

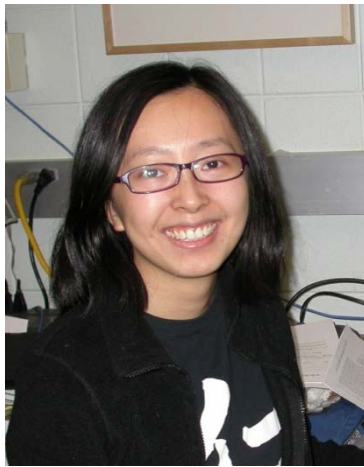
# Parity-Protected Josephson Qubits

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Chernogolovka, June 2016

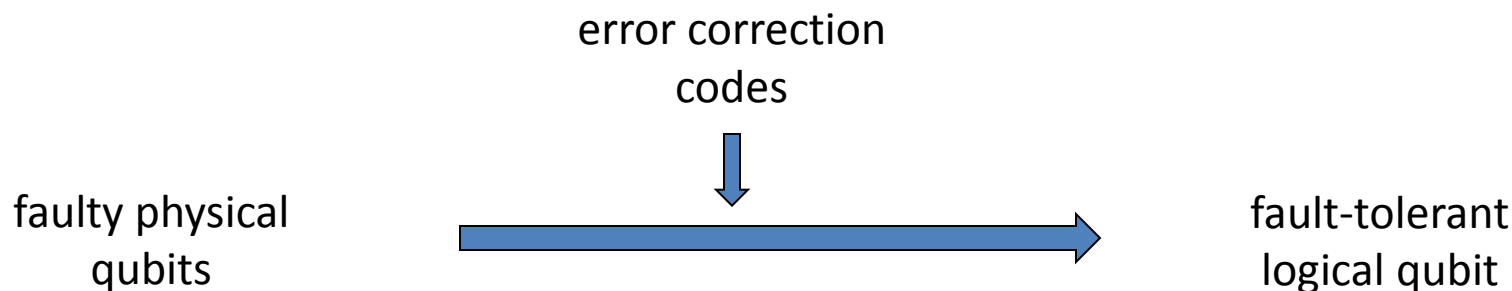
# Outline

- ❑ Qubits: state of the art
- ❑ Idea of Parity-Based Protection
- ❑ Charge-Pairing Qubits
- ❑ Flux-Pairing Qubits
- ❑ Superinductors

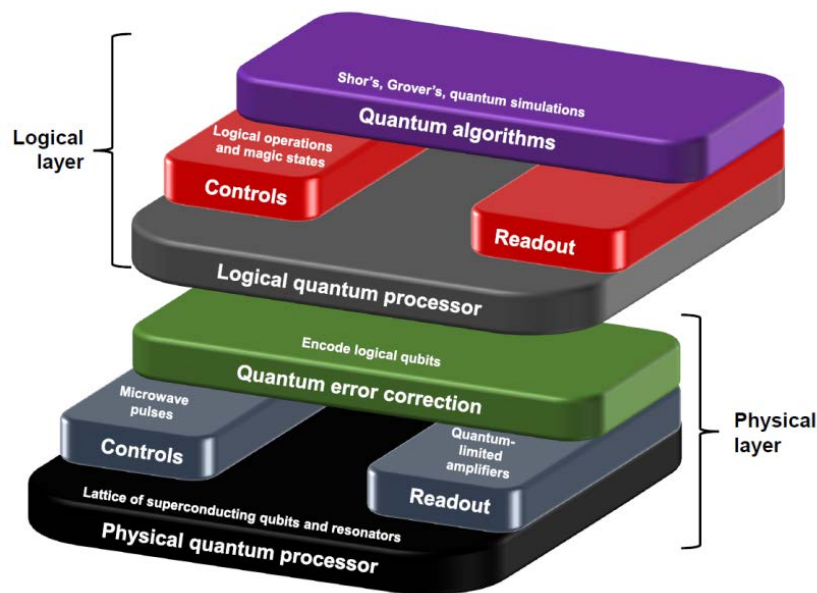
# Main Idea

Physical (faulty) qubit: a two-level quantum system that satisfies DiVincenzo criteria.

Logical (fault-tolerant) qubit: a collection of  $N$  physical qubits that can correct for arbitrary errors in a single qubit by running error correction codes.



*The “fault tolerance” theorem: once a sufficient low error rate of physical qubits is attained, there is a strategy for correcting errors so that one can carry out indefinitely long computations.*



# Physical Qubits: error rate and speed

The error rate :

$$\varepsilon \equiv \frac{\tau_0}{\tau_d}$$

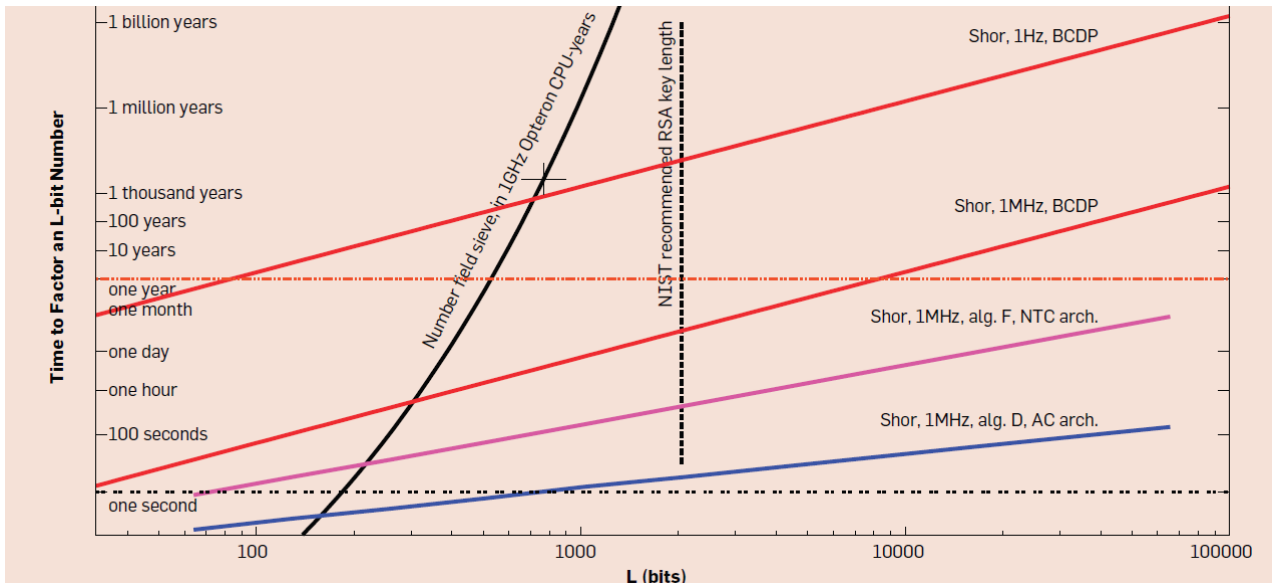
$\tau_0$  - the longest time for one- and two-qubit gates

$$\tau_d \approx \min(T_1, T_\varphi)$$

Threshold values of  $\varepsilon$  for different ECCs:

Earlier codes (Steane, Bacon-Shor):  $\varepsilon \approx 10^{-5}$

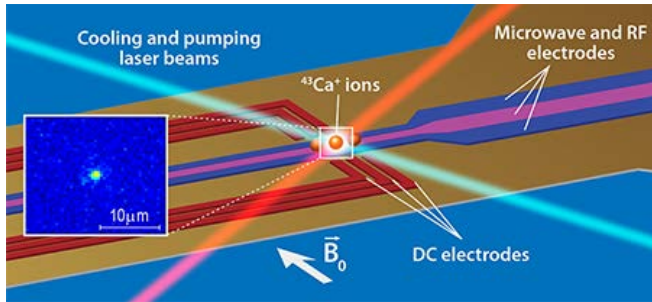
Surface correction code (Kitaev *et al.*):  $\varepsilon \approx 10^{-2}$



QC “speed” below 1MHz is not practical.

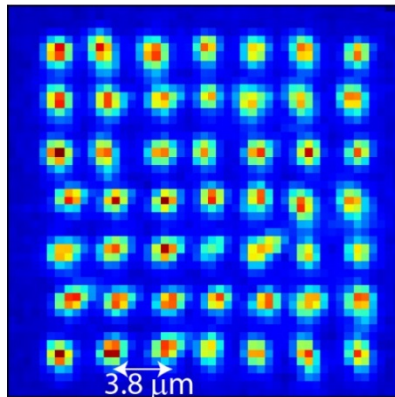
Van Meter & Horsman,  
*Communications of the ACM*  
56, 84 (2013)

# Qubit Implementations (Isolation $\leftrightarrow$ Speed)



Trapped  $^{43}\text{Ca}^+$  ions

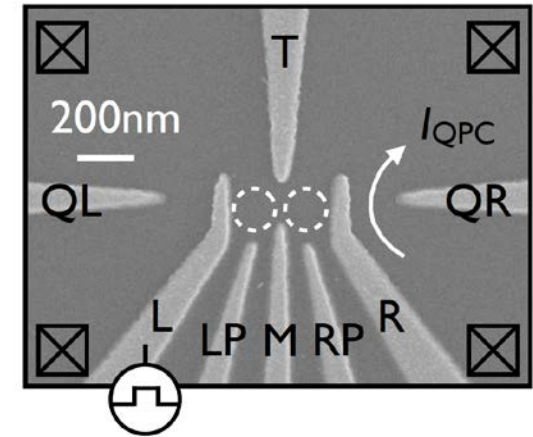
Single-shot readout fidelity 99.93%  
Single-qubit gate fidelity 99.9999%  
Harty *et al.*, *PRL* 113, 220501 (2014)



Trapped Cs atoms

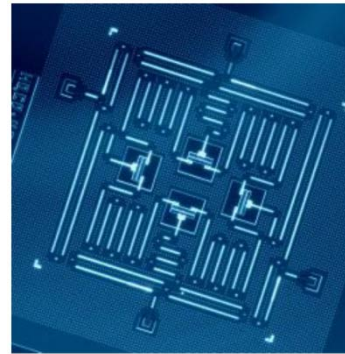
Single-qubit gate fidelity 99.83%  
Xia *et al.*, *PRL* 114, 100503 (2015)

Trapped ions,  
neutral atoms,  
quantum dots,  
superconducting  
qubits,  
etc.



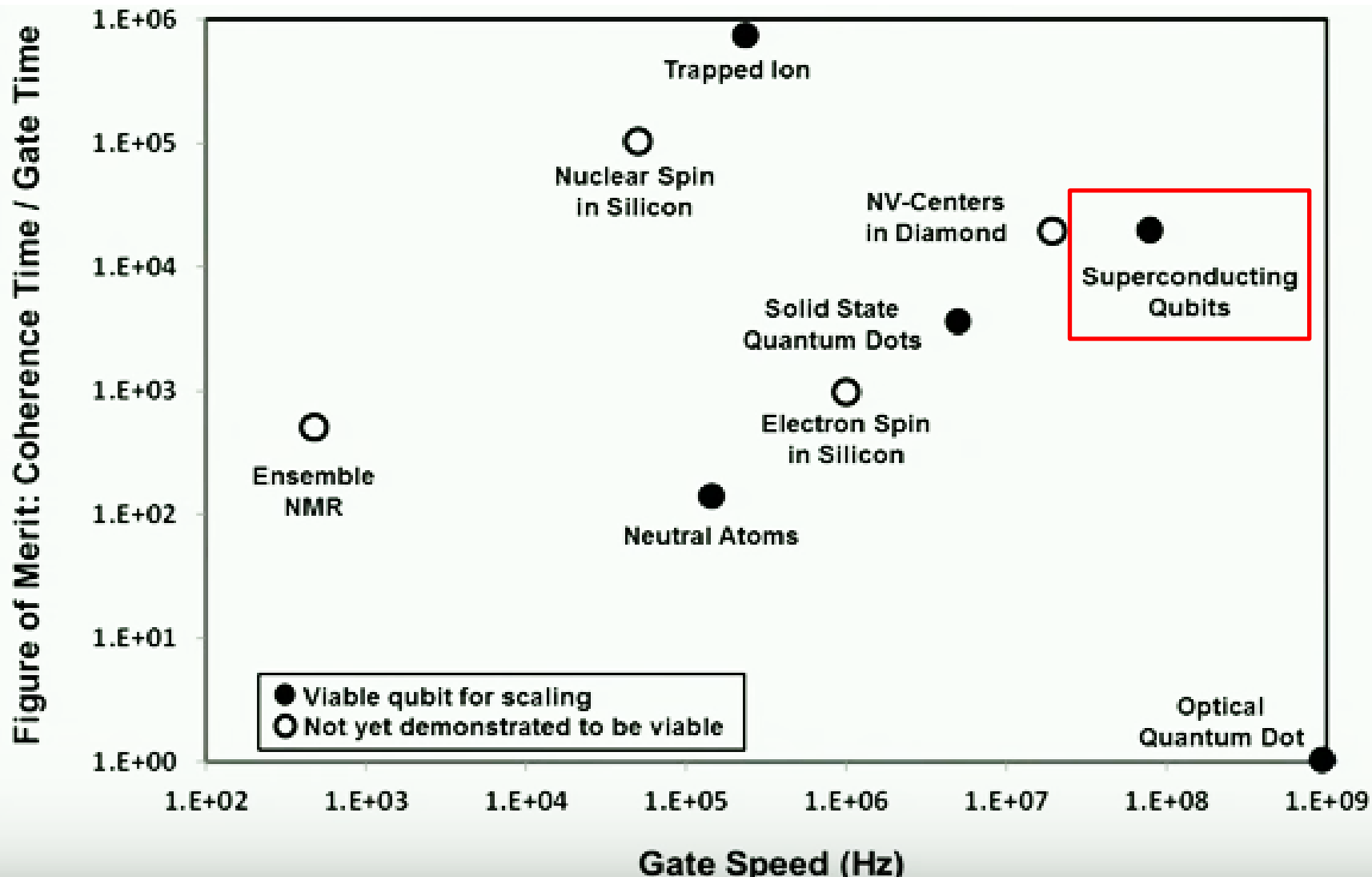
Quantum dots

Single-qubit gate fidelity > 99.5%  
Kim *et al.*, *Nat.QI* 1, 15004 (2015)

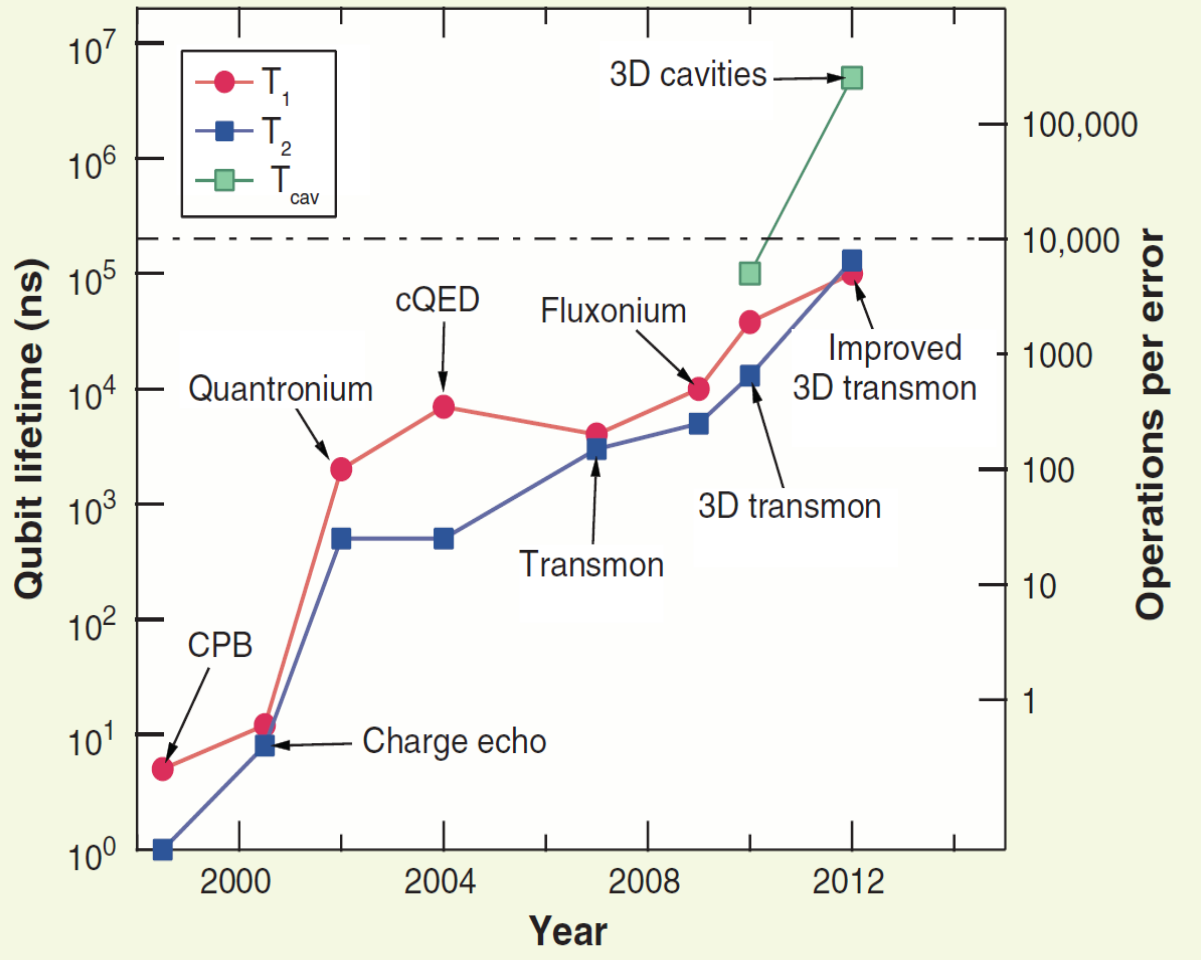


Superconducting qubits

Single-qubit gate fidelity > 99.95%  
Two-qubit gate fidelity 99.5%  
State-of-the-art:  
Gambetta *et al.*, arXiv:1510.0437



# Superconducting Qubits: State of the Art



Devoret & Schoelkopf, Science 2013

Single-qubit gates:

$$\varepsilon \equiv \frac{\tau_0}{T_2} \sim 10^{-4}$$

Two-qubit gates:

$$T_2 \approx 40 \mu s$$

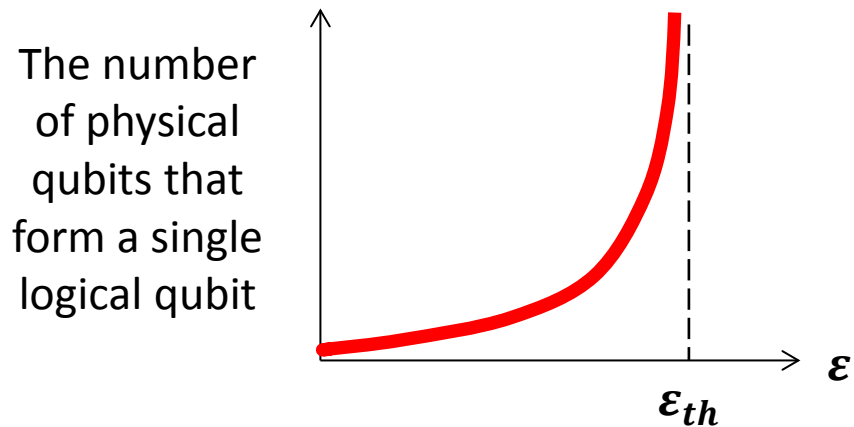
$$\tau_0 \approx 40 ns$$

Martinis' Group, UCSB/Google

$$\varepsilon = 1 \times 10^{-3}$$

# Physical Qubit vs. Logical Qubit

How many physical qubits does it take to build a single logical qubit?



The threshold error rate for *the surface error codes* is  $\sim 1\%$ . If the error rate is  $1/10$  of  $\epsilon_{th}$ , «a reasonably fault-tolerant logical qubit ... takes  $10^3 - 10^4$  physical qubits».



Physical qubits are macroscopic ( $\geq 100 \mu m$ ), areal density  $\sim 10^6$  times less than computer chips - enormous impact on the large scale architecture.

Martinis (UCSB and Google): “We’re somewhere between the invention of the transistor and the invention of the integrated circuit” – **not yet...**



## Main Message

Although simple physical qubit designs (e.g. transmon) are very attractive, they do not eliminate enormous complexity of logical qubits.

Any design that may help to reduce the complexity of the final product (i.e. the logical qubit) is worth considering.

- ❑ Qubits: state of the art
- ❑ **Idea of Parity-Based Protection**
- ❑ Charge-Pairing Qubits
- ❑ Flux-Pairing Qubits
- ❑ Superinductors

# Parity Protected Qubits

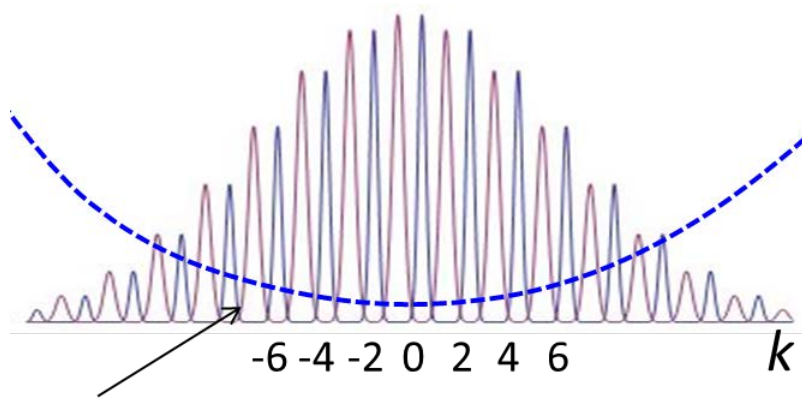
The goal: to engineer two quantum states *indistinguishable* by the environment.

$$H = K (X^n + X^{-n}) + V k^2$$

$k$  – discrete variable

$n=2$  - parity protection

$$X^{\pm 2} |k\rangle = |k \pm 2\rangle$$



$g$  – the number of discrete components of the lowest-energy even (odd) states

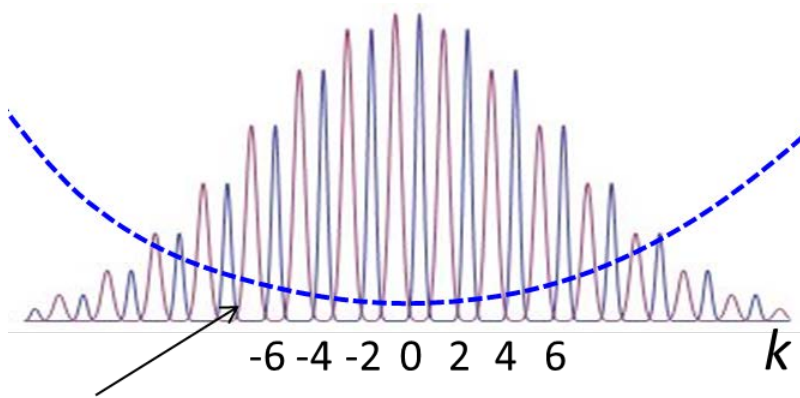
The two-level approximation Hamiltonian

$$H = \frac{E_{01}}{2} \sigma_z + \frac{E^*}{2} \sigma_x$$

the rate of parity violation

Two almost degenerate low-energy states:  $E_{01} \propto \exp(-g)$

Protection against “k” noises  $\propto \exp(-\alpha g)$

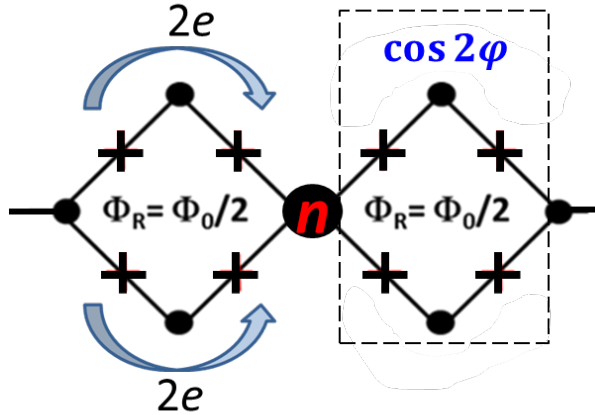


$g$  – the number of discrete components of the lowest-energy even (odd) states

- Decay is suppressed if parity is protected ( $E^* = 0$ ). ←  $T_1 = \infty$
- Coupling to “k” noises is suppressed if the envelope decays slowly (large  $g$ ). ← *long*  $T_2$
- ★ Some fault tolerant rotations can be realized by fast changes of  $V(t)$  – protection from the gate pulse noise.

# Charge Pairing Qubit

Correlated tunneling of TWO Cooper pairs



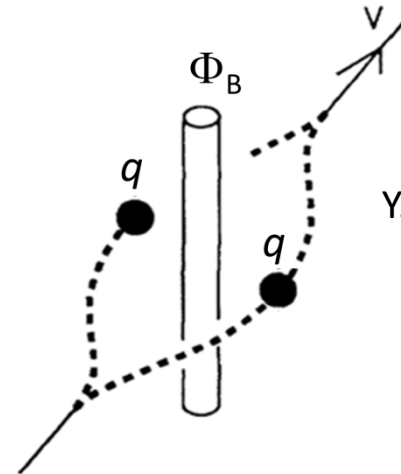
Discrete variable: number of Cooper pairs on the central island

Parity protection (cancellation of “single-particle” tunneling) due to destructive AB interference

Gladchenko *et al.* *Nat. Phys.* **5**, 48 (2009)

Bell *et al.* *PRL* **112**, 167001 (2014)

## Aharonov-Bohm phase



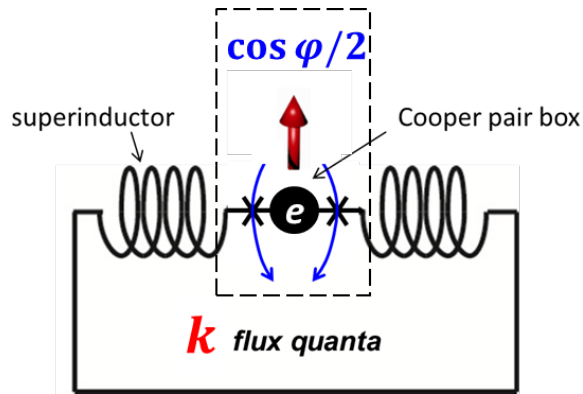
Y. Aharonov & D. Bohm,  
*PR* **115**, 485 (1959)

The wave function of a charge  $q$  moving around a magnetic flux  $\Phi_B$  acquires a topological phase shift

$$\Delta\varphi_{AB} = \frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{2e}{\hbar} \Phi_B$$

# Flux(on) Pairing Qubit

Correlated  
tunneling of  
TWO  
fluxons



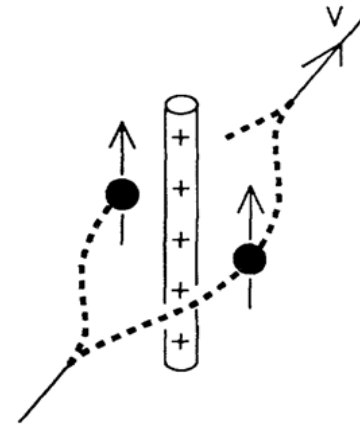
Discrete variable: number of  
flux quanta in the loop

Parity protection (cancellation of  
“single-particle” tunneling)  
due to destructive AC interference

Bell *et al.* *PRL* **109**, 137003 (2012)

Bell *et al.* *PRL* **116**, 107002 (2016)

## Aharonov-Casher phase



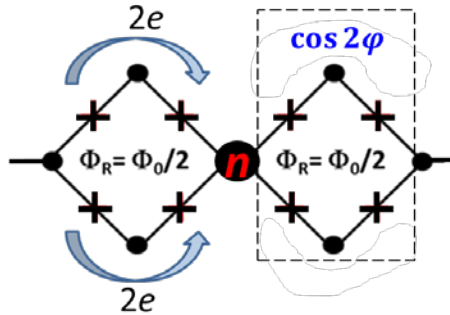
Y. Aharonov & A. Casher,  
*PRL* **53**, 319 (1984)

The wave function of a particle with magnetic dipole moment  $\mu$  moving in 2D around a line charge acquires a topological phase shift proportional to the line charge density.

- ❑ Qubits: state of the art
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- ❑ **Charge-Pairing Qubits**
- ❑ Flux-Pairing Qubits
- ❑ Superinductors

# Charge-Pairing Qubits (discrete variable - Cooper pairs number)

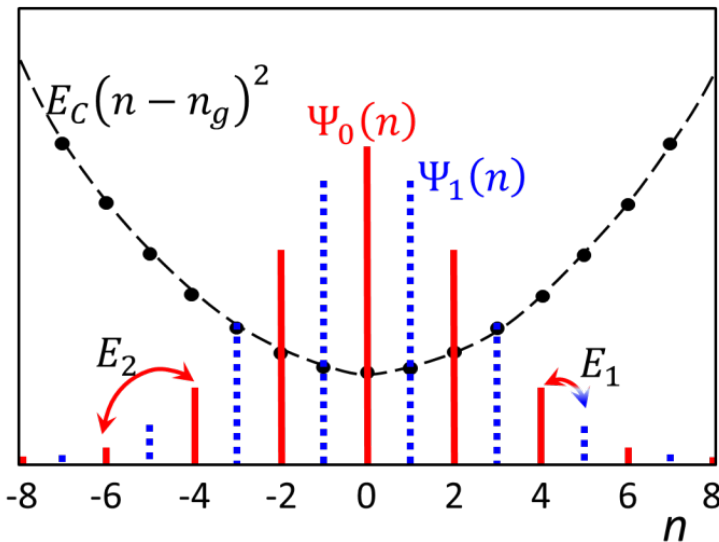
Ioffe, Doucot, Feigel'man *et. al.* (2002 –present)



$$H = -2E_2 \cos 2\varphi - 2E_1 \cos \varphi + E_C (\hat{n} - n_g)^2$$

The two-level approximation Hamiltonian

$$H = \frac{E_{01}}{2} \sigma_z + \frac{E^*}{2} \sigma_x$$



$$g = 4\sqrt{E_2/E_C}$$

Required:  $E_2 \gg E_C$

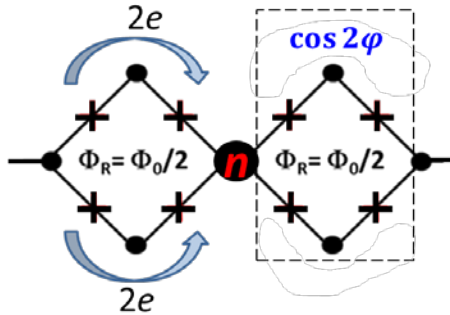
Perfect symmetry ( $E_1 = 0$ ):

$$E_{01} \propto \omega_p \exp(-g) \cos(\pi n_g)$$

$$\omega_p = 4\sqrt{E_2 E_C} \quad \text{- plasma frequency}$$



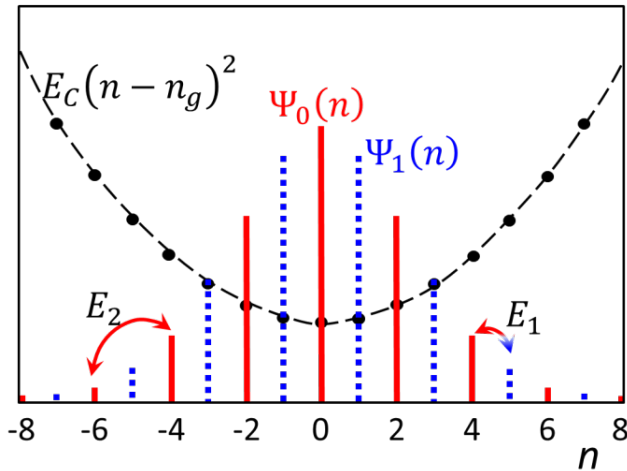
# Decoupling from noises



The two-level approximation Hamiltonian

$$H = \frac{E_{01}}{2} \sigma_z + \frac{E^*}{2} \sigma_x$$

Perfect symmetry ( $E_1 = 0$ ):



Energy relaxation is suppressed  
(very long  $T_1$ )

Coupling to the charge noise is exp. small  
 $\propto \exp(-g) \sin(\pi n_g)$

Parity violation: single Cooper pair hopping (non-zero  $E_1$ ).

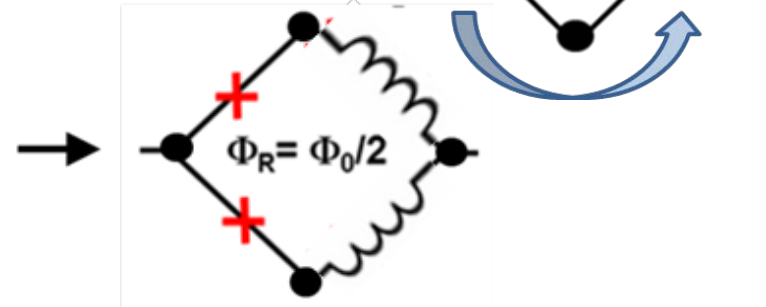
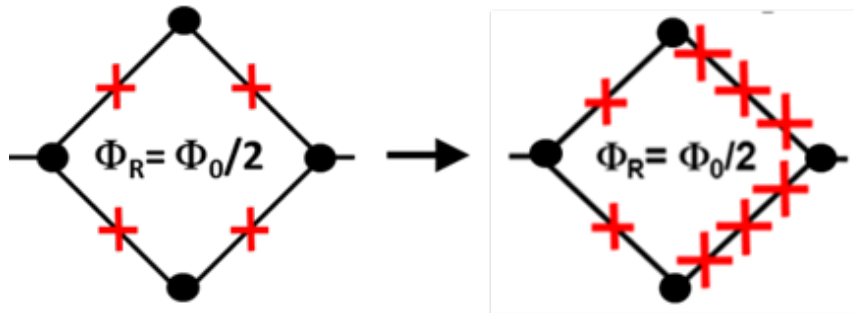
Sources of parity violation: rhombus asymmetry, flux noise.

To suppress coupling to the flux noise, the rhombi chain should be longer.

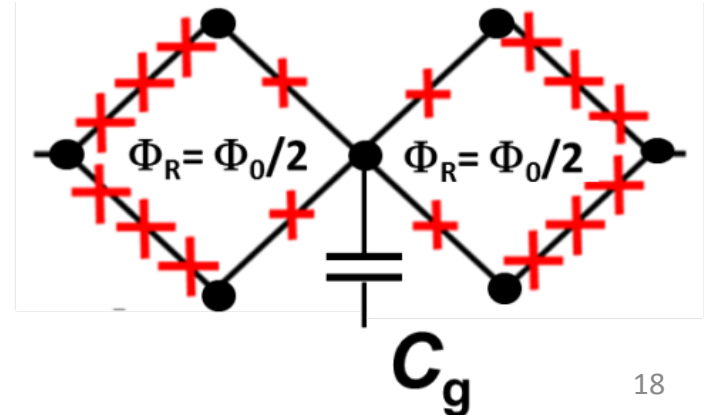
# Problem of Offset Charges

Optimal  $\frac{E_J}{E_C} \approx 3 - 4$       Phase slip rate  $\propto \exp\left(-\alpha \sqrt{\frac{E_J}{E_C}}\right)$

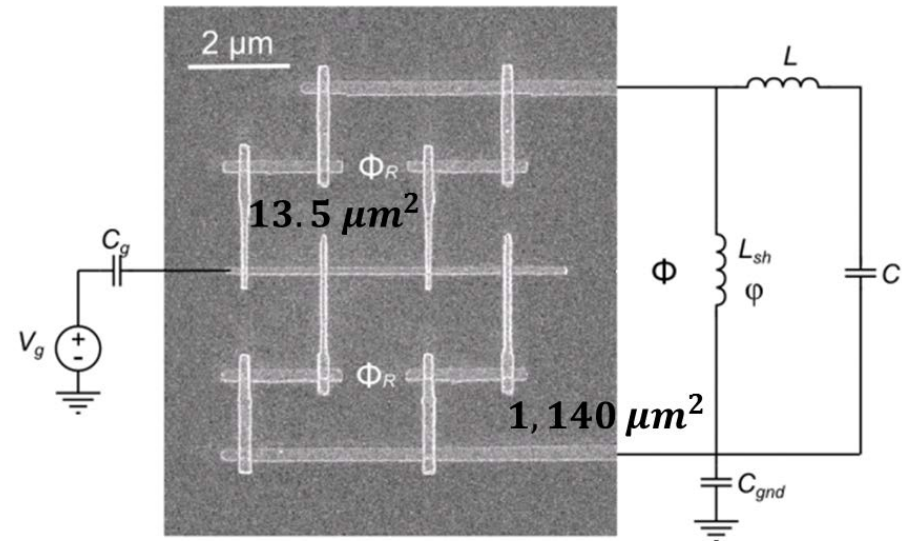
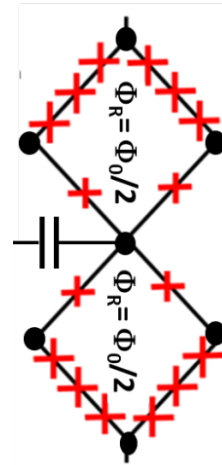
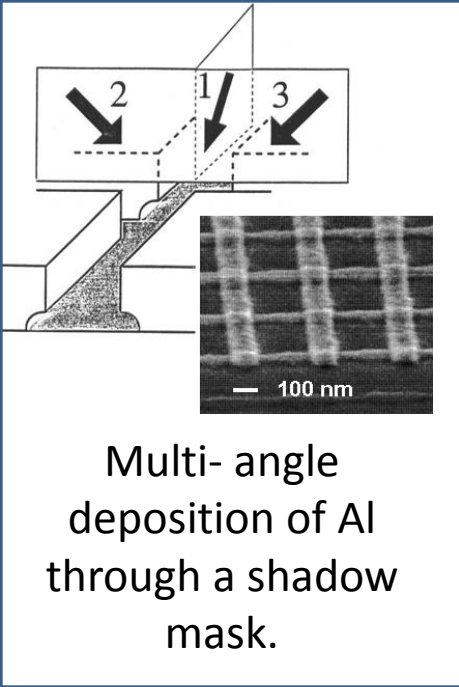
Phase slips + uncontrollable offset charges on unbiased islands  $\Rightarrow$  Symmetry breaking



$E_{JL}/E_{JS} \sim 3-4$  implies  $E_{JL}/E_{CL} \sim 30-50$   
 no phase slips across larger junctions  
 chain of 3-4 larger junctions is equivalent to a (classical) inductor



# Device and Readout



Small JJ:  $0.13 \times 0.14 \mu\text{m}^2$   
 Large JJ:  $0.25 \times 0.25 \mu\text{m}^2$

Global magnetic field:

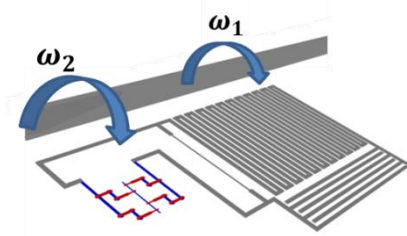
$$\left\{ \begin{array}{l} \Phi_R \approx \Phi_0/2 \\ \varphi = 2\pi \frac{\Phi}{\Phi_0} \end{array} \right. \quad E_1 \approx 0$$

- phase difference across the chain

Two experimental “knobs”:

- the gate voltage controls  $n_g$  on the central island;
- the flux in the “phase” loop controls  $\varphi$  across the chain.

# Spectroscopic Measurements



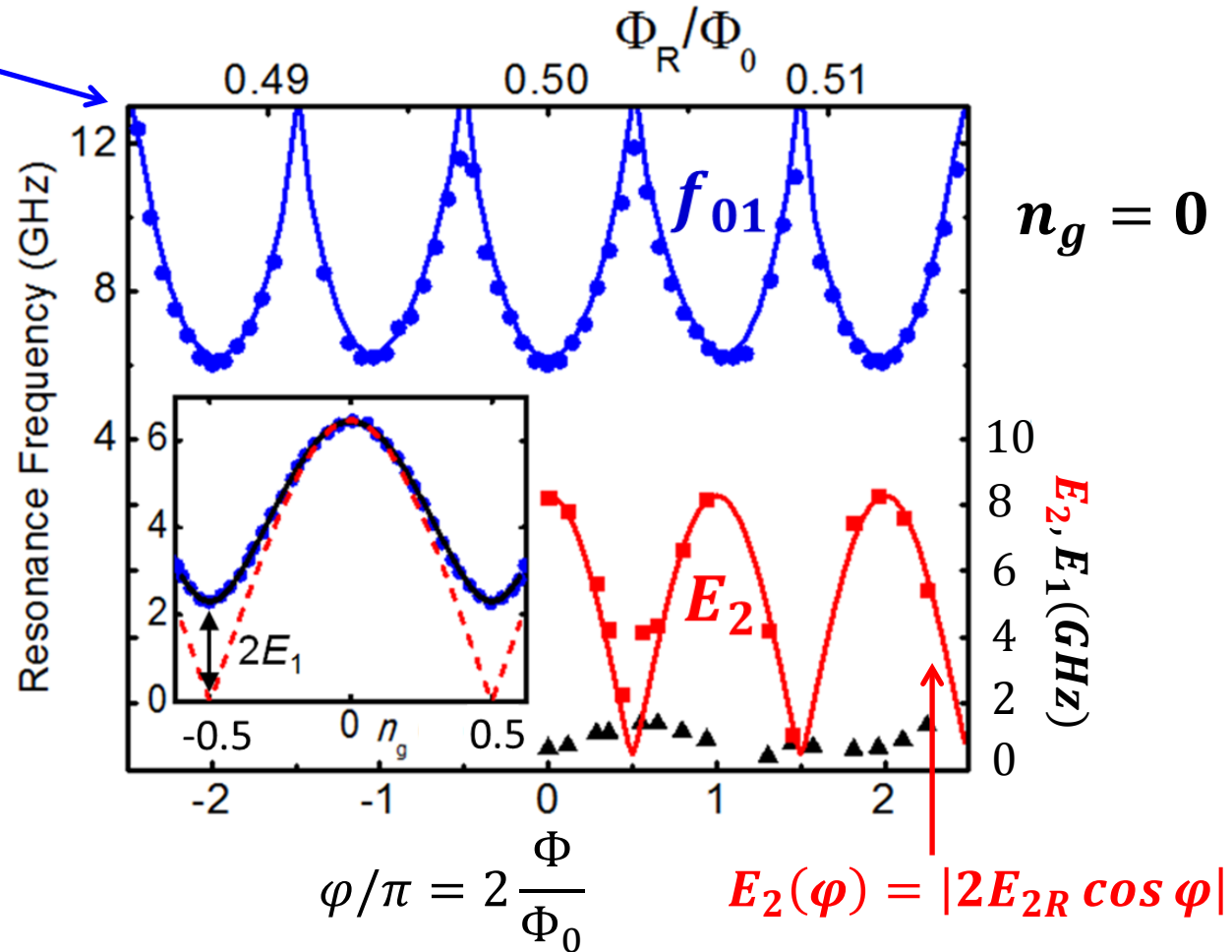
$$E_{01}^* = \sqrt{E_{01}^2 + (2E_1)^2}$$

$$E_2(max) = 8.5GHz$$

$$E_C \approx 15GHz$$

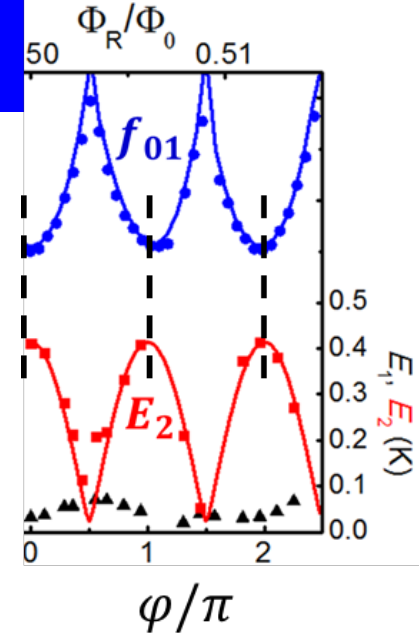
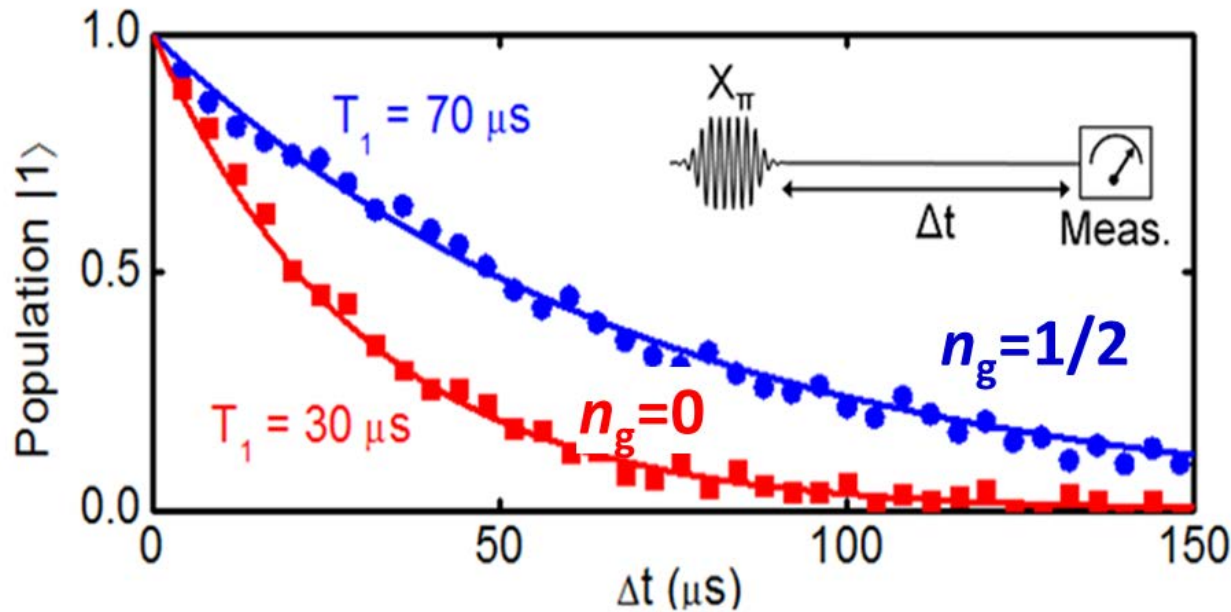
$$E_1 = 0.75GHz$$

$$\Delta E_{2R} \leq 0.1 E_{2R}$$



# Time-Domain Measurements

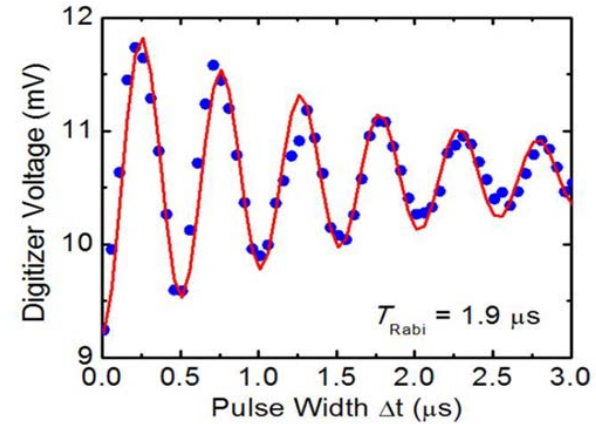
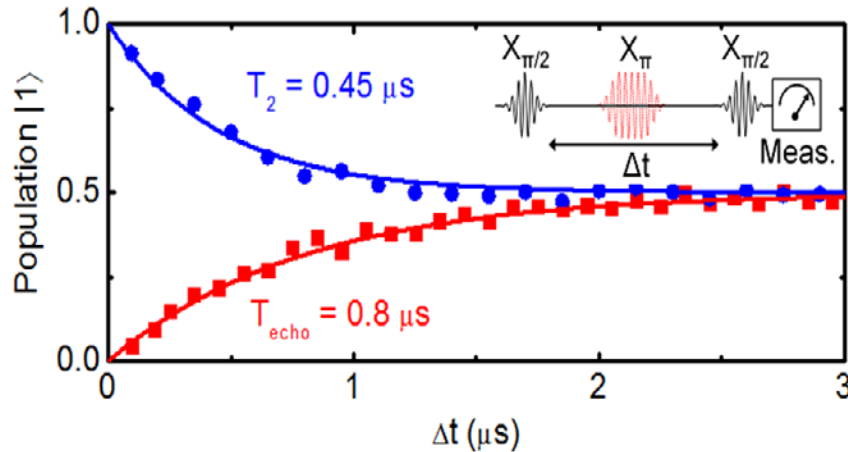
Optimal regime:  $\max E_2, \min E_1 \rightarrow \varphi = n\pi$  (min  $E_{01}$ )



$$Q \equiv \omega_{01} T_1 \approx 1 \cdot 10^6$$

At  $n_g = 0$  the effect of charge noise and dielectric losses is negligibly small.  
 Primary source of decay: coupling to the transmission line and readout resonator.

# Dephasing Time



$$\Gamma_2^\Phi = \sqrt{\frac{\log(E_1/\Omega_0)}{2\pi}} \left(\frac{2E_1}{E_{01}}\right) (2\delta E_1) \quad \delta E_1 - \text{fluctuations of } E_1 \text{ caused by the } 1/f \text{ flux noise}$$

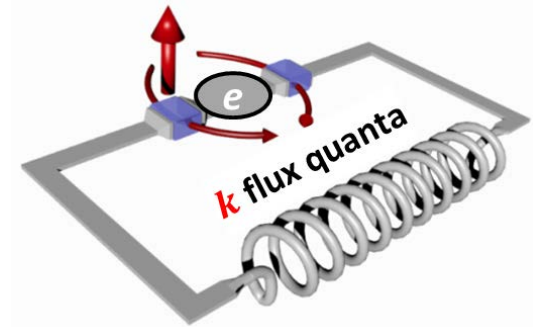
Primary source of dephasing:  
flux noise in the rhombi loops.



- Reduction of asymmetry
- Increase of  $E_2/E_C$
- Alternative implementation of  $\cos 2\varphi$  elements<sub>22</sub>

# The toolbox for protected qubits: $\cos(2\varphi)$ , $\cos(\varphi/2)$ , super- $L$

- ❑ Idea of Parity-Based Protection
- ❑ Flux-Pairing Qubits
- ❑ Charge-Pairing Qubits
- ❑ Superinductors

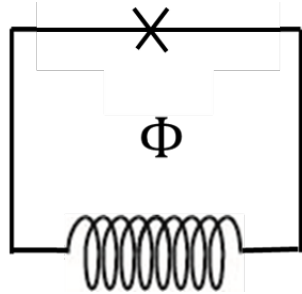


- ❑ Qubits: state of the art
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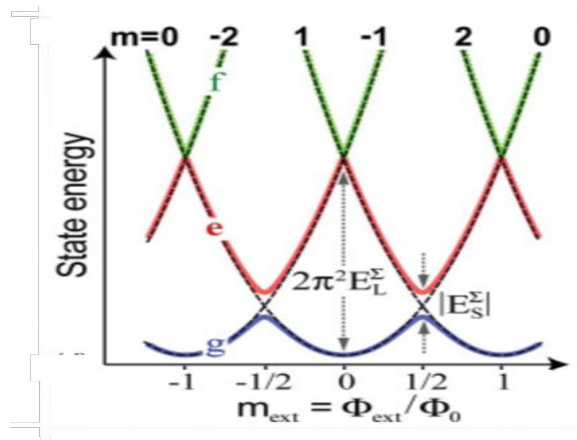


# "cos( $\varphi$ ) - L" and "cos( $\varphi/2$ ) - L" Qubits

**Fluxonium** (Manucharyan *et al.* 2009)



$$E_L = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L}$$

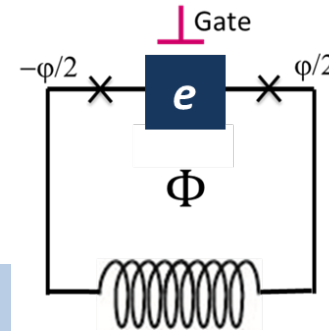


single fluxon tunneling

$$E_{sps} \propto \omega_p \exp\left(-\sqrt{\frac{8E_J}{E_C^*}}\right)$$

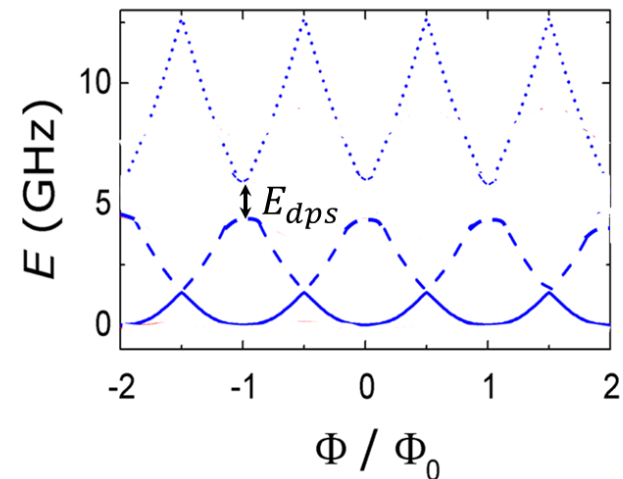
$$\omega_p = \sqrt{8E_J E_C^*}$$

**Flux-pairing Qubit**



$$q = e$$

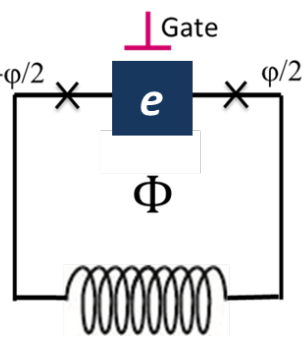
$$E_L \approx E_{dps}$$



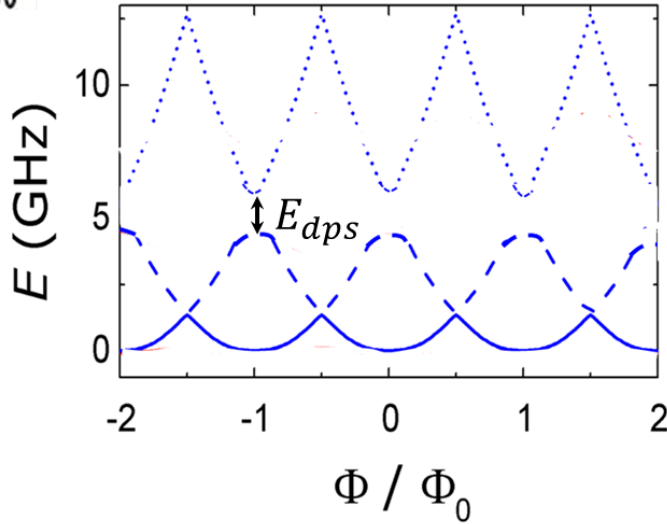
double fluxon tunneling

$$E_{dps} \propto \omega_p \exp\left(-2\sqrt{\frac{8E_J}{E_C^*}}\right)$$

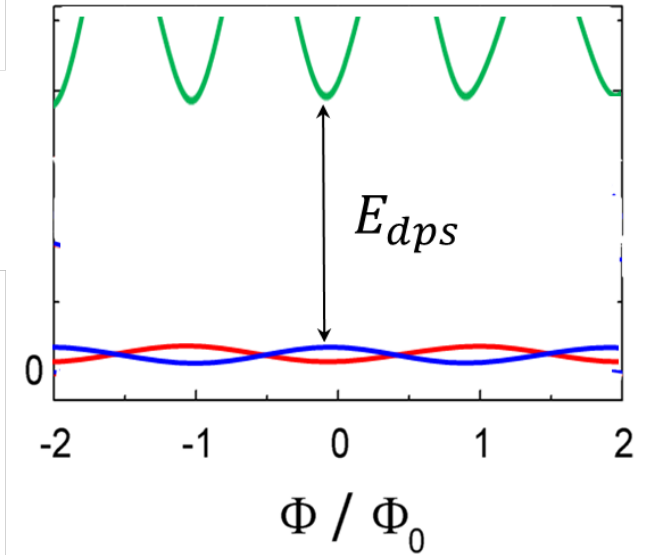
# Flux-Pairing Qubit ( $q=e$ )



$$E_L \gtrsim E_{dps}$$



$$E_L \ll E_{dps}$$

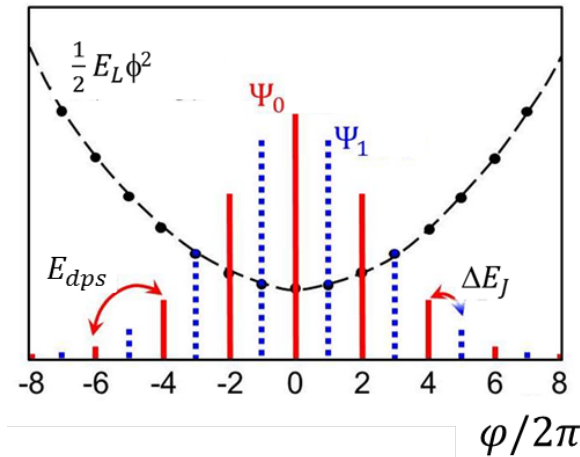
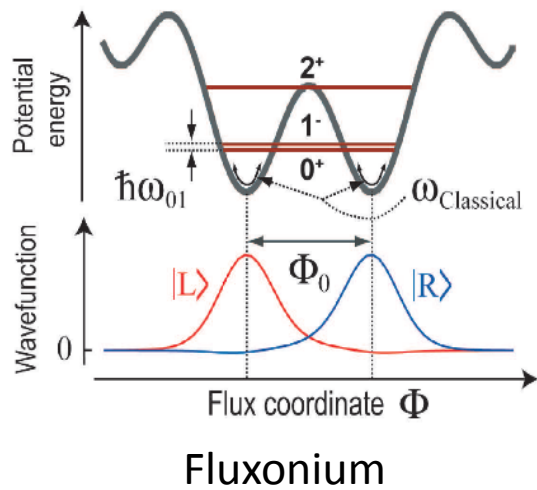


$$H = -\left(\frac{E_L^*}{2}\sigma_z + \frac{E_{sps}}{2}\sigma_x\right)$$

$$E_L^* = E_L(1 - 2\Phi/\Phi_0)$$

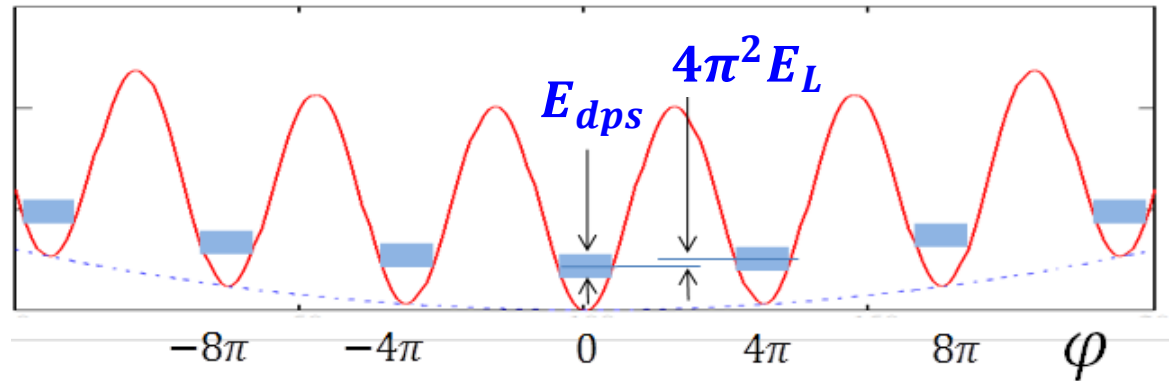
$$E_L^* = E_L \exp\left(-\sqrt{\frac{E_{dps}}{E_L}}\right)$$

# Flux-Pairing Qubit ( $E_L \ll E_{dps}$ )



Large quantum fluctuations of phase ( $\langle k^2 \rangle \gg 1$ ):

$$E_{dps} \gg 4\pi^2 E_L$$

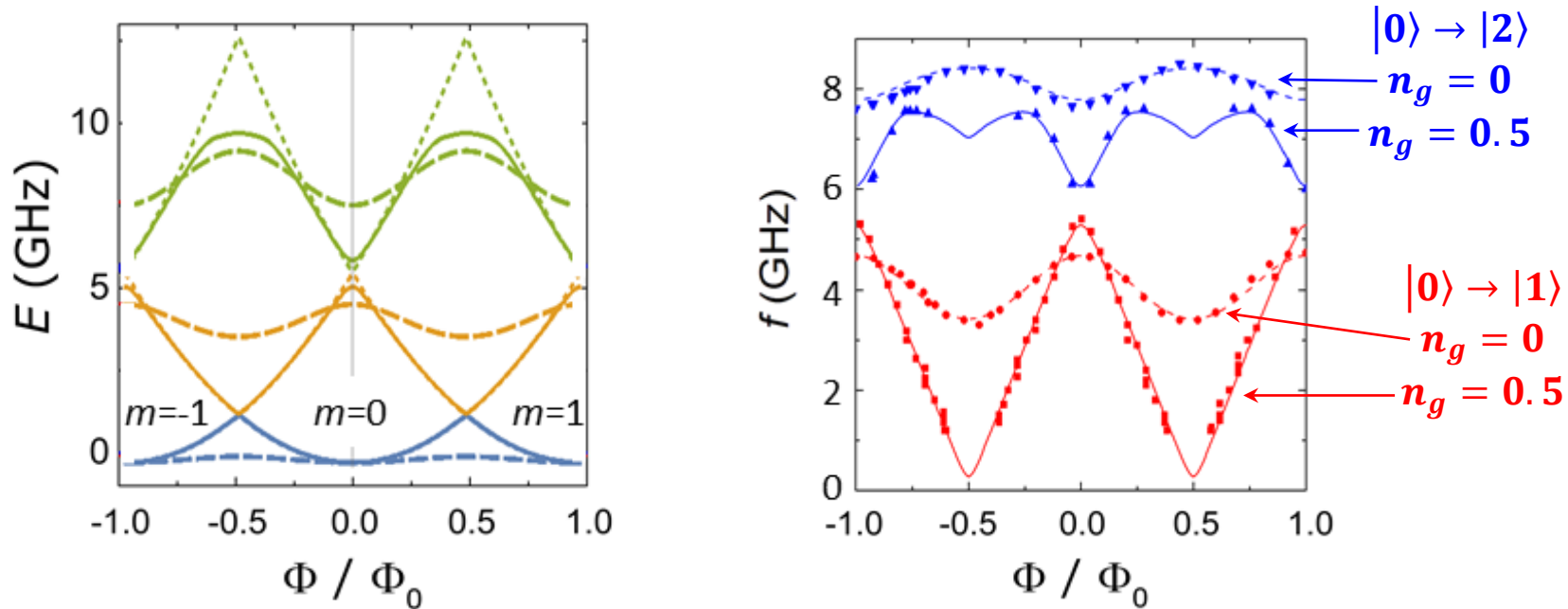


Implementation of the fault tolerant flux-pairing qubit:

1. Superinductance  $\sim 10 \mu\text{H}$  (!)
2. High rate of double phase slips ( $\frac{E_J}{E_C} \leq 0.2$ )

Fluxonium:  
 $L = 0.3 \mu\text{H}$

# Spectroscopic Evidence of Aharonov-Casher Effect



Bell *et al.* PRL **116**, 107002 (2016)

Complete suppression of single phase slips at  $n_g = 0.5$  -  
clear spectroscopic evidence of the Aharonov-Casher effect



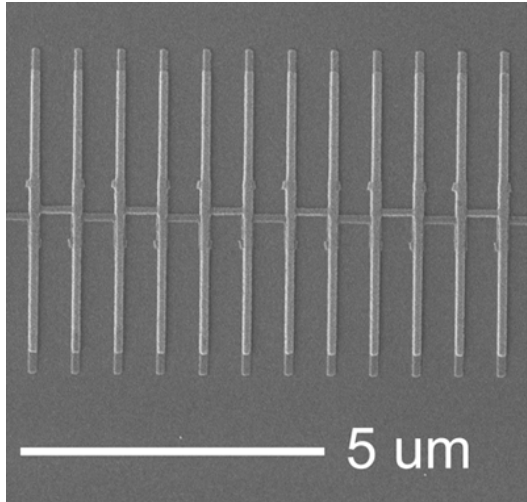
- Decrease  $E_L$  down to  $\sim 1$  mK ( $L \geq 10 \mu\text{H}$ ) without decreasing the superinductor self-mode frequency .
- Increase the rate of double phase slips ( $\frac{E_J}{E_C} \leq 0.2$ ).

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# Superinductors

$$\text{Impedance } Z \gg R_Q \equiv \frac{h}{(2e)^2} \approx 6.5k\Omega$$

No dissipation and dephasing

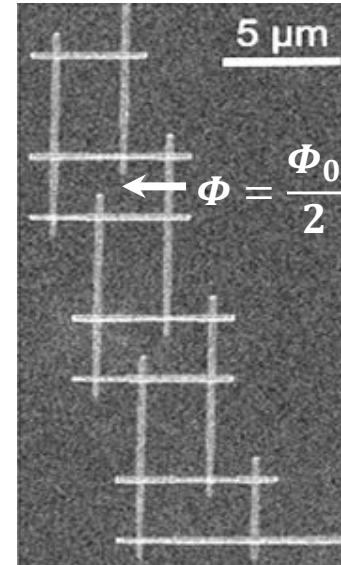


Chain of junctions with  $\frac{E_J}{E_C} \gg 1$

$$L = 0.3\mu\text{H}$$

Manucharyan *et al.*  
*Science* 326, 113 (2009)

Masluk *et al.*  
*PRL* 109, 137002 (2012).



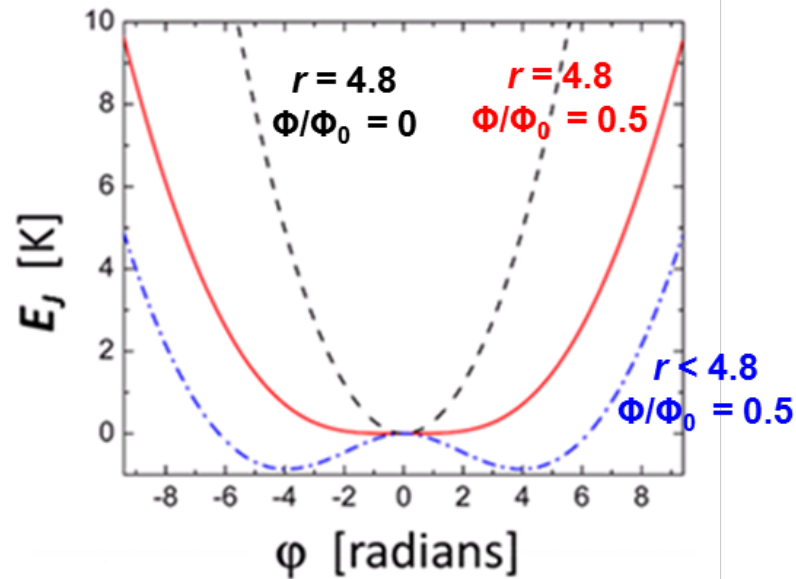
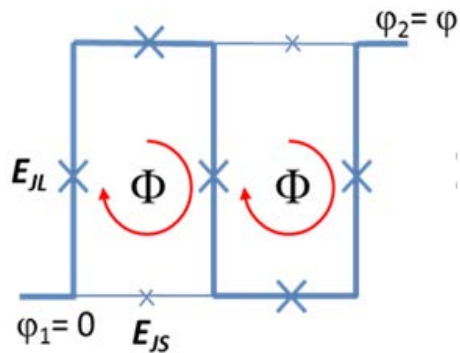
Chain of asymmetric SQUIDs

$$L(\Phi = \Phi_0/2) = 3\mu\text{H}$$

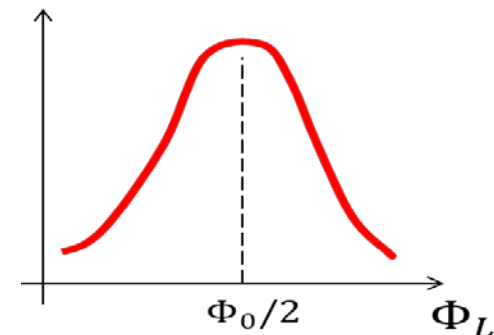
Bell *et al.*  
*PRL* 109, 137003 (2012)

Specific to our design: *tunable inductance and non-linearity.*

# Tunable Nonlinear Superinductor



$$L_J \propto \left( \frac{d^2 E_J(\varphi)}{d\varphi^2} \right)^{-1}$$



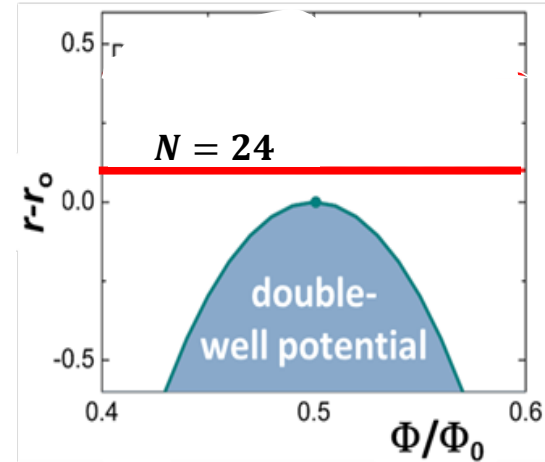
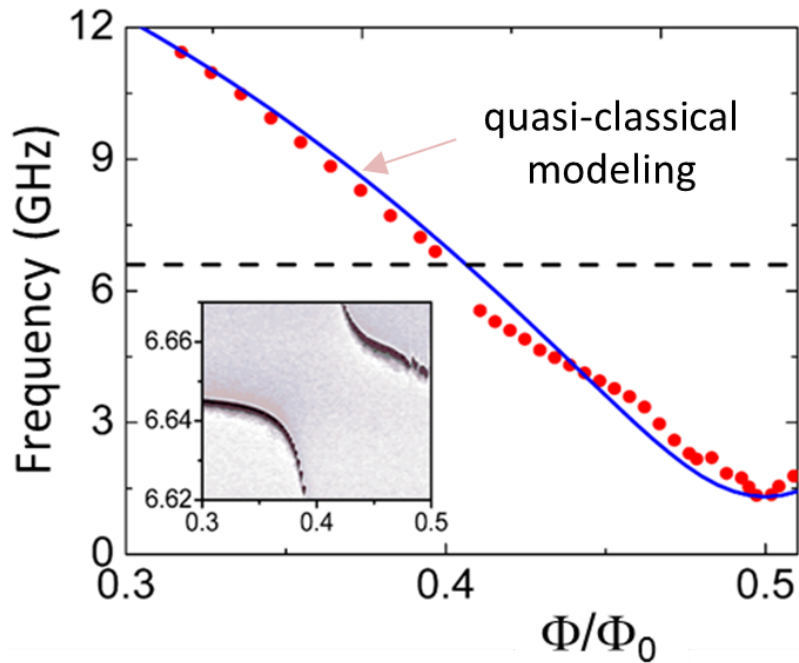
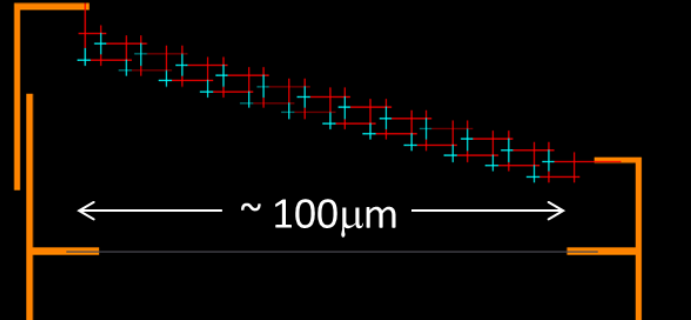
$$r \equiv \frac{E_{JL}}{E_{JS}}$$

For the optimal  $r$  value,  $E_J(\varphi)$  becomes “flat” at  $\Phi=1/2\Phi_0$ , and the “classical”  $L_J$  diverges.

- the ladder backbone is formed by the junctions with  $E_{CL} \ll E_{JL}$  - the phase slip rate is exponentially small;
- Potential can be varied from a single well to double well either by changing  $E_{JL}/E_{JS}$  or by changing the flux.

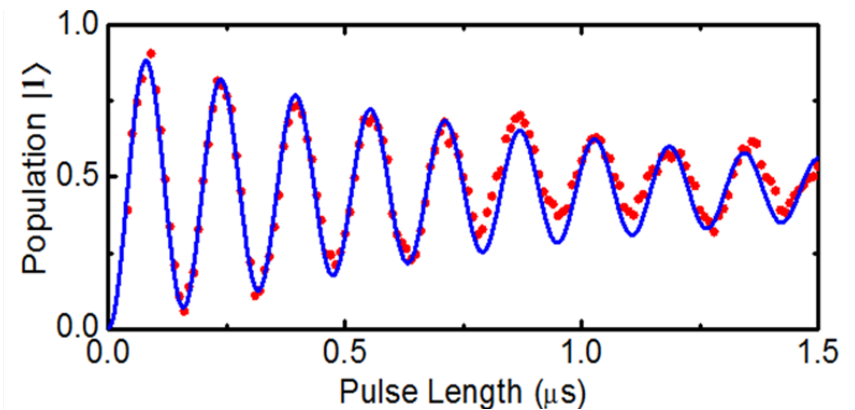


# Ladders with 24 unit cells



$$L_K(\Phi = \Phi_0/2) = 3\mu\text{H}$$

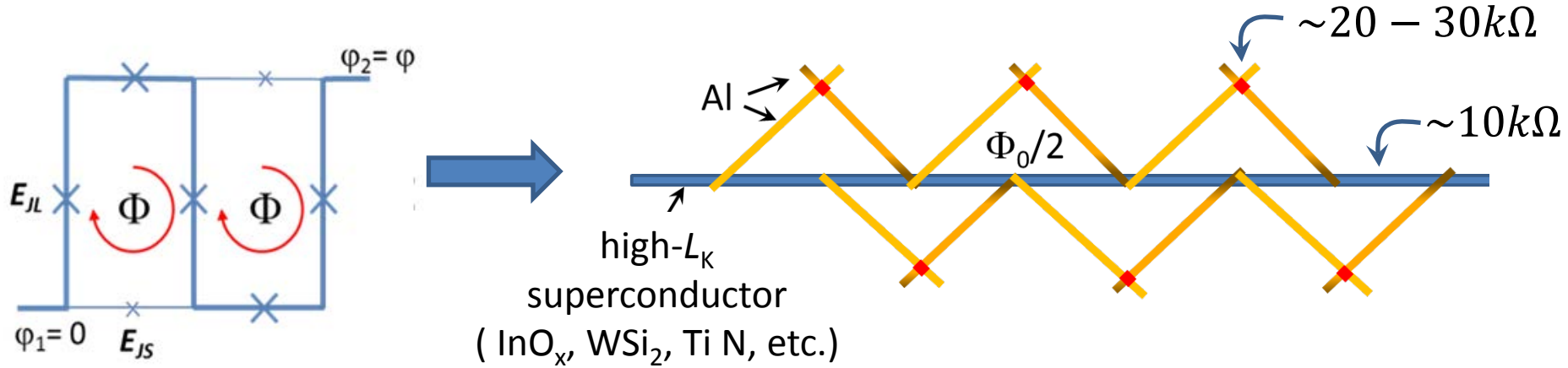
- this is the inductance of a 3-meter-long wire!



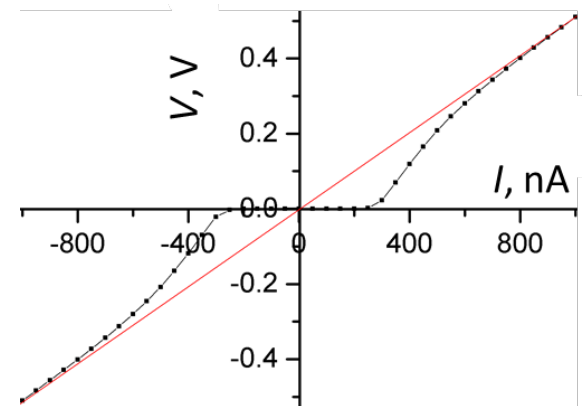
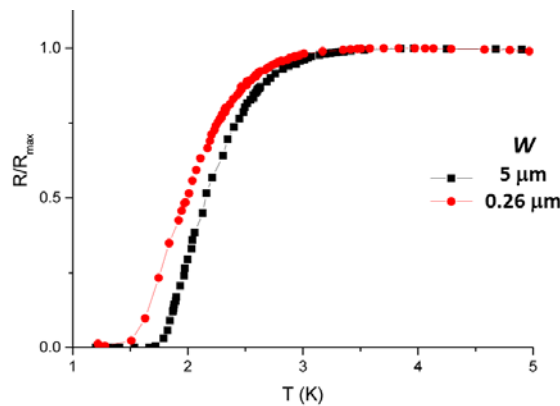
$$Z(3\text{GHz}) = 50\text{k}\Omega > R_Q \equiv \frac{h}{(2e)^2}$$

Bell *et al.* PRL 109, 137003 (2012).

# Collaboration with Sacépé Lab (Grenoble)



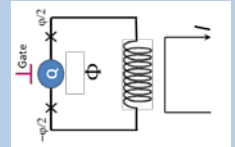
InO<sub>x</sub> films:  $R_{\square} = 2.5 k\Omega$ ,  $W = 0.26 \mu m$ ,  $T_C \approx 1.5 K$





- increase the SL inductance without decreasing the resonance frequency of the superinductor

- fast control of the superinductance for fault-tolerant operations



- implement superinductor-based traveling wave parametric amplifiers;

*Bell and Samolov, Phys. Rev. Applied 4, 024014 (2015)*

- use 1D chains of asymmetric SQUIDs as a platform for the study of 1D quantum phase transitions.

# Summary

## The toolbox for protected qubits: **$\cos(2\varphi)$ , $\cos(\varphi/2)$ , super- $L$**

- Charge- (flux-) pairing qubits offer the possibility of coherence protection and fault-tolerant operations.
- Observed:
  - suppression of energy relaxation in a minimalistic rhombi chain;
  - spectroscopic evidence of Aharonov-Casher effect in flux-pairing devices.
- Current work:
  - optimization of the parameters of the flux-pairing qubits (smaller  $E_L$  and larger  $E_{dps}$ );
  - development of better superinductors (for applied projects as well as a novel tool to study 1D QPTs).