MAX-PLANCK-GESELLSCHAFT

Anomalous Hall effect with massive Dirac fermions

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in collaboration with

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arXiv:1504.03658 and more to follow

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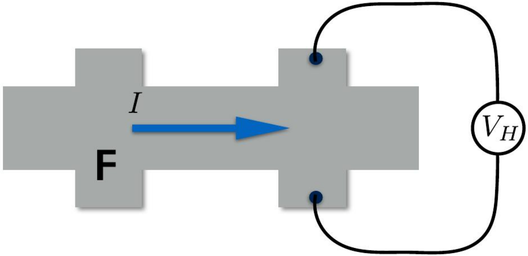
Anomalous Hall effect

Hall effect due to magnetization rather then B field [Edwin Hall (1881)]



- Ordinary Hall: $\rho_{xy} = \frac{B}{nec}$ **disorder independent**
- Anomalous Hall (requires spin-orbit coupling):

$$\sigma_{xy} = \text{const} \quad \Longrightarrow \quad \rho_{xy} \approx \frac{\sigma_{xy}}{\sigma_{xx}^2} \sim \rho_{xx}^2$$

Anomalous Hall effect: experiment

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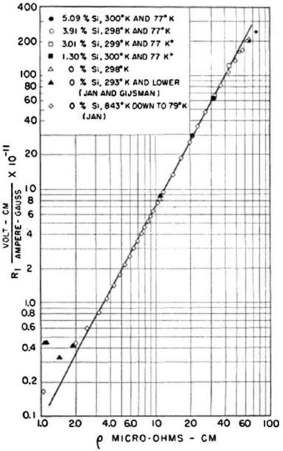
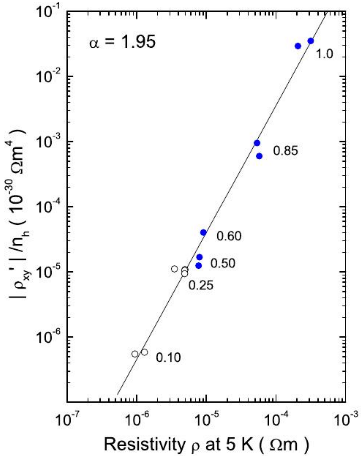
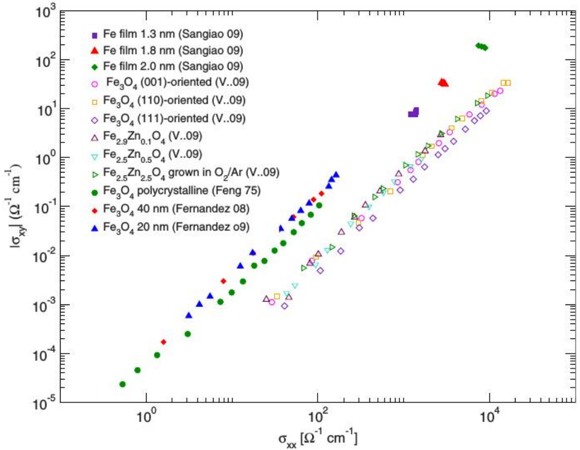


FeSi [Kooi '54]



$CuCr_2Se_{4-x}Br_x$ [Lee et al '04]

Anomalous Hall effect: experiment



Fe on MgO [Sangiao et al ’09]

Fe_3O_4 [Feng et al ’75, Venkateshvaran et al ’08, Fernandez-Pacheco et al ’08]

Quasiclassical picture

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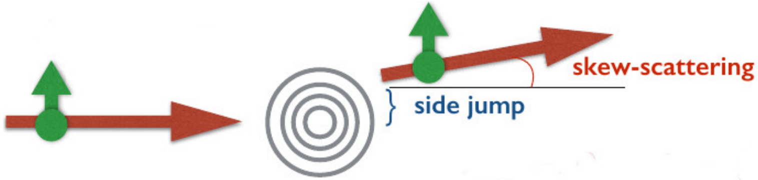
- Intrinsic mechanism (clean limit) [Karplus, Luttinger '54]:
  - Spin-orbit coupling $\Longrightarrow$ Berry curvature
  - Ferromagnetism $\Longrightarrow$ breaks spin symmetry

$$\mathbf{v} = \frac{\partial E}{\partial \mathbf{p}} + e\mathbf{E} \times \mathbf{b}$$

- Extrinsic mechanism (impurities involved):
  - Skew scattering [Smit '55, '58]
  - Side jump [Berger '70]



Minimal model: massive Dirac

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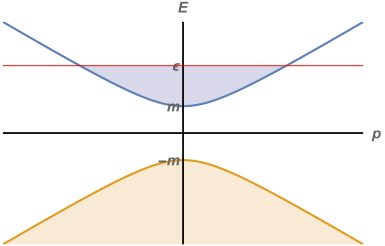
Required ingredients:

- spin-orbit coupling
- magnetisation

$$H = \underbrace{\sigma \mathbf{p}}_{\text{SO}} + \underbrace{m\sigma_z}_{\text{F}} + V(\mathbf{r})$$



Example: HgTe/CdTe quantum well [Bernevig, Hughes, Zhang '06]

$$H = \begin{pmatrix} h(\mathbf{p}) & 0 \\ 0 & h^*(-\mathbf{p}) \end{pmatrix}, \quad h = \begin{pmatrix} m + bp^2 & p_x - ip_y \\ p_x + ip_y & -m - bp^2 \end{pmatrix}$$

Spin Hall effect!

Kubo formalism

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Longitudinal conductivity: $\sigma_{xx} = -\frac{1}{2h}\mathrm{Tr}\left\langle j_x(G^R - G^A) j_x(G^R - G^A)\right\rangle$

Hall Conductivity [Streda '82]:

$$\sigma^{\mathrm{I}}_{xy} = \frac{1}{h}\mathrm{Tr}\left\langle j_x G^R j_y G^A \right\rangle \qquad \sigma^{\mathrm{II}}_{xy} = ec\frac{\partial n}{\partial B}$$

- σ^{I}_{xy} – "normal contribution" coming from Fermi surface
- $\sigma^{\mathrm{II}}_{xy}$ – "anomalous contribution" coming from entire Fermi see
  responsible for chiral anomaly and quantum AHE
  exists in the gap!

Disorder: Gaussian white noise

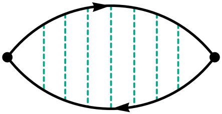
$$\langle V(\mathbf{r})\rangle = 0, \qquad \langle V(\mathbf{r})V(\mathbf{r}')\rangle = 2\pi\alpha\delta(\mathbf{r}-\mathbf{r}')$$

$\alpha \ll 1$ – small parameter of the theory (quasiclassical approximation)

How to solve?

Textbook knowledge: **intersecting impurity lines are small**

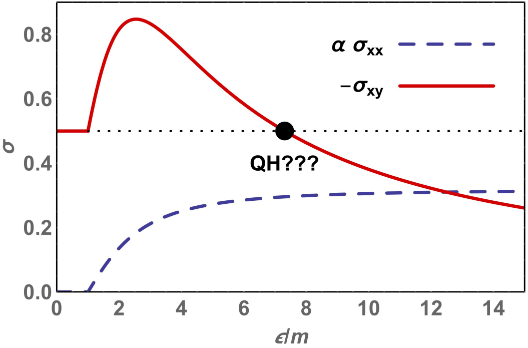
Quasiclassical approximation (Drude):



$$\sigma_{xx} = \frac{1}{\pi\alpha}\,\frac{\epsilon^2 - m^2}{\epsilon^2 + 3m^2}$$

$$\sigma_{xy} = -\frac{4\epsilon m(\epsilon^2 + m^2)}{(\epsilon^2 + 3m^2)^2}$$

[Sinitsyn et al ’06, ’07]



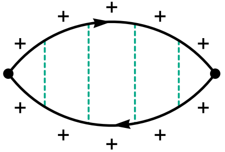
Parsing the result

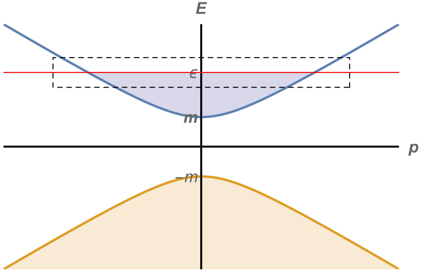
Separate electrons and “positrons”:

$$G = (\epsilon - \boldsymbol{\sigma}\mathbf{p} - m\sigma_z)^{-1} = \underbrace{\frac{P_+}{\epsilon - \sqrt{m^2 + p^2}}}_{G_+} + \underbrace{\frac{P_-}{\epsilon + \sqrt{m^2 + p^2}}}_{G_-}$$

$$P_\pm = \frac{1}{2}\left[1 \pm \frac{\boldsymbol{\sigma}\mathbf{p} + m\sigma_z}{\sqrt{m^2 + p^2}}\right]$$

Project on to upper band:





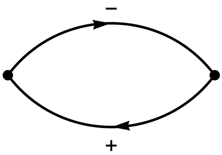
$$\sigma_{xx} = \frac{1}{\pi\alpha}\frac{\epsilon^2 - m^2}{\epsilon^2 + 3m^2}$$ – **correct**

$\sigma_{xy} = 0$ – **wrong!**

Include one “positron” line!

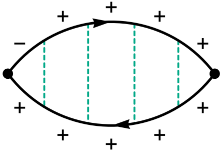
Parsing the result

Intrinsic:



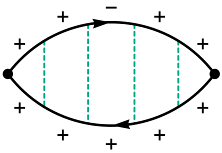
$$\sigma_{xy}^{\text{int}} = -\frac{m}{2\epsilon}$$

Side jump:



$$\sigma_{xy}^{\text{side}} = -\frac{2m(\epsilon^2 - m^2)}{\epsilon(\epsilon^2 + 3m^2)}$$

Skew scattering:



$$\sigma_{xy}^{\text{skew}} = -\frac{3m(\epsilon^2 - m^2)^2}{2\epsilon(\epsilon^2 + 3m^2)^2}$$

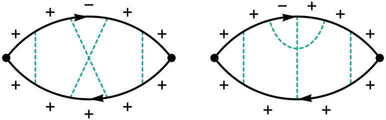
"Positron" introduces smallness $\Longrightarrow$ $\sigma_{xy} \sim \alpha^0$

Missing contribution

Virtual "positron" state is very short in real space

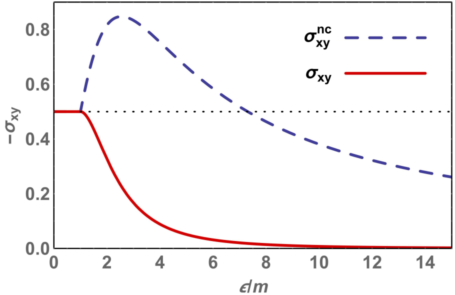
$\Longrightarrow$ **no cost for intersecting impurities!**

Extra skew scattering:



Correct Hall conductivity:

$$\sigma_{xy} = -\frac{8\epsilon m^3}{(\epsilon^2 + 3m^2)^2}$$



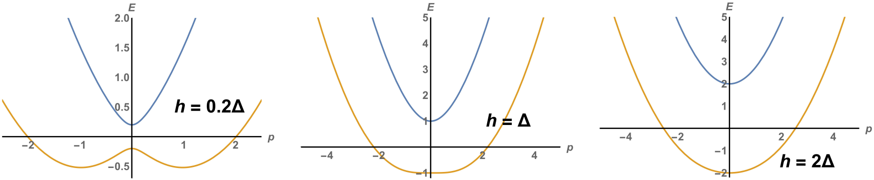
- Faster decay: $\sigma_{xy} \sim (m/\epsilon)^3$ at $\epsilon \gg m$
- No spurious QH transition

Bychkov-Rashba model

Hamiltonian: $H = \dfrac{p^2}{2m} + \lambda \boldsymbol{\sigma} \times \mathbf{p} + h\sigma_z$

Parameters: $\Delta = m\lambda^2, \quad E = \epsilon/\Delta, \quad H = h/\Delta$

Spectrum:

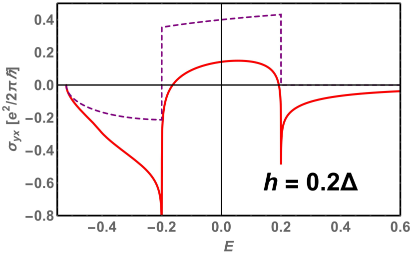


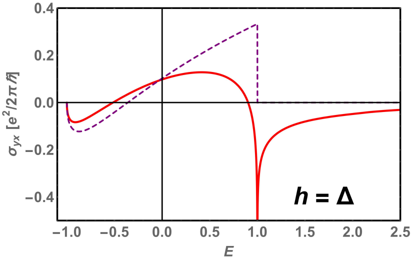
Three energy ranges:

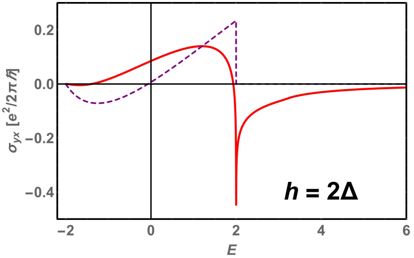
- $\epsilon > h \implies$ "conduction band"
- $-h < \epsilon < h \implies$ "gap"
- $\epsilon < -h \implies$ "valence band"

AHE for Bychkov-Rashba

Nunner et al ’07







$\sigma_{xy}^{\mathrm{nc}} = 0$ for $\epsilon > h$

But the correct result is finite!

$$\sigma_{xy} = \frac{H}{8\pi}\left[\frac{1}{E^2} - \frac{1}{E^3} + \dots\right]$$

Exact expression involves complete elliptic integrals

Summary and outlook

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- **Non-crossing approximation does not hold for AHE!**
- Cross diagrams
  - suppress AHE for massive Dirac particles
  - **the only** source of AHE for Rashba model in conduction band
- Many spintronics quantities are affected by cross diagrams:
  - anomalous Hall effect
  - spin or valley Hall effect
  - anti-damping-like spin-orbit torque
  - polar Kerr effect in $p_x + ip_y$ superconductors
  - etc...

A good part of spintronics calculations has to be reconsidered