Low-temperature anomalies in disordered superconducting films close to upper critical field

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Collaboration: Benjamin Sacepe, Neel Inst., Grenoble

- Strongly disordered superconductors:
  phase diagram in (H-T) plane with $dH_{c2}/dT > 0$ at $T=0$
- Critical current v/s H at low T
- BKT transition in strong magnetic field
- How to find a relation between all the above features
- Theory problems to be solved:
  3D quantum glass transition
  2D BKT transition with background vortices

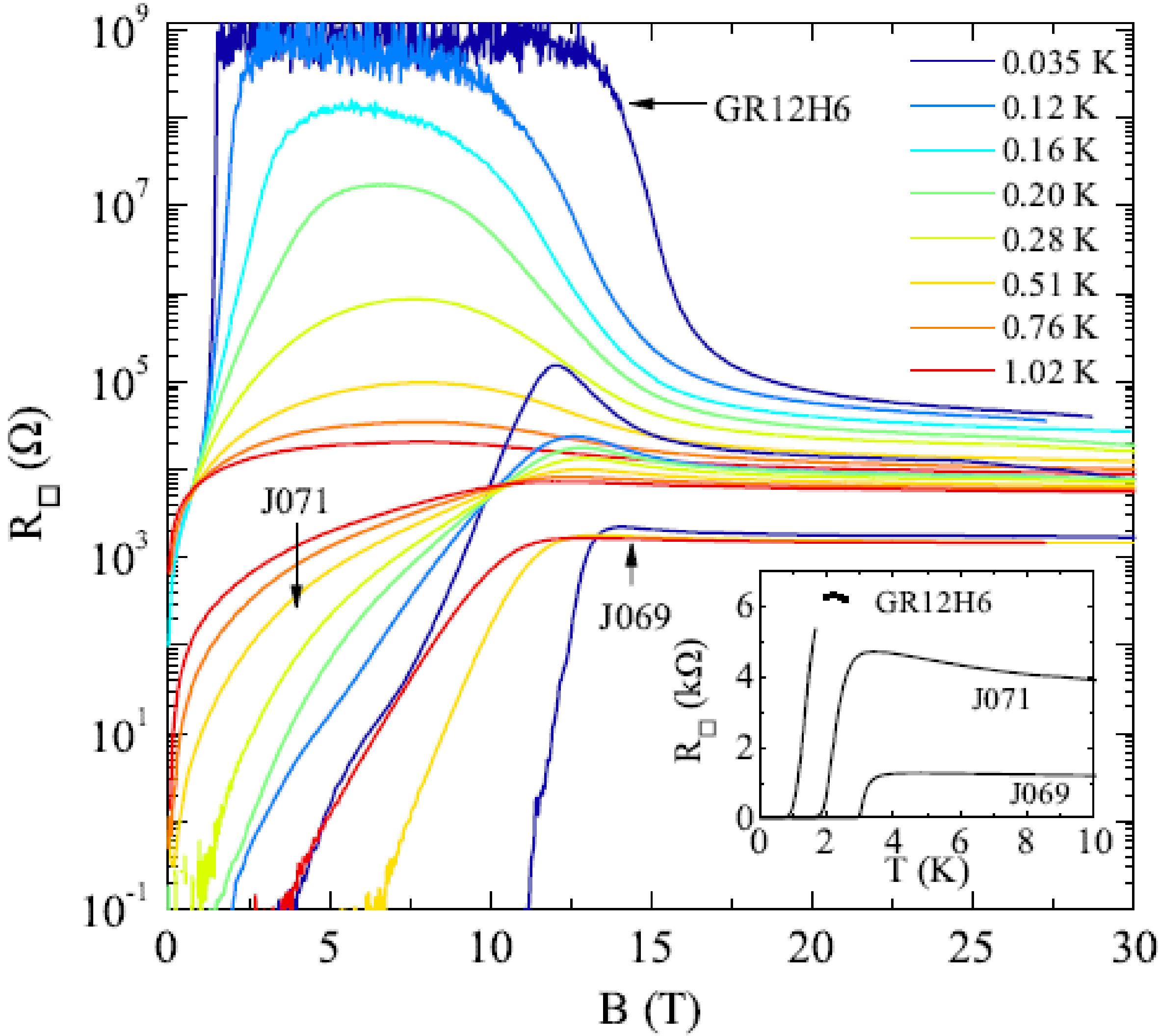
The object: amorphous SC films
which are not in proximity to SIT

1. Well-defined H_{c2}

2. No (strong) peak in magnetoresistance above H_{c2}

Examples: *moderately* disordered InO_x
Mo_xGe_{1-x} films
Very thin Ga films

Evolution of SMT into SIT



B.Sacepe et al, Phys. Rev. B (2015)

Phase Diagram

InO_x magnetoresistance data

(B.Sacepe & J.Seidemann, Grenoble 2013)

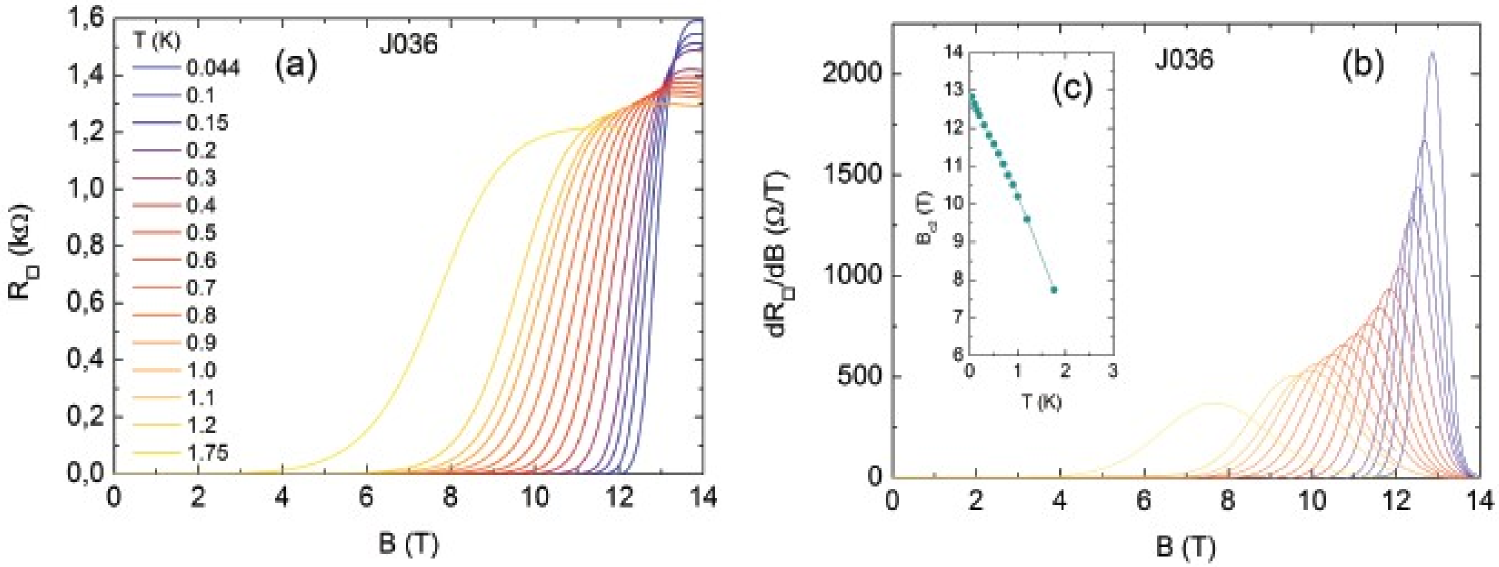


FIGURE 4.8.: (a) resistance versus external field at different temperatures of one InO_x sample, (b) derivatives of the curves shown in (a), (c) upper critical field versus temperature, where the field values correspond to the maxima in (b).

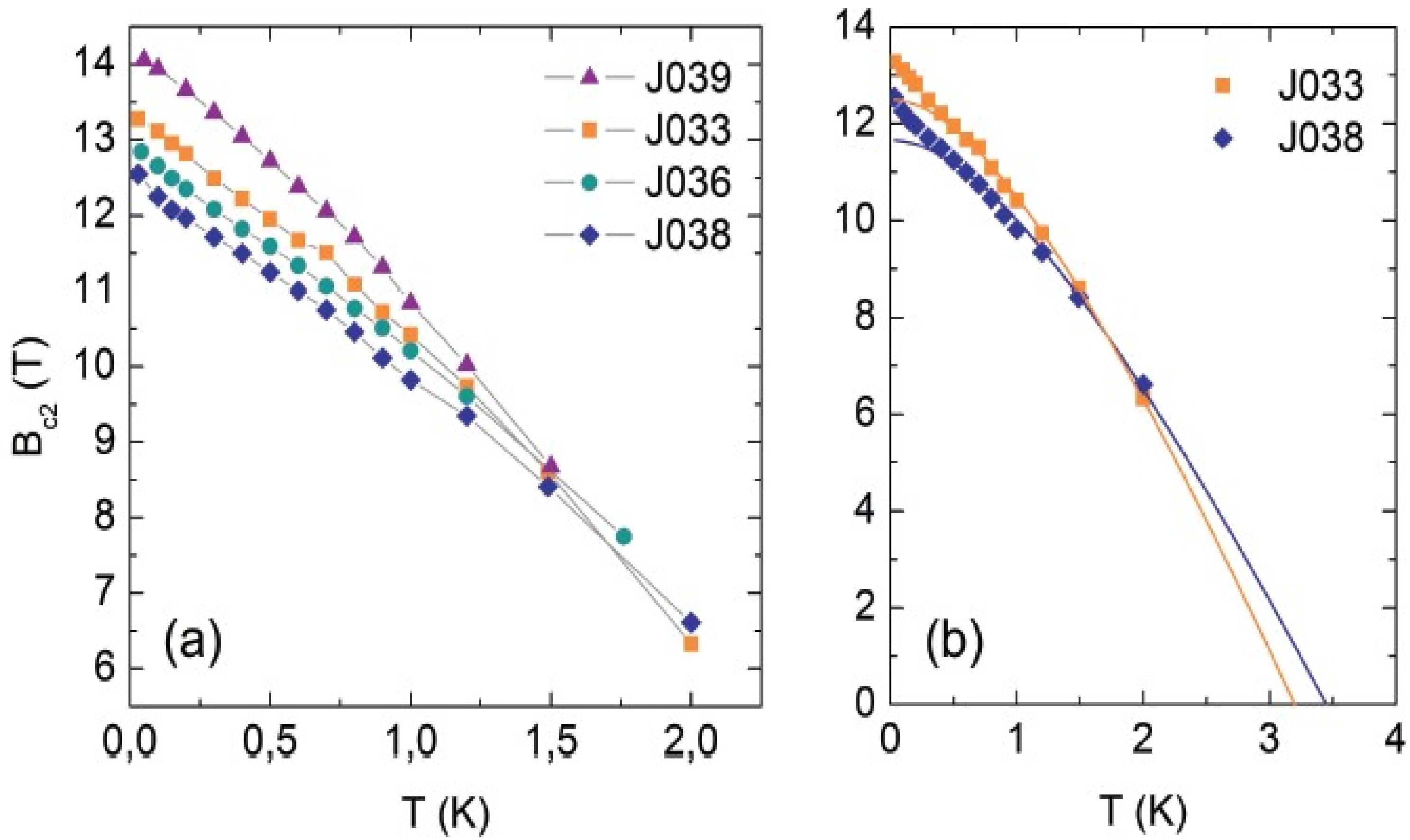


FIGURE 4.9.: (a) B_{c2} versus T curves at low temperature for four samples with different disorder, (b) two of the four samples, with the BCS-fit as solid line.

there is no saturation when zero temperature

is approached, as predicted in the theory for conventional superconductors.

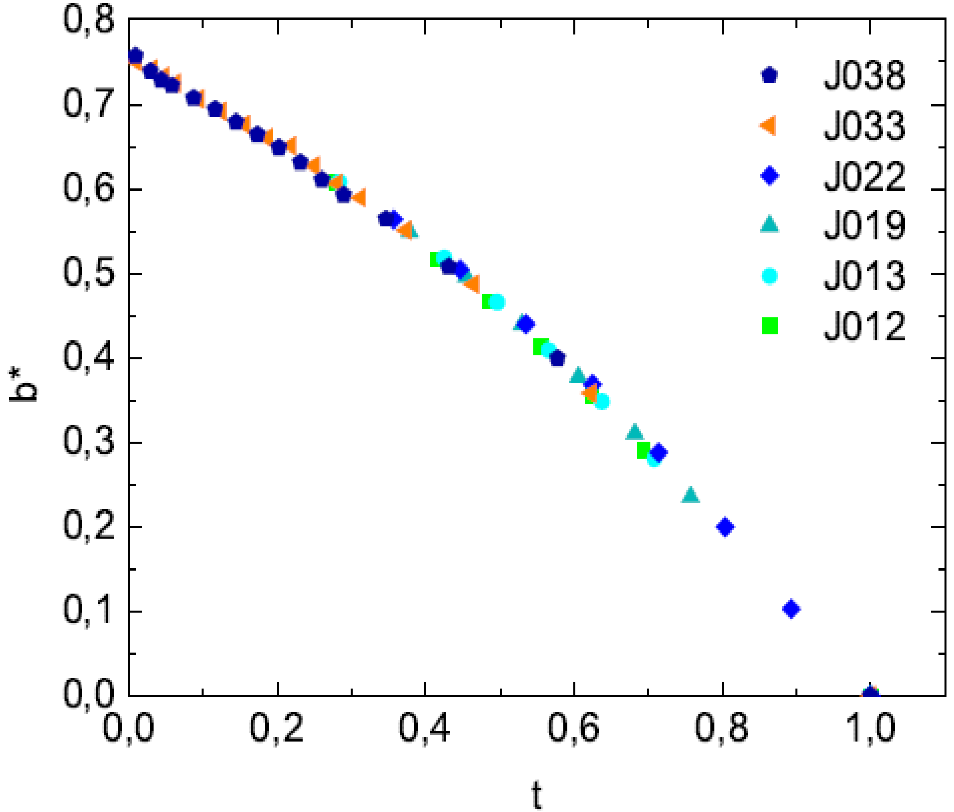


Figure 4.10.: Reduced upper critical field versus reduced temperature of the six different InO_x samples of the figure above in one plot.

How can one understand a nonzero slope dH_{c2}/dT at $T=0$?

V. M. Galitski and A. I. Larkin, Phys. Rev. Lett. 87, 087001 (2001)

B. Spivak and F. Zhou, Phys. Rev. Lett. 74, 2800 (1995).

F. Zhou and B. Spivak, Phys. Rev. Lett. 80, 5647 (1998).

Explanations in terms of mesoscopic fluctuations

for the “upturn” of the $H_{c2}(T)$ curve at low T

Previous experiments of this kind:

S. Okuma et al., J. Phys. Soc. Jpn. 52, 3269 (1983);

A. F. Hebard and M. A. Paalanen, Phys. Rev. B 30, 4063 (1984).

A. Nodrstrom, U. Dahlborg, O. Rapp, Phys. Rev. B48, 12866 (1993)

A. F. Hebard and M. A. Paalanen

A. NORDSTRÖM, U. DAHLBORG, AND Ö. RAPP

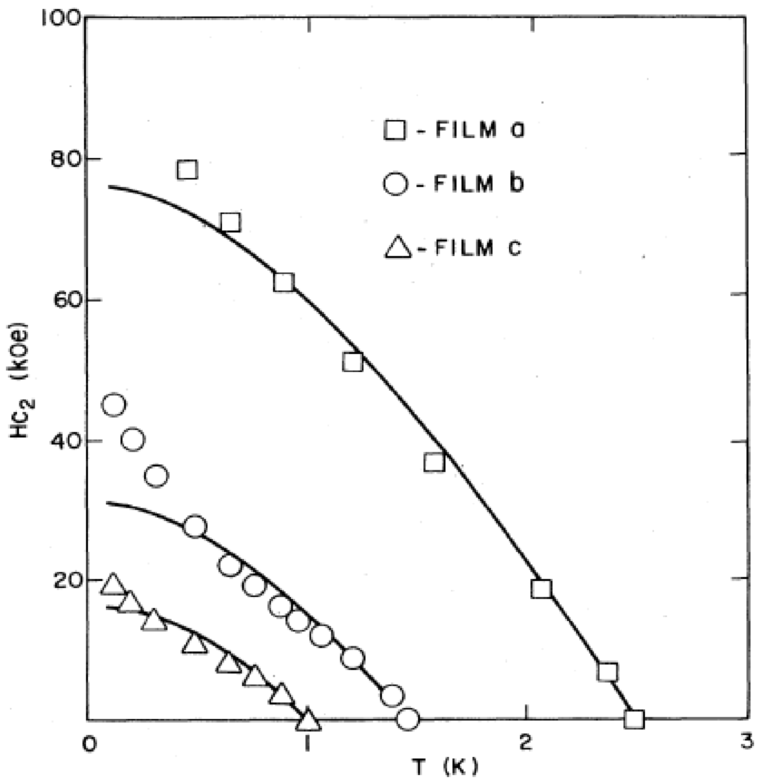


FIG. 2. Temperature dependence of H_{c2} for films (a)–(c). The solid lines are theory.

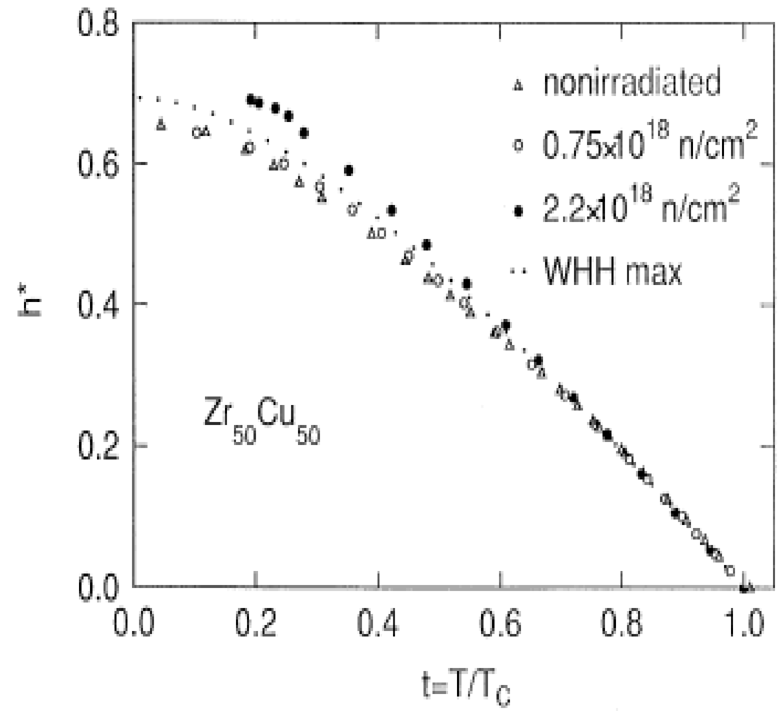


FIG. 5. Critical field data for $Zr_{50}Cu_{50}$ in reduced parameters. h^* increases with irradiation. After full neutron dose data reach above the WHH maximum.

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SCIENCE

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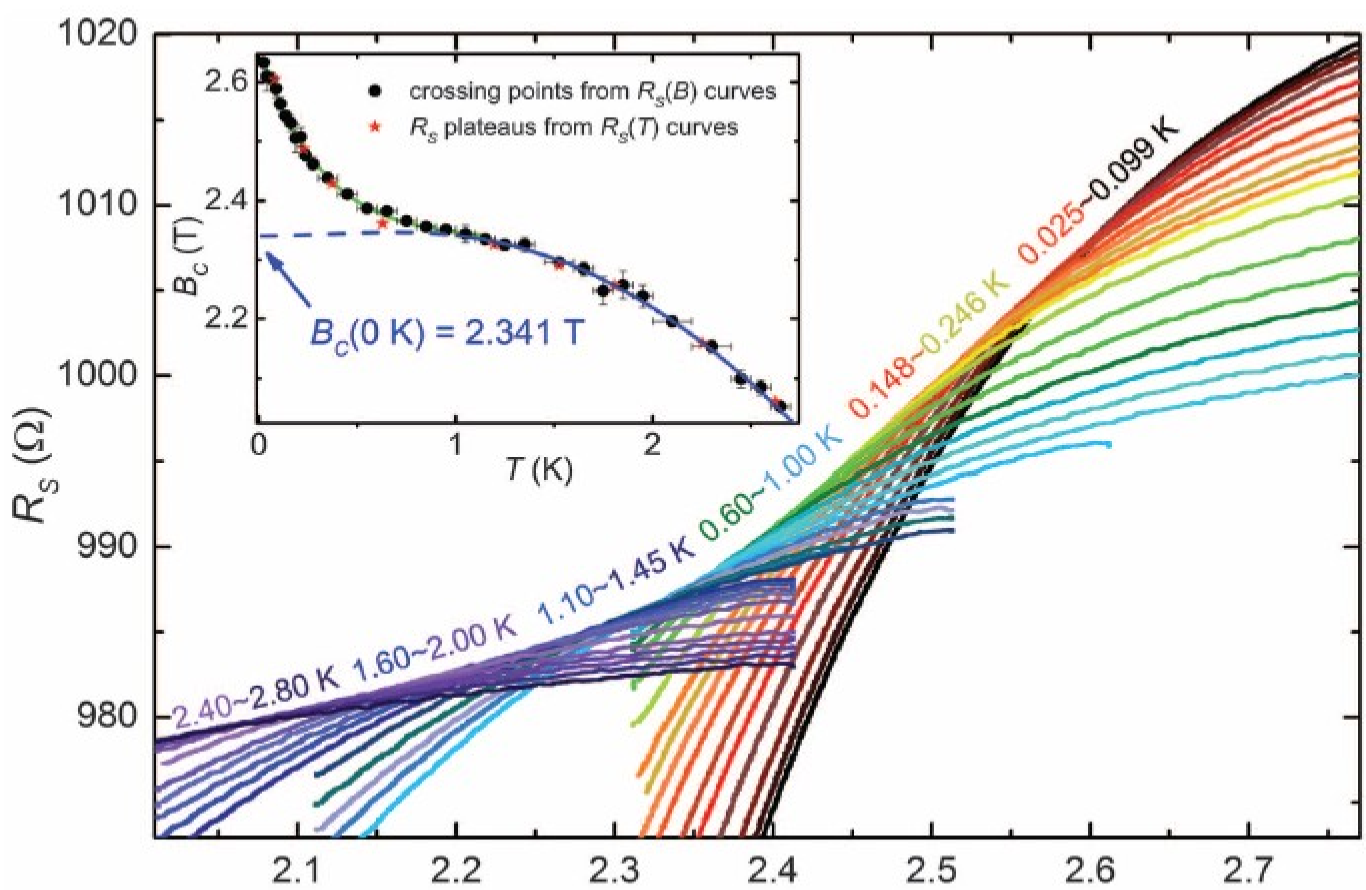
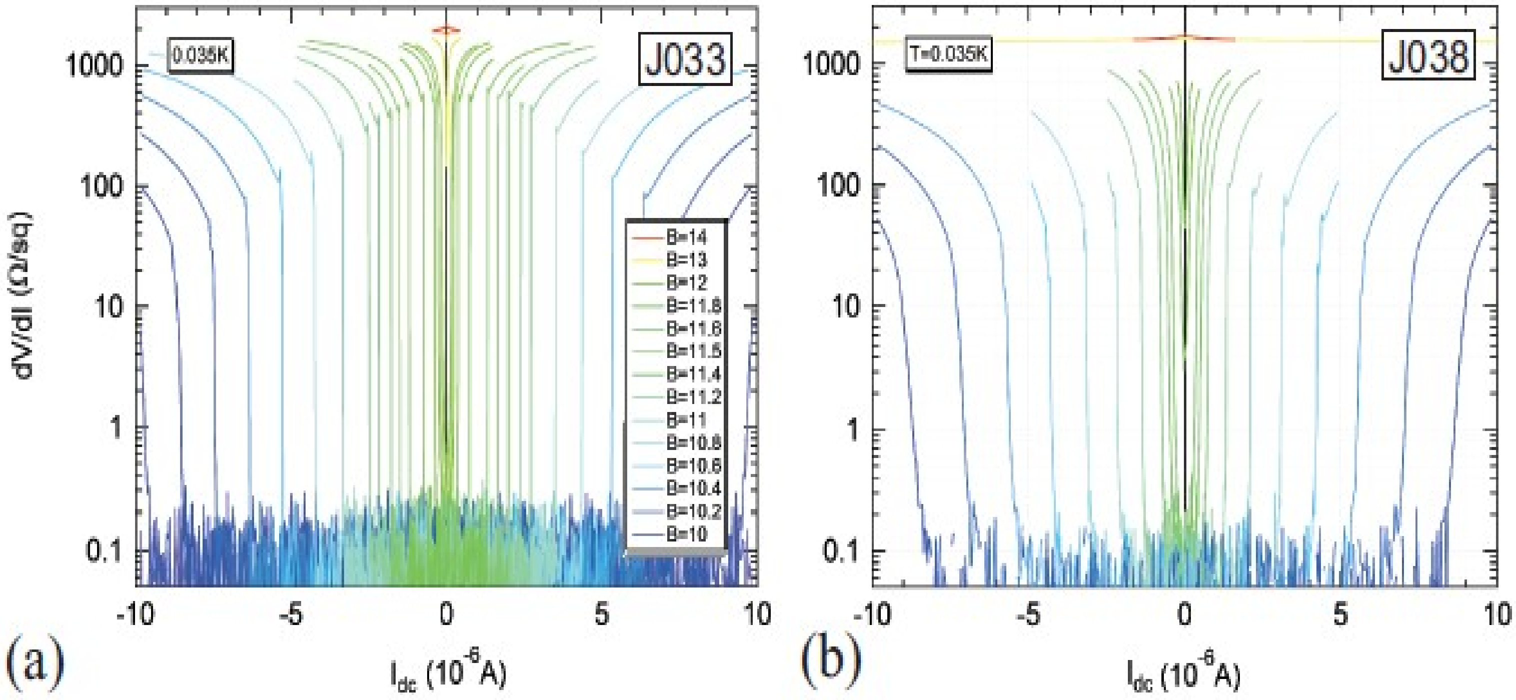
Quantum Griffiths singularity of superconductor-metal transition in Ga thin films

Ga films with a nominal thickness of three monolayers (3 ML) were epitaxially grown on a 3-μm GaN(0001) substrate in an ultrahigh-vacuum molecular beam epitaxy (MBE) chamber (*21*). Scan-



Critical current v/s magnetic field at very low $T << T_c$

Critical current density near T=0



J.Seidemann, Thesis

J.Seidemann, Thesis

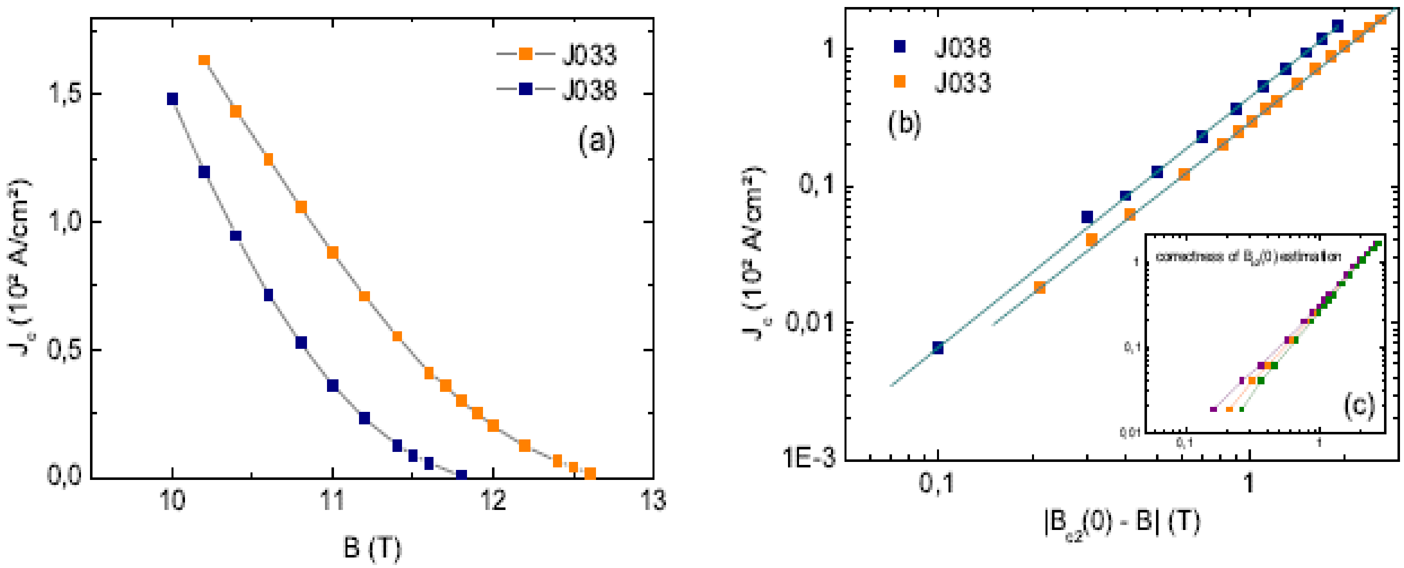


FIGURE 4.12.: Critical current density as (a) a function of the external magnetic field and (b) a function of the absolute value of $B_{c2}(0) - B$. The added straight lines in (b) underline the linear dependence. Inset (c) shows the deviation from a straight line for a variation of $B_{c2}(0)$ of 0.1 T for sample J033.

$$J_c \propto |B_{c2}(0) - B|^{\upsilon}$$

“Mean-field theory” value:

$$\upsilon = 3/2.$$

the measurements lead to a power of 1.62 and 1.65 in sample J033 and J038

BKT transition in strong magnetic field

Vortex depairing on top of large vortex density ??

New experiment: BKT transition in a strong magnetic field (A.Yazdani et al, PRL 2013)

“Evidence for a universal minimum superfluid response in field-tuned disordered superconducting films measured using low frequency ac conductivity”

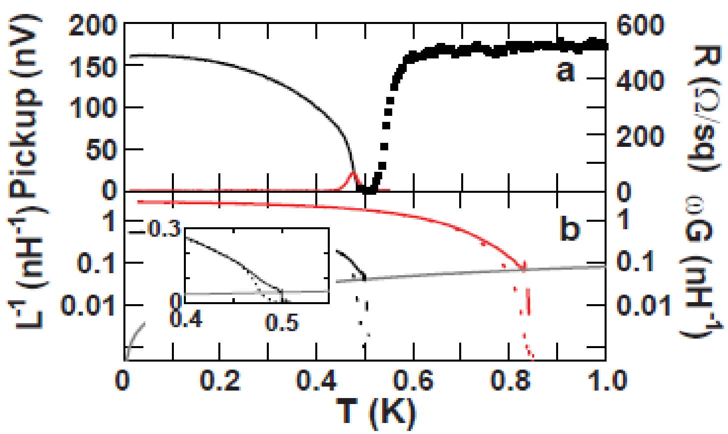


FIG. 1. (a) The in-phase (black) and out-of-phase (red) voltage on the pickup coil ($f = 20$ kHz and $I = 20\,\mu$A on the drive coil), plotted along with the resistivity from conventional dc transport (black squares), is shown as a function of temperature at zero field for a MoGe film. (b) Plotted here is L^{-1} derived from the data in MoGe film in part (a) (black line), and a second less disordered film (red). Also shown are the imaginary part of ωG (dashed lines) and the BKT prediction for L^{-1} (gray). The inset shows a close-up of the data from the more disordered film on a linear scale.

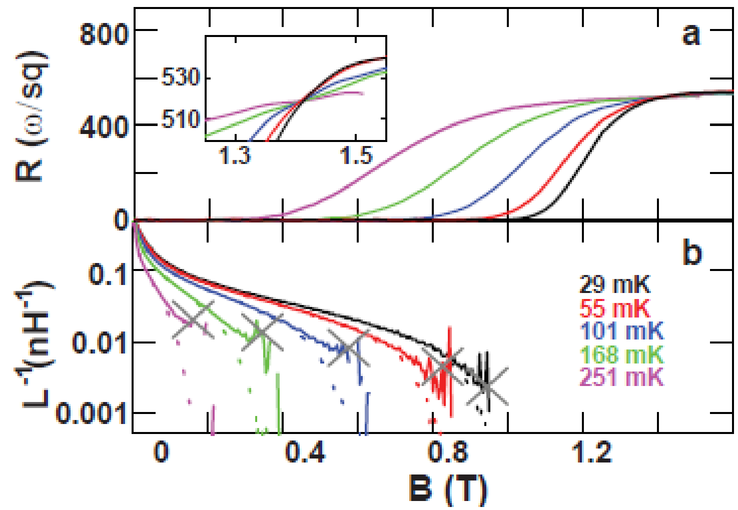


FIG. 2. (a) Measurements of dc resistivity isotherms, shown here for the MoGe sample in Figure 1(a) and taken at the indicated temperature, shown as a function of field. (inset) A close-up indicates that the isotherms cross at a field of $B_X = 1.41 \pm 0.02$T. (b) Measurements of L^{-1} isotherms for the same film as (a) taken at 50 kHz show a discontinuous jump which moves to larger values of the field at lower temperatures. Also shown are the imaginary part of ωG (dashed lines) and the BKT prediction for L^{-1} (gray crosses).

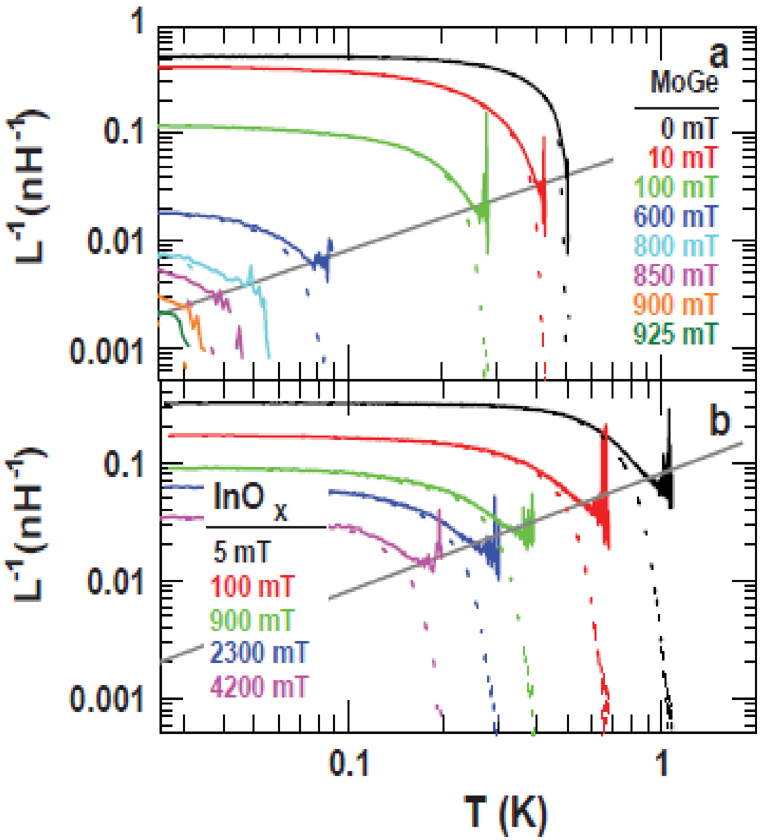


FIG. 3. Temperature sweeps of L^{-1}, taken here at 20 kHz, in the presence of an applied magnetic field show a discontinuous jump to zero in both (a) MoGe and (b) InO_x thin film samples. Also shown are the imaginary part of ωG (dashed lines) and the BKT prediction for L^{-1} (gray).

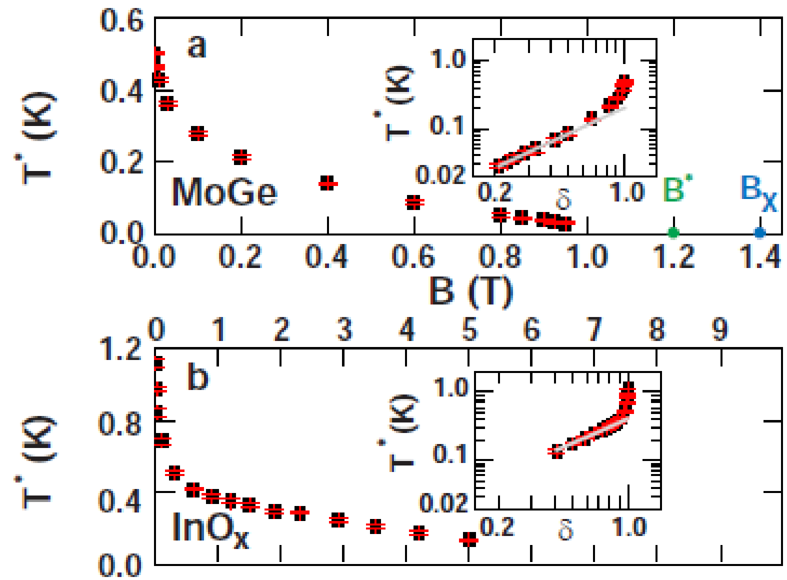


FIG. 4. Shown are the temperatures (T^*), along with error bars (red lines), at which L^{-1} is measured to jump to zero in our temperature sweeps at fixed field in the (a) MoGe and (b) InO_x samples (f = 20 kHz). The inset shows a fit of the data to the power law form $T^* \sim \delta^{\nu z}$, where $\delta = (1 - B/B^*)$ on a log-log scale. In part (a), the fitted critical field B^* from the ac measurements is shown in green, along with the crossing field B_X from the resistivity data of Figure 2 in blue.

$$T^* \sim (1 - B/B^*)^{\nu z}$$

νz = 1.25 ± 0.25 (MoGe)

1.3 ± 0.4 for InO_x

Close to 1

How can one reconcile these three types of data ?

1) Finite slope of $H_{c2}(T)$ at T close to zero

2) $J_c \sim (H_{c2} - B)^{3/2}$ at $T = 0$

3) BKT transition with $\Theta(T_c) = (2/\pi)\, T_c$

where $\Theta = (\hbar/2e)^2 \rho_s \cdot d.$ Even at strong magnetic field !

General idea: the observed effects are due to **combination** of 3D quantum (T=0) phase transition and 2D finite-temperature BKT transition

$J_c \sim (H_{c2} - B)^{3/2}$ at $T = 0$ is the feature of

3D quantum glass transition

and is analogous to the usual

behavior of depairing current in superconductor

$j_c(T) \sim (T_c - T)^{3/2}$ $\rho_s \sim (T_c - T)$

In a similar way, we may expect:

$J_c = \rho_s v_{s\,max}$

$\rho_s \sim (H_{c2} - B)$

$v_{s\,max} \sim (H_{c2} - B)^{1/2}$

Temperature of the BKT transition in strong field is determined by the property of the T=0 transition

$$\rho_s \sim (H_{c2} - B)$$

Universal ratio ρ_s/T at BKT leads to the scaling

$$T_c(B) = \mathcal{A}\,(H_{c2} - B)$$

with $\mathcal{A} \sim$ d (film thickness)

H_{c2} is the feature of the bulk (3D) problem ,

it is d-independent at fixed bulk resistivity

What about theory ?

Two approaches look possible:

1. *vortex pinning* and critical current in type-II SC

2. *transverse stiffness* in a Gauge Glass, or Phase Glass or Superconducting Glass

Vortex (or flux) pinning

A. Larkin and Yu. Ovchinnikov (multiple papers and reviews)

1. Vortex pinning theories always start from the Abrikosov solution for a single vortex or vortex lattice, i.e. some "usual" short-range order is assumed to exist

2. Close to H_{c2} ***collective pinning*** transforms into a ***single-vortex pinning*** due to vanishing of the vortex lattice shear modulus, leading to a "peak-effect" in $j_c(B)$

3. For a region above the peak, a dependence $j_c(B) \sim (H_{c2} - B)$ is mentioned in L&O review paper Physica B+C **126**, 187 (1984)

Depairing current and strong pinning

1. Deparing: $F_{sc}[B] \sim (H_{c2} - B)^2$

$\delta F_{sc}[B, v_s] = \rho_s v_s^2/2$ $\rho_s \sim \Delta^2 \sim H_{c2} - B$

$\Rightarrow$ $v_{s\,max} \sim (H_{c2} - B)^{1/2}$ $\Rightarrow$ $j_c \sim (H_{c2} - B)^{3/2}$

2. Strong pinning: $(h/e)\, j_c\, \xi \sim F_{sc}[B]$

Here ξ is the effective correlation length near B – controlled phase transition into broken-symmetry state

If $\xi^{-2} \sim (H_{c2} - B)$ then $j_c \sim (H_{c2} - B)^{3/2}$

However, an U(1) – breaking order parameter seems to be of *glassy* nature in our problem.

What about spin/gauge glass theory ?

PHYSICAL REVIEW B **68**, 104427 (2003)

Absence of phase stiffness in the quantum rotor phase glass

Philip Phillips Denis Dalidovich

$$H=-E_C\sum_i\left(\frac{\partial}{\partial\theta_i}\right)^2-\sum_{\langle i,j\rangle}J_{ij}\cos(\theta_i-\theta_j-A_{ij}),$$

The major result: $\rho_s = 0$ and conductivity is finite:

$$\sigma_{\text{bos}}(\omega=0,T\to 0)=\frac{4}{3}\frac{e^2\eta q_0}{hm^4}$$

It is not clear what is the origin and magnitude of η

Evidently, $\rho_s = 0$ does not agree with experimental data

How about previous theories ?

$$\sigma^{(1)}(i\omega_n)=\frac{16e^2}{\hbar\omega_n}\frac{4q_{\text{EA}}\Delta_q}{3}\Pi(i\omega_n),$$

$$\Delta_q=q_{\text{EA}}-\int_0^1 q(s)\,ds=q_{\text{EA}}s_1/2$$

is the broken ergodicity parameter

$$\Pi(i\omega_n)=\int_0^\beta d\tau e^{i\omega_n\tau}[\beta\delta(\tau)-\cancel{1}].$$

$$s_1=2y_1q_{\text{EA}}T/\kappa$$

Vanishes at T=0

βs_1 = const >0 at T =0

Classical XY or gauge glass

H. Sompolinsky, G. Kotliar, and A. Zippelius, Phys. Rev. Lett. 5 392 (1984).

Phys. Rev. B 35, 311 (1987).

$\rho_s \sim \underline{q_{EA}\Delta_q} \sim (T_c - T)^3$ (replica method)

System of Josephson junctions as a model of a spin glass

V. M. Vinokur, L. B. Ioffe, A. I. Larkin, and M. V. Feĭgel'man

Zh. Eksp. Teor. Fiz. **93**, 343–365 (July 1987)

The same result by means of *dynamic slow cooling* approach

Theory of Diamagnetism in Granular Superconductors

M. V. Feigelman,[1] and L. B. Ioffe[1,2] Phys. Rev. Lett. **74**, 3447 (1995)

$$\mathbf{j} = -\rho_s^g \delta\mathbf{A}, \quad \rho_s^g = \frac{4\pi^2 c \xi_0^2 n T_g}{\Phi_0^2}\tau^3 \quad \mathbf{j}(\mathbf{a}) = -\mathbf{e_a}(2\pi c \xi_0 n T_g/\Phi_0)\tau^4 Y(a/\tau)$$

Within continuous Parisi RSB scheme $\underline{\rho_s \sim (T_c - T)^3}$

QUANTUM GLASS TRANSITION IN A PERIODIC LONG-RANGE JOSEPHSON ARRAY

D. M. Kagan[1], L. B. Ioffe[1,2], M. V. Feigel'man[1]*
ЖЭТФ, 1999,

$$\mathcal{H} = \mathcal{H}_J + \mathcal{H}_C = -E_J \sum_{m,n} \cos\left(\phi_n - \phi_m - \frac{2e}{\hbar c}\int \mathbf{A}\cdot d\mathbf{l}\right) + \frac{(2e)^2}{2}\sum_{m,n} \hat{C}^{-1}_{m,n} \frac{\partial}{\partial \phi_m}\frac{\partial}{\partial \phi_n}$$

$$s_m = e^{i\phi_m} \qquad G_{m,n}(\tau) = -\langle T_\tau s_m(\tau) s^\dagger_n(0)\rangle$$

Vanishes at QPT

$$\mathcal{G}(\tilde{t}) = \frac{1}{\sqrt{\tilde{t}}} f\left(\frac{\tilde{t}}{\tau_1}\right) + \mathcal{G}_1 \exp\left(-\frac{\tilde{t}}{\tau_0}\right) \qquad \tau_0 = \frac{\sqrt{32/27}}{\tilde{g}^3}\frac{1}{b^2} \qquad \mathcal{G}_1 = \sqrt{\frac{27}{2}}\tilde{g}^3\, b.$$

This solution is more similar to 1-step RSB

Results for diamagnetic response

$$\chi_{\mathcal{M}}(\omega) \propto \sqrt{i\omega}\ln\omega. \qquad \omega \gg (J/J_c - 1)^2 \alpha^{-3/2}$$

$$\chi_{\mathcal{M}}(\omega) \propto \frac{J_c^3}{(J_c - J)^3}\,\omega^2, \qquad \omega \ll \frac{C_l (J - J_c)^2}{e^2 \alpha^{5/2}}.$$

In disordered phase

Scaling in glassy phase: $\rho_s \propto (J - J_c)$

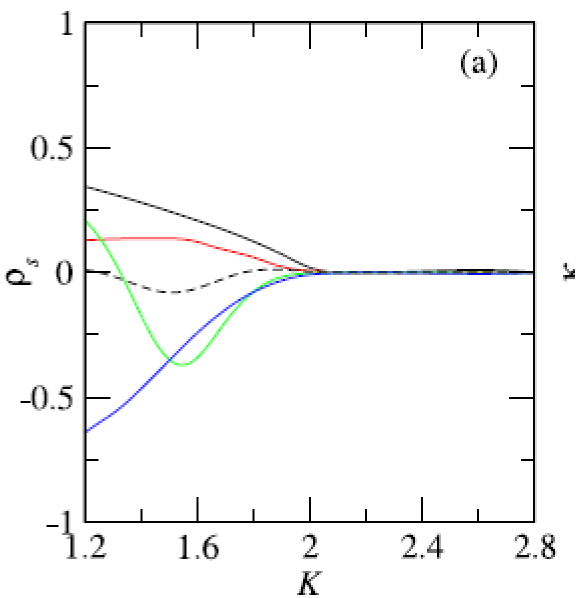
Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

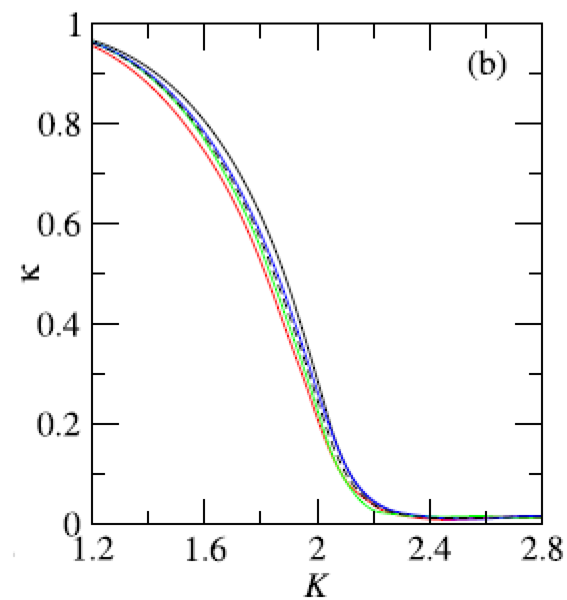
Phase glass and zero-temperature phase transition in a randomly frustrated two-dimensional quantum rotor model

Lei-Han Tang[1] and Qing-Hu Chen[2]

doi:10.1088/1742-5468/2008/04/P04003

$$\hat{H} = \frac{E_Q}{2}\sum_i \hat{n}_i^2 - E_J\sum_{\langle ij\rangle}\cos(\theta_i - \theta_j - a_{ij}).$$





The helicity modulus against $K = \sqrt{E_Q/E_J}$ for four disorder realizations at $L = L_z = 16$. The dashed line in each case indicates an average over the four samples.

Conclusions

1) Anomaly of nonzero dH_{c2}/dT slope at $T=0$ is due to

a) BKT nature of the transition at $T>0$ and

b) linear dependence of $\rho_s \sim (H_{c2} - B)$

which is a feature of a 3D (or 2D) quantum glass transition

2) For the same SC material, the slope dH_{c2}/dT

grows as $1/d$

3) Theory of phase stiffness near quantum glass transition is to be developed