \quad \text{for } |E| \ll T_c$$

$$t_c = 3t_0^2/(2|\gamma_0|)$$




LDOS fluctuations are particularly strong in the range of parameters where enhancement of $T_c$ occurs

$$|E| \sim T \sim T_c\text{:}\ \langle \rho^q \rangle / \langle \rho \rangle^q \sim (t_0/|\gamma_0)^{q(q-1)} \gg 1$$

# Coulomb interactions in 2D: Suppression of $T_c$

- RG equations:

$$\frac{dt}{dy} = \frac{t^2}{2}, \qquad \frac{d\gamma_c}{dy} = \frac{t}{2} - 2\gamma_c^2.$$

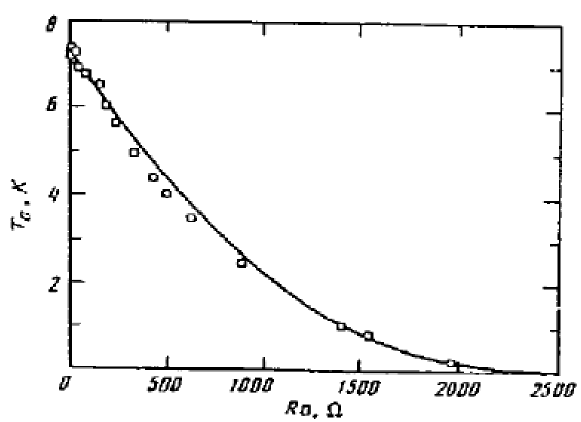
[Castelani et al. (1984); Ma, Fradkin (1986)]

- suppression of superconducting transition temperature:

$$T_c \sim \tau^{-1} e^{-2y_F}, \qquad y_F = \frac{1}{2\sqrt{t_0}} \ln \frac{2|\gamma_{c,0}| + \sqrt{t_0}}{2|\gamma_{c,0}| - \sqrt{t_0}}.$$

[Finkel'stein (1987)]

- separatrix: $t_0 = 4\gamma_{c,0}^2$



# Coulomb interactions in 2D: Average LDOS

$$\frac{\langle \rho(E)\rangle}{\rho_0} = Z_c^{1/2}(T_c)[1 - 8(1-\ln 2)t_c(T_c\tau_{\rm GL})^2] \qquad \text{for } |E| \ll \tau_{GL}^{-1},$$

$$\frac{\langle \rho(E)\rangle}{\rho_0} = Z_c^{1/2}(T_c)\left[1 + 2\frac{t_c T_c^2}{E^2}\ln(|E|\tau_{\rm GL})\right] \qquad \text{for } \tau_{GL}^{-1} \ll |E| \ll T_c,$$

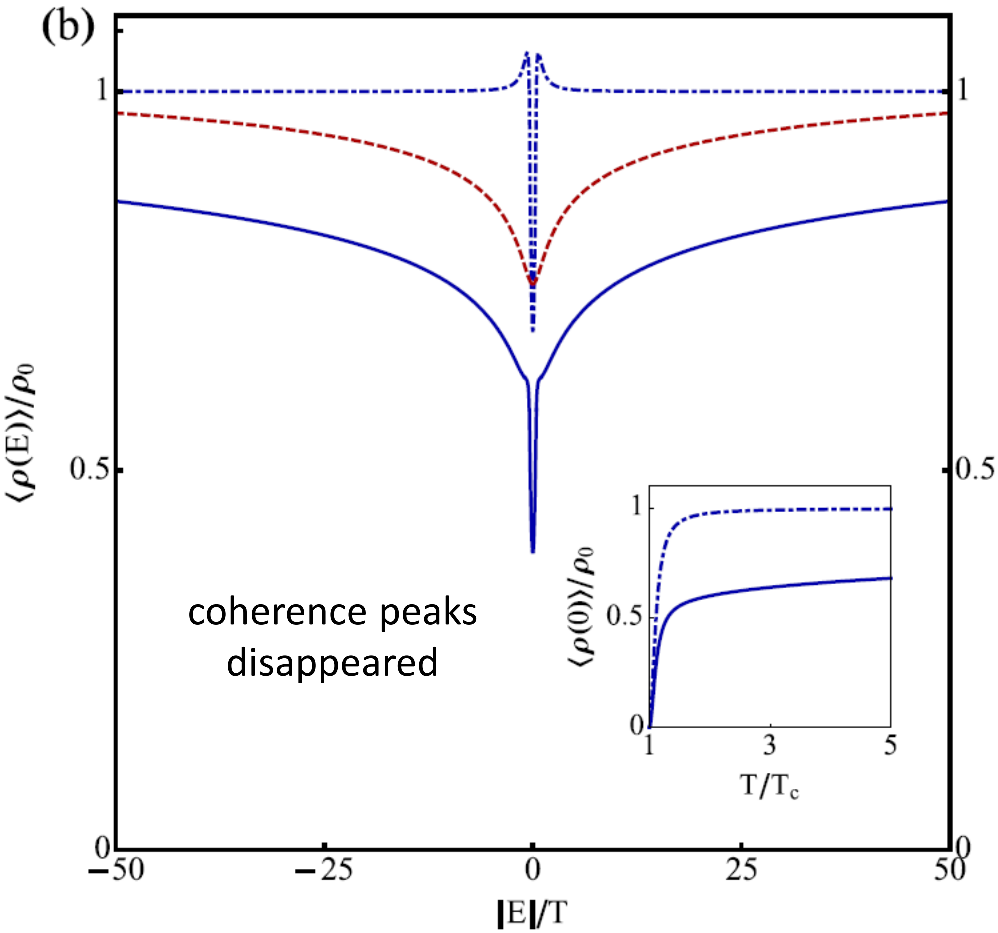
$$\frac{\langle \rho(E)\rangle}{\rho_0} = Z_c^{1/2}(|E|)\left[1 + 2\frac{t(L_E)T_c^2}{E^2}\ln(T_c\tau_{\rm GL})\right] \qquad \text{for } T_c \ll |E| \ll 1/\tau$$

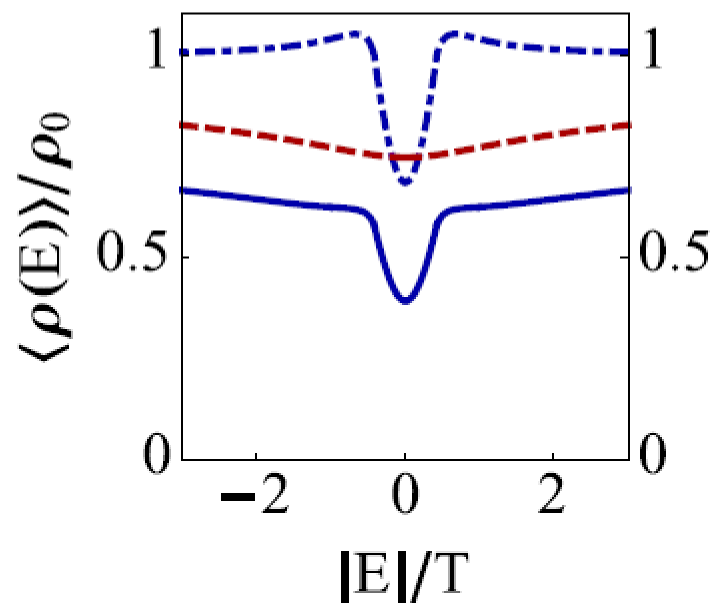
$$Z_c^{1/2}(|E|) = \exp\left[-\frac{t_0}{4}\ln(E\tau)\ln\frac{E\tau}{\varkappa^2 l^2}\right], \qquad Z_c^{1/2}(T_c) = \exp\left[-\frac{1}{4}\ln\left(\frac{2|\gamma_{c,0}|-\sqrt{t_0}}{2|\gamma_{c,0}|+\sqrt{t_0}}\right) \times \ln\left(\frac{2|\gamma_{c,0}|-\sqrt{t_0}}{2|\gamma_{c,0}|+\sqrt{t_0}}(\varkappa l)^{-2\sqrt{t_0}}\right)\right] \ll 1$$

Enhancement of the superconducting dip by the Coulomb ZBA

# Coulomb interactions in 2D: Average LDOS

$$T = 1.2T_c \qquad t_0 = 0.03,\ T_c\tau_{\mathrm{GL}} = 2.5,\ \text{and}\ \varkappa l = 0.2$$



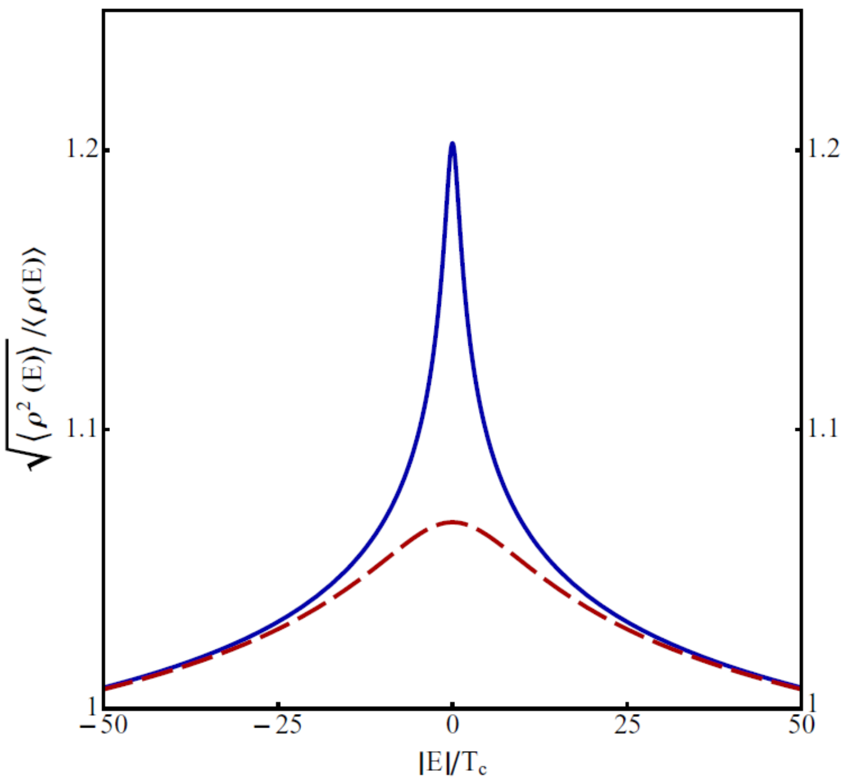




Dotted-dashed blue curve ignores renormalization; dashed red curve: $T$=10$T_c$

# Coulomb interaction in 2D: LDOS fluctuations

$$\frac{\langle \rho^q(E)\rangle}{\langle \rho(E)\rangle^q} = \left(\frac{t_c}{t_0}\right)^{\frac{q(q-1)}{2}} \left(1 + \frac{t_c}{4}\ln\frac{E}{T}\right)^{\frac{q(1-q)}{2}} \quad \text{for } T_c \ll |E|$$

$$\frac{\langle \rho^q(E)\rangle}{\langle \rho(E)\rangle^q} = \left(\frac{t_c}{t_0}\right)^{\frac{q(q-1)}{2}} \quad \text{for } |E| \ll T_c$$




$$t_c = t_0\left(1 + \frac{\sqrt{t_0}}{4}\ln\frac{2|\gamma_{c,0}| + \sqrt{t_0}}{2|\gamma_{c,0}| - \sqrt{t_0}}\right) \sim t_0 \ll 1$$

LDOS fluctuations may be strong
near the separatrix $t_0 = 4\gamma_{c,0}^2$

# SIT near Anderson transition (RG approach)

Burmistrov, IG, Mirlin, PRL´12

- RG on BCS line $\gamma_s = -\gamma_t = -\gamma_c = -\gamma$

$$\frac{dt}{dy} = \nu^{-1}(t - t_*) + \eta\gamma, \qquad \frac{d\gamma}{dy} = -\Delta_2\gamma - a\gamma^2$$

we assume $|\gamma_0| \ll t_* \ll 1$ and $|\Delta_2|\nu \neq 1$.

- $T_c$ as a function of $\xi = |t - t_*|^{-\nu}$ $(\delta_\xi = \tau^{-1}(\xi/l)^{-d})$:

metallic side $t < t_*$

$$\delta_\xi < T_c^*, \qquad T_c(\xi) \sim T_c^* = \tau^{-1}(a|\gamma_0|/|\Delta_2|)^{d/|\Delta_2|},$$

$$T_c^* < \delta_\xi < \tau^{-1}, \quad T_c(\xi) \sim \delta_\xi \exp\left[-\frac{a}{|\Delta_2|}\left(\frac{T_c^*}{\delta_\xi}\right)^{|\Delta_2|/d}\right],$$

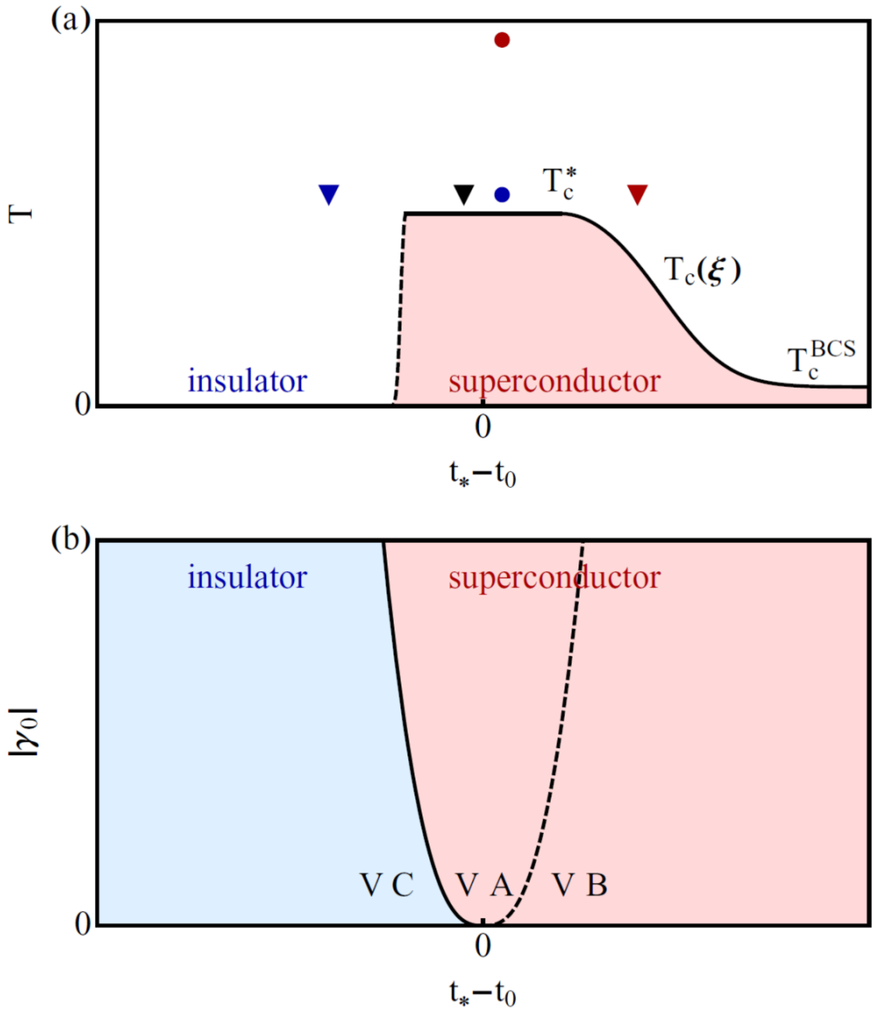
insulating side $t > t_*$

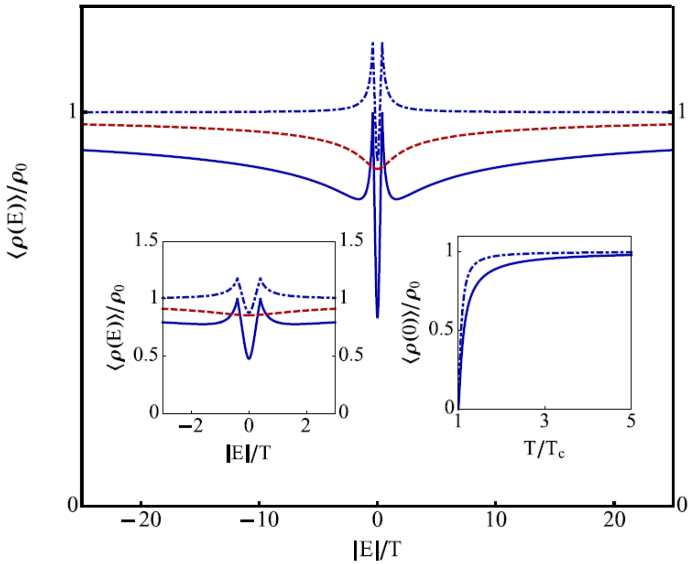
$$\delta_\xi < T_c^*, \quad T_c(\xi) \sim T_c^* = \tau^{-1}(a|\gamma_0|/|\Delta_2|)^{d/|\Delta_2|},$$

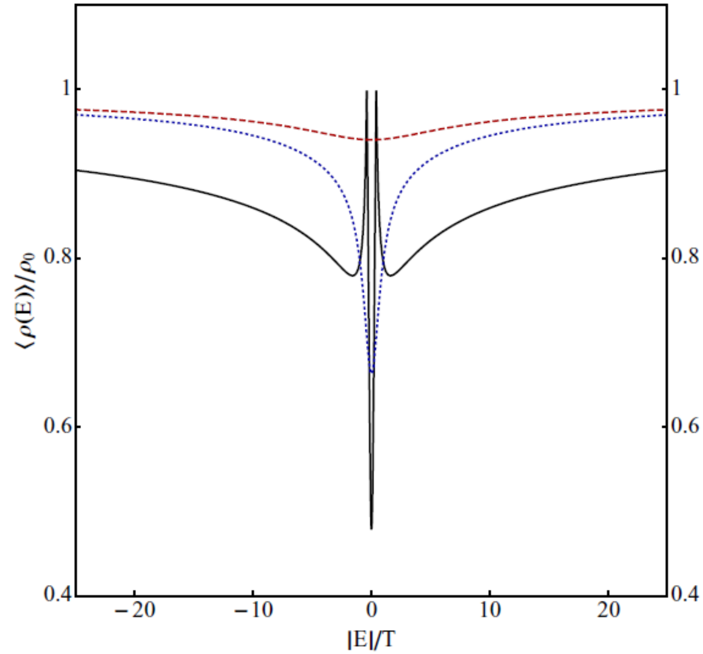
$$T_c^* < \delta_\xi, \qquad T_c(\xi) = 0.$$

$T_c^*$ agrees with Feigel'man, Ioffe, Kravtsov, Yuzbashyan (2007)

# SIT near Anderson transition



# LDOS in the critical region: $\delta_\xi \ll T_c^*$

- disorder-averaged LDOS above $T_c^*$, $|E|, T > T_c^*$:

$$\frac{\langle \rho(E) \rangle}{\rho_0} = 1 - \frac{\mu}{a} \left( \frac{T_c^*}{\max\{|E|, T\}} \right)^{|\Delta_2|/d} + \frac{\mu}{a} (T_c^* \tau)^{|\Delta_2|/d}$$

- disorder-averaged LDOS near $T_c^*$, $T - T_c^* \ll T_c^*$:

$$\frac{\langle \rho(E) \rangle}{\rho_0} = \begin{cases} 1 - c_d t_c (T_c^* \tau_{GL})^{\frac{6-d}{2}}, & |E| \ll \tau_{GL}^{-1}, \\ 1 + \tilde{c}_d t_c (T_c^* / |E|)^{\frac{6-d}{2}}, & \tau_{GL}^{-1} \ll |E| \ll T_c^*. \end{cases}$$

- RG for LDOS moments $\langle \rho^q \rangle = \langle \rho \rangle^q m_q$

$$\frac{d \ln \rho}{dy} = \mu \gamma, \qquad \frac{d \ln m_q}{dy} = -\Delta_q + b_q \gamma \,.$$

- fluctuations of LDOS above $T_c^*$, $|E|, T > T_c^*$:

$$\frac{\langle \rho^q(E) \rangle}{\langle \rho(E) \rangle^q} = (\tau \max\{T, |E|\})^{\frac{\Delta_q}{d}} \left[ 1 + \left( \frac{T_c^*}{\max\{T, |E|\}} \right)^{\frac{|\Delta_2|}{d}} - (T_c^* \tau)^{\frac{|\Delta_2|}{d}} \right]^{-x_q}$$

where $x_q = b_q/a + \Delta_q/\Delta_2$.

strong fluctuations of LDOS at $|E| \sim T \sim T_c^*$: $\frac{\langle \rho^q \rangle}{\langle \rho \rangle^q} \sim (|\gamma_0|)^{\Delta_q/|\Delta_2|} \gg 1$

# LDOS in the insulating phase $\delta_\xi \gtrsim T_c^*$ :

- high energies $|E| \gg \delta_\xi$ (critical states):

$$\frac{\langle \rho(E) \rangle}{\rho_0} = 1 + \frac{\mu}{|\Delta_2|} \gamma(\xi) \left( \frac{\delta_\xi}{|E|} \right)^{\frac{|\Delta_2|}{d}}, \qquad \frac{\langle \rho^q(E) \rangle}{\langle \rho(E) \rangle^q} = \left( \frac{\gamma(\xi)}{\gamma_0} \right)^{\frac{\Delta_q}{\Delta_2}} \left( \frac{\delta_\xi}{|E|} \right)^{\frac{|\Delta_q|}{d}}$$

- low energies $|E| \ll \delta_\xi$ (localized states):

$$\frac{\langle \rho(E) \rangle}{\rho_0} = 1 - a_1 |\gamma(\xi)| \left( \ln \frac{\delta_\xi}{|E|} \right)^{d+2} - a_2 \gamma^2(\xi) \frac{\delta_\xi}{|E|},$$

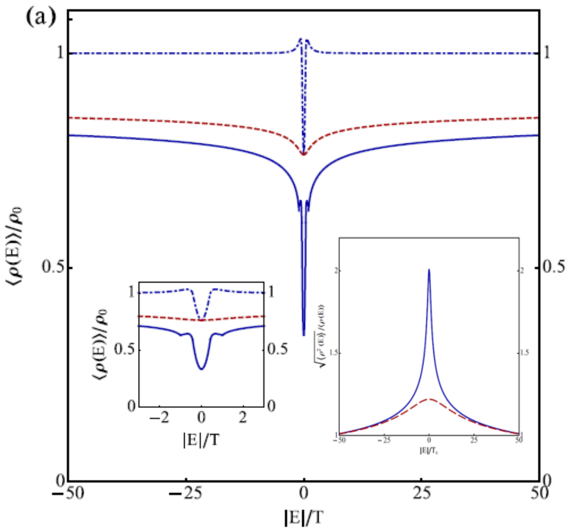
$$\frac{\langle \rho^q(E) \rangle}{\langle \rho(E) \rangle^q} = \left( \frac{\gamma(\xi)}{\gamma_0} \right)^{\frac{\Delta_q}{\Delta_2}} \left( \frac{L}{\xi} \right)^{d(q-1)}$$

- the scale $\Delta_P = |\gamma(\xi)| \delta_\xi \sim \xi^{-d-\Delta_2}$ (cf. Feigel'man, Ioffe, Kravtsov, Yuzbashyan (2007))

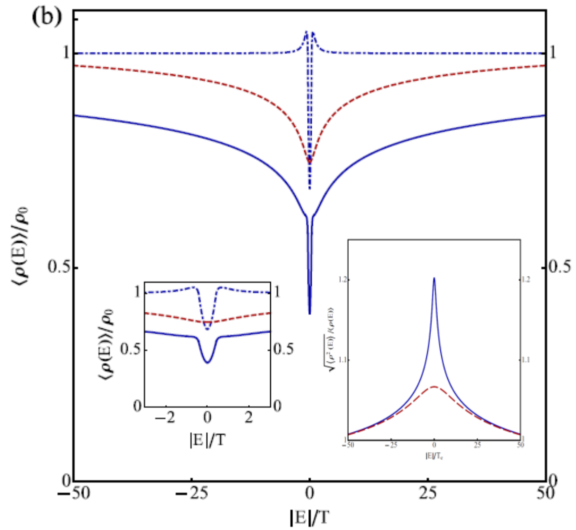
“pseudogap energy scale”

# Summary:

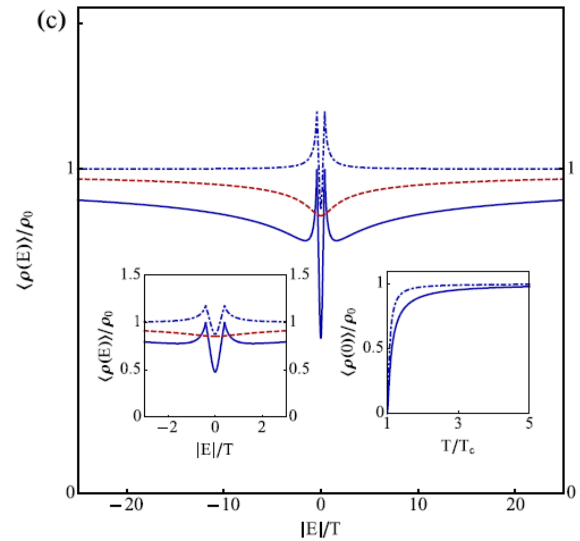
2D short-range




2D Coulomb




Anderson criticality




- Depletion of averaged LDOS: **interplay** of renormalization and real processes, **not** just a simple **superposition** of ZBA and SC DOS
- **LDOS fluctuations** within "**fermionic picture**" (no emergent granularity)
- **Short-range interaction:** strong mesoscopic fluctuations of LDOS on top of enhancement of $T_c$ due to multifractality
- **Coulomb interaction:** not so strong mesoscopic fluctuations of LDOS near superconducting transition; precursors of coherence peaks suppressed