One-Dimensional Spin Glass with Oscillating Long-Range Interaction

Z. Phys. B - Condensed Matter 51, 237-249 (1983) M.V. Feigel'man and L.B. Ioffe

and Chapter 3.2 in

SPIN GLASSES AND RELATED PROBLEMS

V.S. DOTSENKO, M.V. FEIGEL'MAN and L.B. IOFFE

Sov. Sci. Rev. A. Phys. Vol. 15, 1990, pp. 1–250
Photocopying permitted by license only

$$V(\mathbf{p}) = \frac{V_0}{\left(\frac{p^2 - Q^2}{2Q\kappa}\right)^2 + 1}; \qquad \kappa \ll Q.$$

$$V_1(x) = \kappa V_0 e^{-\kappa|x|} \cos Qx$$

$$V_3(\mathbf{x}) = \frac{Q}{2\pi} \kappa V_0 e^{-\kappa x} \frac{\sin Qx}{x}$$

$$\kappa \ll c \ll Q.$$

chain of Ising spins $\sigma_i = \pm 1$ with the oscillating interaction:

$$H = \frac{1}{2} \sum_{ij} \sigma_i V_{ij} \sigma_j - \sum_i \sigma_i h_i$$

$$V_{ij} = \kappa e^{-\kappa|x_i - x_j|} \cos Q(x_i - x_j). \qquad (1)$$

$$H=\frac{1}{2}\sum_{ij}\sigma_i V_{ij}\sigma_j-\sum_i \sigma_i h_i$$
$$V_{ij}=\kappa\, e^{-\kappa|x_i-x_j|}\cos Q(x_i-x_j).$$

$$Z=\sum_{\{\sigma_i=\pm 1\}}\int \mathscr{D}\psi\, \mathscr{D}\psi^* \exp\left\{-\frac{1}{T}\int\left[\frac{1}{\kappa^2}\left|\left(i\frac{d}{dx}+Q\right)\psi\right|^2\right.\right.$$
$$\left.\left.+|\psi|^2\right]dx+\frac{1}{T}\sum_i \sigma_i(\psi(x_i)+\psi^*(x_i)+h)\right\}$$
$$=\int \mathscr{D}\psi\, \mathscr{D}\psi^* \exp\left\{-\frac{1}{T}H_{\mathrm{eff}}[\psi]\right\} \tag{2}$$

where

$$H_{\mathrm{eff}}[\psi]=\int\left[\kappa^{-2}\left|\left(i\frac{d}{dx}+Q\right)\psi\right|^2+|\psi|^2\right.$$
$$\left.-T\sum_i \delta(x-x_j)\ln \mathrm{ch}\frac{\psi+\psi^*+h}{T}\right]dx. \tag{3}$$

Expand in powers of ψ

$$H_{\text{eff}}[\psi]=\int\left\{\kappa^{-2}\left|\left(i\frac{d}{dx}+Q\right)\psi\right|^2\right.$$

$$\gamma=\kappa/c\ll 1$$

$$+\left(1-\frac{1}{T}\sum_j\delta(x-x_j)\right)|\psi|^2$$

$$\left.+\sum_j\delta(x-x_j)\left[-\frac{1}{2T}(\psi^2+\psi^{*2})+\frac{1}{2T^3}(\psi+\psi^*)^4\right]\right\}dx.$$

At $T<T_c=c$ the averaged value of the coefficient at $|\psi|^2$, which is equal to $\tau=1-c/T$, becomes negative, which implies the instability of the state with $\langle\psi\rangle=0$.

$t \ll \quad t_m \sim \exp(\gamma^{-1}|\tau|^{3/2})$. Phase slips are absent

At $|\tau|\gg\gamma^{2/3}$ it is convenient to pass over to the new variables: the amplitude and phase of the field ψ:

$$\psi=\rho\exp(i\varphi+iQx). \tag{5}$$

The free energy $H_{\text{eff}}[\psi]$ from (3) can be minimized over ρ, which leads to the phase-dependent energy

$$H[\varphi]=\int\left\{\frac{\rho^2}{\kappa^2}\left(\frac{d\varphi}{dx}\right)^2-T\sum_i \delta(x-x_i)\right.$$

$$\cdot\left[\ln\operatorname{ch}\left(\frac{2\rho}{T}\cos(Qx+\varphi)\right)\right.$$

$$\left.\left.-\left\langle\ln\operatorname{ch}\left(\frac{2\rho}{T}\cos\alpha\right)\right\rangle_\alpha\right]\right\}dx \tag{6}$$

where $\langle F(\alpha)\rangle_\alpha=\int\limits_0^{2\pi}F(\alpha)\,d\alpha/2\pi$, and ρ is approximately determined by the equation

$$\rho=c\left\langle\cos\alpha\cdot\operatorname{th}\left(\frac{2\rho}{T}\cos_\alpha\right)\right\rangle. \tag{7}$$

Equation (7) is obtained if $H[\varphi]$ is neglected (the validity of it is discussed below) in comparison with

$$F_{\text{MFA}}=H_{\text{eff}}[\psi]-H[\varphi]=\rho^2-cT\left\langle\ln\operatorname{ch}\frac{2\rho\cos_\alpha}{T}\right\rangle_\alpha. \tag{8}$$

II. The Vicinity of the Transition Point $\gamma^{2/3} \ll |\tau| \ll 1$

$$F_{\mathrm{MFA}} = \tau \rho^2 + \frac{c h^2}{2 T^3} \rho^4 - \frac{c h^2}{2T} + \frac{c \rho^2}{T^3} h^2. \tag{9}$$

For the specific heat $C(T)$ and magnetic susceptibility $\chi(T)$ we obtain:

$$C(T) = c\,\theta(-\tau) \qquad (|\tau| \gg \gamma^{2/3}) \tag{10}$$

$$\chi(T) = c/T \qquad (\tau \gg \gamma^{2/3}) \tag{11a}$$

$$\chi(T) = 1 - |\tau| \qquad (-\tau \gg \gamma^{2/3}). \tag{11b}$$

Formula (11b) is valid at observation times that are not too large (see below). Formula (10) and (11) are similar to those in the Mattis model [12] of spin glasses without frustrations, which is quite natural, since the averages $\langle \sigma_i \rangle$ are also expressed through slow variables in our model. The difference is that we have two variables, ρ and φ:

$$\langle \sigma_i \rangle = -\frac{\partial F}{\partial h_i} = \mathrm{th}\left[\frac{2\rho}{T} \cos(Q x_i + \varphi_i)\right]$$

Phase-dependent Hamiltonian with "pinning"

$$H[\varphi(x), Q] = |\tau| \int dx \left[\gamma^{-2}\left(\frac{d\varphi}{dx}\right)^2 - c\sum_j \delta(x-x_j) \cdot \cos 2(Qx+\varphi)\right].$$

$$\langle \cos(\varphi(x) - \varphi(0))\rangle \sim \exp(-xL_\varphi^{-1}),$$

$$L_\varphi = A c^{-1}\gamma^{-4/3} \gg c^{-1} \qquad (A \sim 1)$$

Hamiltonian (13) coincides with the one studied in [9–11] in connection with the problem of charge-density wave pinning by impurities.

9. Fukuyama, H., Lee, P.A.: Phys. Rev. **B17**, 535 (1977)
10. Feigel'man, M.V.: Zh. Eksp. Theor. Fiz. **79**, 1095 (1980)
11. Vinokur, V.M., Mineev, M.B., Feigelman, M.V.: Zh. Eksp. Theor. Fiz. **81**, 2142 (1981)

Stochastic Transfer Matrix method [10]

$$\Psi_N(\varphi_N) = \text{const} \cdot \exp\left[-\frac{\varepsilon_N(\varphi_N)}{T}\right].$$

$$\varepsilon_{N+1}(\varphi) - \varepsilon_N(\varphi) = -\frac{1}{2}\frac{l_{N+1}}{v_F}\left(\frac{\partial \varepsilon_N}{\partial \varphi}\right)^2 + \frac{T}{2}\frac{l_{N+1}}{v_F}\frac{\partial^2 \varepsilon_N}{\partial \varphi^2} - V\cos(\varphi + Qx_{N+1}).$$

a Hamilton-Jacobi equation (with a discrete imaginary "time" N) corresponding to the "equation of motion"

$$v_F\varphi'' + V\delta(x-x_k)\sin(\varphi+Qx) = 0.$$

(

$$\varepsilon_N(\varphi) = -\beta_N \cos(\varphi - \gamma_N)$$

$$\beta_0 = (cv_F V^2)^{1/3}.$$

Moderately Low Temperature Range $\kappa \ll T \ll T_c = c$

$$\varepsilon_{n+1}(\varphi) - \varepsilon_n(\varphi) = -\frac{\kappa^2}{4\rho_0^2} l_{n+1} \left(\frac{\partial \varepsilon_{n+1}}{\partial \varphi}\right)^2 + \frac{T}{4}\frac{l_{n+1}}{\rho_0^2}\kappa^2 \frac{\partial^2 \varepsilon_n}{\partial \varphi^2} + V(\varphi + Q x_n)$$

At $T \ll c$ the phase-pinning potential

$$V(\varphi + Q x) = -T \ln \operatorname{ch}\left[\frac{2\rho}{T}\cos(\varphi + Q x)\right]$$

Cusp singularity at low T

replace l_n by $\langle l_n \rangle = c^{-1}$, (at $T \gg \kappa$).

$$\varepsilon_{n+1}(\varphi) - \varepsilon_n(\varphi) = -\frac{\kappa}{4\rho_0^2 c}\left(\frac{\partial \varepsilon_n}{\partial \varphi}\right)^2 + V(\varphi + \alpha_n)$$

Generating functional

$$P(j) = \int \mathscr{D}\mu\, \mathscr{D}\varepsilon \exp\left\{ i \sum_n \int d\varphi \left[\mu_n \left(\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c}\left(\frac{\partial \varepsilon_n}{\partial \varphi}\right)^2 - V(\varphi + \alpha_n) \right) \right] \right\}$$

Let us pass from integrating over μ to integrating over M, so that $M''_{\varphi\varphi} = \mu$.

$$P(j) = \int \mathscr{D}M\, \mathscr{D}\varepsilon \exp\left(S - \sum_n \int d\varphi\, j_n \varepsilon_n\right)$$

$$S = i \sum_n \int d\varphi \left\{ M_n \frac{\partial^2}{\partial \varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c}\left(\frac{\partial \varepsilon_n}{\partial \varphi}\right)^2 - V(\varphi + \alpha_n) \right] \right\}.$$

Average the functional over α_n

$$S = \sum_n \left\{ \int i M_n \frac{\partial^2}{\partial\varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial\varphi} \right)^2 \right] d\varphi \right.$$

$$\left. + \ln \left[\int_0^\pi d\varphi_n \exp(2i\rho_0 M_n(\varphi_n)) \right] \right\}$$

$$= \sum_n \left[\int i M_n \frac{\partial^2}{\partial\varphi^2} \left[\varepsilon_{n+1} - \varepsilon_n + \frac{\kappa^2}{4\rho_0^2 c} \left(\frac{\partial \varepsilon_n}{\partial\varphi} \right)^2 \right] d\varphi \right.$$

$$\left. - 2\rho_0^2 \int M_n^2(\varphi)\, d\varphi + \underline{\sum_{K=3}^{\infty} a_K \rho_0^K \int M_n^K(\varphi)\, d\varphi} \right\}$$

small

$$S_0 = -\int \left\{ \frac{\partial^2}{\partial\varphi^2} \left[\frac{\partial\varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial\varepsilon}{\partial\varphi} \right)^2 \right] \right\}^2 \frac{1}{8\rho_0^2 c} d\varphi\, dx$$

$$T/c \ll \Phi \ll 1;$$

$$E X^{-1} \sim \kappa^2 \rho_0^{-2} E^2 \Phi^{-2}$$

$$E^2 X^{-1} \Phi^{-3} \rho_0^{-2} c^{-1} \sim 1$$

$\longrightarrow$

$$X \sim \gamma^{-4/3} \Phi^{1/3} c^{-1}$$

$$E \sim \gamma^{-2/3} \Phi^{5/3} c$$

Estimate for the neglected terms

$$\Delta S = \sum_{K=3}^{\infty} \tilde{a}_K \frac{c}{(\rho_0 c)^K} \int d\varphi\, dx \left\{ \frac{\partial^2}{\partial \varphi^2} \left[\frac{\partial \varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial \varepsilon}{\partial \varphi} \right)^2 \right] \right\}^K$$

$$\Delta S / S_0 \sim \left(\frac{\kappa}{T} \right)^{2/3} \ll 1$$

Correlation length

The total scale of the $\varepsilon(\varphi)$ variation is determined by fluctuations with $\Phi \sim 1$. Its value $E \sim c\gamma^{-2/3}$ is the same as at $T \sim T_c$.

Let's find the phase correlation length L_φ. For this purpose we estimate the phase variation by each step ($|\Delta \bar{\varphi}| = |\bar{\varphi}_n - \bar{\varphi}_{n-1}|$ where $\bar{\varphi}_n$ is the thermal average of φ_n. Then L_φ will be determined by the condition:

$$L_\varphi c \langle (\Delta \bar{\varphi})^2 \rangle = 1$$

Since thermal fluctuations are small, $\bar{\varphi}_n$ is determined by the position of the $\varepsilon(\varphi)$ minimum (perhaps, a local one): $\varepsilon_n'(\bar{\varphi}) = 0$. Using the recurrent equation (23) we get

$$\langle (\Delta \bar{\varphi})^2 \rangle \sim \beta^{-2} = \left\langle \left(\frac{\partial^2 \varepsilon}{\partial \varphi^2} \right)^2 \right\rangle^{-1}$$

$$\beta \sim c\gamma^{-2/3} (T/c)^{-1/3}$$

$$L_\varphi \sim c\gamma^{-4/3} (T/c)^{-2/3}$$

$$X_{\min} \sim c^{-1} \gamma^{-4/3} (T/c)^{1/3}, \quad (\Phi \sim T/c)$$

Observables

First of all, we find the free energy of the system in a certain metastable state. As was shown at the end of the previous section, the barrier height at $T \gg \kappa$ is much larger than T; therefore, when calculating the characteristics of the system corresponding to short time scales (quasiequilibrium I), the phase φ_i can be considered constant at a given point i and as satisfying the condition $\varepsilon_i'(\varphi_i)=0$. Thermal fluctuations of $\rho(x)$ can always be neglected; therefore, the free energy F_I coincides with the Hamiltonian H_{eff} for the given configuration of $\{\varphi_i\}$.

H_{eff} is given by Formula (3). For future purposes it is convenient to rewrite it as:

$$F_I = H_{\text{eff}} = \int \{\kappa^{-2}[(\nabla\rho)^2 + \rho^2(\nabla\varphi)^2] + \rho^2\}\, dx$$
$$-T\sum_j \ln\text{ch}\left(\frac{h + 2\rho\,\text{Cos}(\varphi + \alpha_j)}{T}\right) \qquad (33)$$

$$\chi_{\text{eq}} = \frac{c}{T}$$

$$\langle\sigma_i\rangle = -\frac{\partial F}{\partial h_i} = \text{th}\left[\frac{2\rho}{T}\cos(Qx_i + \varphi_i)\right]$$

Distribution function of phases at local minimum in nearly uniform

$$\rho(\varphi_i + \alpha_i) = \frac{1}{2\pi} + O\left(\left(\frac{\kappa}{T}\right)^{2/3}\right)$$

The behavior of the magnetic susceptibility $\chi = -\frac{\partial^2 F}{\partial h^2}$ is different at different times of observation.

Let's first consider χ in the region where $t_0 \ll t \ll t_1$ (for quasiequilibrium I, see the end of the previous section). Inserting Formula (33) for F we obtain:

$$\chi_I = \frac{c}{T}\left\langle \text{ch}^{-2}\left(\frac{2\rho_0\,\text{Cos}(\varphi_i + \alpha_i)}{T}\right)\right\rangle = \tfrac{1}{2} \qquad (42)$$

Slowly time-dependent behavior at $t_0 \ll t \ll t_1$

$$t_0 \sim \exp(E_0/T) \sim \exp[(T/\gamma)^{2/3}]$$

$$t_1 \sim \exp(\gamma^{-2/3}/T)$$

One of the major characteristics: dissipative response

$$\langle \sigma_i \rangle \approx \text{sign}\,[(\cos(\phi_i + Qx_i)]$$

$$\text{Im}\,\chi(\omega) \sim \frac{1}{T}\int dE\, d\Delta\, R(E,\Delta)\,\frac{\omega\tau(E)}{1+[\omega\tau(E)]^2}\,N(E)\,\text{sech}^2\frac{\Delta}{2T}.$$

$\tau(E) \sim e^{E/T}$ E is the free-energy barrier between two states

Δ is the energy difference between these states

$R(E,\Delta)$ is the joint probability density

$N(E) \sim \Phi(E)X(E)$ is the number of spins that flip in the course of the transition between two metastable states

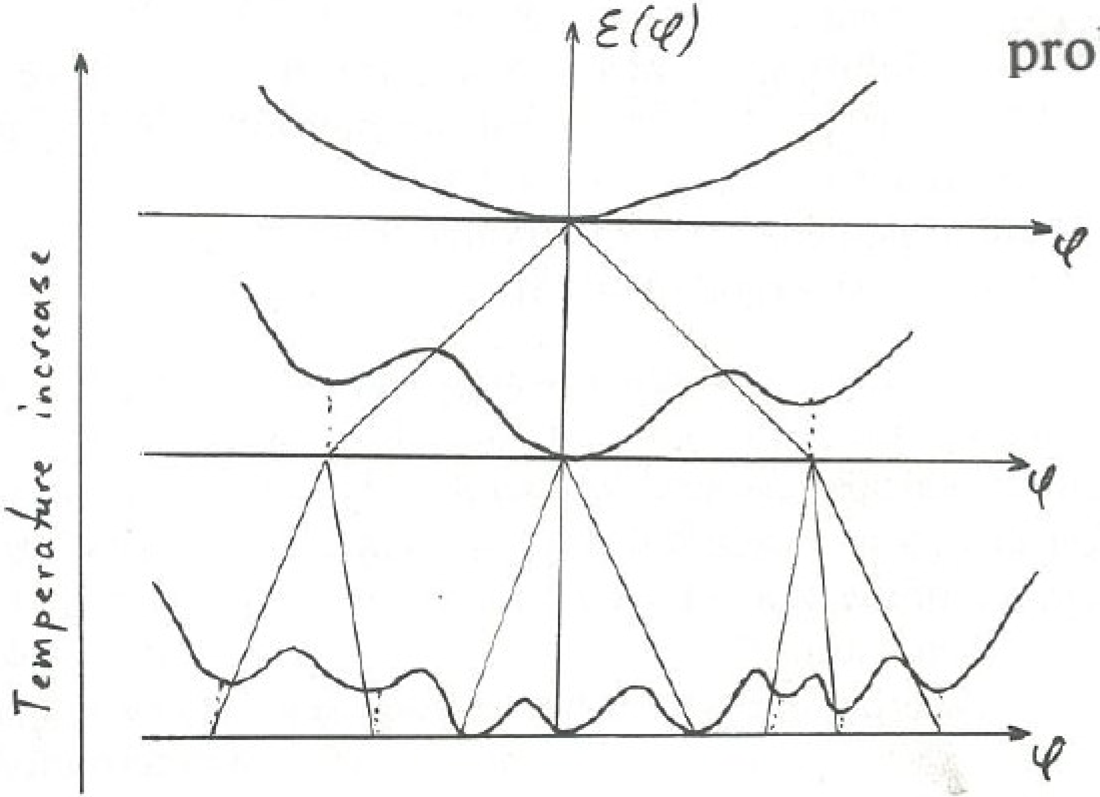

Figure 11 Hierarchical structure of metastable states.

probability of finding a zero of $f(\phi^{\mathrm{I}}+\Phi)$ within an interval of width Φ_0

$$P(\Phi) \sim \frac{\Phi_0 \beta}{F(\Phi)} = \left(\frac{\Phi_0}{\Phi}\right)^{2/3}$$

the total number of zeros

$$M_0 \sim \int_{\Phi_0}^{\pi} \frac{\mathrm{d}\Phi}{\Phi_0} P(\Phi) \sim \left(\frac{\pi}{\Phi_0}\right)^{1/3} \sim T^{-1/3}$$

The minima of $\epsilon(\phi)$ constitute a fractal set with fractal dimension $D_f = \frac{1}{3}$

consider the behaviour of the function $f(\phi) = \partial\epsilon/\partial\phi$ at $\phi = \phi^{\mathrm{I}} + \Phi$, where ϕ^{I} is any zero of $f(\phi)$. The characteristic scale of $f(\phi^{\mathrm{I}}+\Phi)$ is $F(\Phi)$

$$F(\Phi) \sim E(\Phi)/\Phi \sim (\Phi/\gamma)^{2/3}$$

$$X \sim \gamma^{-4/3}\Phi^{1/3}c^{-1}$$

$$E \sim \gamma^{-2/3}\Phi^{5/3}c$$

the scale β of $\partial f/\partial\phi$ is determined by the smallest-scale fluctuations with $\Phi \sim \Phi_0$:

$$\beta \sim \gamma^{-2/3}\Phi_0^{-1/3}$$

$$\operatorname{Im} \chi(\omega) \sim \frac{1}{T} \int \mathrm{d}E\, \mathrm{d}\Delta\, R(E, \Delta) \frac{\omega\tau(E)}{1+[\omega\tau(E)]^2} N(E) \operatorname{sech}^2 \frac{\Delta}{2T}.$$

$$N(E) \sim \Phi(E) X(E) \sim (E/\gamma)^{4/5}$$

$$M(E) \sim \frac{1}{\gamma^{2/15} E^{1/5}} \qquad X(E) \equiv X(\Phi(E)) \sim \gamma^{-6/5} E^{1/5}$$

Thus the linear density of relaxation modes with barriers in the interval $(E, E+\mathrm{d}E)$ is given by

$$W(E)\, \mathrm{d}E \sim \frac{\mathrm{d}}{\mathrm{d}E}\left(\frac{M(E)}{X(E)}\right) \mathrm{d}E \sim \gamma^{16/15} \frac{\mathrm{d}E}{E^{7/5}}. \qquad (3.2.16)$$

characteristic scale of Δ is of order E, so that $\int \mathrm{d}\Delta\, R(E, \Delta) \operatorname{sech}^2 \frac{\Delta}{2T} \sim W(E) \frac{T}{E}$

Combining all above estimates:

$$\operatorname{Im} \chi(\omega) \sim \int \frac{\mathrm{d}E}{E} W(E) N(E) \frac{\omega\tau(E)}{1+[\omega\tau(E)]^2} \sim \frac{\gamma^{4/15}}{(T \ln \omega^{-1})^{3/5}},$$

In the time domain: $c(t) \sim \frac{\gamma^{4/5}}{(T \ln t)^{3/5}}$.

$\chi(t) = -(1/T)\, dC(t)/dt \quad (t>0)$

Fluct-diss. relation

Two problems to solve

1. Generating functional (defined in slide 7) and corresponding action

$$S_0 = -\int \left\{ \frac{\partial^2}{\partial \varphi^2} \left[\frac{\partial \varepsilon}{\partial x} + \frac{\kappa^2}{4\rho_0^2} \left(\frac{\partial \varepsilon}{\partial \varphi} \right)^2 \right] \right\}^2 \frac{1}{8\rho_0^2 c} d\varphi\, dx$$

refer to the free energy $\varepsilon^{>}_{n}(\varphi)$ defined for the recursion > from the left end of the chain to the reference site n

Actually one should consider the sum $\varepsilon_n(\varphi) = \varepsilon^{>}_{n}(\varphi) + \varepsilon^{<}_{n}(\varphi)$ of two parts of free energy since this is the physical free energy in the middle of the chain

The problem: a) to produce scaling estimates similar to those presented in slide 8 , but the total $\varepsilon_n(\varphi)$;

b) to check if the distribution of zeros of the function $d\varepsilon_n(\varphi)/d\varphi$ is the same as was derived for $d\varepsilon^{>}_{n}(\varphi)/d\varphi$ (slide 12) ; if it is not the same, to derive the correct one.

2. Imaginary part of response Im $\chi(\omega)$ was calculated (slide 12) assuming Gibbs distribution for different metastable states, each of them defined by some minimum of function $\varepsilon_n(\varphi)$. However, this assumption of full equilibrium is not valid if aging dynamics on timescales $t_a << t_1$ is considered (for definition of t_1 see slide 11). Namely, thermodynamic Gibbs distribution will be established for modes separated by energy barriers $E << T \ln (t_a/t_0)$, whereas modes with barriers $E >> T \ln (t_a/t_0)$ will be populated just randomly.

As a result, function $1/\cosh^2(\Delta/2T)$ in the integral on slide 13 should be replaced by some non-equlibrium function, dependent on the value of t_a

The problem: to find (approximately) this non-equilibrium and non-stationary distribution function for the range of $E \sim T \ln (t_a/t_0)$ and then calculate aging part of the response function Im $\chi(\omega)$.