Thermal conductivity of the disordered Fermi and electron liquids

Georg Schwiete

Johannes Gutenberg Universität Mainz

Alexander Finkel'stein

Texas A&M University, Weizmann Institute of Science,

and Landau Institute of Theoretical Physics

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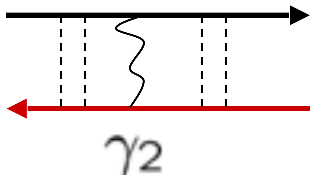
Perturbation theory – interacting systems

$$\delta\sigma(T) = -\frac{e^2}{\pi h}\left[1 + \left(1 - \frac{3}{2}\gamma_2\right)\right]\log(1/T\tau)$$

WL correction (Gang of 4)

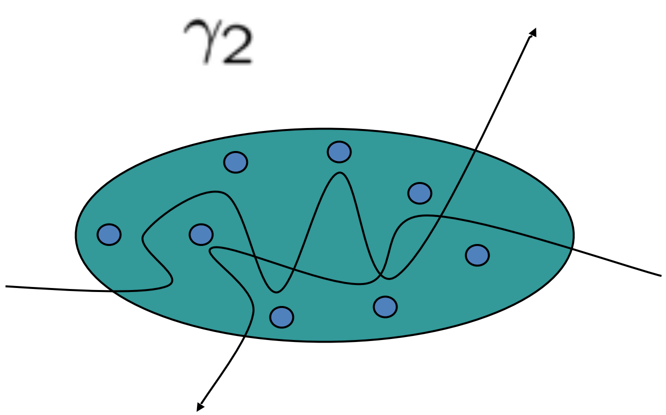
1-singlet contribution

3-triplet contribution



$\boxed{\text{For} \quad \gamma_2 \ll 1, \quad \delta\sigma(T) < 0}$ **(insulating)**

Altshuler, Aronov and Lee (1980), Finkel'stein (1983)



Finkel'stein (1983)

Disorder makes the interaction scale-dependent

$$\boxed{\delta\gamma_2(T) \sim \rho\log(1/T\tau) > 0}$$

In the presence of disorder
γ_2 is considerably enhanced at low T

Non-linear Sigma model (NLσM)

Non-linear Sigma model: Effective low energy ($T < 1/\tau < E_F$) action for the disordered Fermi/electron liquid- *Finkel'stein (1983)*
[noninteracting case: Wegner, Efetov Larkin Khmelnitskii,… (1979-)]

$$\boxed{S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 4iz\hat{\epsilon}Q\right] + Q(\Gamma_1 + \Gamma_2 + \Gamma_c)Q}$$

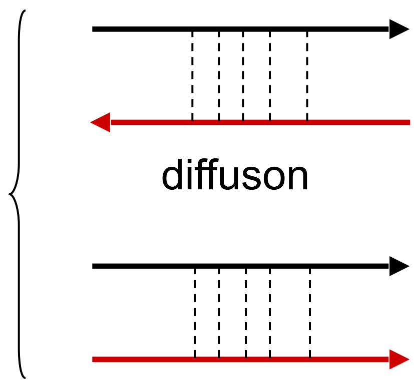
$\rho \sim 1/D$

frequency renormalization

$$\Gamma_2 = -\frac{F_0^\sigma}{1+F_0^\sigma}$$

triplet-channel

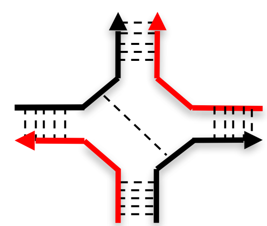
$$\langle QQ \rangle \sim \frac{1}{D\mathbf{q}^2 - iz\omega}$$

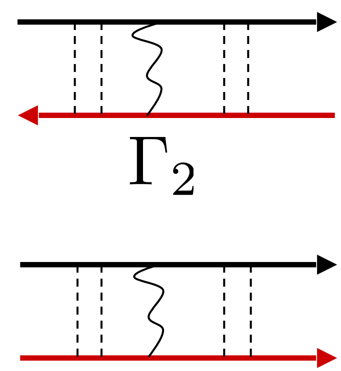


diffuson

cooperon

$$Q^2(\mathbf{r}) = 1$$





Γ_2

Γ_c

Different methods: Replica/Keldysh

Structure of the RG equations

The interplay of disorder and interactions is captured by a set of coupled Renormalization Group (scaling) equations for ρ and γ_2

$$\frac{d\ln\rho}{d\xi} = \beta_\rho(\rho,\gamma_2)$$

$$\frac{d\gamma_2}{d\xi} = \beta_2(\rho,\gamma_2)$$

$$\xi = \ln(1/T\tau)$$

$$\gamma_2 = \frac{\Gamma_2}{z}$$

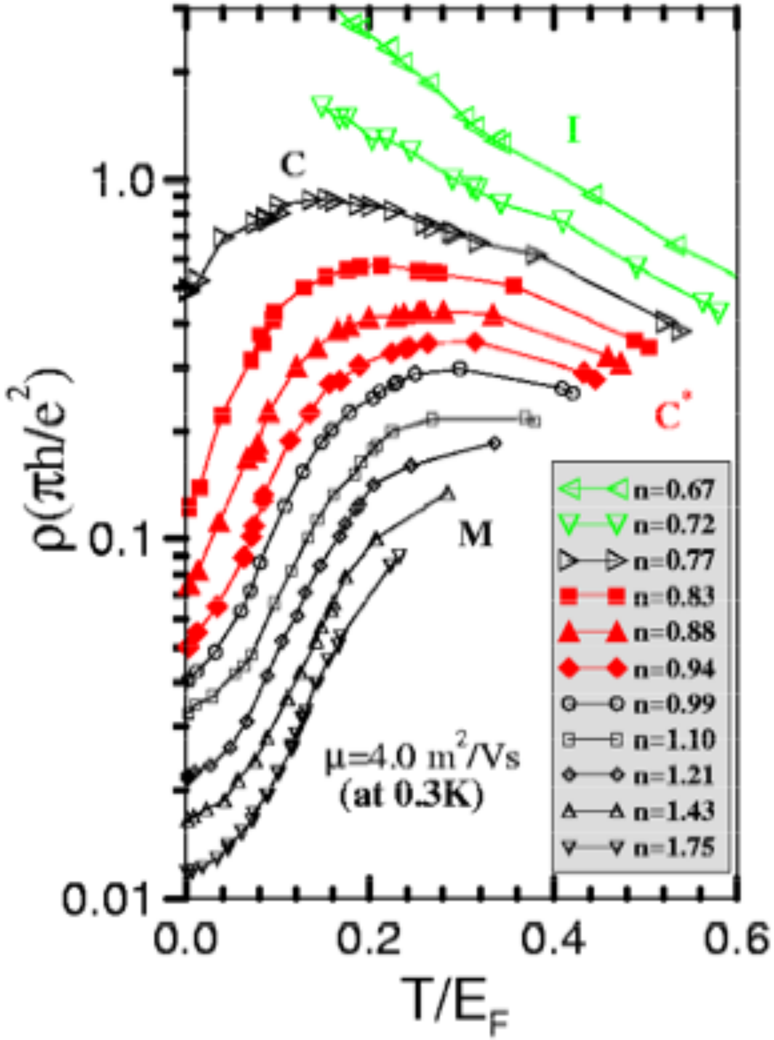
One more equation:

$$\frac{d\ln z}{d\xi} = \beta_z(\rho,\gamma_2)$$

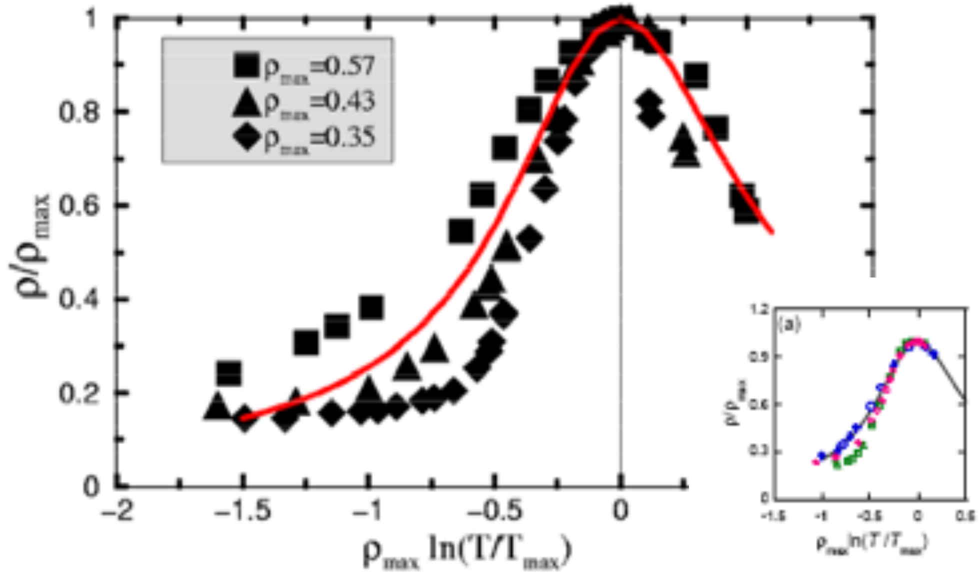
does not affect the flow of ρ and γ_2,
important to understand thermodynamic properties

1-loop: leading order in ρ, all orders in the interaction.

Analysis of high-mobility sample with RG for two valleys



Pudalov, et al., ('98)



Data from the region **C*** in a high-mobility sample. **No adjustable parameters** are used.

A.Punnoose and A. Finkelstein, PRL (2002)
S. Anissimova et al., Nature Physics (2007)

Thermal transport and the Wiedemann Franz law

Transport coefficients

$$\begin{pmatrix} \mathbf{j}_e \\ \mathbf{j}_k \end{pmatrix} = \begin{pmatrix} \sigma & \alpha\sigma \\ \Pi\sigma & \tilde{\kappa} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}$$

Electric current

$$\mathbf{j}_e$$

Heat current

$$\mathbf{j}_k = \mathbf{j}_\varepsilon - \mu \mathbf{j}_n$$

Energy current

Particle current

Electric conductivity σ

Seebeck coefficient α

Peltier coefficient Π

Thermal conductivity $\tilde{\kappa}$

Onsager relation

$$\Pi = T\alpha$$

The Wiedemann-Franz law

Electric conductivity	σ	α	Seebeck coefficient
Peltier coefficient	Π	$\tilde{\kappa}$	Thermal conductivity

$$\mathbf{j}_e = 0$$

$$\kappa = \tilde{\kappa} + \mathcal{O}\left((T/\varepsilon_F)^2\right)$$

$$\mathbf{E} = 0$$

The Wiedemann-Franz "law"

$$\kappa = \mathcal{L}_0 \sigma T$$

Lorenz number

$$\mathcal{L}_0 = \frac{\pi^2}{3}\frac{k_B^2}{e^2} = \frac{c_{FL}}{2\nu e^2 T}$$

The Wiedemann Franz law is an approximate low-temperature relation for itinerant electron systems. What is the range of validity?

Heat transport and the Wiedemann-Franz law in disordered electron systems - History of the problem

- **Wiedemann-Franz law ($\kappa/\sigma T$=const.) holds**
  for noninteracting disordered electron systems - *Chester, Thellung (1961)*.

- **Wiedemann-Franz law holds**
  for a Fermi liquid - *Langer (1962).*

After the development of the scaling theory of localization for interacting electrons
[Finkel'stein 83, Castellani et al. 84]:

- **Wiedemann-Franz law holds**
  for the disordered electron liquid (renormalized perturbation theory, Ward Identities)
  - *Castellani, di Castro, Kotliar, Lee, Strinati (1987-).*

- **Wiedemann-Franz law violated**
  for the disordered electron liquid (perturbation theory)

  Kubo-formula – *Arfi (1992), Niven, Smith (2005).*

  Kinetic equation approaches - *Livanov et al. (1991), Raimondi et al. (2004), Catelani, Aleiner (2005), Michaeli, Finkelstein (2009)*.

While approaches differ, the result is common: Additional corrections, Wiedemann-Franz law violated

Can one resolve the contradiction and construct a comprehensive theory (including RG and additional log corrections) ?

Can one generalize the RG approach to thermal transport?

- How is heat transported through the system?
- What are the consequences of replacing the electric field by a temperature gradient

How to approach the problem?
How to do RG including a temperature gradient?

Perturbative calculations for κ were (mostly) based on kinetic equation approaches. Including a temperature gradient is straightforward, but how to do RG?

The scaling theory for σ was developed on the basis of a field theory (NlσM) with source fields. How to account for a temperature gradient?

$$\mathrm{Nl}\sigma\mathrm{M} \quad \xrightarrow{source\ \varphi} \quad \langle nn \rangle \quad \xrightarrow{Einstein} \quad \sigma$$

Our approach: Renormalize the NlσM with source fields (Luttinger‘s „gravitational potential“ mimics temperature variation).

$$\mathrm{Nl}\sigma\mathrm{M} \quad \xrightarrow{source\ ??} \quad \langle kk \rangle \quad \xrightarrow{Einstein} \quad \kappa$$

Source fields for the heat density correlation function

Action:

$$\boxed{S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - k[\psi^*,\psi]\right)}$$

$$\mathcal{Z} = \int D(\psi,\psi^*)\, \mathrm{e}^{iS} \qquad\qquad k = h_0 + h_{int} - \mu n$$

$$\boxed{S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - (1+\eta) k[\psi^*,\psi]\right)}$$

$$\boxed{\chi_{kk} = \frac{i}{2} \frac{\delta^2 \mathcal{Z}}{\delta\eta_{\mathbf{r}_1 t_1} \delta\eta_{\mathbf{r}_2 t_2}}}$$

Gravitational potential

Luttinger (1964)

Source fields for the heat density correlation function

$$S[\psi^*, \psi] = \int_{\mathbf{r},t} (\psi^* i\partial_t \psi - (1+\eta)k[\psi^*, \psi])$$

Problem: $$S_{dis} = -\int_{\mathbf{r},t} (1+\eta)\psi^* u_{dis}\psi$$

Change of variables: $$\psi \to \frac{1}{\sqrt{1+\eta}}\psi \qquad \psi^* \to \psi^* \frac{1}{\sqrt{1+\eta}}$$

After this transformation, the derivation of the NLσM is straightforward:

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\}Q\right] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

$$\lambda = \frac{1}{1+\eta} \approx 1 - \eta + \eta^2 + \ldots$$

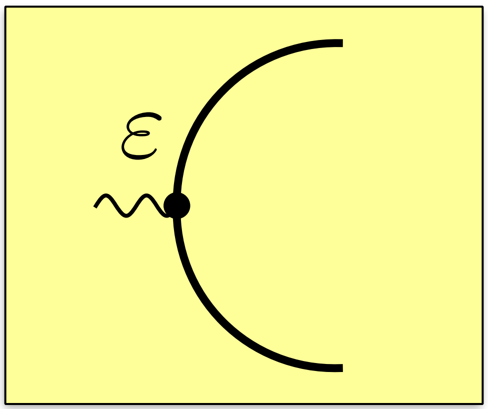
nonlinear in η!

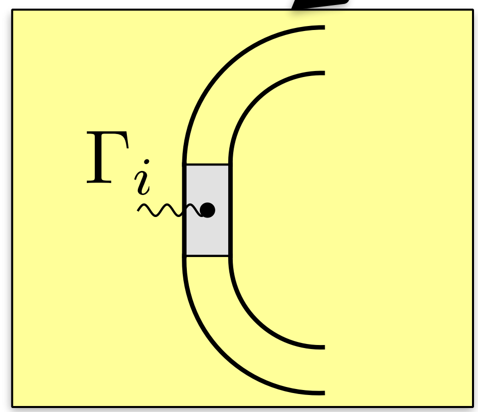
NIσM with "gravitational potentials"

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\}Q\right] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

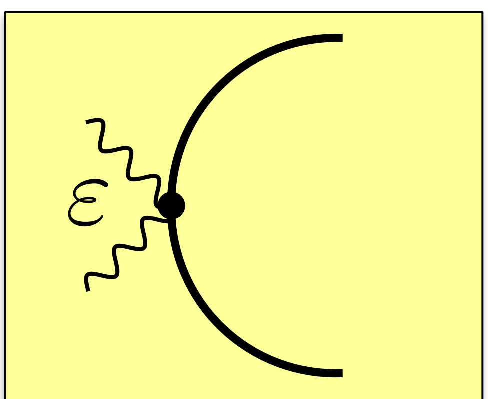
$$\lambda \approx 1 - \eta + \eta^2$$

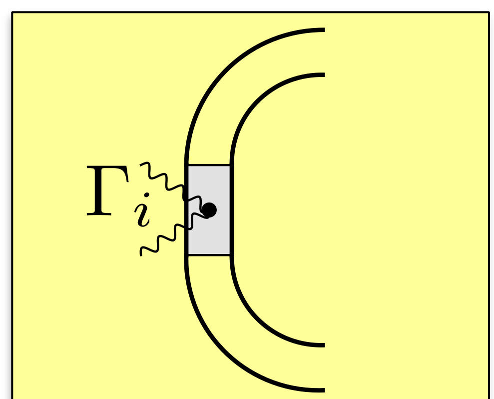
η





η^2





The correlation function

Heat density correlation function in the diffusive limit

$$\boxed{\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega}}$$

Static limit

$$\chi_{kk}(\mathbf{q}\to 0, \omega = 0) = -cT$$

Conservation law

$$\chi_{kk}(\mathbf{q} = 0, \omega \to 0) = 0$$

Thermal conductivity

$$\boxed{\kappa = cD_k}$$

Specific heat

1. Direct calculation: Heat density - specific heat
(linear terms in η)
2. Static part of the correlation function
(quadratic terms in η)

Heat density and specific heat

$$S[Q] \sim \int d\mathbf{r}\,\mathrm{tr}[D(\nabla Q)^2 - 2z\{\hat{\varepsilon}, 1-\eta+\eta^2\}Q] + Q(1-\eta+\eta^2)(\Gamma_1+\Gamma_2)Q$$

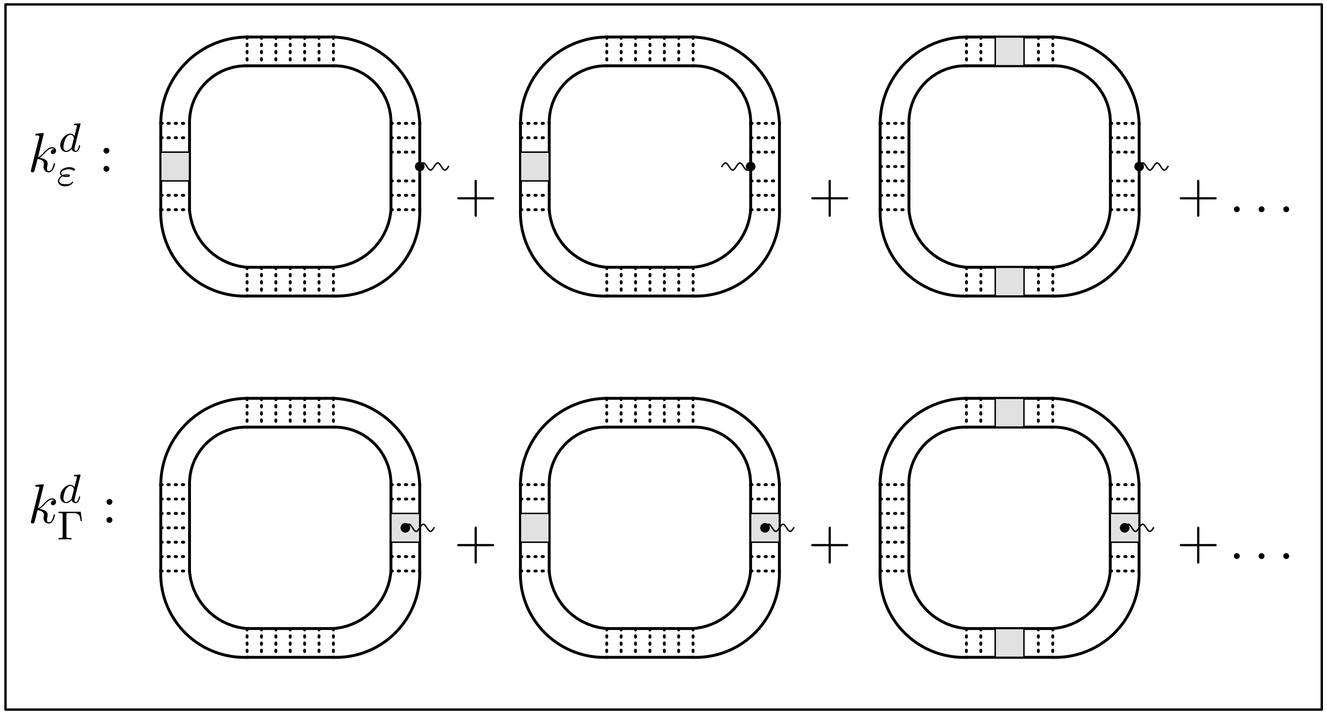
Heat density

$$k^d_{\eta=0} = \frac{i}{2}\,\frac{\delta \mathcal{Z}}{\delta\eta}\bigg|_{\eta=0}$$

Specific heat

$$\delta c = \partial_T k^d_{\eta=0} = \delta z c_{FL}$$

Castellani, Di Castro (1986)

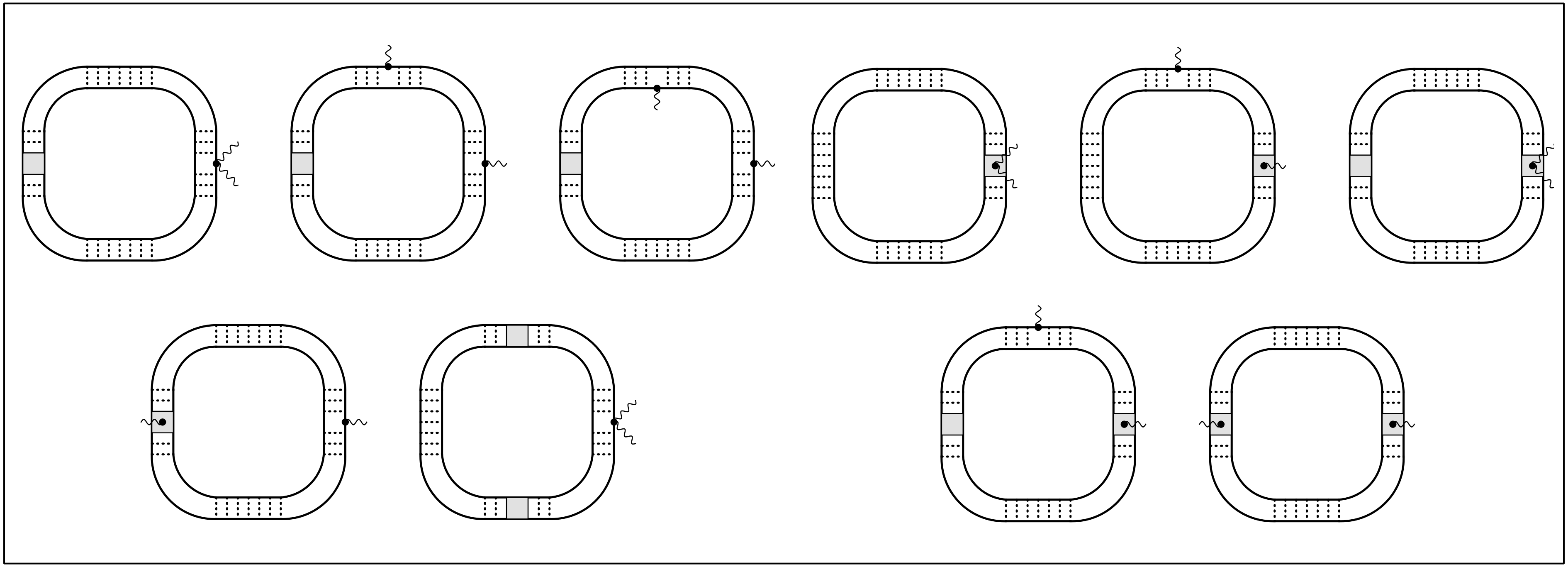


Specific heat and the static part of the correlation function

$$S[Q] \sim \int d\mathbf{r}\,\mathrm{tr}[D(\nabla Q)^2 - 2z\{\varepsilon, 1-\eta+\eta^2\}Q] + Q(1-\eta+\eta^2)(\Gamma_1+\Gamma_2)Q$$

$$\chi_{kk}^{st} = \frac{i}{2}\frac{\delta^2 \mathcal{Z}}{\delta\eta^2} = -Tc$$

$$c = zc_{FL}$$



RG and the dynamical part of the correlation function

RG and the dynamic part of the correlation function

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

Initial conditions: $\zeta_D = 0 \quad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$

Parameterization: $Q = u\, U_s Q_f U_s^{-1}\, u^{-1} \qquad Q^2 = 1$

$Q_f = U_f \Lambda U_f^{-1}$ fast

$Q_s = U_s \Lambda U_s^{-1}$ slow

u slowest: distribution function

$$U^{-1}_{\varepsilon_1\varepsilon_2}\zeta_i(\varepsilon_2-\varepsilon_3)U_{\varepsilon_3\varepsilon_4} \neq \zeta_i(\varepsilon_1-\varepsilon_4)$$

$$\Delta(D\zeta_D) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta D$$

$$\Delta(z\zeta_z) = \left(\zeta_D D\frac{\partial}{\partial D} + \zeta_z z\frac{\partial}{\partial z} + \zeta_{\Gamma_1}\Gamma_1\frac{\partial}{\partial \Gamma_1} + \zeta_{\Gamma_2}\Gamma_2\frac{\partial}{\partial \Gamma_2}\right)\Delta z$$

$$\vdots$$

RG and the dynamic part of the correlation function

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

Result: Fixed point

$$\Delta\zeta_D = \Delta\zeta_z = \Delta\zeta_{\Gamma_1} = \Delta\zeta_{\Gamma_2} = 0$$

$$\zeta_D = 0 \qquad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$$

$$S = \int \mathrm{tr}[D(\nabla Q)^2 + 2iz[\{\hat{\varepsilon}, 1-\eta\}Q] + Q(1-\eta)(\Gamma_1+\Gamma_2)Q$$

$$\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega} \qquad D_k = \frac{D}{z}$$

Conductivities and the Wiedemann Franz law

(Generalized) Einstein relations:

$$\sigma = e^2 \lim_{\omega\to 0} \lim_{q\to 0} \frac{\omega}{q^2} \mathrm{Im}\chi_{nn}(\mathbf{q},\omega) = e^2 \frac{\partial n}{\partial \mu} D_n = 2\nu e^2 D$$

$$\boxed{\kappa = -\frac{1}{T} \lim_{\omega\to 0} \lim_{q\to 0} \frac{\omega}{q^2} \mathrm{Im}\chi_{kk}(\mathbf{q},\omega) = c D_k = c_{FL} D}$$

$$D_n = \frac{D}{\frac{\partial n}{\partial \mu}/2\nu} \qquad D_k = \frac{D}{z} = \frac{D}{c/c_{FL}}$$

$$\boxed{\kappa = \frac{c_{FL}}{2\nu e^2}\sigma}$$

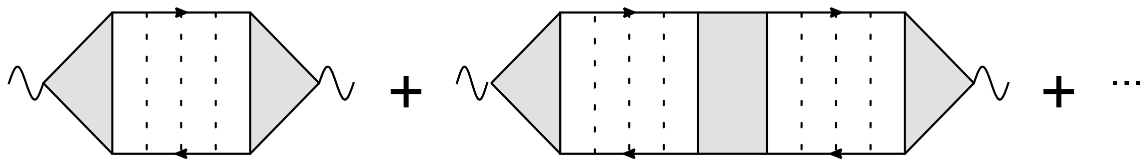
The structure immediately implies:
Wiedemann Franz law is not violated within the RG regime ($T<\varepsilon<1/\tau$), neither for short-range nor for long-range (Coulomb) interaction.

Something is missing in this treatment!

Beyond RG – the low temperature regime

General structure of the correlation functions

Density-density correlation function:	Heat density-heat density correlation function:
$\chi_{nn} = \frac{dn}{d\mu} + 2\nu\Lambda_n^2 \frac{i\omega}{\mathcal{L}^{-1} + 2i\tilde{\Gamma}_s\omega}$	$\chi_{kk} = -cT - c_{FL}T\Lambda_k^2 \frac{i\omega}{\mathcal{L}^{-1}}$
	Castellani et al. (1987)

Dynamical part: 

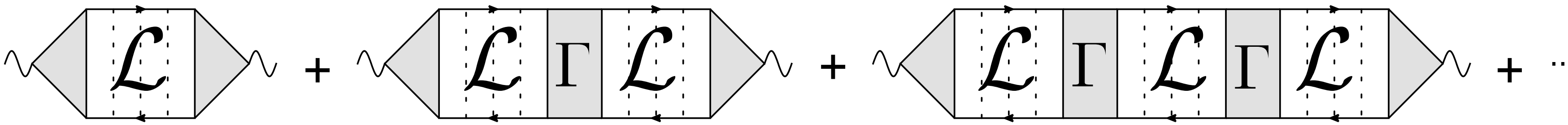
$$\Lambda \ \mathcal{L} \ \Lambda \qquad \Lambda \ \mathcal{L} \ \Gamma \ \mathcal{L} \ \Lambda$$

$$\mathcal{L} = \frac{\xi^2}{D\mathbf{q}^2 - iz\omega}$$

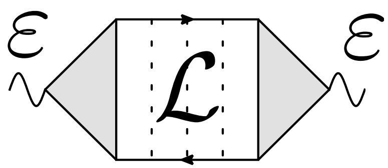
Renormalization of D, ξ, z, Λ, Γ

Why heat transport is different

χ^{dyn}_{nn} :

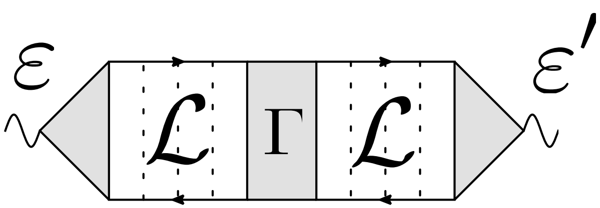


χ^{dyn}_{kk} :



$$\varepsilon^2 \to T^2$$

WF



$$\varepsilon\Gamma\varepsilon' \to 0$$

Rescattering is suppressed for constant Γ!

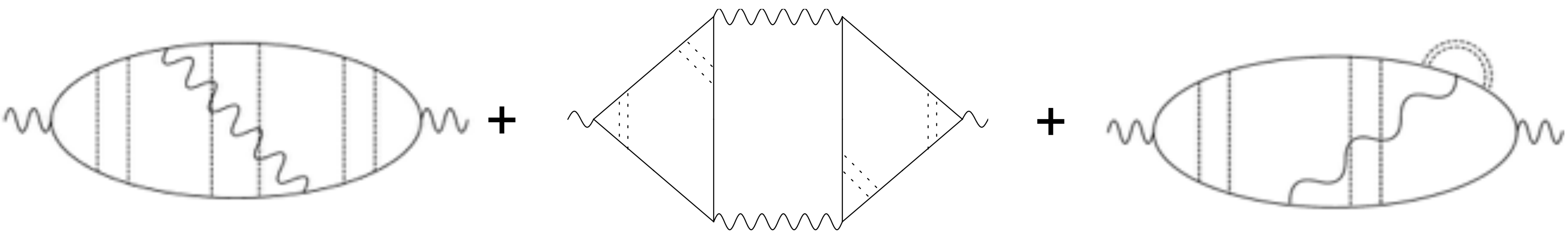
Finite result only for $\Gamma = \Gamma(\varepsilon, \varepsilon')$

Such contributions are not part of the traditional RG scheme!

Additional logarithms

For **short-range interactions** there are no additional (logarithmic) corrections.

For the **electron liquid (Coulomb interaction)** there are **additional logarithmic contributions** from scattering processes with sub-T frequency transfer.



+…

All contributions are proportional to Im(U^R): Decay into particle-hole pairs.

Example:

$$\delta\chi_{kk} \propto \int_{\mathbf{k},\varepsilon,\nu} \varepsilon\nu\partial_\varepsilon F_\varepsilon (F_{\varepsilon+\nu} + F_{\varepsilon-\nu}) \mathrm{Re}\mathcal{D}^2(\mathbf{k},\nu) \mathrm{Im}U^R(\mathbf{k},\nu)$$

Corrections to heat conductivity

Additional logarithmic correction (not related to c!):

$$\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega}$$

$$D_k = \frac{1}{z}\left(D_n + \delta D^h\right)$$

Consistent with conservation law!

Additional logarithmic correction to κ:

$$\delta\kappa = \frac{T}{12}\log\frac{D\kappa_s^2}{T}$$

κ_s: screening radius

WF law is violated!

Agrees with the result of (recent) kinetic equation approaches

From the regime: $\boxed{\frac{T^2}{D\kappa^2} < D\mathbf{k}^2 < T}$

Results: Thermal transport and the WFL

$$S[Q] \sim \int d\mathbf{r}\, \mathrm{tr}[D(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, \lambda\}Q] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

Energy scales

$$\lambda \approx 1 - \eta + \eta^2$$

GS & Finkel'stein, with Keldysh NLσM
PRB **89** (2014); PRB **90** (2014)(R); PRB **90** (2014); forthcoming

ε_F

$\frac{1}{\tau}$

RG regime

WFL -disordered Fermi liquid

-disordered electron liquid

despite strong renormalizations

T

sub-T regime

WFL -disordered Fermi liquid

~~WFL~~ -disordered electron liquid

$\frac{T^2}{D\kappa_s}$

Additional (delocalizing) logarithmic corrections!

Summary

- We developed a field theoretic model with „gravitational potentials“ suitable for the analysis of heat density correlation function in the disordered electron liquid.

- The terms linear and quadratic in the gravitational potentials are consistent with each other and also with thermodynamics.

- For short range interactions the renormalization of κ and of σ are linked through the WF law.

- For long-range (Coulomb) interaction there are additional logarithmic corrections originating from outside of the RG regime. They lead to a violation of the WF law.

Georg Schwiete & Alexander Finkel'stein,
Phys. Rev. B **89** (2014); RG with Keldysh NLσM;
Phys. Rev. B **90** (2014)(R); Wiedemann Franz law
Phys. Rev. B **90** (2014); RG for Keldysh NLσM with grav. potentials
forthcoming: Analysis of low temperature regime

Thank you!