Dynamical phase transition into quantum chaos in a solvable many-body model

Sumilan Banerjee

Weizmann Institute of Science

Landau Institute for Theoretical Physics
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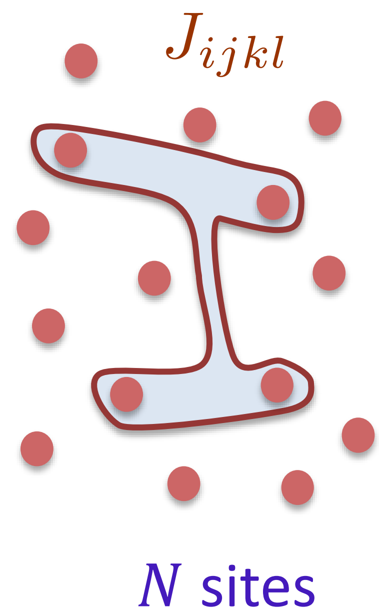
Ehud Altman
(Weizmann Institute & UC Berkeley)

A. Kitaev's Talk: Sachdev-Ye-Kitaev model

Sachdev & Ye, PRL (1993)
Kitaev, KITP (2015)
Sachdev, PRX (2015)

$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_i^N c_i^\dagger c_i \qquad P(J_{ijkl}) \sim e^{-\frac{|J_{ijkl}|^2}{J^2}}$$



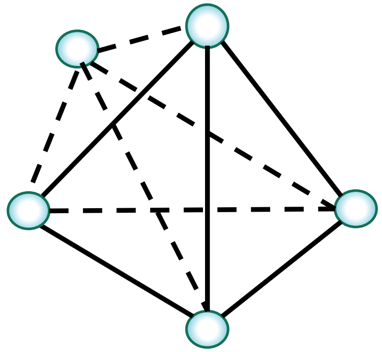
- Solvable in strong coupling for large N
- 'Maximally chaotic'. $\lambda_L = \frac{2\pi}{\beta}$
- Emergent conformal symmetry at low-energy

Integrable model for thermalization and quantum chaos.

* A. Kitaev → Solvable model for holography

Contrast with quadratic infinite range model (model for quantum dot)

$$H = \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} c_i^\dagger c_j$$



$$P(t_{ij}) \sim e^{-\frac{|t_{ij}|^2}{t^2}}$$

- Fermions occupying states of a $N \times N$ random matrix.
- No thermalization or chaos in the many-body sense.

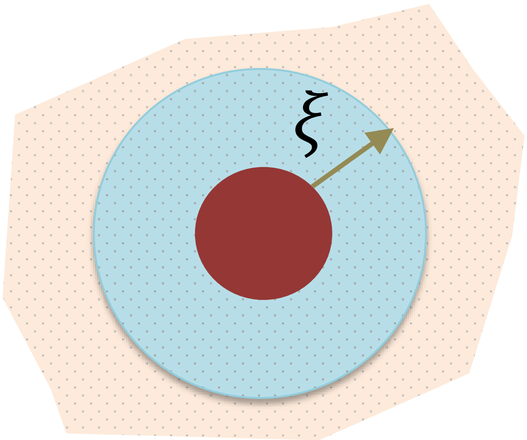
Add weak interaction →

- Fermi liquid state at infinite N. Quasi-particle lifetime $\sim 1/T^2$
- Many body localization at low T for finite N.

Altshuler, Gefen, Kamenev & Levitov, PRL (1997)

This talk: Solvable model with a quantum critical point separating the quadratic and the SYK fixed points.

Physical motivation and connection to many-body localization (MBL)



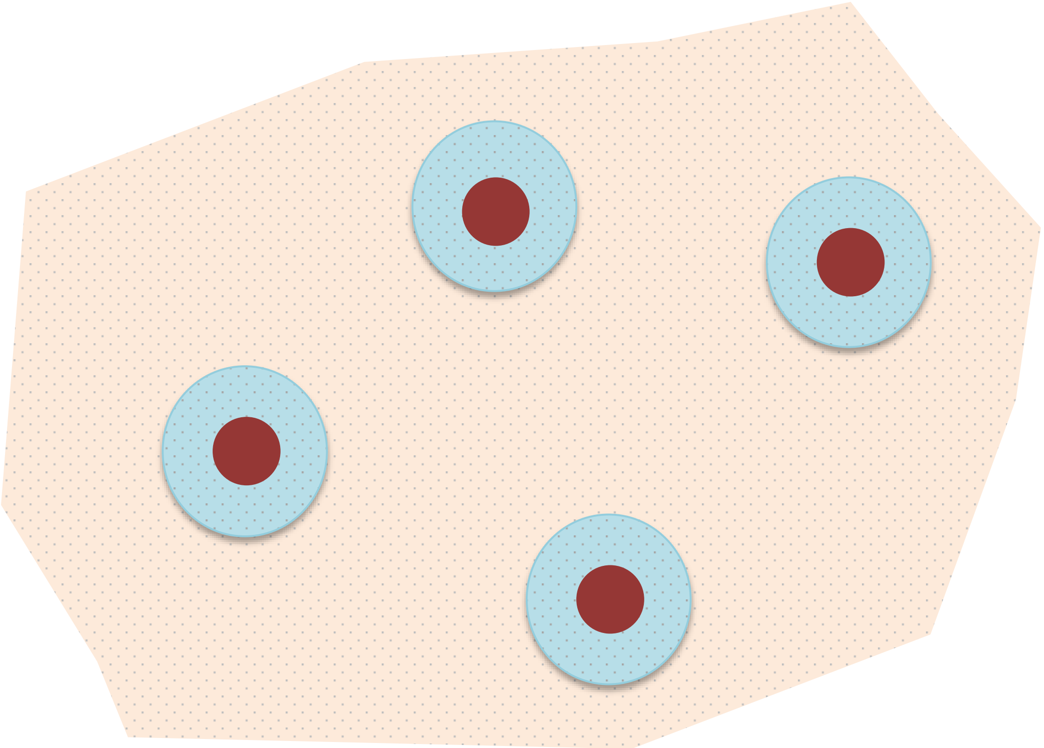
Ergodic 'bubble' within insulator

How does the 'bubble' affect the insulator and *vice versa*?

* A single 'bubble' can destabilize the insulator for $D \geq 2$!!!

W. D. Roeck, KU Leuven (Unpublished)

Stability of MBL to non-perturbative effects (isolated ergodic bubbles) in higher dimension.

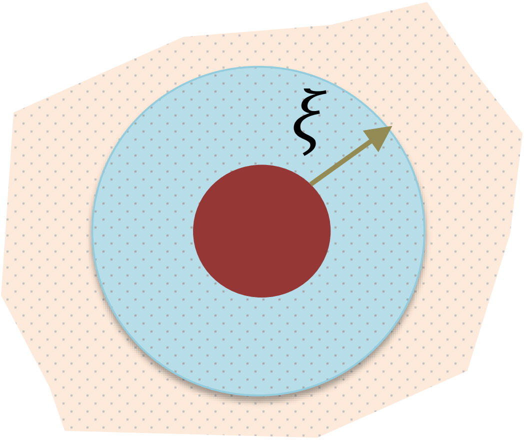


Outline

- Sachdev-Ye-Kitaev (SYK) model.

- Solvable model with quantum critical point separating quadratic and SYK fixed points.
  → Fermi liquid to non-Fermi liquid transition.

- Ergodic bubble in an Anderson insulator.
→ Potential solvable model for MBL transition.

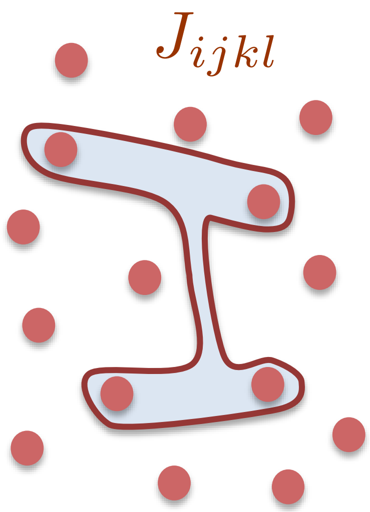


- Conclusions and open questions.

Review of SYK model

Sachdev & Ye PRL (1993), Kitaev, KITP (2015), Georges & Parcollet PRB (1999)

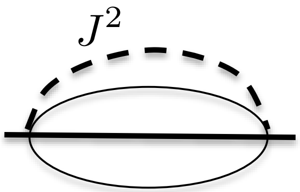
$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$



- Large-N (disorder averaged) saddle point

$$G^{-1}(\omega) = \cancel{\omega} - \Sigma(\omega)$$

$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$



- Conformal symmetry at low-energy ($\omega, T \ll J$)

$$\int d\tau\, G(\tau, \tau_1) \Sigma(\tau_1, \tau') = -\delta(\tau - \tau')$$

$$\tau = f(\sigma)$$

$$f'(\sigma) = \frac{\partial f}{\partial \sigma}$$

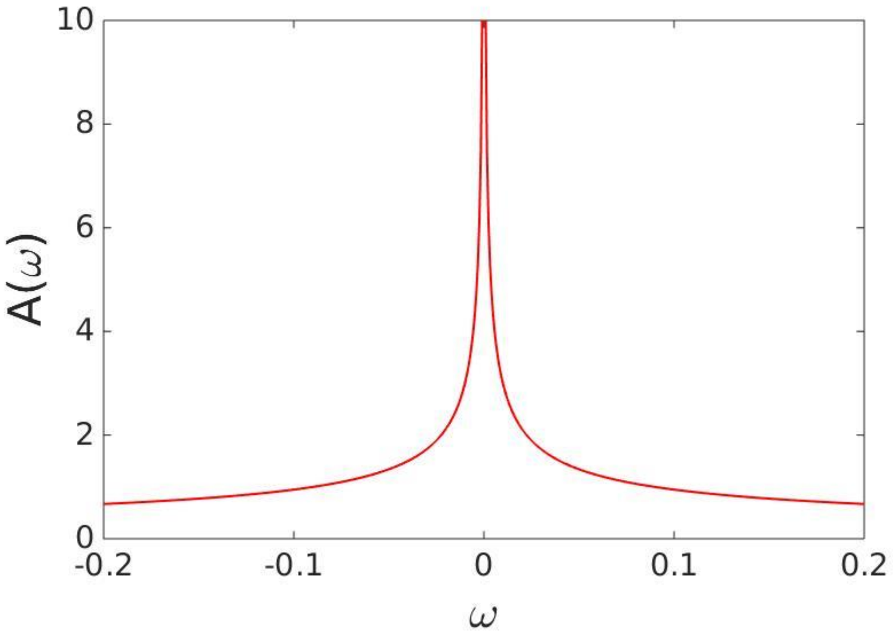
$$\tilde{G}(\sigma_1, \sigma_2) = [f'(\sigma_1) f'(\sigma_2)]^{1/4} G(f(\sigma_1), f(\sigma_2))$$

Sachdev, PRX (2015)

○ Diverging DOS for $\omega \to 0$ at $T = 0$

$$G(\omega) \sim (\mathrm{sgn}(\omega) - i)/\sqrt{J|\omega|}$$

$$\Sigma(\omega) \sim (-\mathrm{sgn}(\omega) + i)\sqrt{J|\omega|}$$



○ Non-Fermi liquid state – Strange metal.

○ Intriguing feature

Extensive ground-state entropy for $T \to 0$ (for infinite N)

→ Quantum chaos and thermalization in SYK model.

Quantum chaos in SYK model

$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

Kitaev, KITP (2015)
Polchinski & Rosenhaus (2016)
Maldacena & Stanford (2016)

→ Out-of-time-order correlation

$$\langle c_i^\dagger(t) c_i(0) c_j^\dagger(t) c_j(0) \rangle \sim 1 - \left(\frac{\beta J}{N}\right) e^{\lambda_L t}$$

$$\lambda_L = \frac{2\pi}{\beta}$$

→ Scrambling time

$$t_* \sim \frac{1}{\lambda_L} \ln N$$

Fastest possible! Maximally chaotic. Like a black hole.

Upper bound to quantum chaos

Maldacena, Shenker & Stanford (2015)

How to drive a phase transition out of this strange-metal chaotic phase?

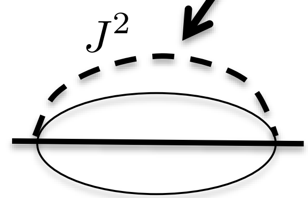
Naive attempt: add a quadratic term

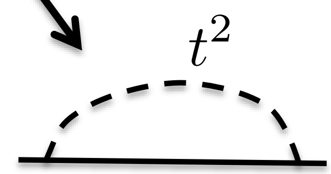
$$H_{SYK} = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{N}} \sum_{ij} t_{ij} c_i^\dagger c_j$$

Parcollet & Georges. (1999)

$$G^{-1}(\omega) = \omega - \Sigma_J(\omega) - t^2 G(\omega)$$

$$\Sigma_J(\tau) = -J^2 G^2(\tau) G(-\tau) =$$





The ansatz $G(\omega) \sim 1/\sqrt{\omega}$ is not self-consistent in the limit $\omega \to 0$

$$G^{-1}(\omega) \sim \omega - \sqrt{J\,\omega} - \frac{t^2}{\sqrt{\omega}}$$

The free fermion ansatz is self consistent: $G(\omega) \sim -i/t$

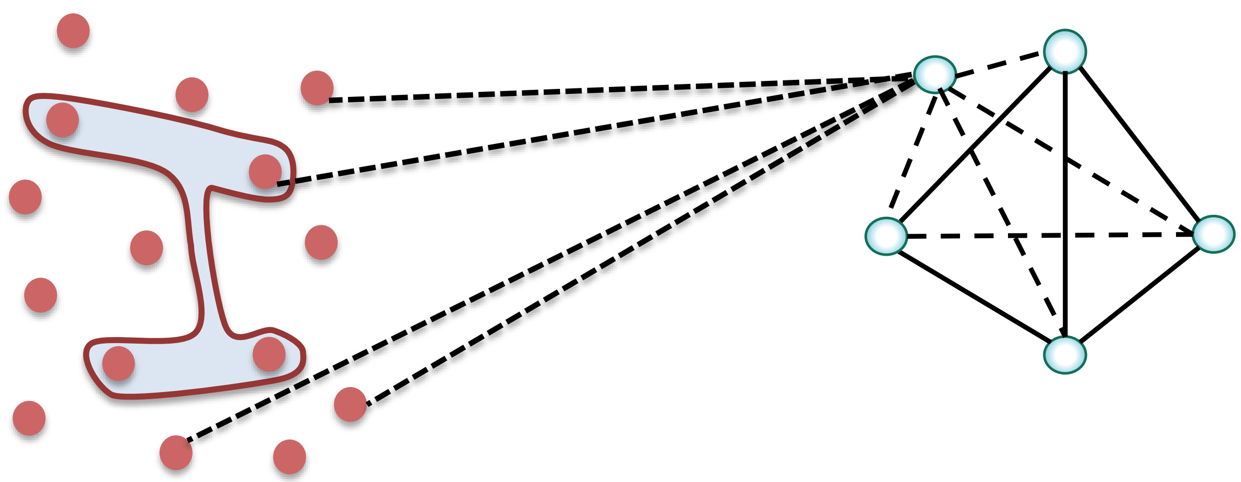
Quadratic interaction is relevant. Always a Fermi liquid. No transition!

Consider a model with two types of sites

<u>N sites:</u>
SYK coupling

<u>M sites:</u>
Random hopping

$$\overline{J_{ijkl}^2} = J^2 \qquad \overline{V_{i\alpha}^2} = V^2 \qquad \overline{t_{\alpha\beta}^2} = t^2$$



$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{M}} \sum_{\alpha\beta} t_{\alpha\beta} a_\alpha^\dagger a_\beta$$

$$+ \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger a_\alpha + h.c.)$$

Physical realization

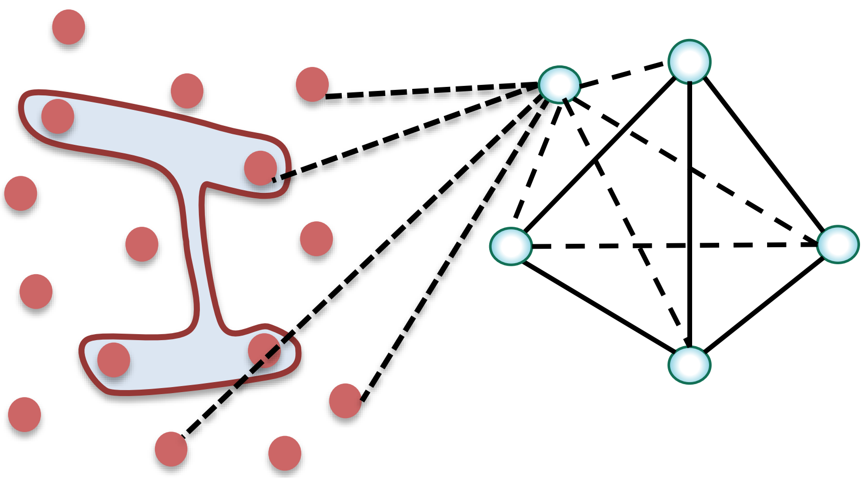
N sites:
SYK coupling

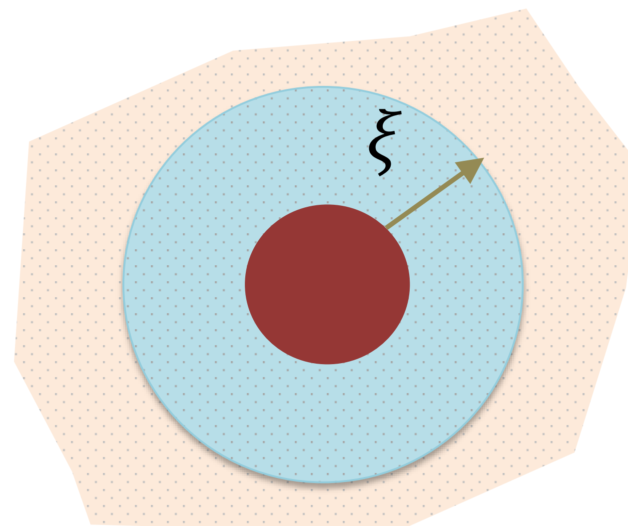
$\overline{J_{ijkl}^2} = J^2$

M sites:
Random hopping

$\overline{V_{i\alpha}^2} = V^2 \quad \overline{t_{\alpha\beta}^2} = t^2$

Ergodic bubble
in an Anderson insulator





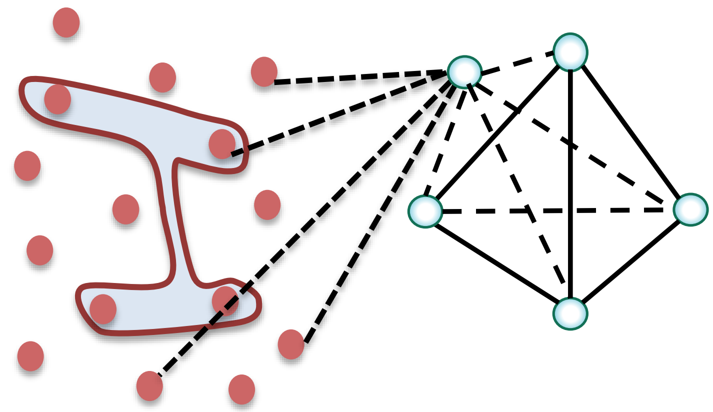
$M \sim \xi^D$

$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{M}} \sum_{\alpha\beta} t_{\alpha\beta} a_\alpha^\dagger a_\beta + \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger a_\alpha + h.c.)$$

Saddle point equations controlled at large M,N

$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \frac{1}{\sqrt{M}} \sum_{\alpha\beta} t_{\alpha\beta} a_\alpha^\dagger a_\beta + \frac{1}{(MN)^{1/4}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger a_\alpha + h.c.)$$

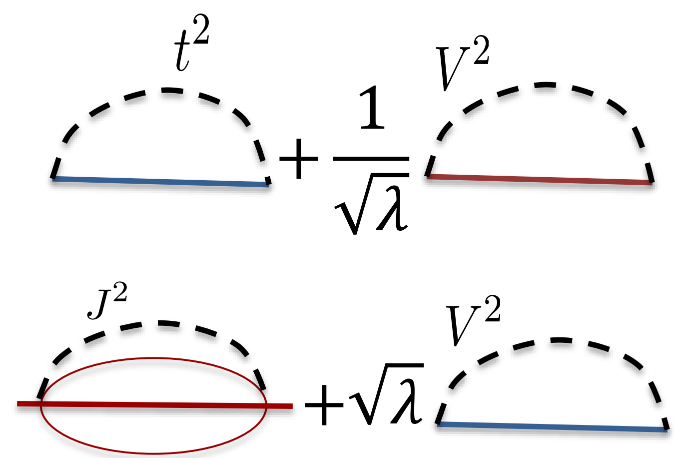
$$\lambda = M/N$$



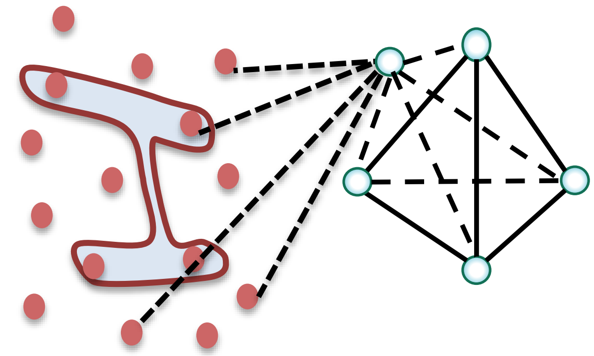
$$\mathcal{G}^{-1}(\omega) = \omega - t^2 \mathcal{G}(\omega) - \frac{V^2}{\sqrt{\lambda}} G(\omega)$$

$$G^{-1}(\omega) = \omega - \Sigma_J(\omega) - V^2 \sqrt{\lambda}\, \mathcal{G}(\omega)$$

$$\Sigma_J(\tau) = -J^2 G^2(\tau) G(-\tau)$$

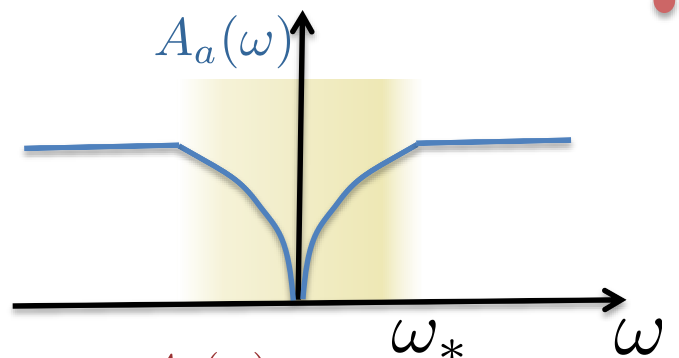


Strange metal phase

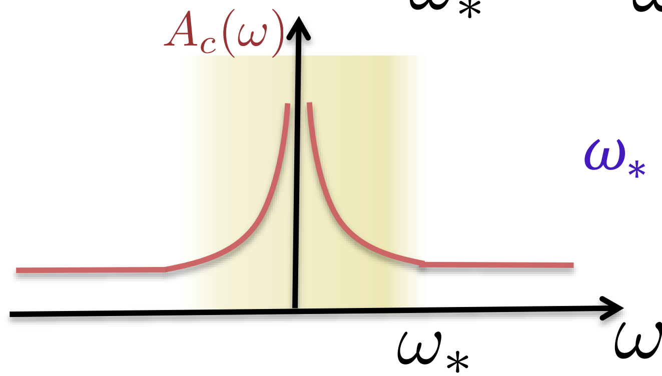


→ Solution for $\lambda = \frac{M}{N} < 1$

$$\mathcal{G}(\omega) \sim \frac{\lambda}{(1-\lambda)^{1/4}} \frac{\sqrt{J|\omega|}}{V^2}$$



$$G(\omega) \sim \frac{(1-\lambda)^{1/4}}{\sqrt{J|\omega|}}$$

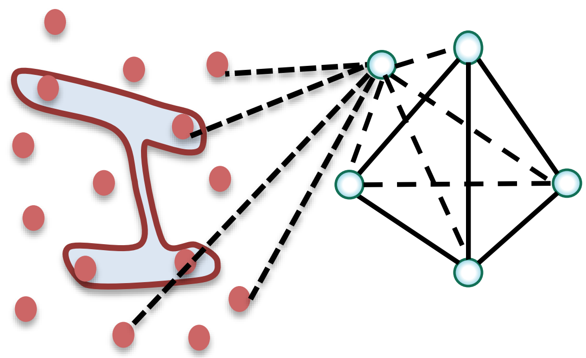


$$\omega_* = \frac{V^4(1-\lambda)^{1/2}}{t^2\lambda^2}$$

Weight and bandwidth of the singularity in G vanishes continuously as $\lambda \to \lambda_c = 1$

Strange metal phase, $\lambda = M/N < 1$

$$\mathcal{G}^{-1}(\omega) = \omega - t^2\mathcal{G}(\omega) - \frac{V^2}{\sqrt{\lambda}}\, G(\omega)$$



$$G^{-1}(\omega) = \omega - \Sigma_{\mathrm{J}}(\omega) - V^2\sqrt{\lambda}\,\mathcal{G}(\omega)$$

$$\Sigma_{\mathrm{J}}(\tau) = -J^2G^2(\tau)G(-\tau)$$

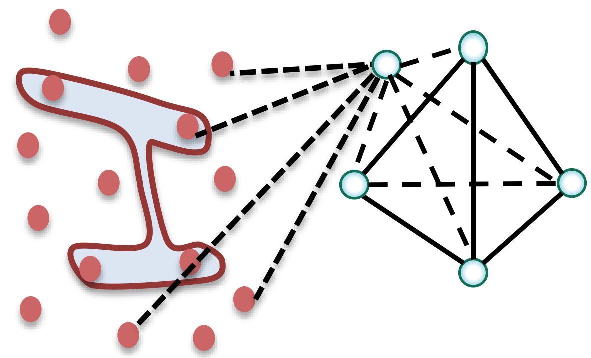
→ Emergent Conformal symmetry for $\omega < \omega_*$

$$\tau = f(\sigma)$$

$$\tilde{G}(\sigma_1,\sigma_2\,) = [f'(\sigma_1)f'(\sigma_2)]^{1/4}\, G(f(\sigma_1),f(\sigma_2))$$

$$\tilde{\mathcal{G}}(\sigma_1,\sigma_2\,) = [f'(\sigma_1)f'(\sigma_2)]^{3/4}\, \mathcal{G}\big(f(\sigma_1),f(\sigma_2)\big) \sim \widetilde{\Sigma}_{\mathrm{J}}(\sigma_1,\sigma_2)$$

"Fermi-liquid" phase



- Solution for $\lambda = M/N > 1$

$$\mathcal{G}(\omega) \sim -i\lambda\sqrt{\lambda-1}\,\frac{t}{V^2}$$

$$G(\omega) \sim -i\,\frac{1}{\sqrt{\lambda-1}}\,\frac{1}{t}$$

→ Constant DOS for $\omega < \omega_* \approx \left(\frac{V^2}{t\lambda}\right)\sqrt{\lambda-1}$

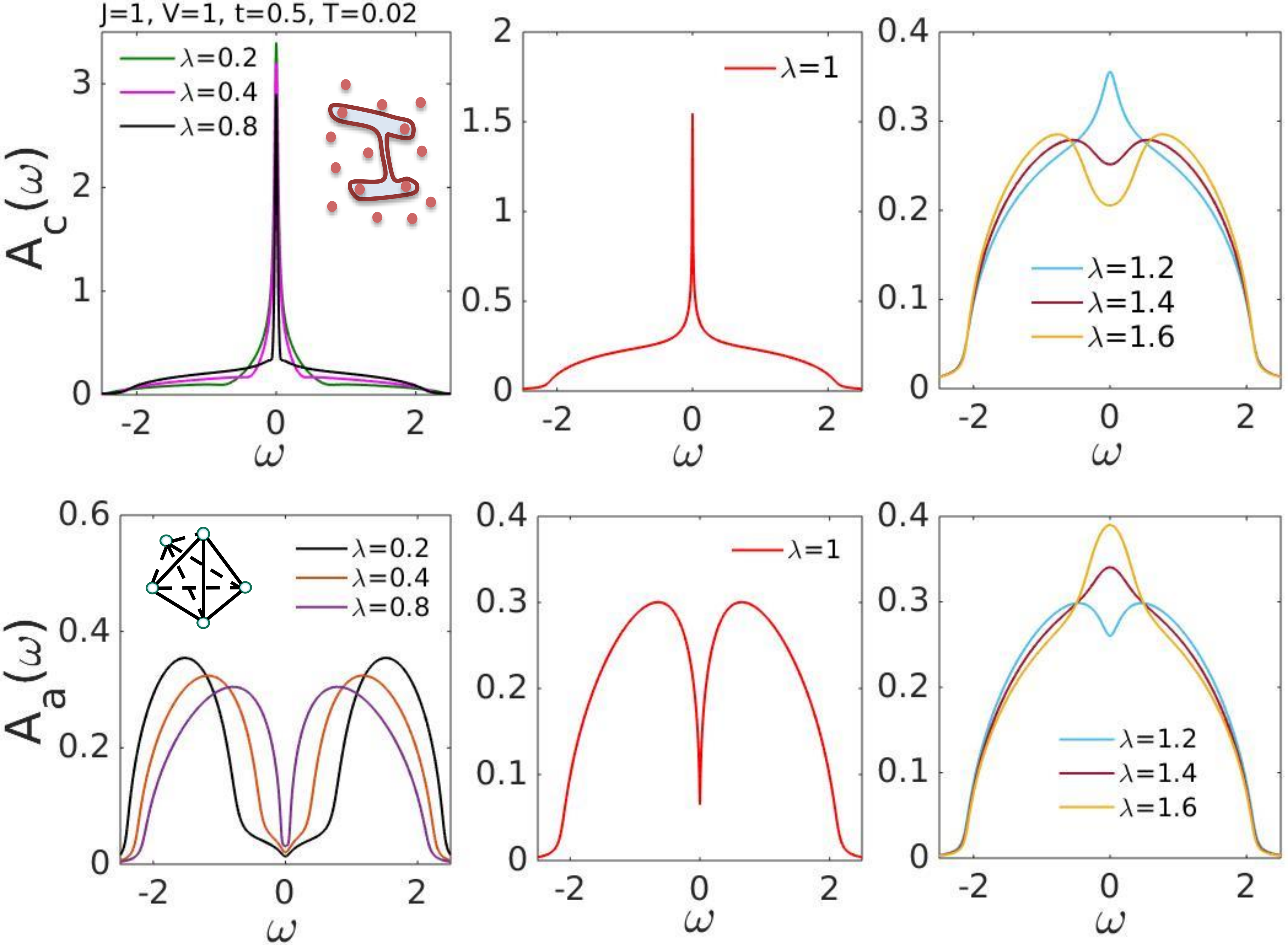
Self-energy, $\mathrm{Im}\Sigma(\omega) \sim -\left(\frac{J^2t^3}{V^6}\right)\frac{\lambda^3}{(\lambda-1)^{3/2}}\,\omega^2$

- Free fermion fixed point, emergent conformal symmetry

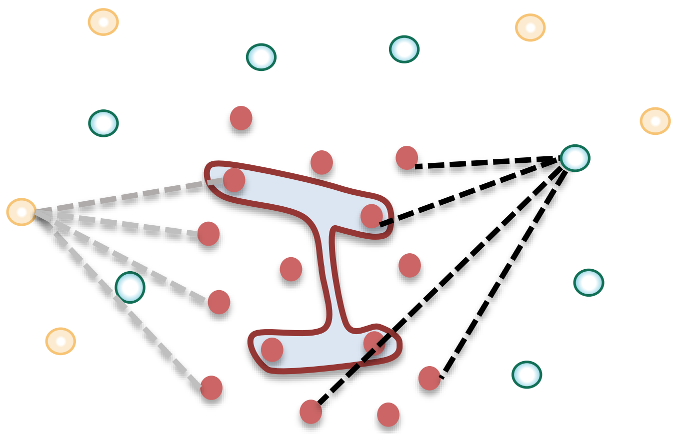
$$\tilde{G}(\sigma_1,\sigma_2) = [f'(\sigma_1)f'(\sigma_2)]^{1/2}\,G\big(f(\sigma_1),f(\sigma_2)\big) \sim \tilde{\mathcal{G}}(\sigma_1,\sigma_2)$$

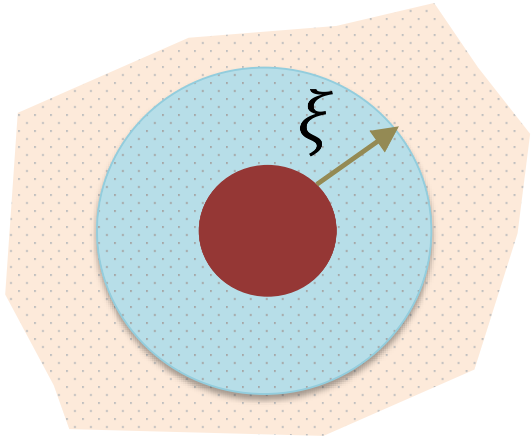
→Critical point at $\lambda = M/N = 1$ separates a strange metal SYK phase from a "trivial phase"

Numerical results



An ergodic bubble in Anderson insulator





$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \sum_\alpha \epsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{\sqrt{N}} \sum_{i\alpha} (V_{i\alpha} c_i^\dagger a_\alpha + h.c.)$$

ϵ_α , energies of localized states ("sites")

Distance r_α from SYK sites.

$$a_\alpha^\dagger = \sum_i \psi_\alpha(i) a_i^\dagger$$

$$\psi_\alpha(r) \sim e^{-r/\xi}$$

$$-W < \epsilon_\alpha < W$$

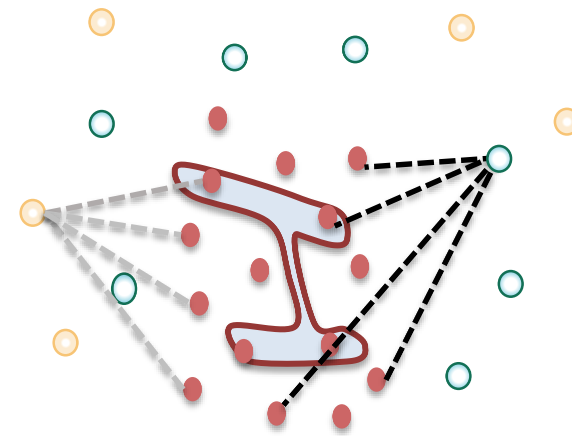
→Exponentially decaying coupling

$$\overline{V_{i\alpha}^2} = V_\alpha^2 = V^2 \exp(-r_\alpha/\xi)$$

Large-N Saddle-point equations

Disorder averaging over $J_{ijkl}, V_{i\alpha}$ →

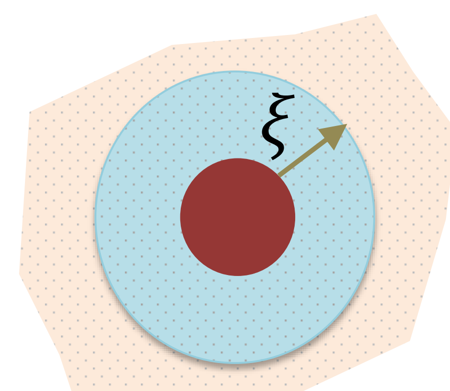
$$G^{-1}(\omega) = \omega - \Sigma_{\mathrm{J}}(\omega) - \frac{V^2}{N}\sum_{\alpha} e^{-\frac{r_\alpha}{\xi}} \mathcal{G}_\alpha(\omega)$$



$$\mathcal{G}_\alpha^{-1}(\omega) = \omega - \epsilon_\alpha - V^2\ e^{-\frac{r_\alpha}{\xi}} G(\omega)$$

- Saddle-point for fixed $\{\epsilon_\alpha\}$ realization.
- Can capture both strong and weak disorder.

How does the ‘bubble’ affect the insulator and *vice versa*?



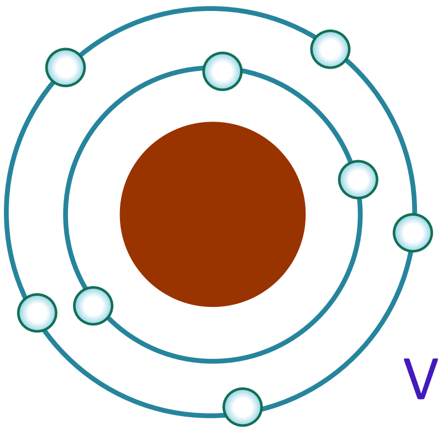
* A single ‘bubble’ can destabilize the insulator for $D \geq 2$!!!

W. D. Roeck, KU Leuven (Unpublished)

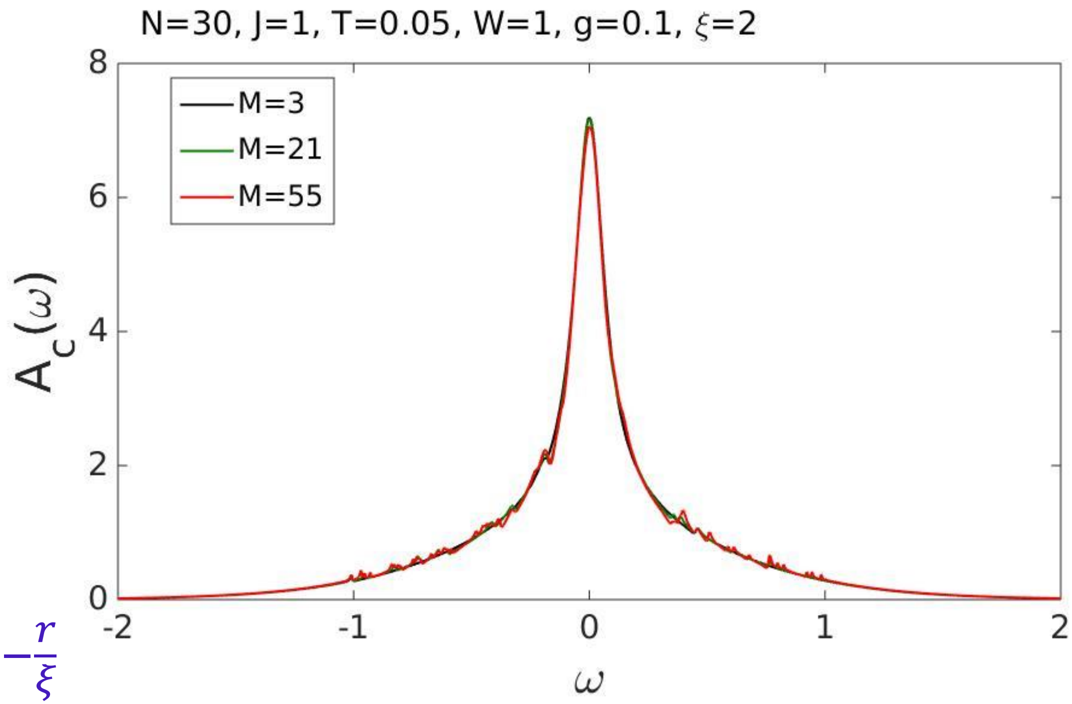
→ SYK site spectral function $A_c(\omega) \sim -\mathrm{Im}G(\omega)$

$V = 0.1, \xi = 2$

Add M localized sites to N SYK sites



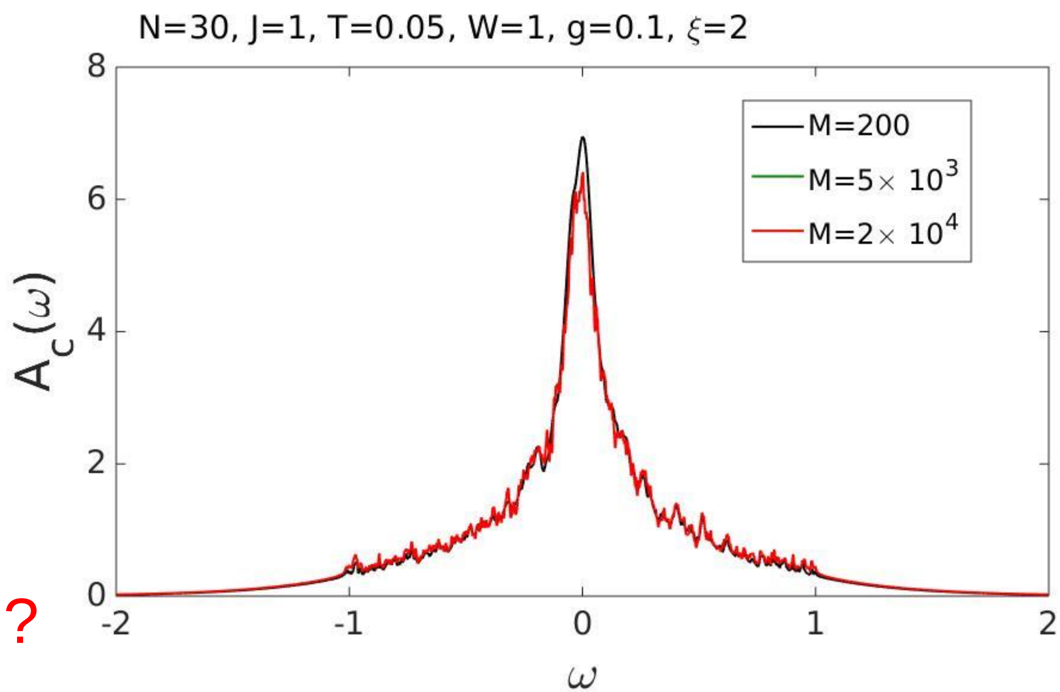
$V(r) = Ve^{-\frac{r}{\xi}}$



Ergodic bubble more or less remains intact.

$M_\xi \simeq \pi\xi^2 < N$

$\rightarrow \lambda < 1$



What happens to the insulator? Delocalized?

Coupling

V= 0.1

$\xi = 10$

Bath is destroyed.

$M_\xi \simeq \pi\xi^2 > N$

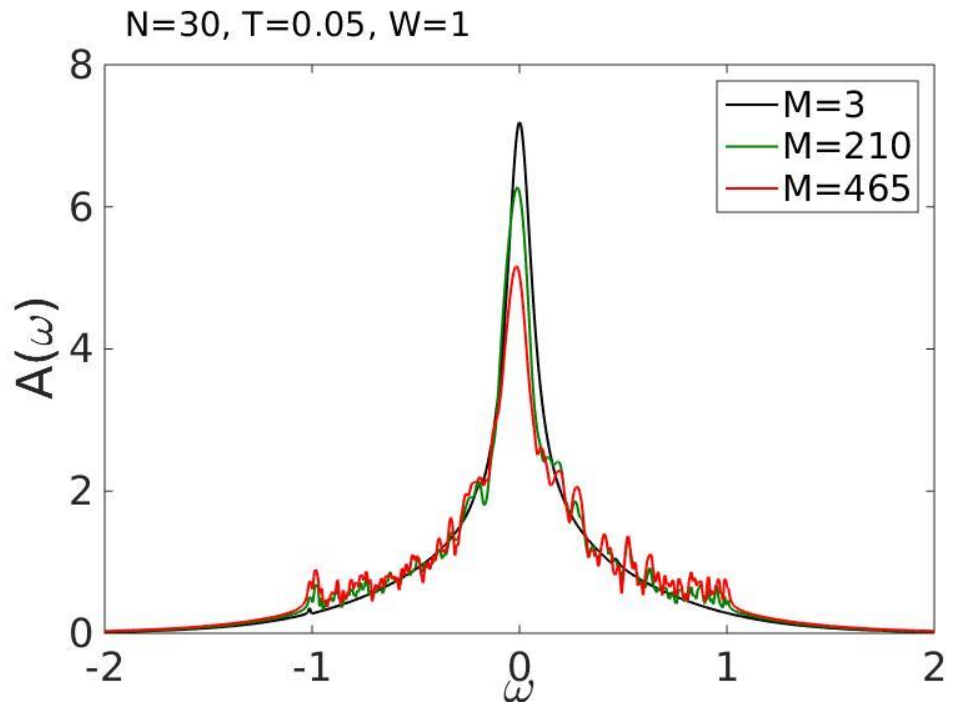
$\rightarrow \lambda > 1$

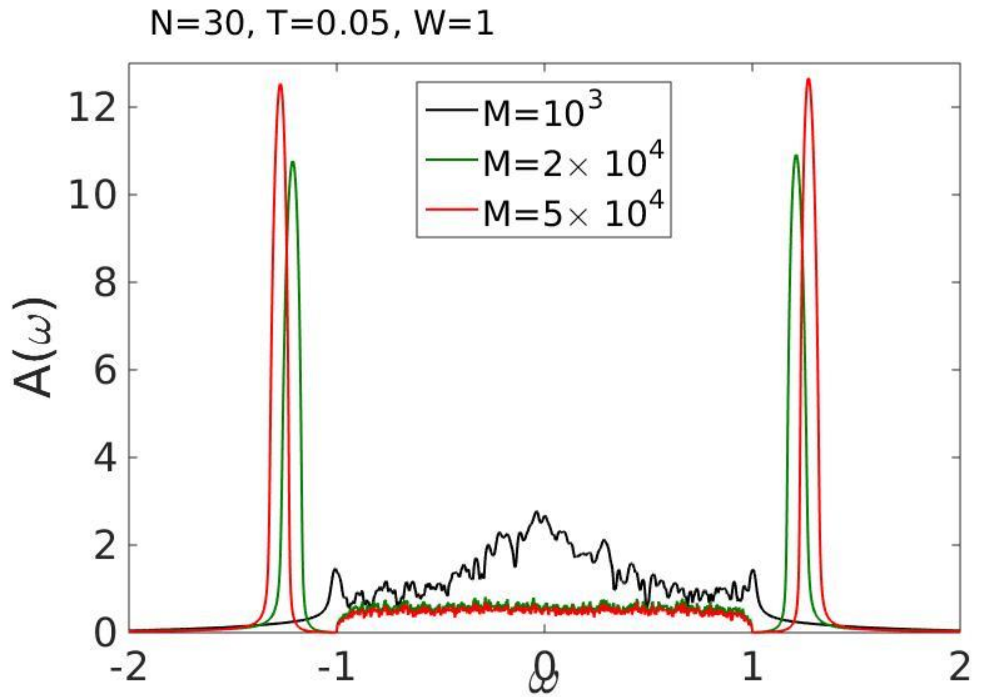
→ Quadratic fixed point.

Bath is localized??

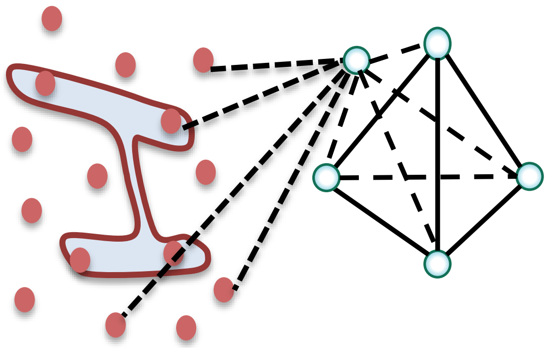
MBL proximity effect

Nandkishore, PRB (2015)





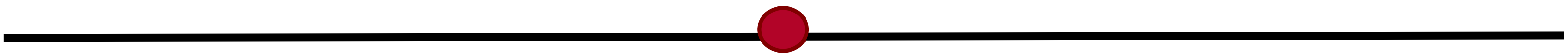
Conclusions



- Solvable model for “Fermi liquid” to strange metal transition.

T=0 critical point

$$\lambda = M/N = \lambda_c = 1$$

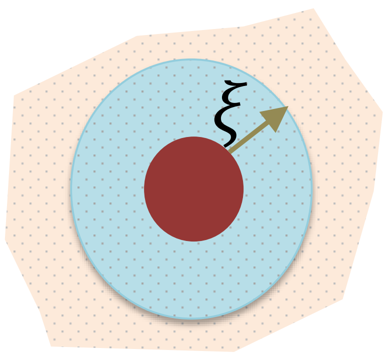


Strange metal

“Trivial” phase

- Non-perturbative effect (ergodic bubble) on MBL.

An ‘ergodic bubble’ might get proximity localized by MBL environment.



→ Not an obstruction to MBL in higher dimension

Many questions remain!

- How does the $T = 0$ (for infinite N) entropy evolve across the transition ?

- Characterize fluctuations around the saddle point.
How does the scrambling time evolve across the transition?

- Transition into a Fermi-liquid ? $1/\tau \sim 1/T^2$
Or a non-ergodic state ? $1/\tau = 0$

- Solvable model for a many-body localization transition?
(Need finite N?)

- Holographic description? Phase transition involving emergence of a black hole in AdS_2 ?

Quantum chaos

- Classical chaos

$$\frac{\partial x(t)}{\partial x(0)} \sim e^{\lambda_L t}$$

λ_L, Lyapunov exponent

- Quantum chaos

→ 'Semiclassical billiards'

Larkin & Ovchinikov (1969)

- Chaos correlator $C(t) = -\langle [x(t), p(0)]^2 \rangle$

$$[x(t), p(0)] \quad \Rightarrow \quad i\hbar\{x(t), p\} = i\hbar \frac{\partial x(t)}{\partial x(0)}$$

←Poisson bracket

$$C(t) \sim \hbar^2\, e^{2\lambda_L t}$$

'Srambling time' $t_* \sim \frac{1}{\lambda_L} \ln\left(\frac{1}{\hbar}\right)$ $C(t_*) \sim 1$

Hubbard model bath

Large D Hubbard model with N sites as bath.

$$H_b = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H_l = \sum_{\alpha} \epsilon_\alpha a_\alpha^{\dagger} a_\alpha \qquad\qquad \epsilon_\alpha \in [-W, W]$$

$$H_{bl} = \frac{1}{\sqrt{N}} \sum_{i\alpha\sigma} (V_{i\alpha} c_{i\sigma}^{\dagger} a_\alpha + h.c.)$$

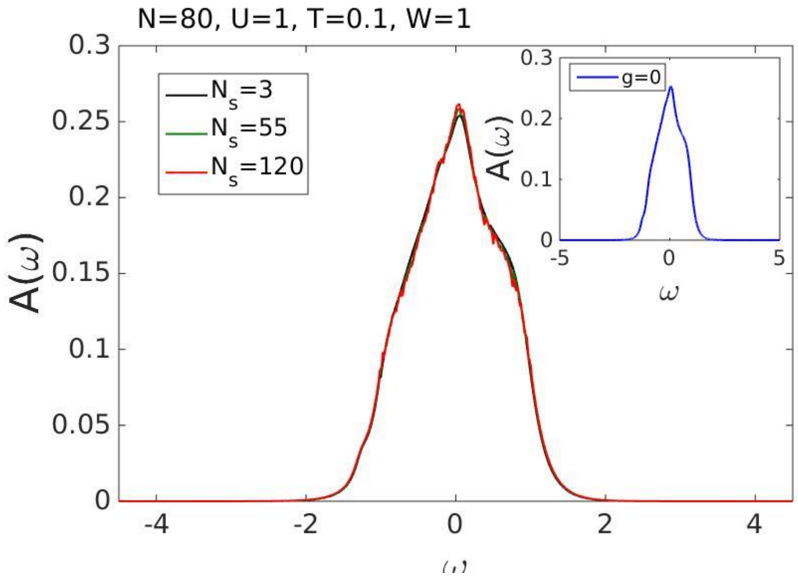
Solved via single-site dynamical mean field theory (DMFT) using iterative perturbation theory (IPT) for large D, N

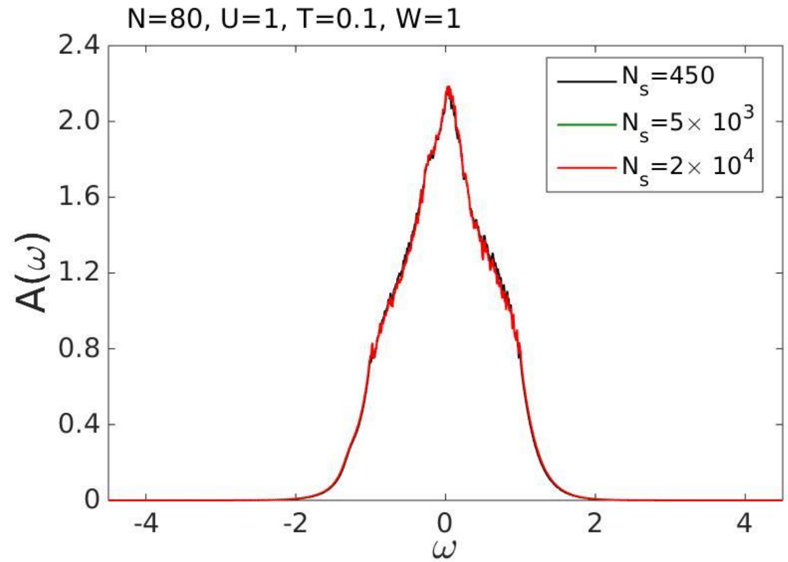
Bath: Large-dimensional
Hubbard model with N sites

Dynamical mean-field theory (DMFT)
Large D, N

$g = 0.1$

$N = 80$





$g = 1$