Particle-hole symmetry without particle-hole symmetry in QHE at ν=5/2

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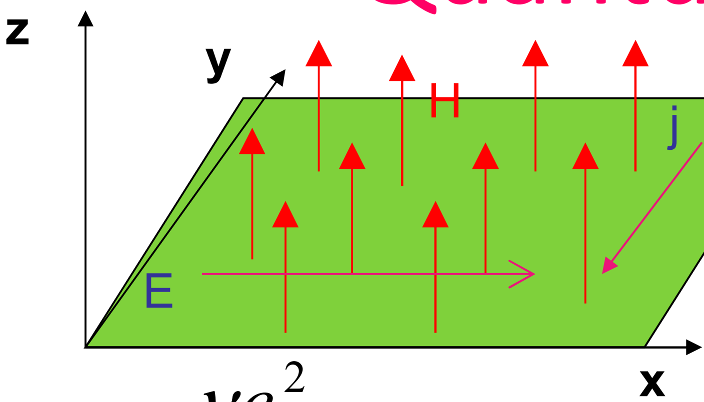
P. T. Zucker and D. E. Feldman, arXiv:1603.03754

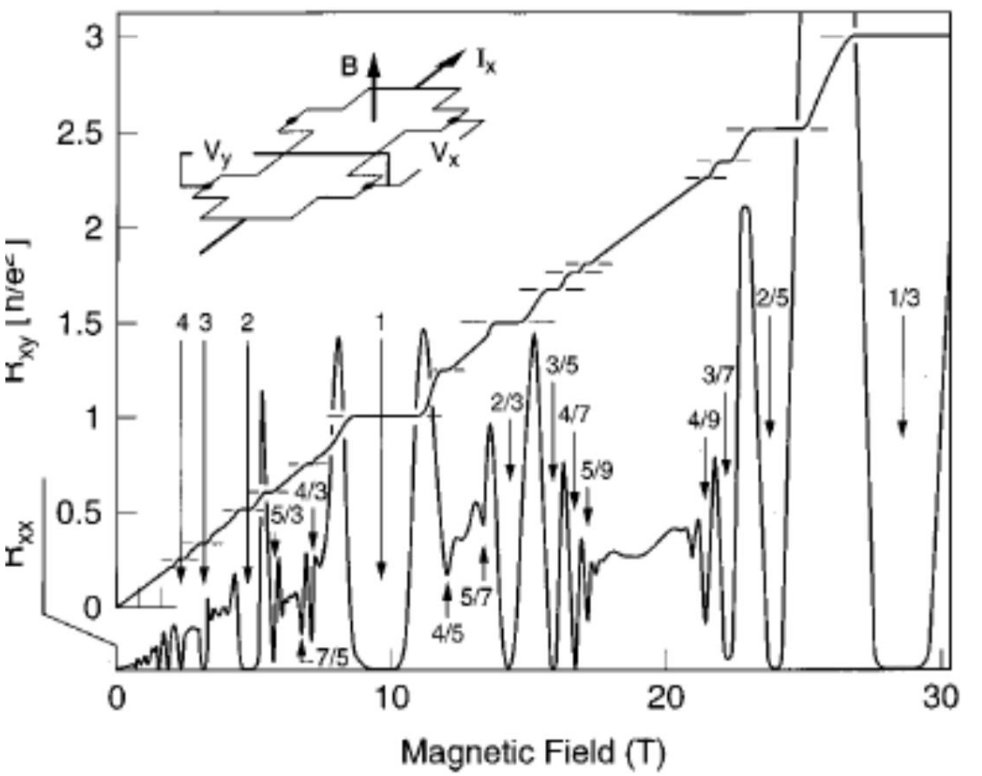
Quantum Hall Effect



$$I_x = \sigma_{xx} V = 0$$

$$I_y = \sigma_{xy} V$$

$$I = \frac{\nu e^2}{h} V; \quad \nu \text{ - rational number (filling factor)}$$



$$\frac{e^2}{h} = (25.8k\Omega)^{-1}$$

$$= 1\,\text{Klitzing}$$

H.L. Stormer *et al*., RMP S298 (1999)

Laughlin State

$$\nu = \frac{1}{2n+1};\ \Psi = \prod (z_i - z_j)^{2n+1} \exp\left(-\sum |z_i|^2/4l^2\right);\ z_k = x_k + iy_k$$

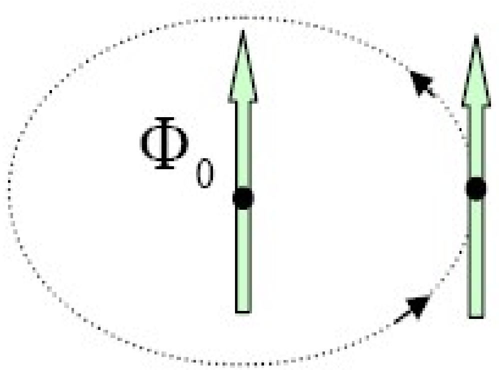
Quasiparticle charge (Laughlin)

$$q = \nu e$$

Quasiparticle statistics

(Arovas, Schrieffer, Wilczek)

$$\theta = 2\pi\nu$$



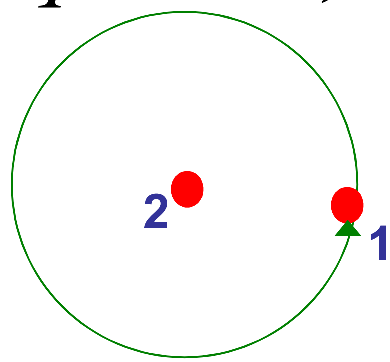
$$\theta = \frac{q}{\hbar c}\oint \vec{A}\,d\vec{r}$$

Many other odd-denominator states,e.g., $\nu = p/(2pn+1)$

Moore-Read (Pfaffian) state

$$\nu = 5/2$$

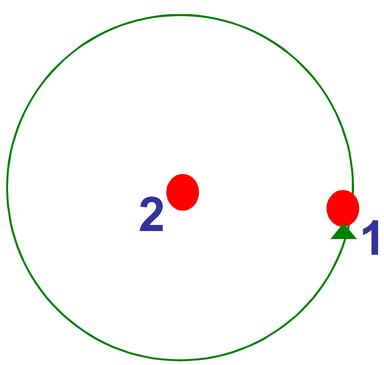
$$q = e/4; \text{non - Abelian statistics}$$



$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

Several states at given quasiparticle positions

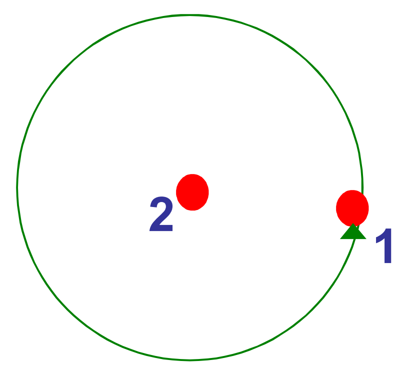
Vacuum superselection sector $\left|1\right\rangle$



$$\psi \rightarrow \psi$$

$$\theta = 0$$

Fermion sector $\left|\varepsilon\right\rangle$



$$\psi \rightarrow -\psi$$

$$\theta = \pi$$

$$\alpha\left|1\right\rangle\left|\;\right\rangle + \beta\left|\varepsilon\right\rangle\left|\;\right\rangle \rightarrow \alpha\left|1\right\rangle\left|\;\right\rangle - \beta\left|\varepsilon\right\rangle\left|\;\right\rangle$$

Theoretical proposals at ν=5/2

FQHE of bosons naturally corresponds to even denominators. Cooper pairing?

Experimental confirmation: charge *e*/4.

Numerous ways to build Cooper pairs:

- Pfaffian state
- 331 state
- K=8 state
- $SU(2)_2$ state
- anti-Pfaffian state
- anti-331 state
- anti- $SU(2)_2$ state

Numerics favors Pfaffian and anti-Pfaffian

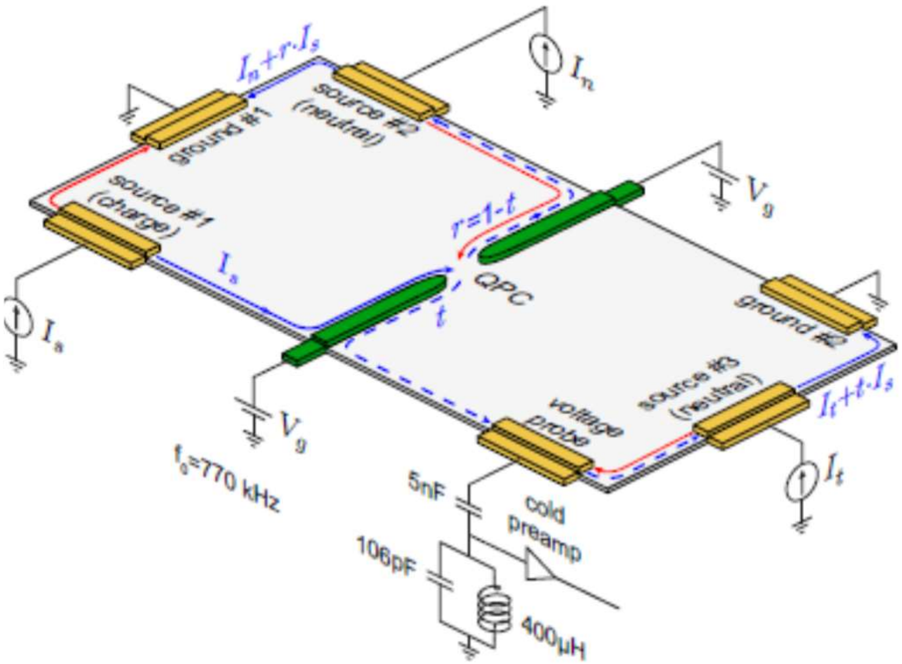
Pfaffian and anti-Pfaffian states

- numerics supports Pfaffain and anti-Pfaffian states in the absence of disorder and Landau level mixing
- poor results for the energy gap
  - strong disorder
  - LLM parameter ~ 1.3
- Small energy differences for proposed states

  [J. Biddle *et al.*, *Phys. Rev. B* **87**, 235134 (2013)]
- no QHE at realistic LLM in numerics

[K. Pakrouski *et al.*, *Phys. Rev. X* **5**, 021004 (2015)]

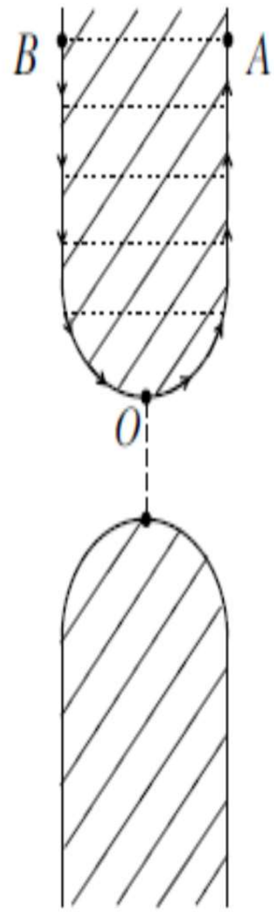
Upstream neutral modes



Observation of a topologically protected upstream neutral mode [A. Bid *et al.,* Nature **466**, 585 (2010)]. Compatible with anti-Pfaffian. Incompatible with Pfaffian.

The observed physics at ν=5/2 is similar to ν=2/3. It differs from the filling factors where no topologically protected upstream modes are present but edge reconstruction is possible [H. Inoue *et al.*, Nature Comm. **5**, 4067 (2014).]

Tunneling



Theory: $G \sim T^{2g-2}$

- Pfaffian: $g = \frac{1}{4}$
- Anti-Pfaffian: $g = \frac{1}{2}$

Experiment: $g_{\text{exp}} > g_{\text{theor}}$

Experiment gives an upper bound on g
The upper bound of 0.4 is consistent with Pfaffian and excludes anti-Pfaffian

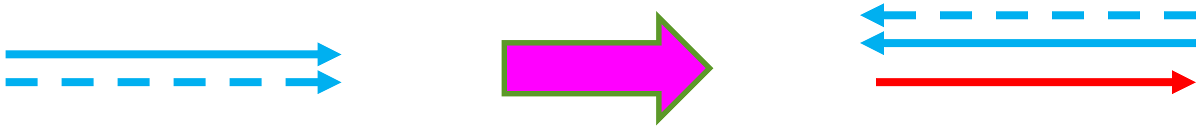
I. P. Radu *et al.*, *Science* **320**, 899 (2008)
X. Lin *et al.*, *Phys. Rev. B* **85**, 165321 (2012)
S. Baer *et al.*, *Phys. Rev. B* **90**, 075403 (2014)

PH-Pfaffian state

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]

s-pairing of Dirac fermions

Particle-hole symmetry:



$$G = \frac{e^2}{2h}; \quad k = \pi^2 T/6h$$

Edge theory:

$$-\frac{2}{4\pi}[\partial_t\varphi\partial_x\varphi + v_c\partial_x\varphi\partial_x\varphi] + i\psi(\partial_t - v_n\partial_x)\psi$$

$$\psi = \psi^+$$

Wave function:

$$\int\{d^2 s_i\}\mathrm{Pf}\left\{\frac{1}{\bar{s}_i - \bar{s}_j}\right\}\prod(s_i - s_j)^2 \exp[-|s_i|^2 + 2\bar{s}_i z_i - |z_i|^2]$$

Fusion and braiding rules

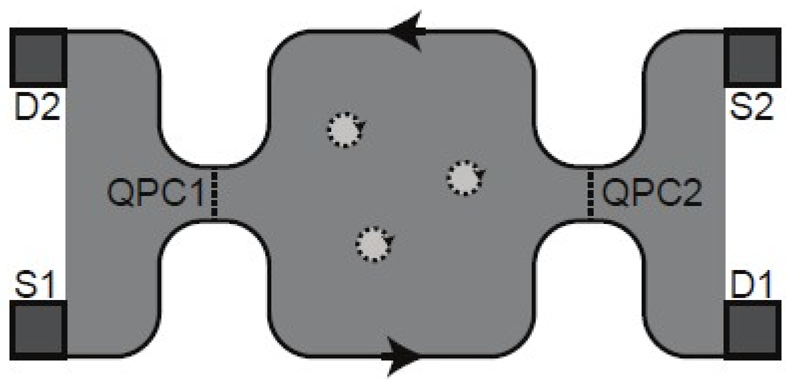
- 6 quasiparticle types:

  topological charge σ and electric charges $\pm e/4$;

  topological charges 1 and ψ and electric charges 0 and $e/2$
- Fusion rules $\psi \times \psi = 1; \psi \times \sigma = \sigma; \sigma \times \sigma = 1 + \psi$
- Braiding rules determine the phase accumulated by a quasiparticle moving around another quasiparticle. The phase depends on the fusion channel.

Comparison with the experiment

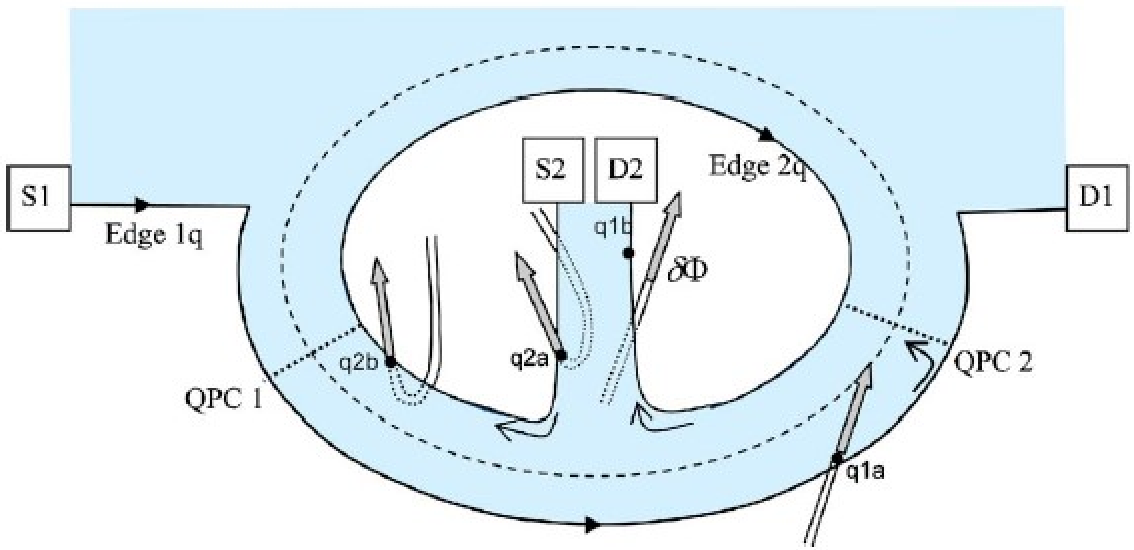
- An upstream neutral mode
- Tunneling exponent $g = \frac{1}{4}$
- Topological even-odd effect



New experimental signatures

Thermal Hall conductance $\frac{\pi^2 k^2 T^2}{6h}$

Mach-Zehnder interferometry

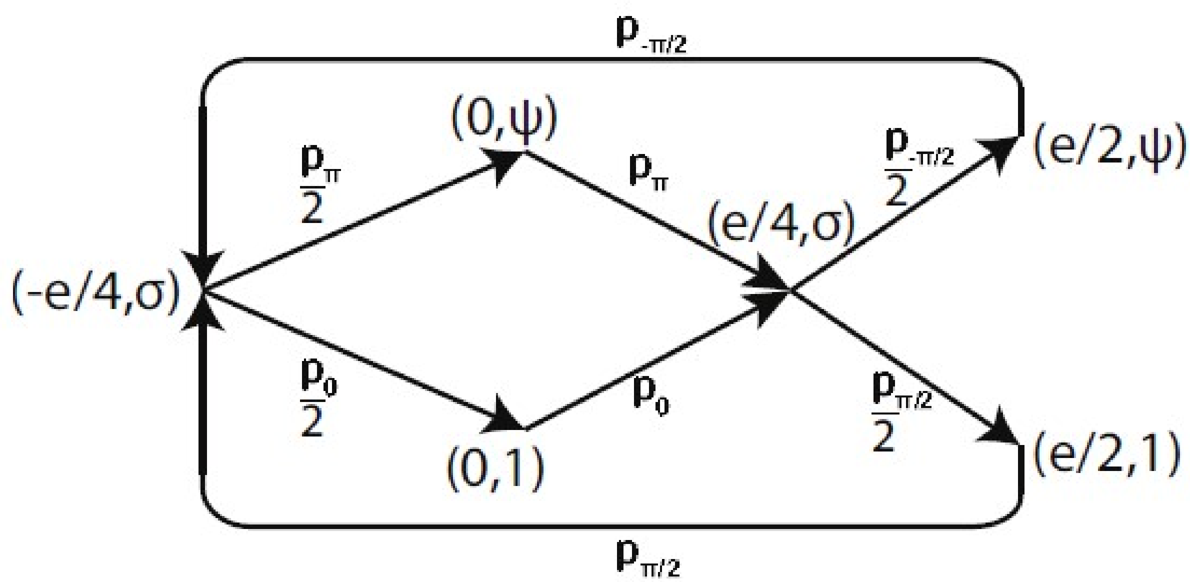


No magnetic field dependence of the current.
Shot noise diverges at some magnetic fields.

Mach-Zehnder interferometry

Tunneling probability depends on the accumulated topological charge

$$P \sim |\Gamma_1|^2 + |\Gamma_2|^2 + 2u|\Gamma_1\Gamma_2|\cos(\varphi_{AB} + \varphi_{stat} + \alpha)$$

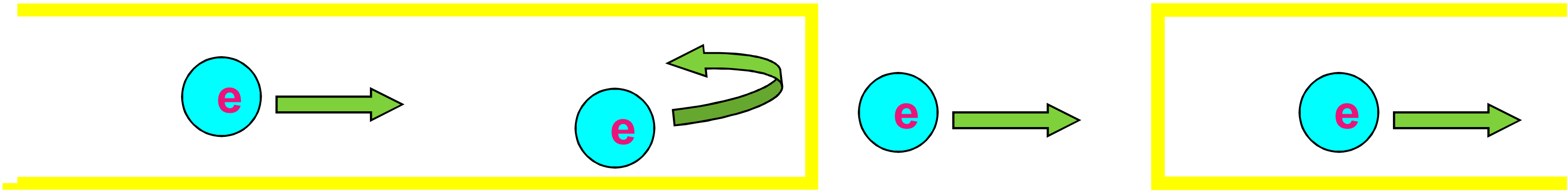


$$I = \text{const}[|\Gamma_1|^2 + |\Gamma_2|^2]$$

Shot Noise

$$S = \int [\langle I(0)I(t)\rangle - \langle I\rangle^2]dt = q\langle I\rangle$$

Walter Shottky



L. Saminadayar et al., R. de Picciotto et al. (1997):q=e/3

Shot noise in MZ interferometer

$$S = q^* \langle I\rangle$$

$$q^* = \frac{e}{64}\sum p_i \sum \frac{1}{p_i} \text{ diverges at some fields}$$

Closing Argument

- PH-Pfaffian topological order is consistent with all experiments
- Numerics with the particle-hole symmetric Hamiltonians supports states that break the particle-hole symmetry
- Realistic Hamiltonians have no symmetry
- The ground state is not symmetric, yet the topological order is compatible with the particle-hole symmetry