Metal-Insulator Transition and Related Phenomena in 2D

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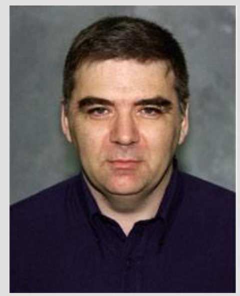
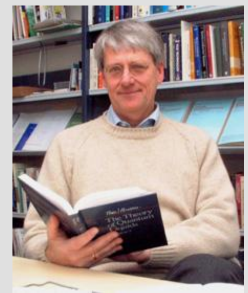


A. M. Finkelstein
Texas A&M



T. M. Klapwijk
TU Delft

V. M. Pudalov
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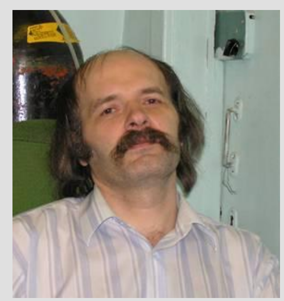
A. Punnoose
CCNY

M. P. Sarachik
CCNY



A. A. Shashkin
ISSP

Outline

Scaling theory of localization: the origin of the common wisdom "all electron states are localized in 2D"

Samples

What do transport experiments show?

Interplay between disorder and interactions in 2D; flow diagram

Spin susceptibility

g-factor or effective mass?

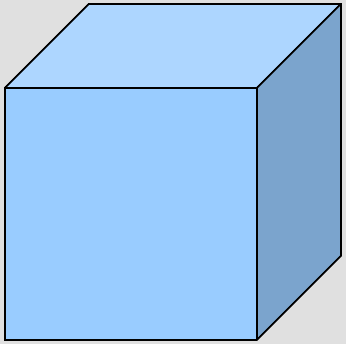
Is it a Wigner crystal?

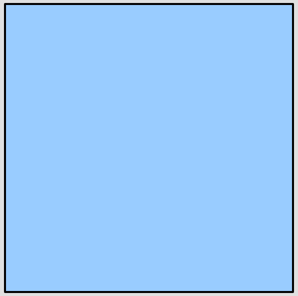
Summary

In 1979, a powerful theory was created by the “Gang of Four” (Abrahams, Anderson, Licciardello, and Ramakrishnan), according to which, there is no conductivity in 2D at zero temperature.

This became one of the most influential paradigms in modern condensed matter physics.

Ohm's law in *n* dimensions

3d:  $G = 1/R = \sigma A/L = \sigma L$

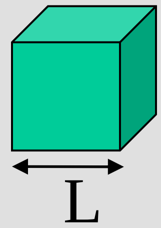
2d:  $G = \sigma L/L = \sigma$

1d: $G = \sigma / L$

n d: $G = \sigma L^{n-2}$

$d(\ln G)/d(\ln L) = \beta(G)$

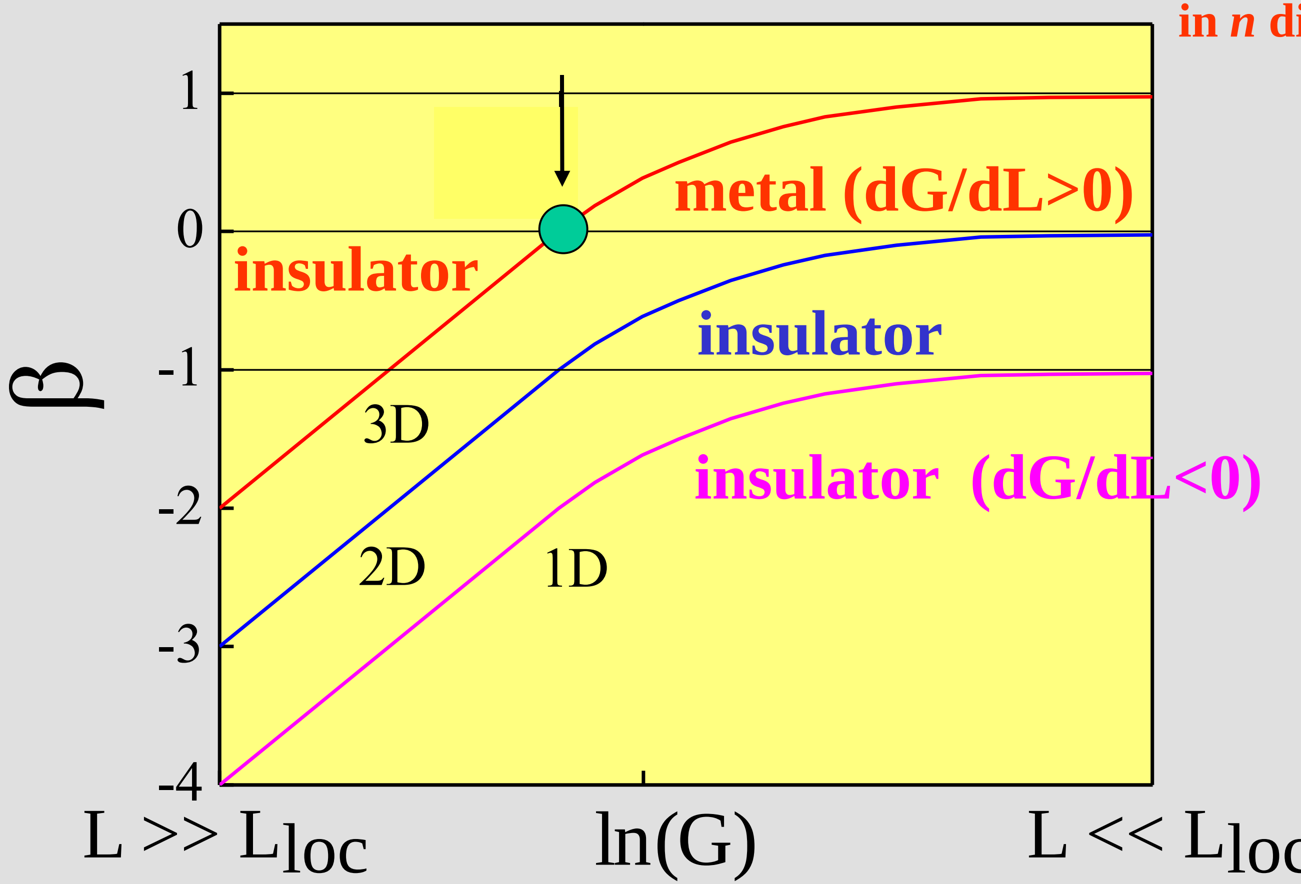
Abrahams, Anderson, Licciardello, and Ramakrishnan, *PRL* 42, 673 (1979)



$$G = 1/R \sim L^{n-2}\, e^{-L/L_{loc}}$$

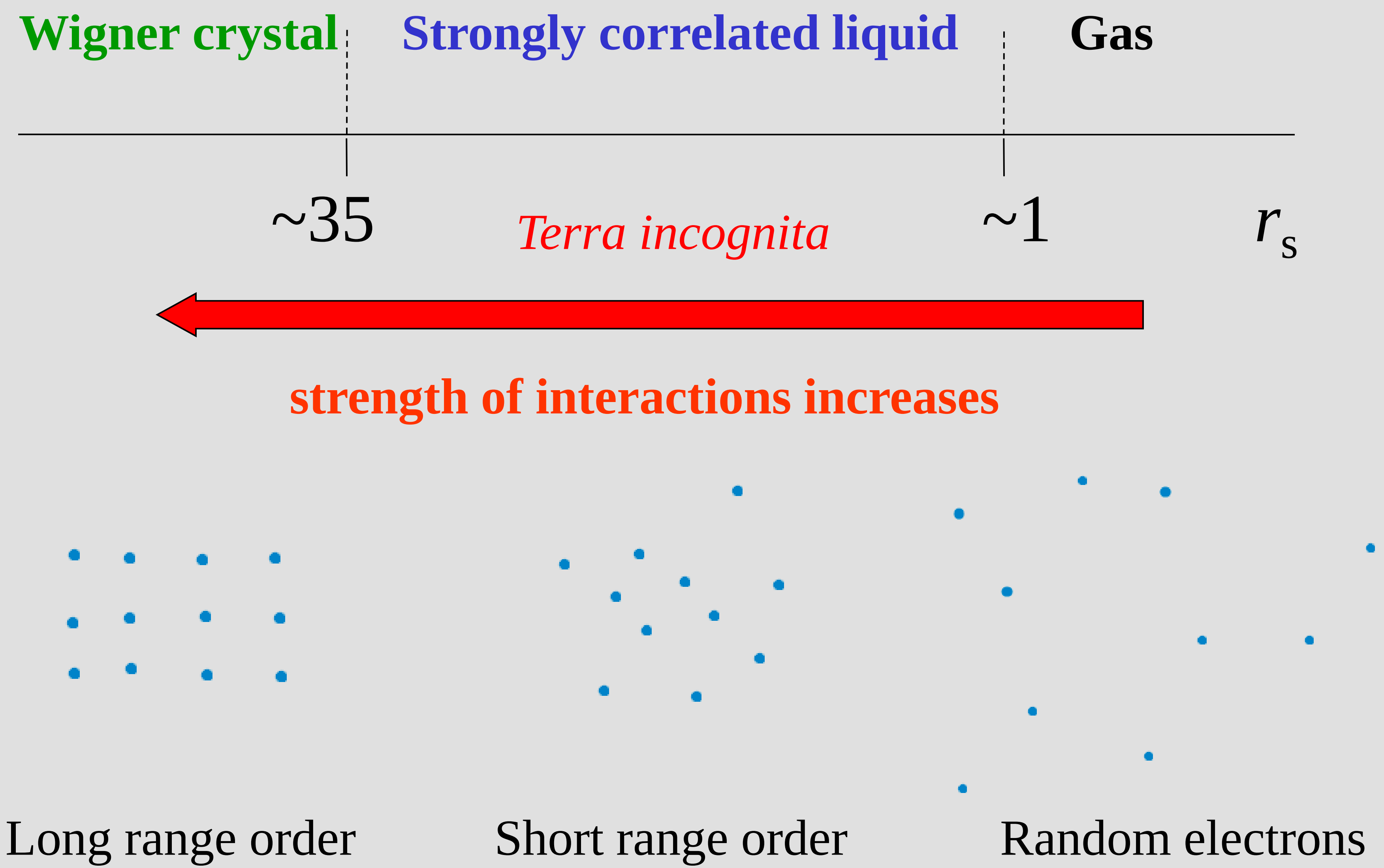
Ohm's law in *n* dimensions

Anderson localization



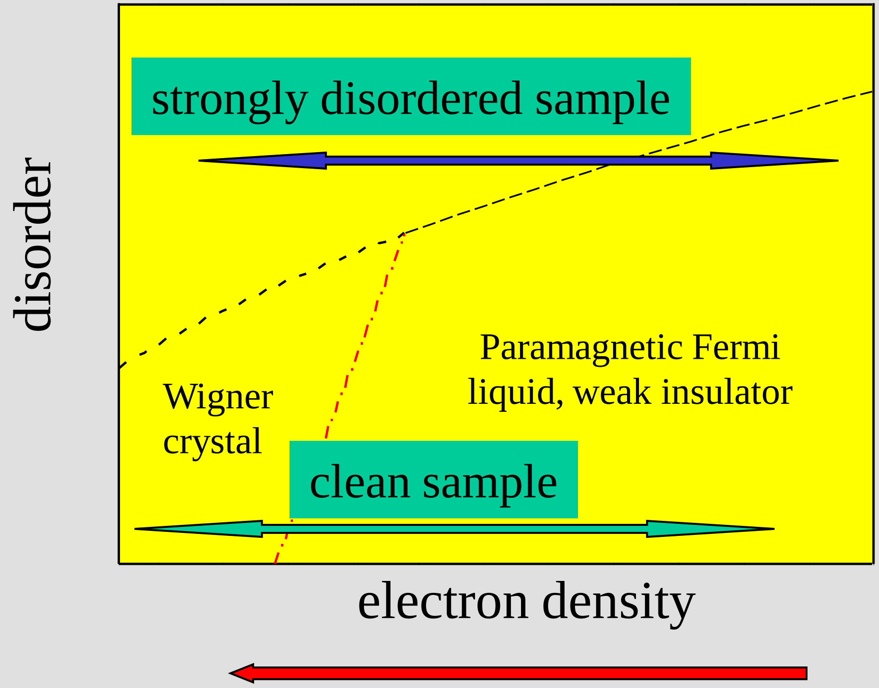
Works for **non-interacting** (!) electrons

$$r_s = \frac{\text{Coulomb energy}}{\text{Fermi energy}}$$



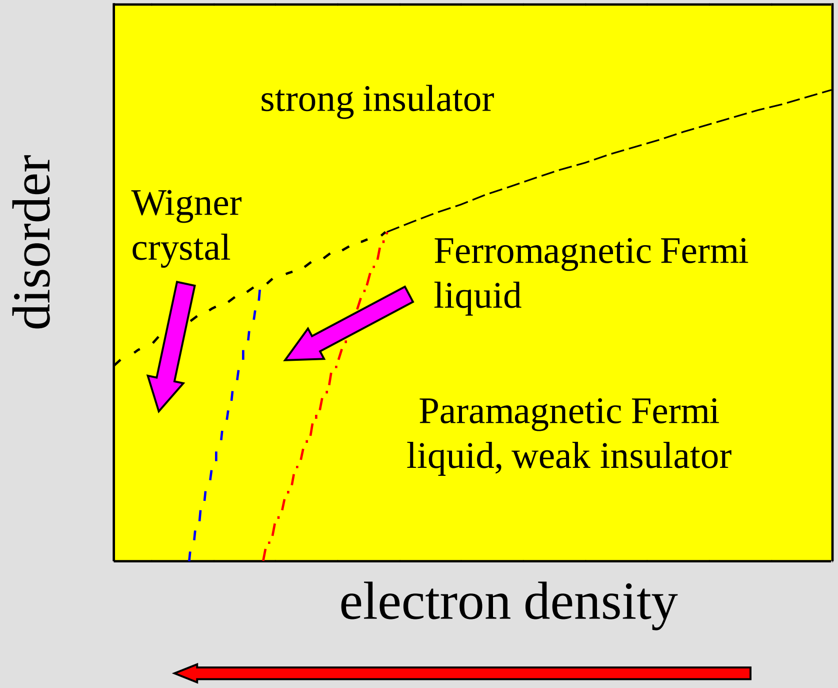
Suggested phase diagrams for strongly interacting electrons in two dimensions

Tanatar and Ceperley, *Phys. Rev*. B **39**, 5005 (1989)



strength of interactions increases

Attaccalite *et al. Phys. Rev. Lett*. **88**, 256601 (2002)



strength of interactions increases

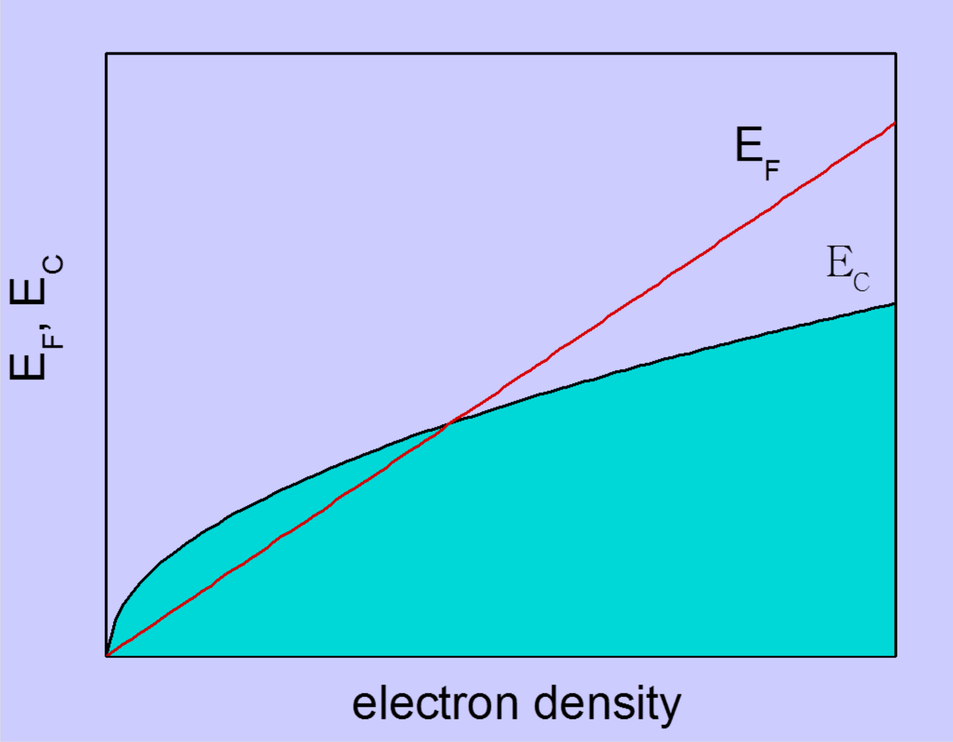
In 2D, the kinetic (Fermi) energy is proportional to the electron density:

$$E_F = (\pi h^2/m)\, N_s$$

while the potential (Coulomb) energy is proportional to $N_s^{1/2}$:

$$E_C = (e^2/\varepsilon)\, N_s^{1/2}$$

Therefore, the relative strength of interactions increases as the density decreases:



Scaling theory of localization: the origin of the common wisdom “all electron states are localized in 2D”

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What do transport experiments show?

Interplay between disorder and interactions in 2D; flow diagram

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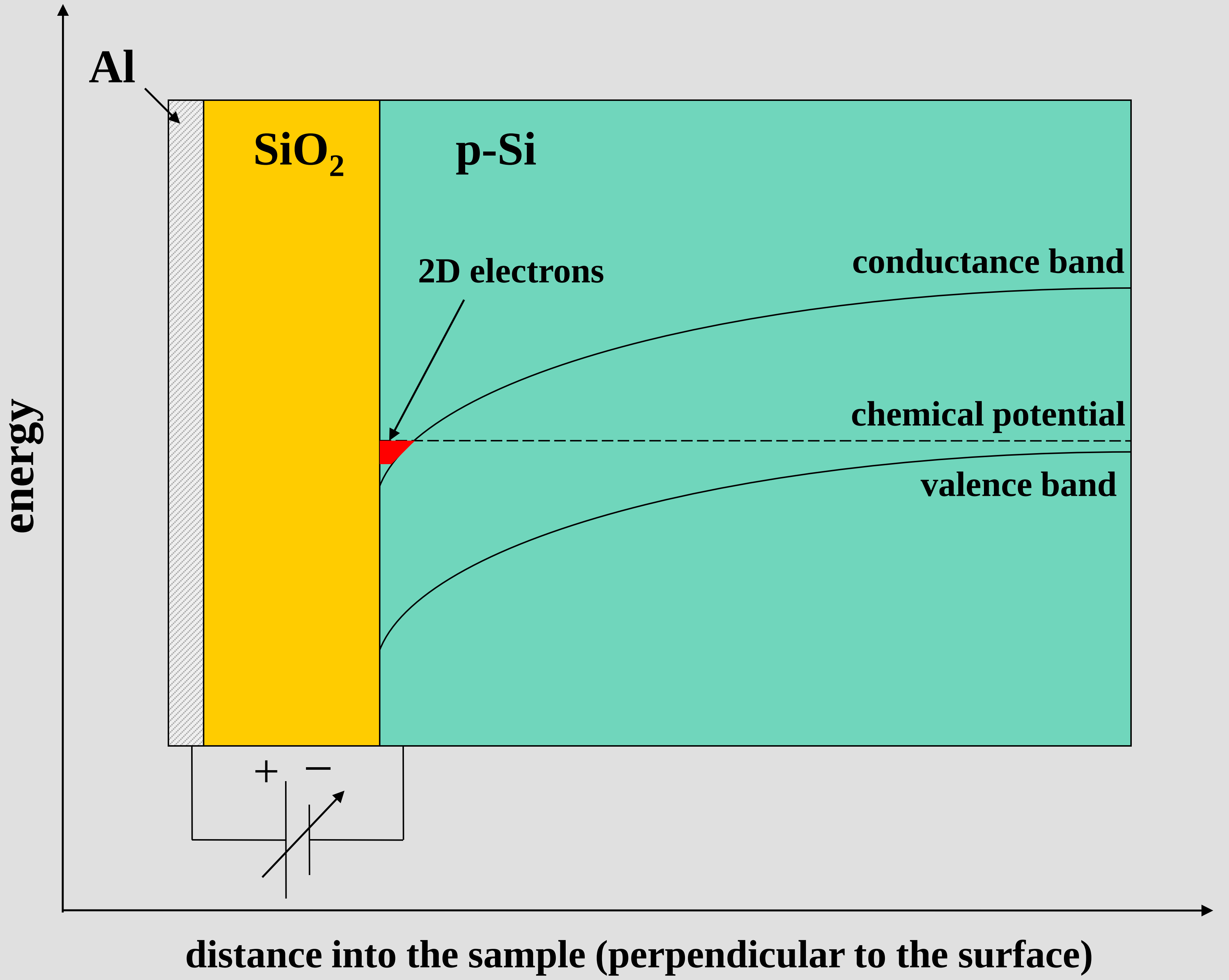
g-factor or effective mass?

Is it a Wigner crystal?

Summary


Al
SiO_2
p-Si
2D electrons
conductance band
chemical potential
valence band
energy
+
−
distance into the sample (perpendicular to the surface)

Why Si MOSFETs?

It turns out to be a very convenient 2D system to study strongly-interacting regime because of:

- Relatively large effective mass ($0.19\ m_0$)
- Two valleys in the electronic spectrum
- Low average dielectric constant ε=7.7

As a result, at low densities, Coulomb energy strongly exceeds Fermi energy: $E_C >> E_F$

$r_s = E_C / E_F$ >10 can be easily reached in clean samples.

For comparison, in n-GaAs/AlGaAs heterostructures, this would require 100 times lower electron densities. Such samples are not yet available.

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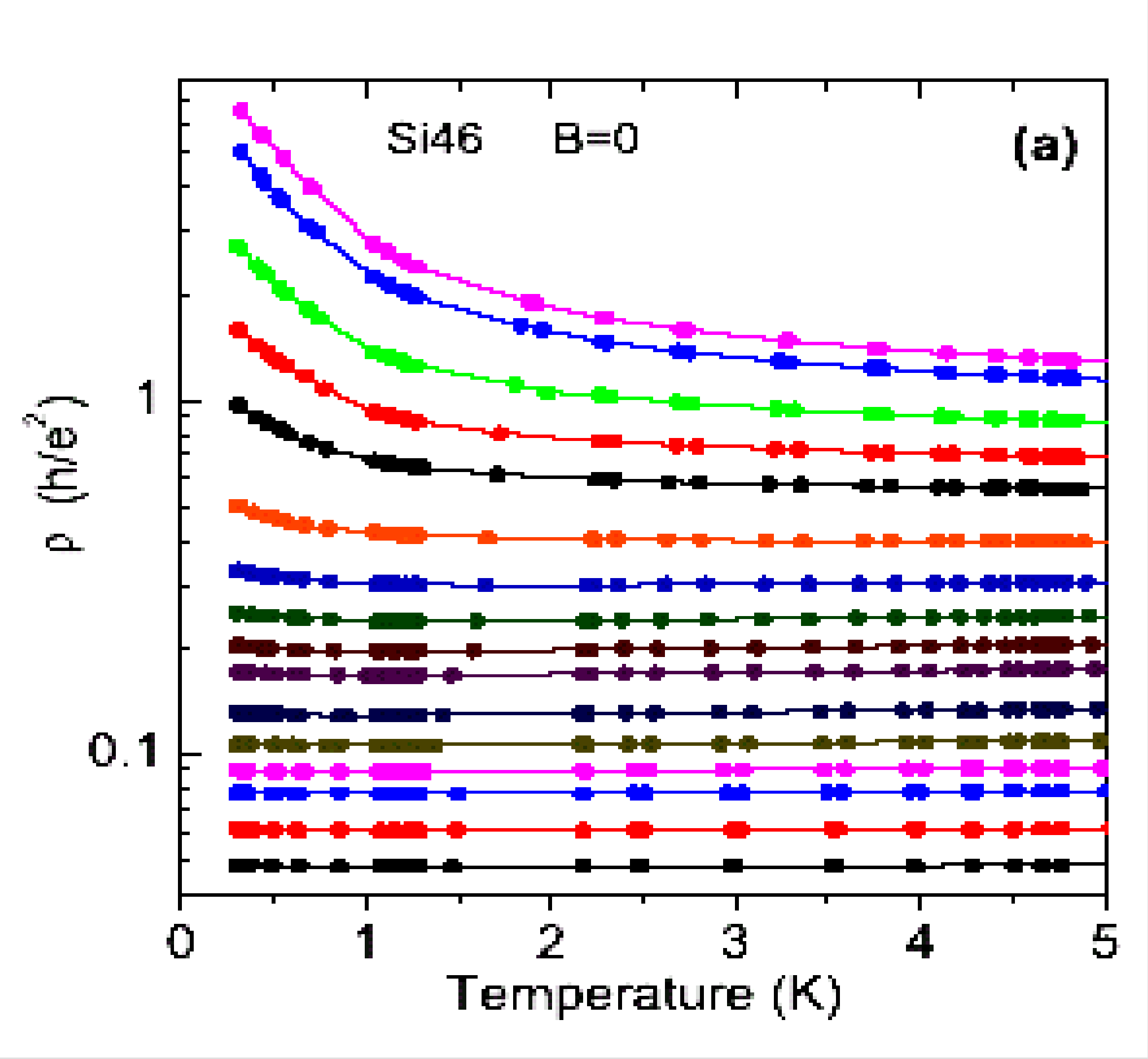
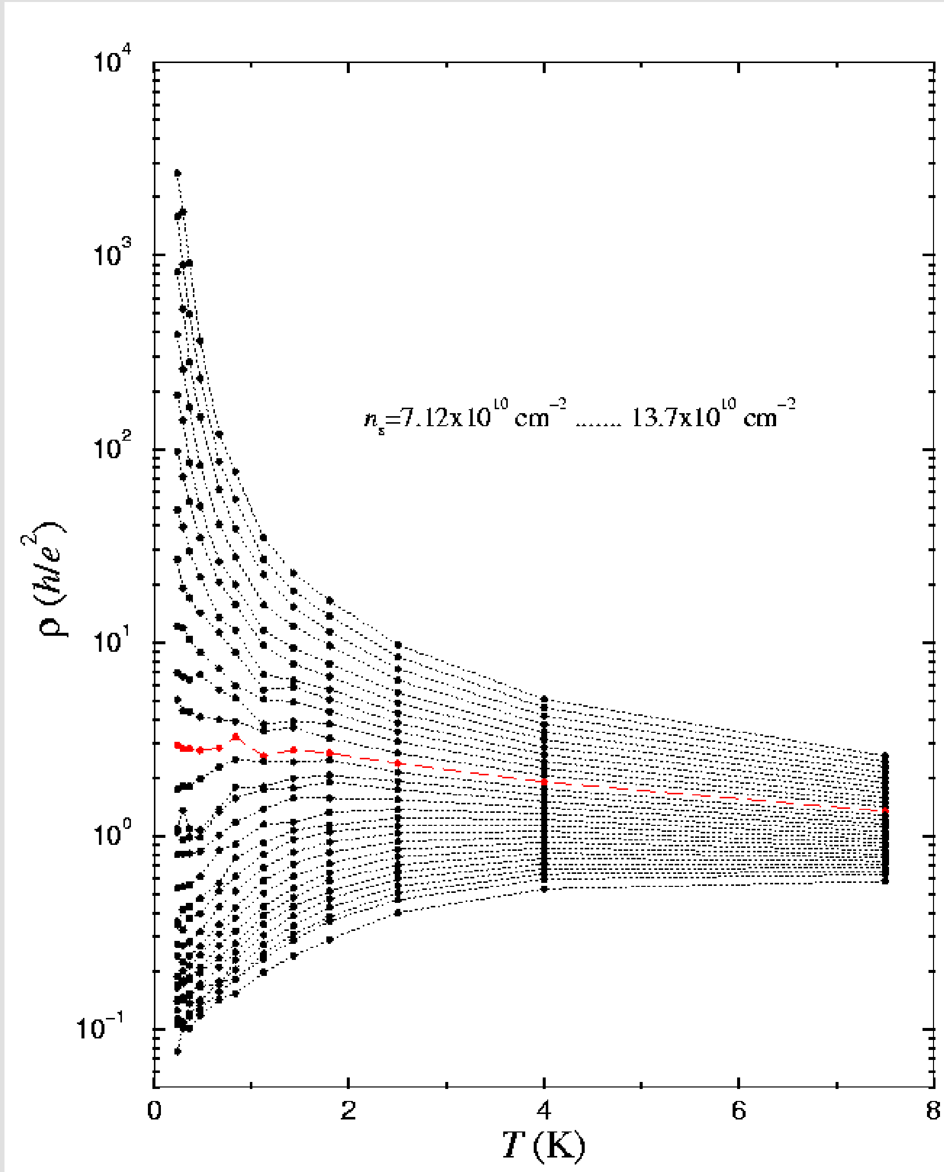
This is what it is expected to look like (weakly-interacting electrons)…



(Pudalov *et al.*)

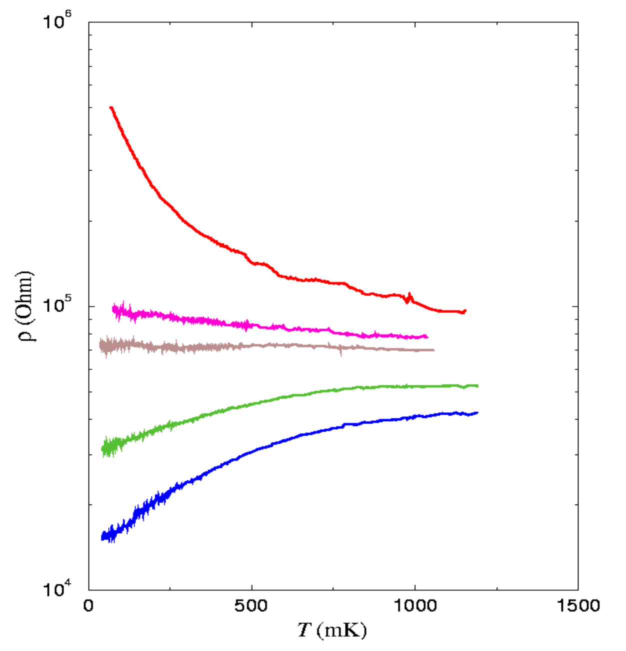
…but this is what it looks like when the electron-electron interactions are strong



S.V.K., Mason, Bowker, Furneaux, Pudalov, and D'Iorio, *PRB* 1995

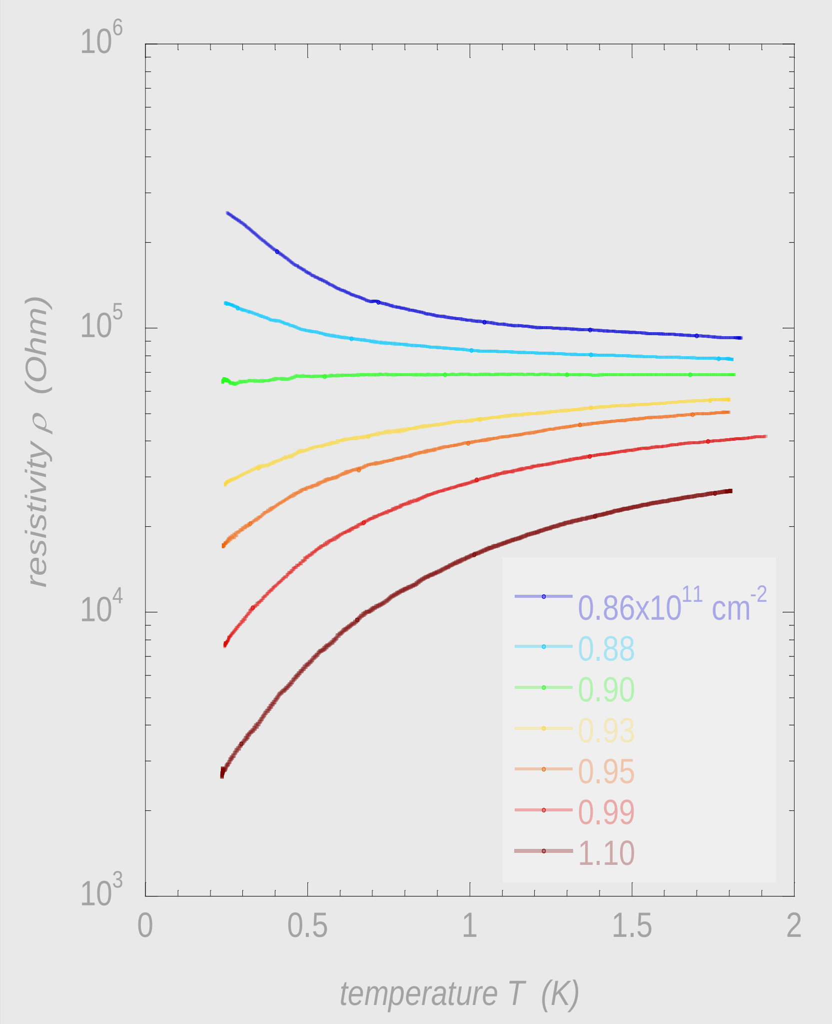
In very clean samples, the transition is practically universal:

S.V.K. and Klapwijk, *PRL* 2000



Klapwijk's sample

Sarachik and S.V.K., *PNAS* 1999



Pudalov's sample

Reaction of referees (1993):

Referee A:
“The paper should not be published in PRL. Everyone knows there is no zero-temperature conductivity in 2-d.”

Referee B:
“The reported results are most intriguing, but they must be wrong. If there indeed were a metal-insulator transition in these systems, it would have been discovered years ago.”

Referee C:
“I cannot explain the reported behavior offhand. Therefore, it must be an experimental error.”

Timeline:

1993: Metal-insulator transition in 2D is discovered.
Paper submitted to *Phys. Rev. Lett*. and rejected.
Proposal submitted to NSF and declined.

1994: Proposal submitted to NSF and declined.

1995: Proposal submitted to NSF and declined.

1996: Proposal submitted to NSF and declined.

1997: Proposal submitted to NSF and declined.

However, also in 1997….

…a similar transition has been observed in other 2D structures:

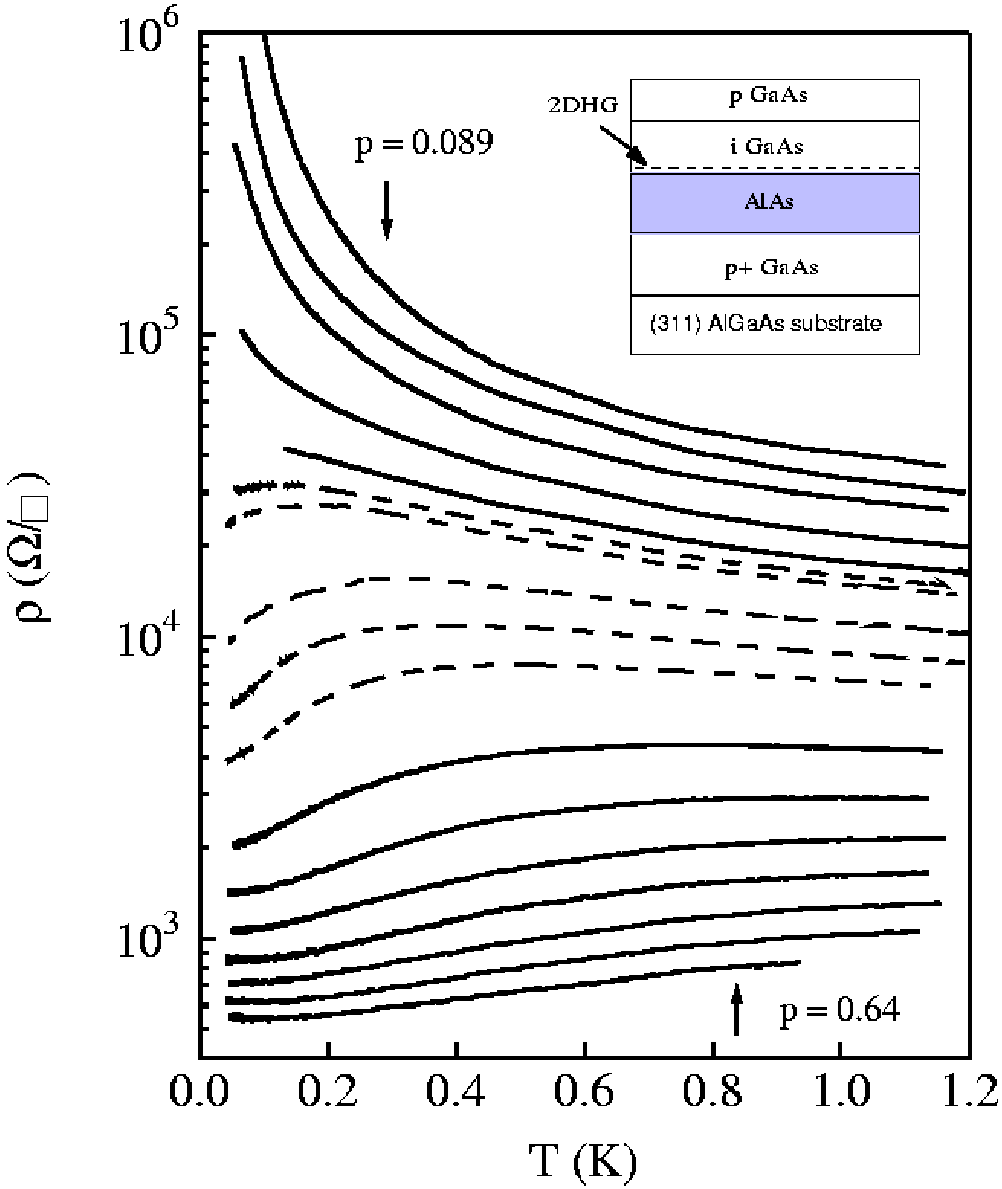
•p-Si:Ge (Coleridge's group; Ensslin's group)

•p-GaAs/AlGaAs (Tsui's group, Boebinger's group)

•n-GaAs/AlGaAs (Tsui's group, Stormer's group, Eisenstein's group)

•n-Si:Ge (Okamoto's group)

•p-AlAs (Shayegan's group)



(Hanein, Shahar, Tsui *et al.*, *PRL* 1998)

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VOLUME 44, NUMBER 19 PHYSICAL REVIEW LETTERS 12 MAY 1980

Interaction Effects in Disordered Fermi Systems in Two Dimensions

B. L. Altshuler and A. G. Aronov
Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188 350, U.S.S.R.

and

P. A. Lee
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 11 February 1980)

Interaction effects in disordered Fermi systems are considered in the metallic regime. In two dimensions, logarithmic corrections are obtained for conductivity, density of states, specific heat, and Hall constant. These results are compared with a recent theory of localization as well as some experiments.

$$\delta\sigma = (e^2/4\pi^2\hbar)(2-2F)\ln(T\tau)$$

- always insulating behavior

Zeitschrift fur Physik B (Condensed Matter) -- 1984 -- vol.56, no.3, pp. 189-96

Weak localization and Coulomb interaction in disordered systems

Finkel'stein, A.M.
L.D. Landau Inst. for Theoretical Phys., Acad. of Sci., Moscow, USSR

$$\delta\sigma = \frac{e^2}{2\pi^2\hbar}\cdot\ln(T\tau)\cdot\left[1+3\cdot\left(1-\frac{\ln\left(1+F_0^\sigma\right)}{F_0^\sigma}\right)\right]$$

Altshuler-Aronov-Lee's result

Finkelstein's term

- Insulating behavior when interactions are weak
- Metallic behavior when interactions are strong
- Effective strength of interactions grows as the temperature decreases

7/17/2015

More recent development: two-loop RG theory

disorder takes over

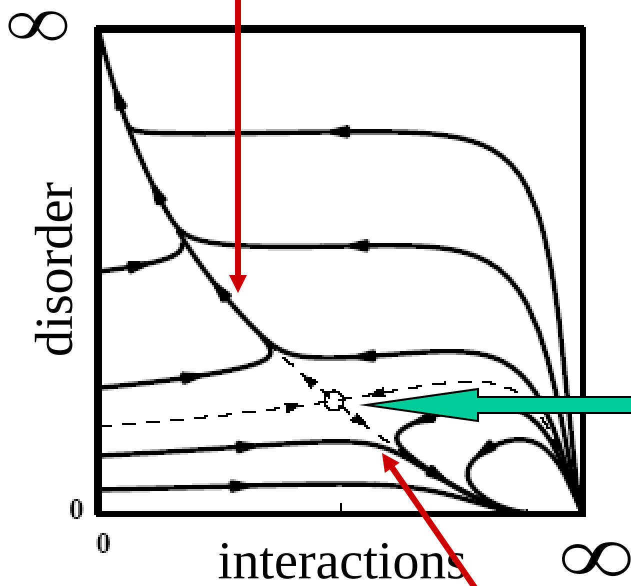
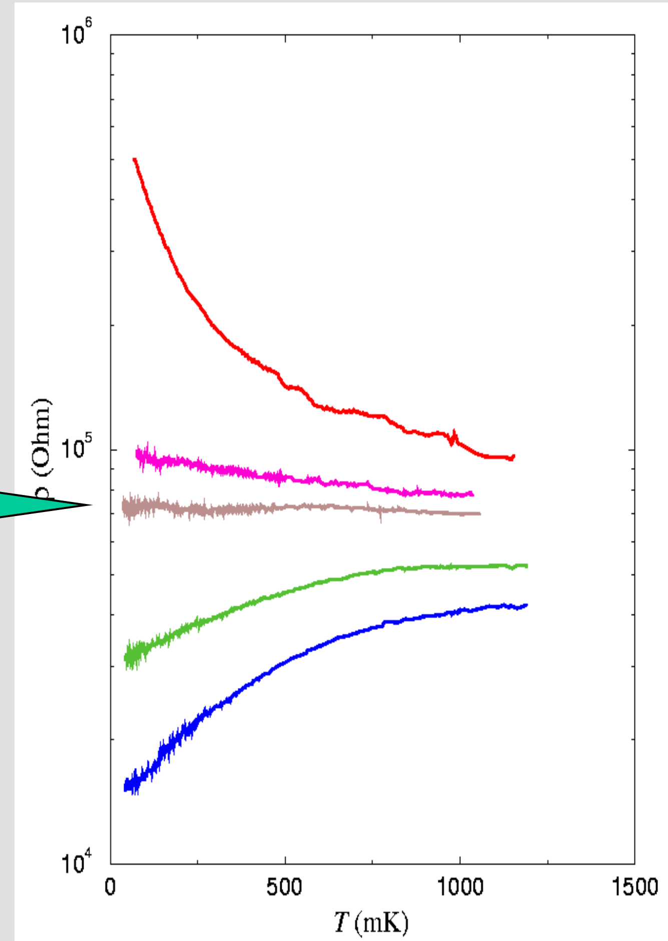


FIG. 1: The disorder-interaction, t-Θ, flow diagram of the 2D electron gas obtained by solving Eqs. (4) and (5) with the Cooper channel included ($\alpha = 1$). Arrows indicate the direction of the flow as the temperature is lowered. The circle denotes the quantum critical point of the metal-insulator transition, and the dashed lines show the separatrices.

QCP



Punnoose and Finkelstein, *Science* **310**, 289 (2005)

metallic phase stabilized by *e-e* interaction

7/17/2015

Experimental test

First, one needs to ensure that the system is in the diffusive regime ($T\tau < 1$).

One can distinguish between diffusive and ballistic regimes by studying magnetoconductance:

$$\Delta\sigma(B,T) \propto \left(\frac{B}{T}\right)^2$$ - diffusive: low temperatures, higher disorder ($Tt < 1$).

$$\Delta\sigma(B,T) \propto \frac{B^2}{T}$$ - ballistic: low disorder, higher temperatures ($Tt > 1$).

The exact formula for magnetoconductance (Lee and Ramakrishnan, 1982):

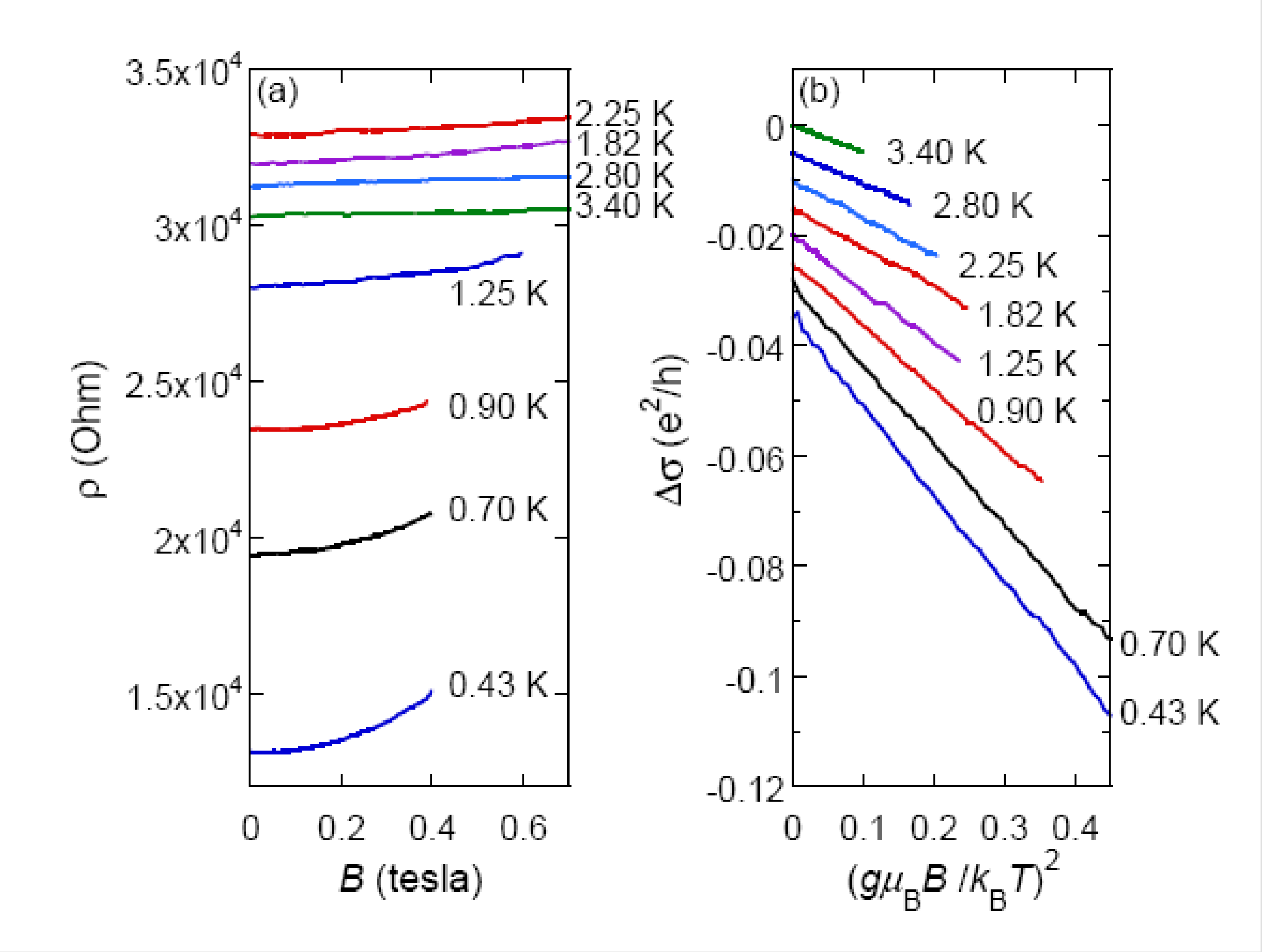
$$\Delta\sigma(B,T) = -4\left[\frac{0.091e^2}{\pi \cdot h}\right] \cdot \gamma_2(\gamma_2 + 1) \cdot \left(\frac{g\mu_B}{k_B}\right)^2 \left(\frac{B}{T}\right)^2$$

2 valleys

for $$\left(\frac{g\mu_B B}{k_B T}\right)^2 << 1$$

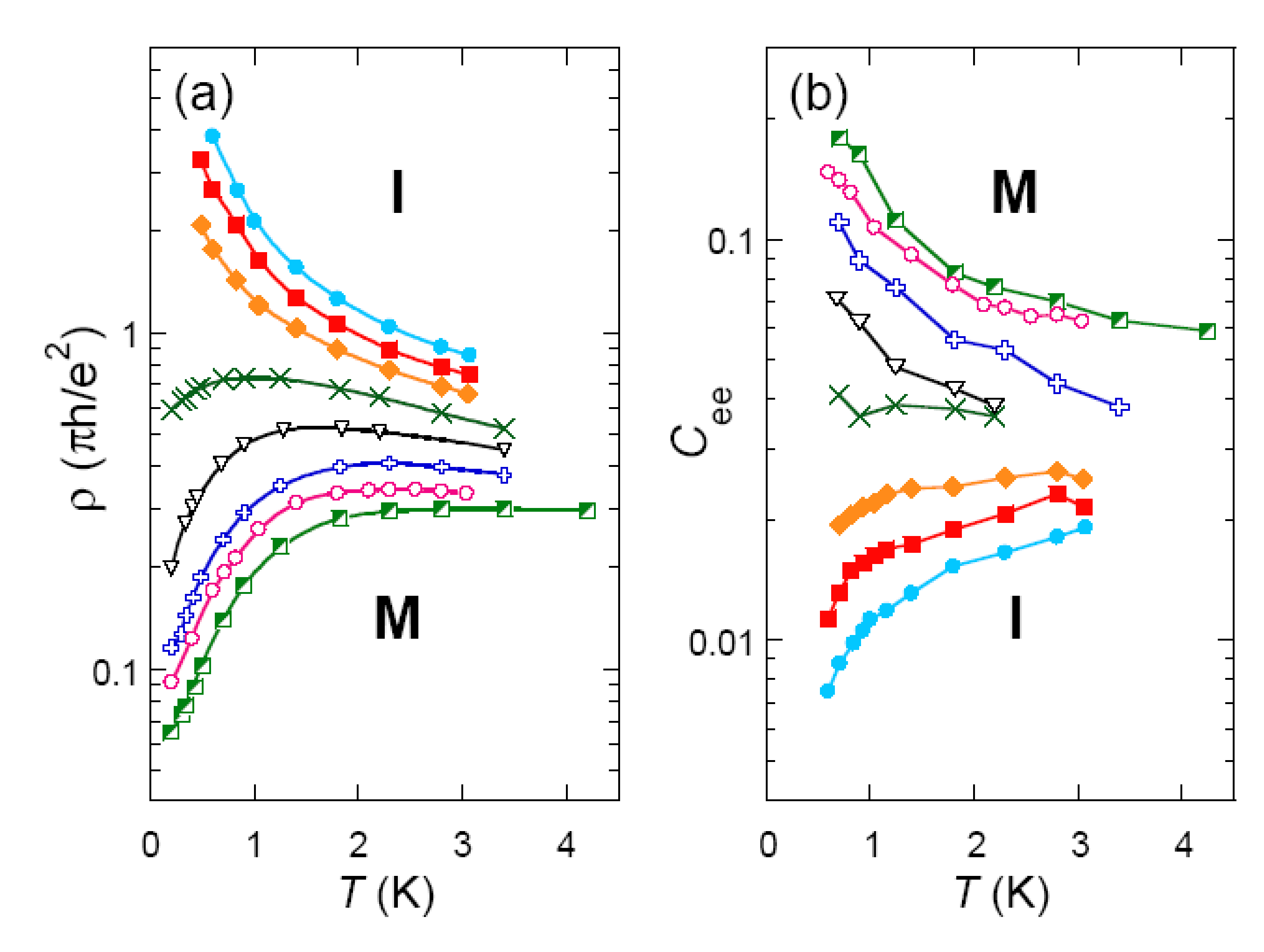
In standard Fermi-liquid notations, $$\gamma_2 = -\frac{F_0{}^a}{1 + F_0{}^a}$$

Experimental results (low-disordered Si MOSFETs; "just metallic" regime; $n_s = 9.14 \times 10^{10}$ cm^{-2}):



S. Anissimova *et al*., *Nature Phys*. 3, 707 (2007)

Temperature dependences of the resistance (a) and strength of interactions (b)



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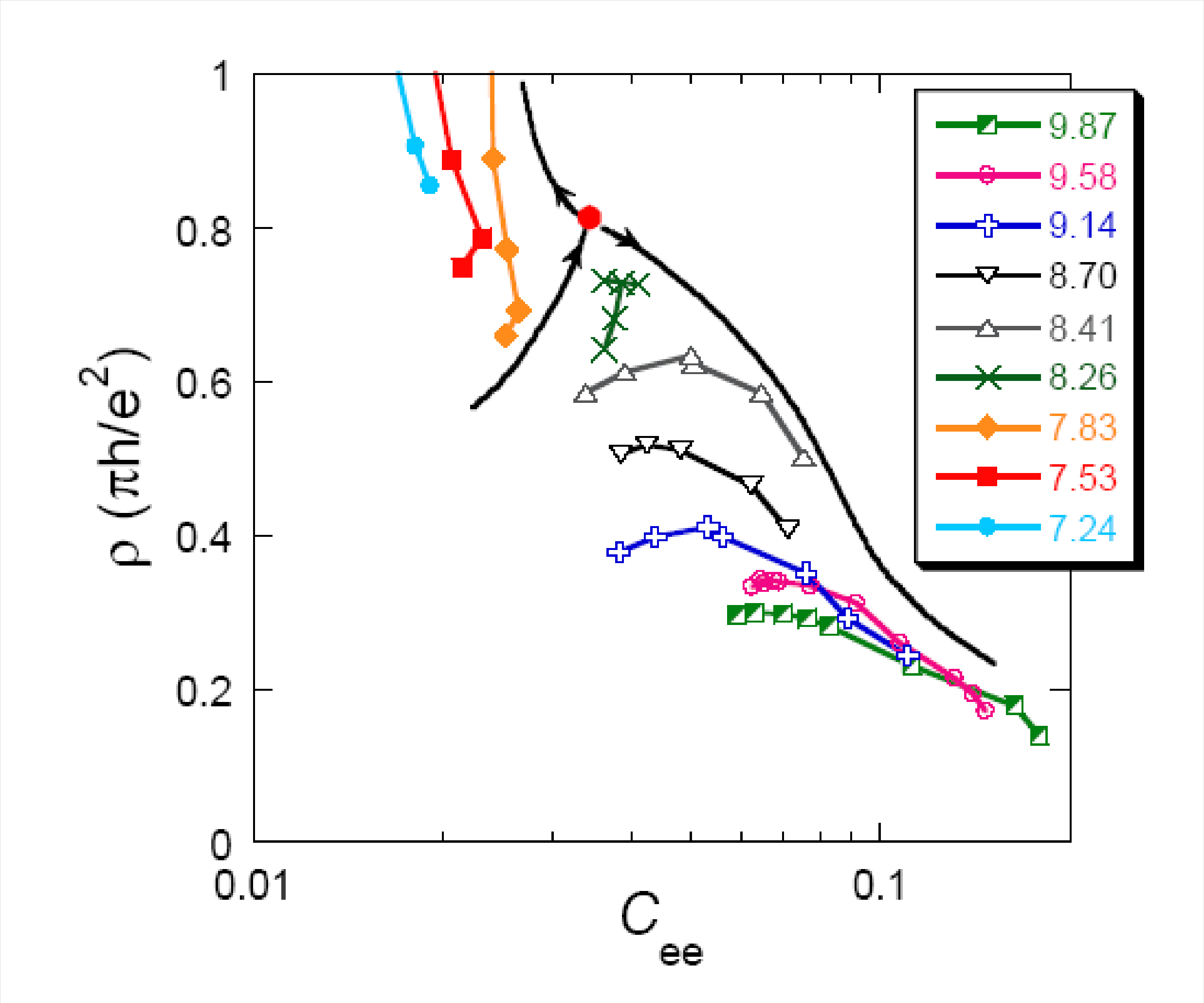
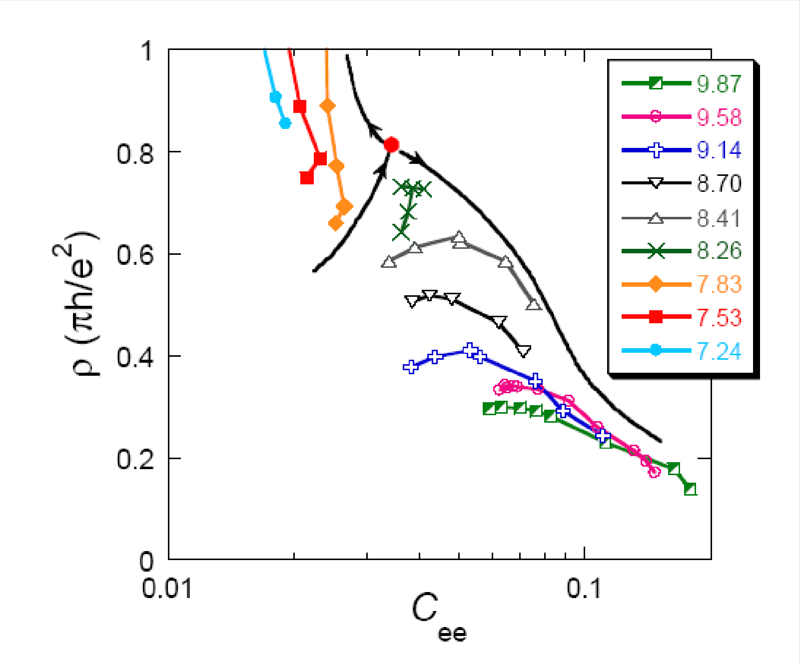
Experimental disorder-interaction flow diagram of the 2D electron liquid

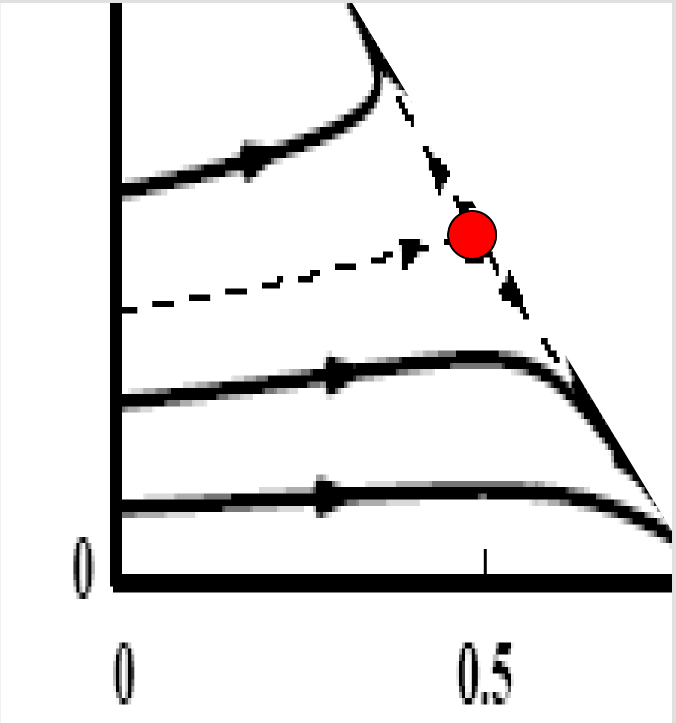


S. Anissimova *et al.*, *Nature Phys*. 3, 707 (2007)

Experimental *vs*. theoretical flow diagram

(qualitative comparison b/c the 2-loop theory was developed for multi-valley systems)





S. Anissimova *et al*., *Nature Phys*. 3, 707 (2007)

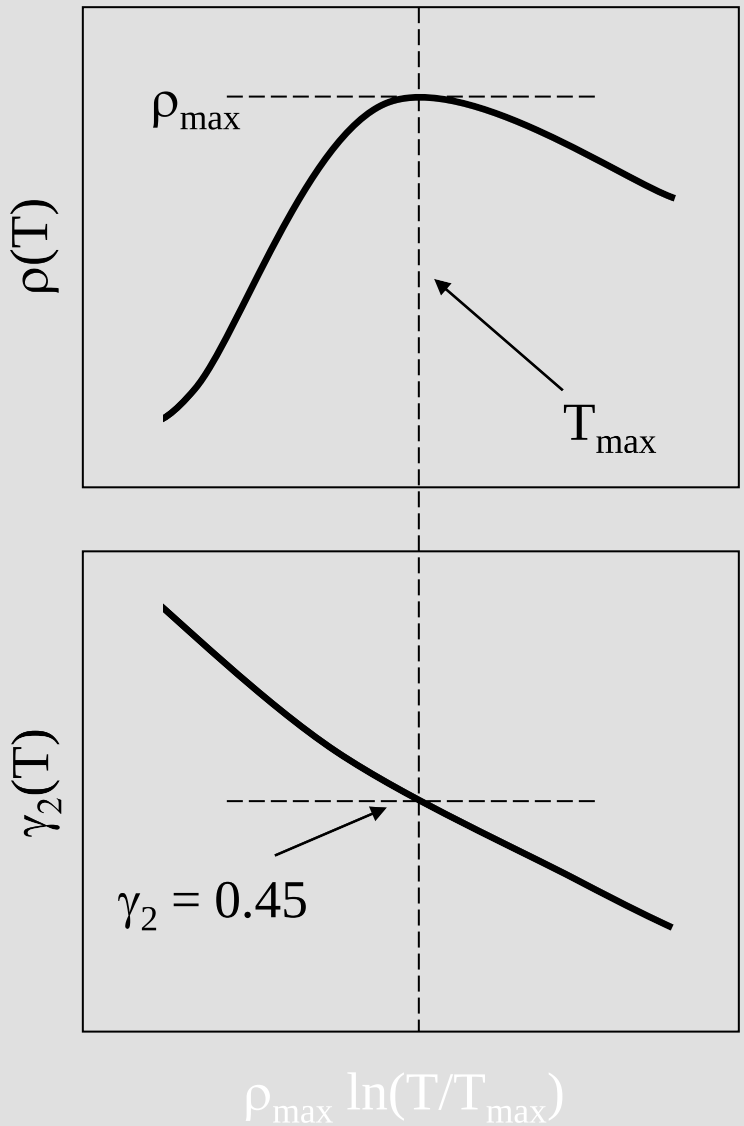
Quantitative predictions of the one-loop RG for 2-valley systems

(Punnoose and Finkelstein, *Phys. Rev. Lett*. 2002)

Solutions of the RG-equations for $\rho << \pi h/e^2$:
a series of non-monotonic curves $\rho(T)$. After rescaling, the solutions are described by a *single universal curve:*

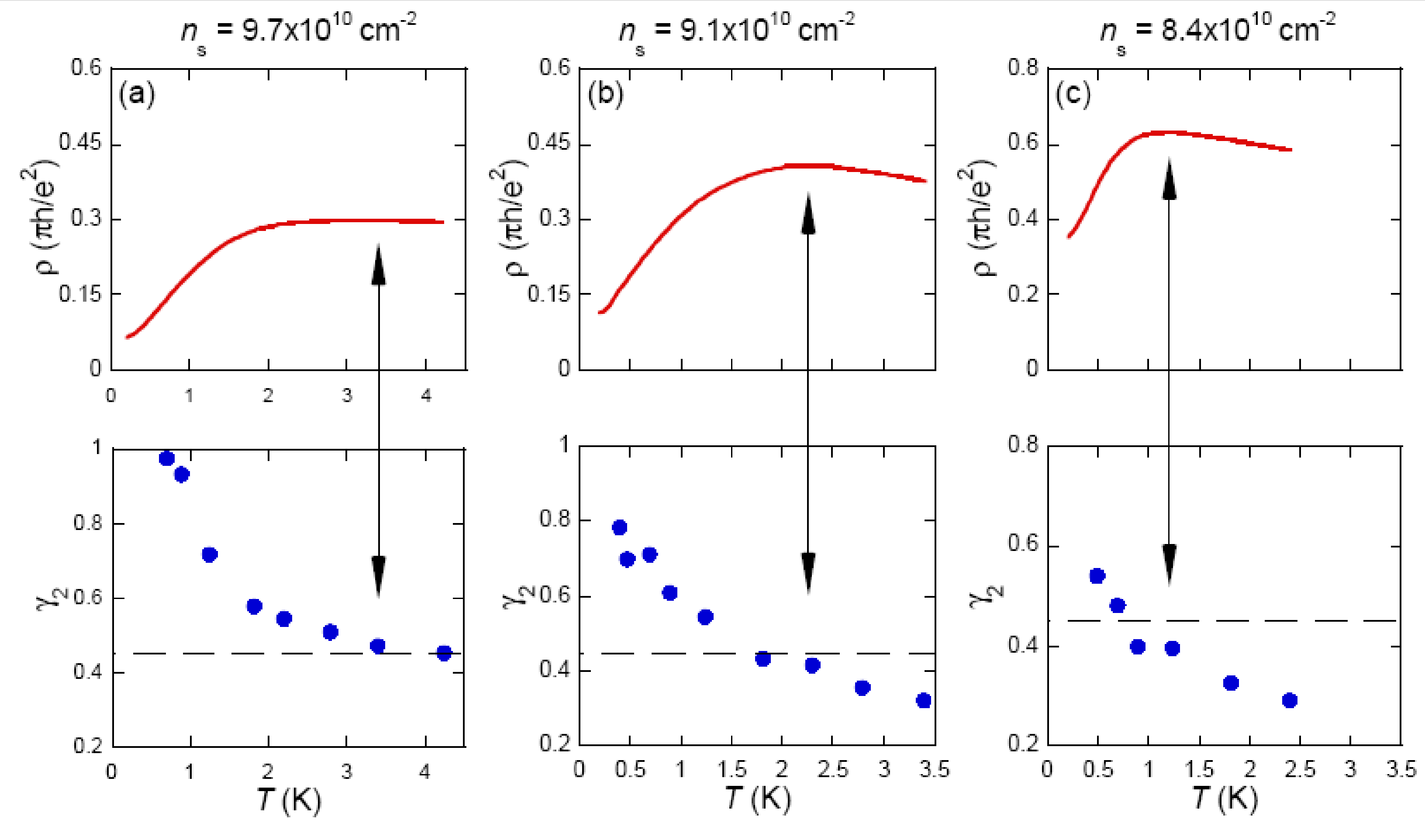
$$\rho(T) = \rho_{max} R(\eta)$$

$$\eta = \rho_{max} \ln(T_{max}/T)$$



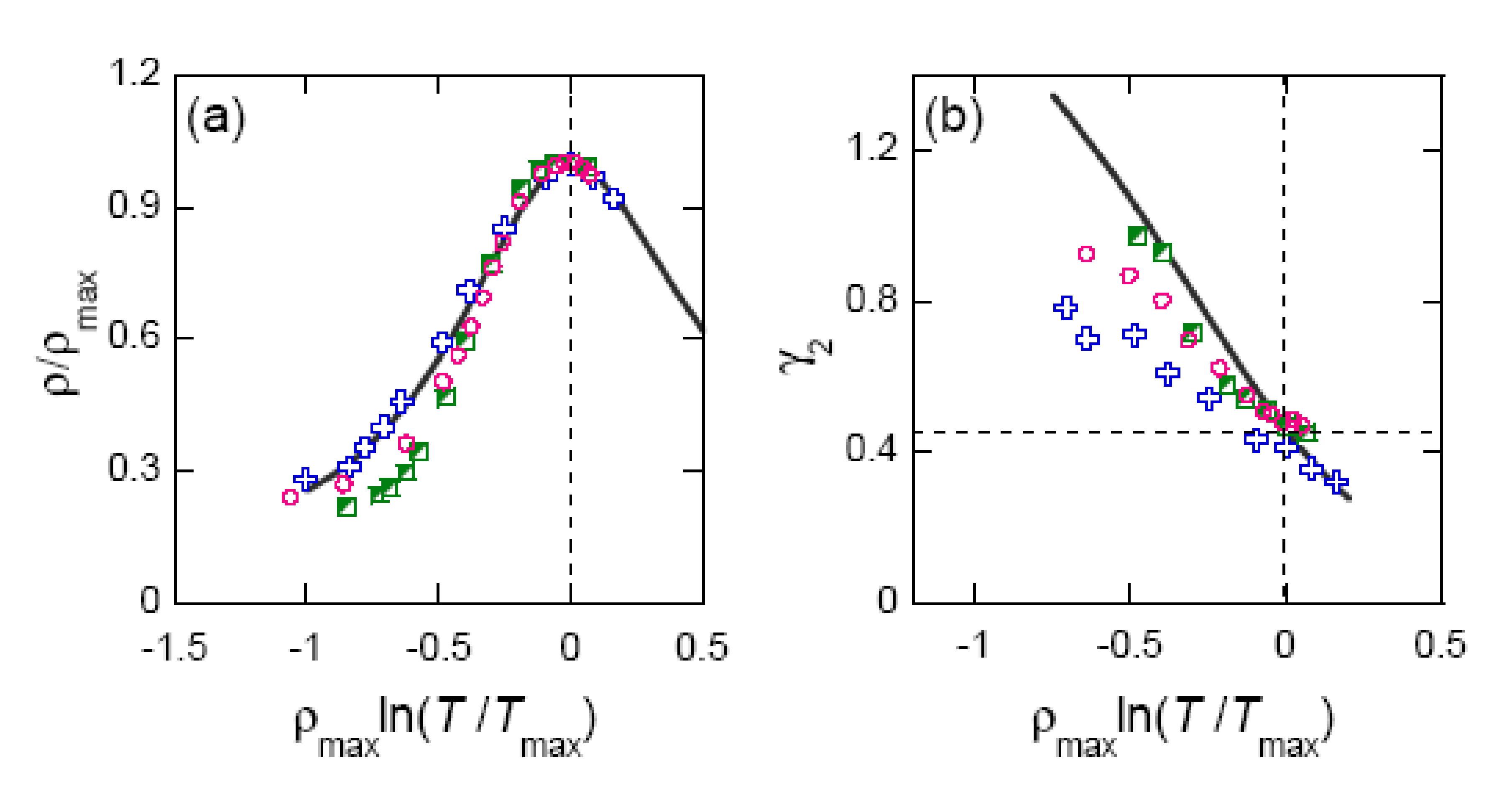
For a 2-valley system (like Si MOSFET), metallic $\rho(T)$ sets in when $\gamma_2 > 0.45$

Resistance and interactions *vs. T*



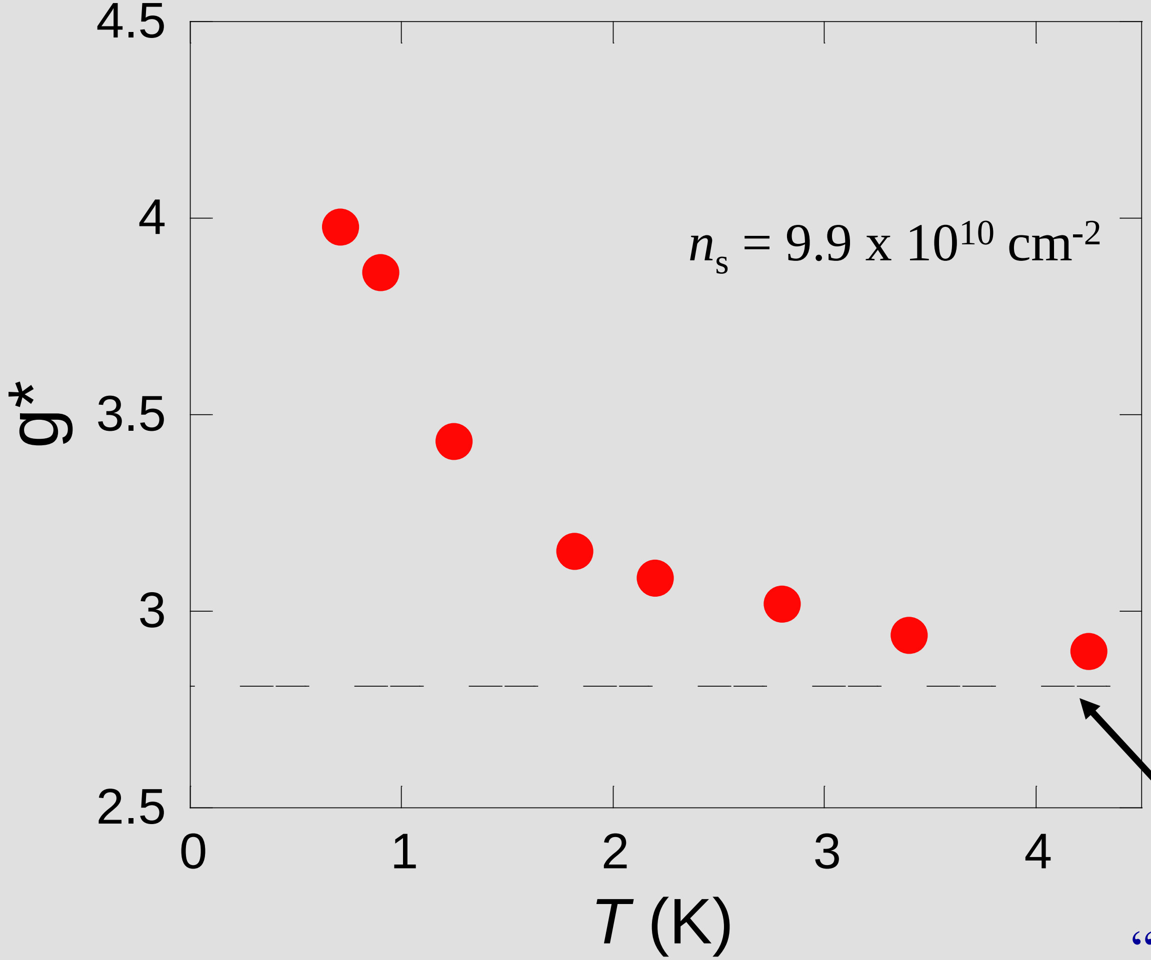
Note that the metallic behavior sets in when $\gamma_2 \sim 0.45$, exactly as predicted by the RG theory

Comparison between theory (lines) and experiment (symbols) (no adjustable parameters used!)



S. Anissimova *et al.*, *Nature Phys*. 3, 707 (2007)

g-factor grows as T decreases



$$g^* = 2(1+\gamma_2)$$

"ballistic" value

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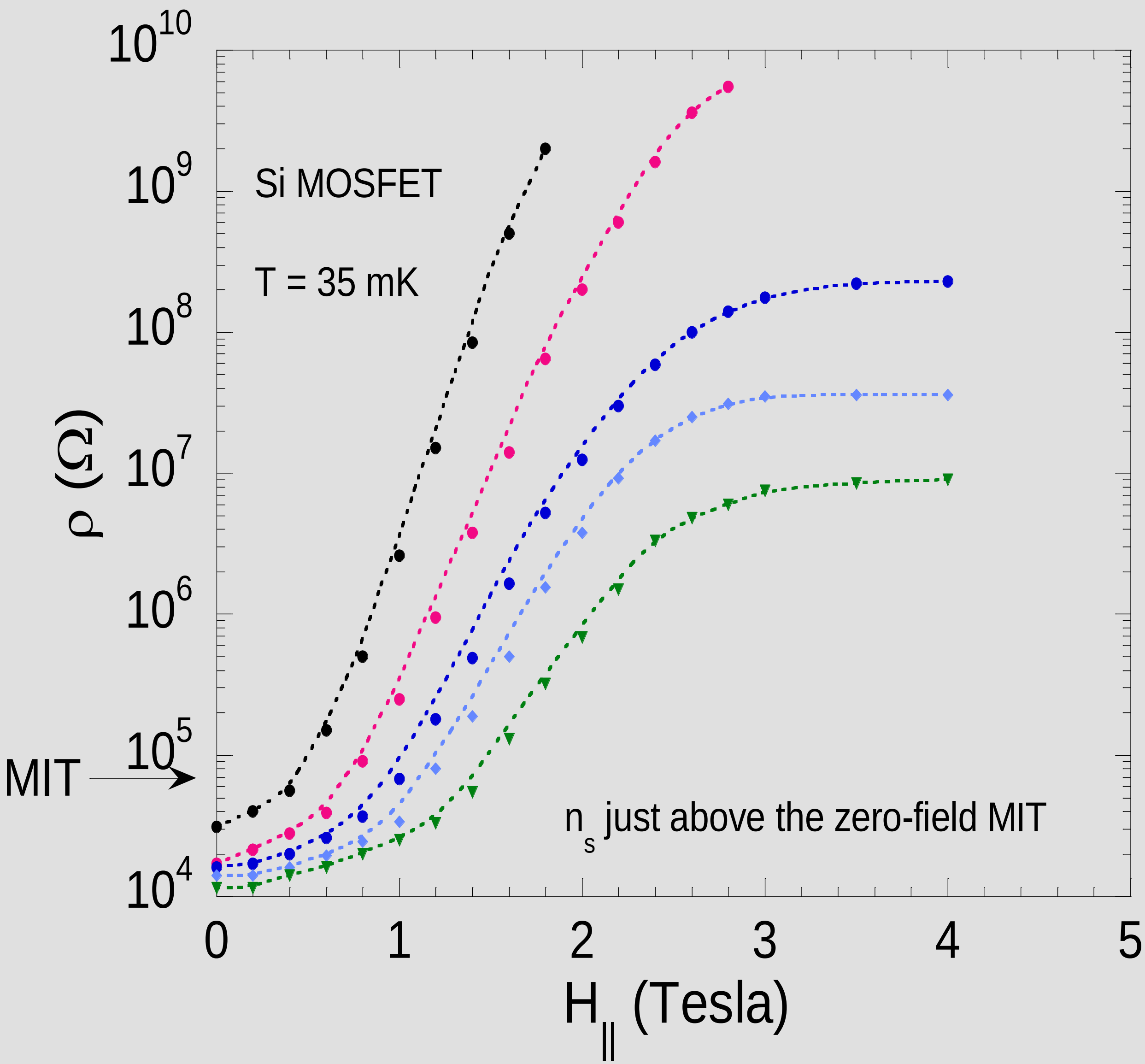
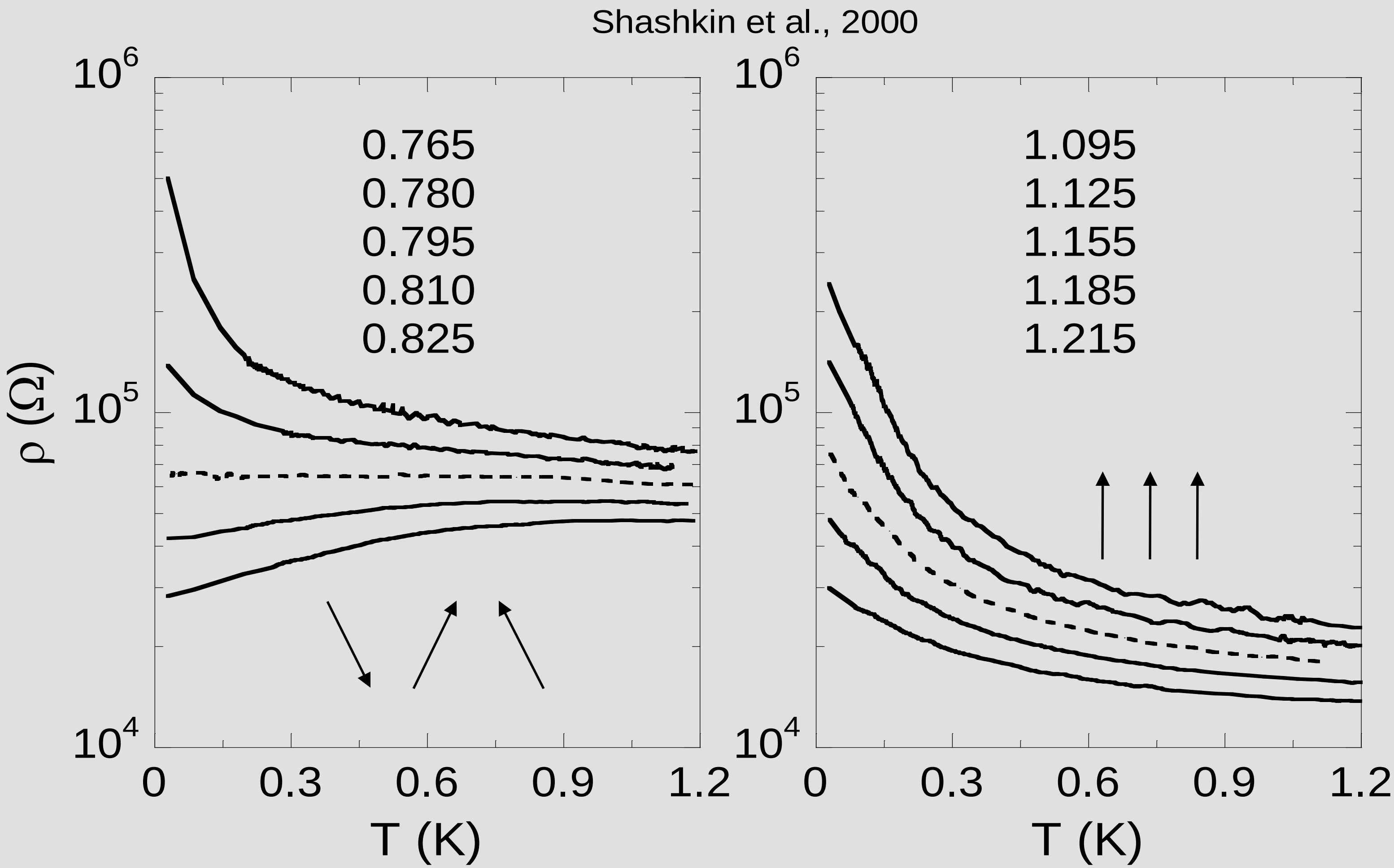
Summary

The effect of the *parallel* magnetic field:

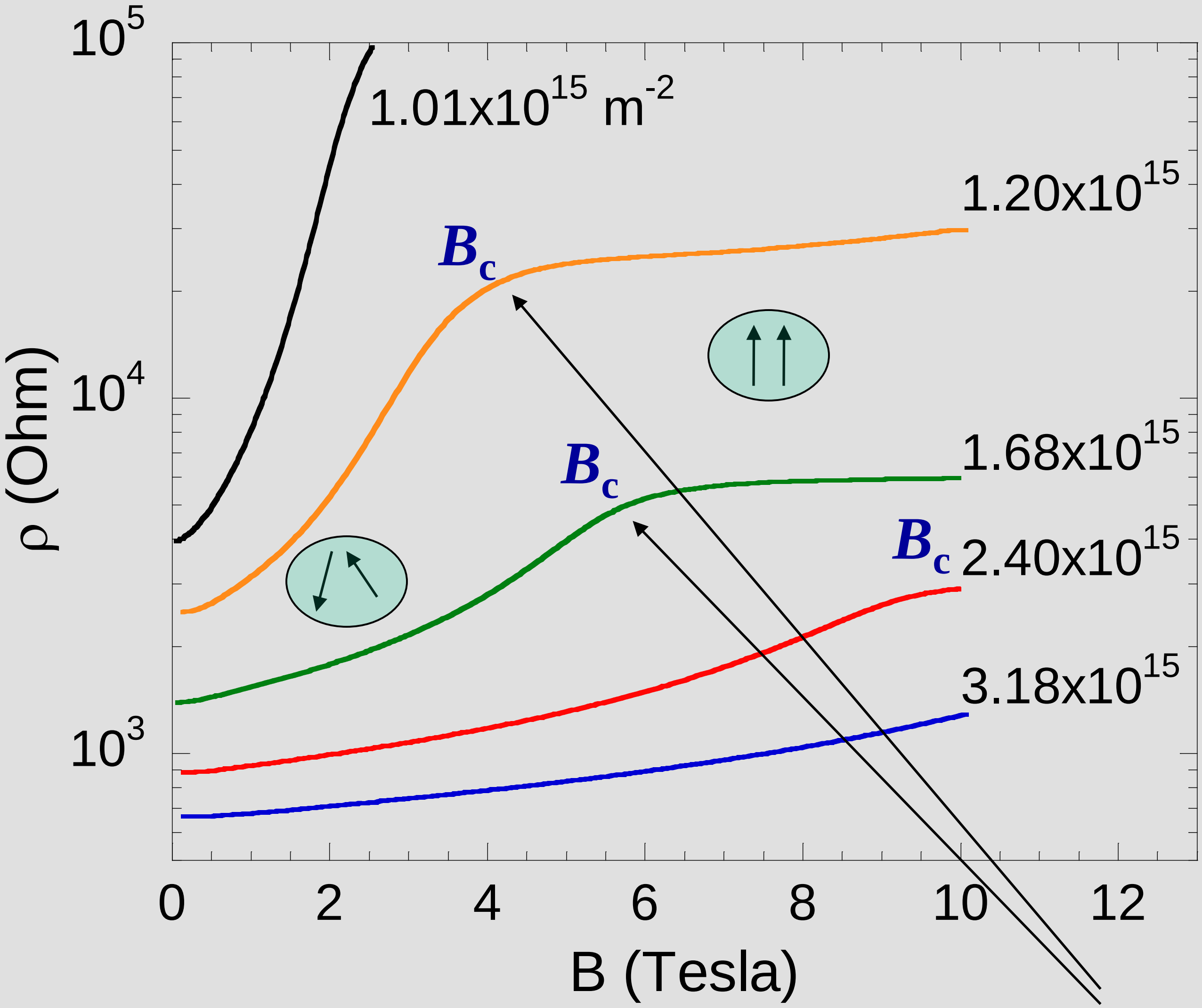


Magnetic field, by aligning spins, changes metallic R(T) to insulating:



Such a dramatic reaction on parallel magnetic field suggests unusual spin properties

Magnetoresistance in a parallel magnetic field

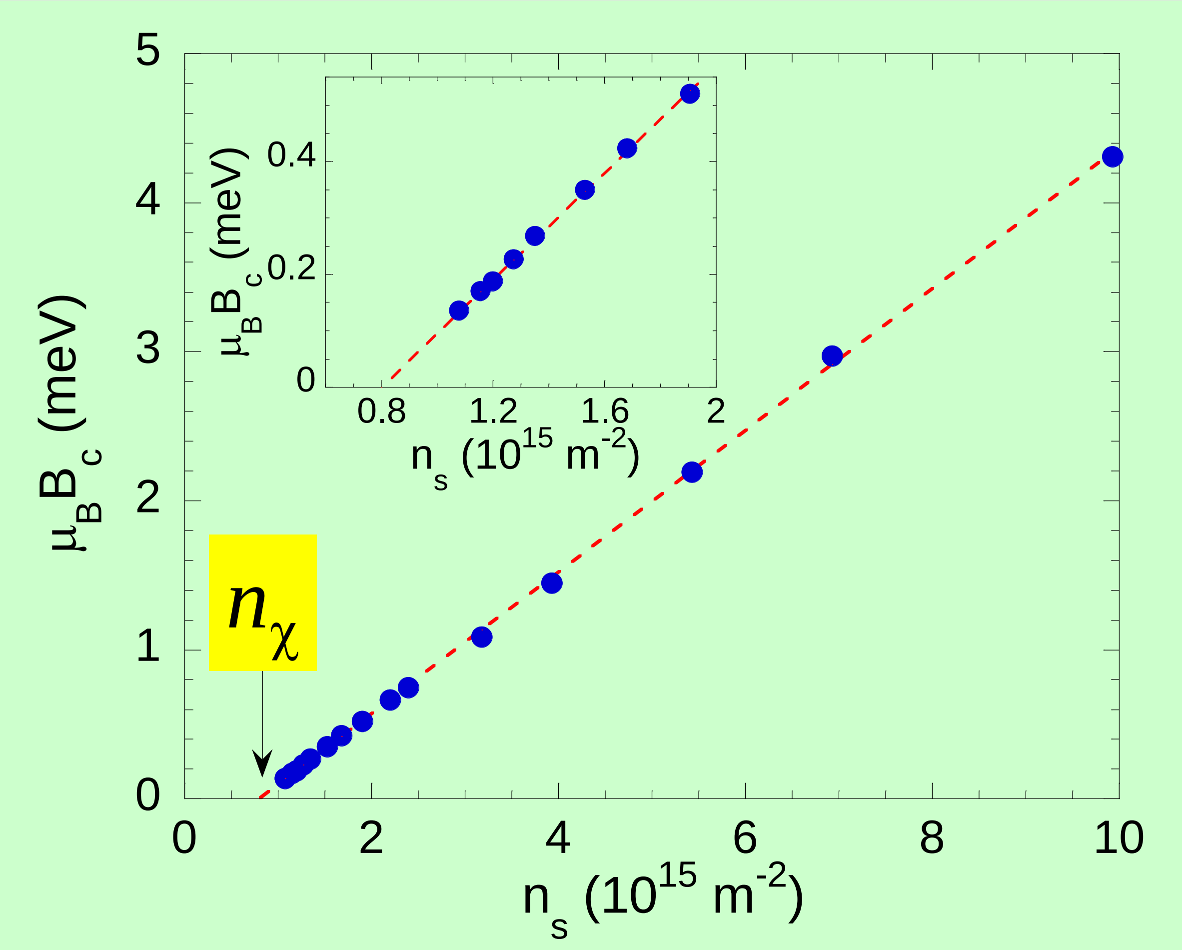


T = 30 mK

Shashkin, Kravchenko, Dolgopolov, and Klapwijk, *PRL* 2001

Spins become fully polarized
(Okamoto et al., *PRL* 1999; Vitkalov et al., *PRL* 2000)

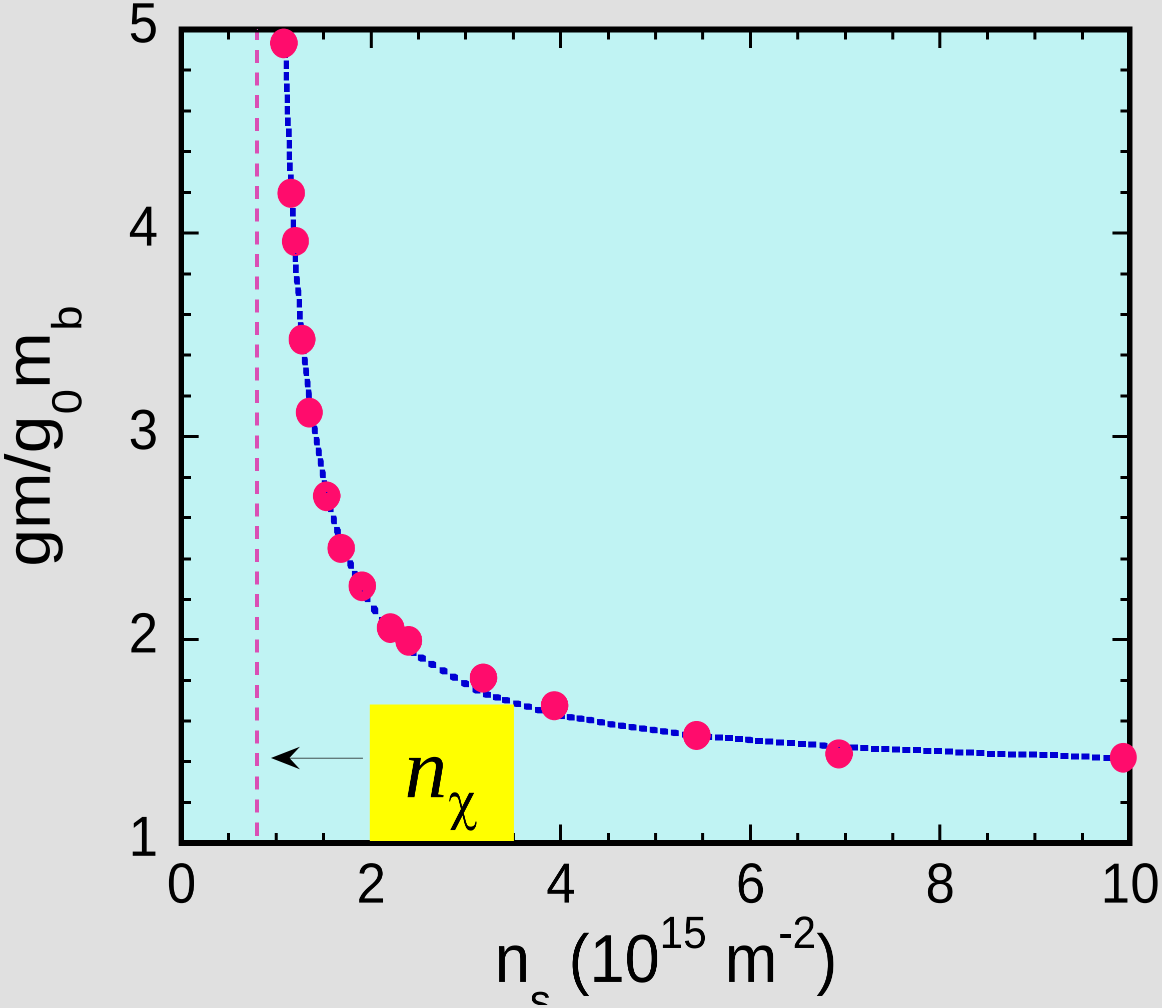
Extrapolated polarization field, B_c, vanishes at a finite electron density, n_χ



Shashkin, S.V.K., Dolgopolov, and Klapwijk, *PRL* 2001

Spontaneous spin polarization at n_χ?

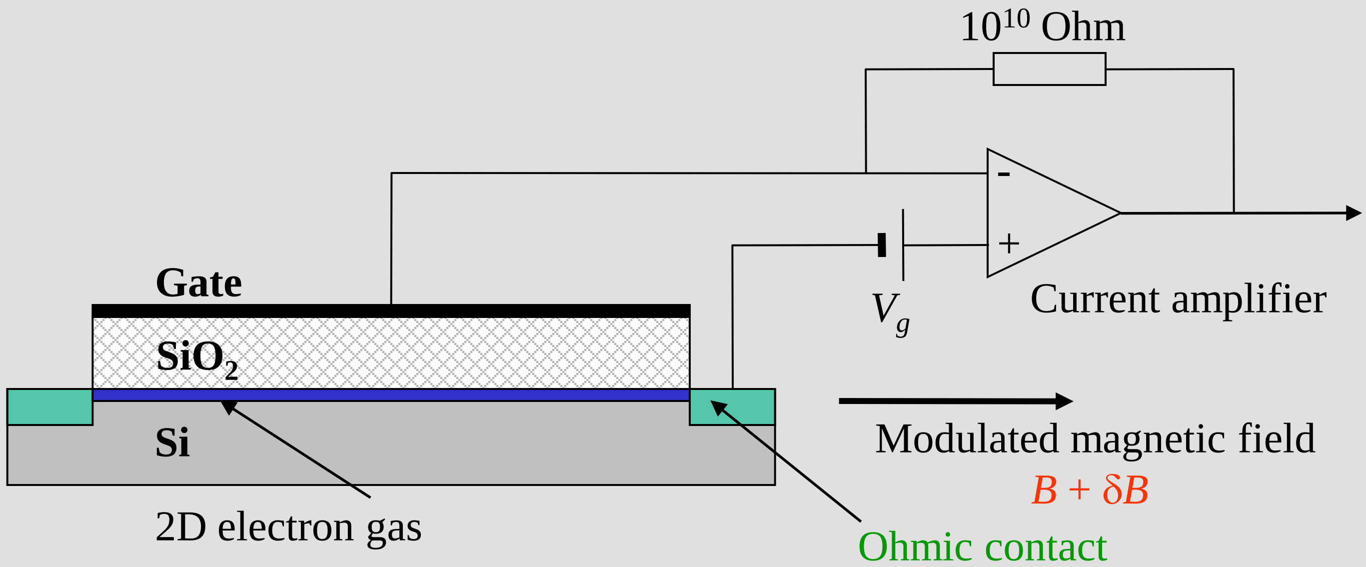
$\chi \sim gm$ as a function of electron density calculated using $g^*m^* = \pi\hbar^2 n_s / B_c\mu_B$



(Shashkin et al., *PRL* 2001)

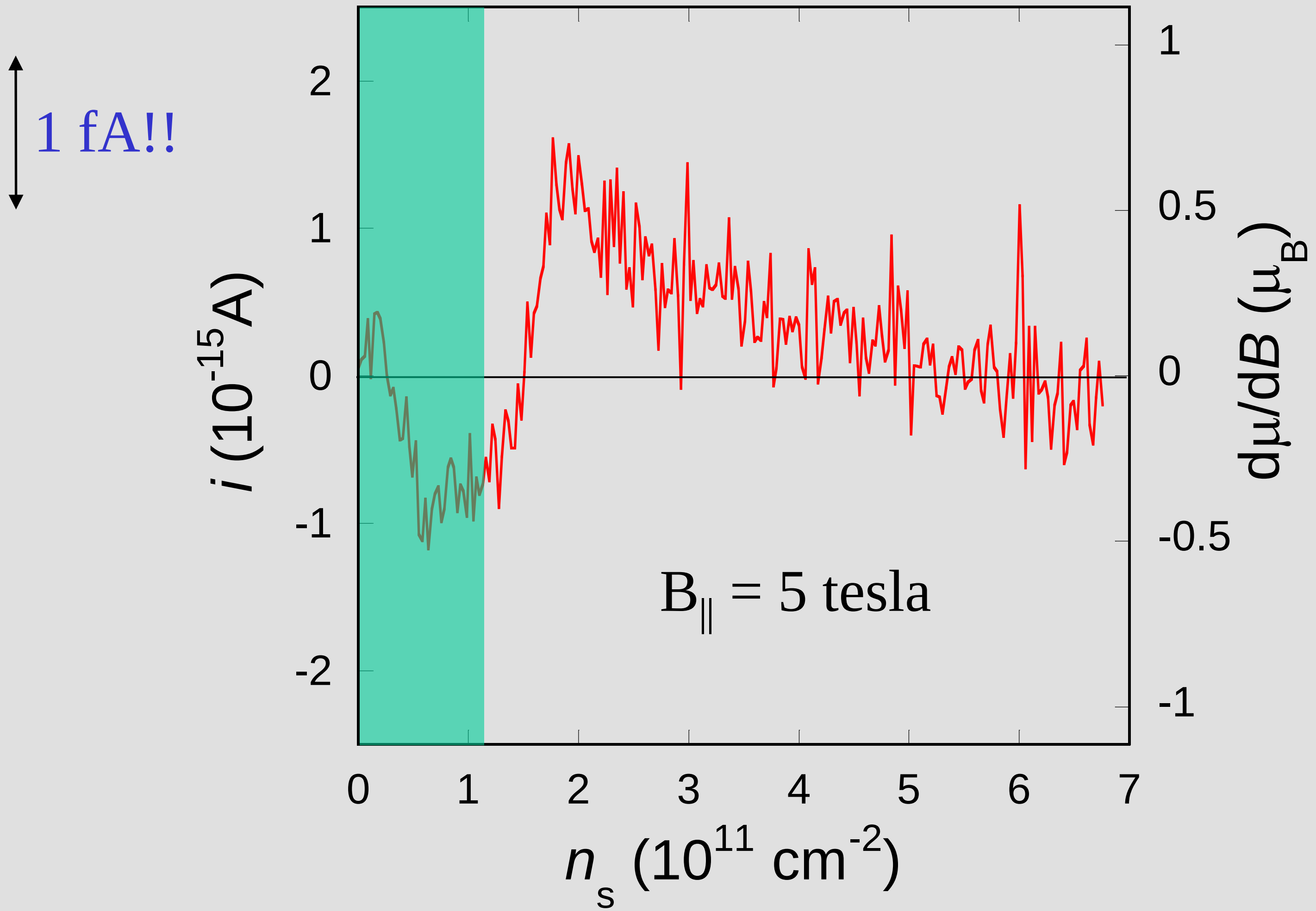
Magnetic measurements without magnetometer

suggested by B. I. Halperin (1998); first implemented by O. Prus, M. Reznikov, U. Sivan *et al.* (2002)



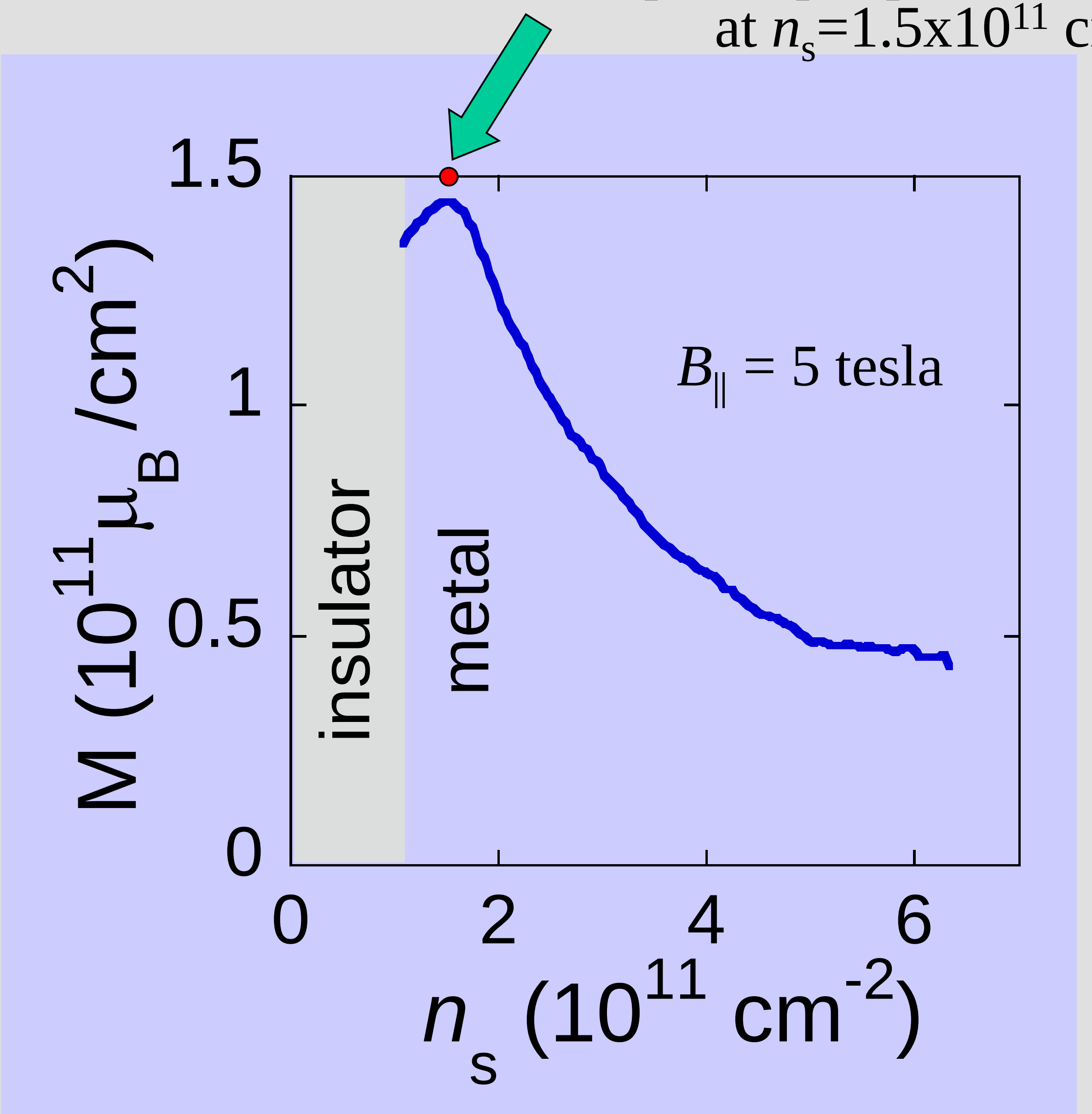
$$i \propto d\mu/dB = -\, dM/dn_s$$

dμ/dB = - dM/dn


1 fA!!
2
1
0
-1
-2
i (10^-15 A)
1
0.5
0
-0.5
-1
dμ/dB (μB)
B|| = 5 tesla
0
1
2
3
4
5
6
7
ns (10^11 cm^-2)

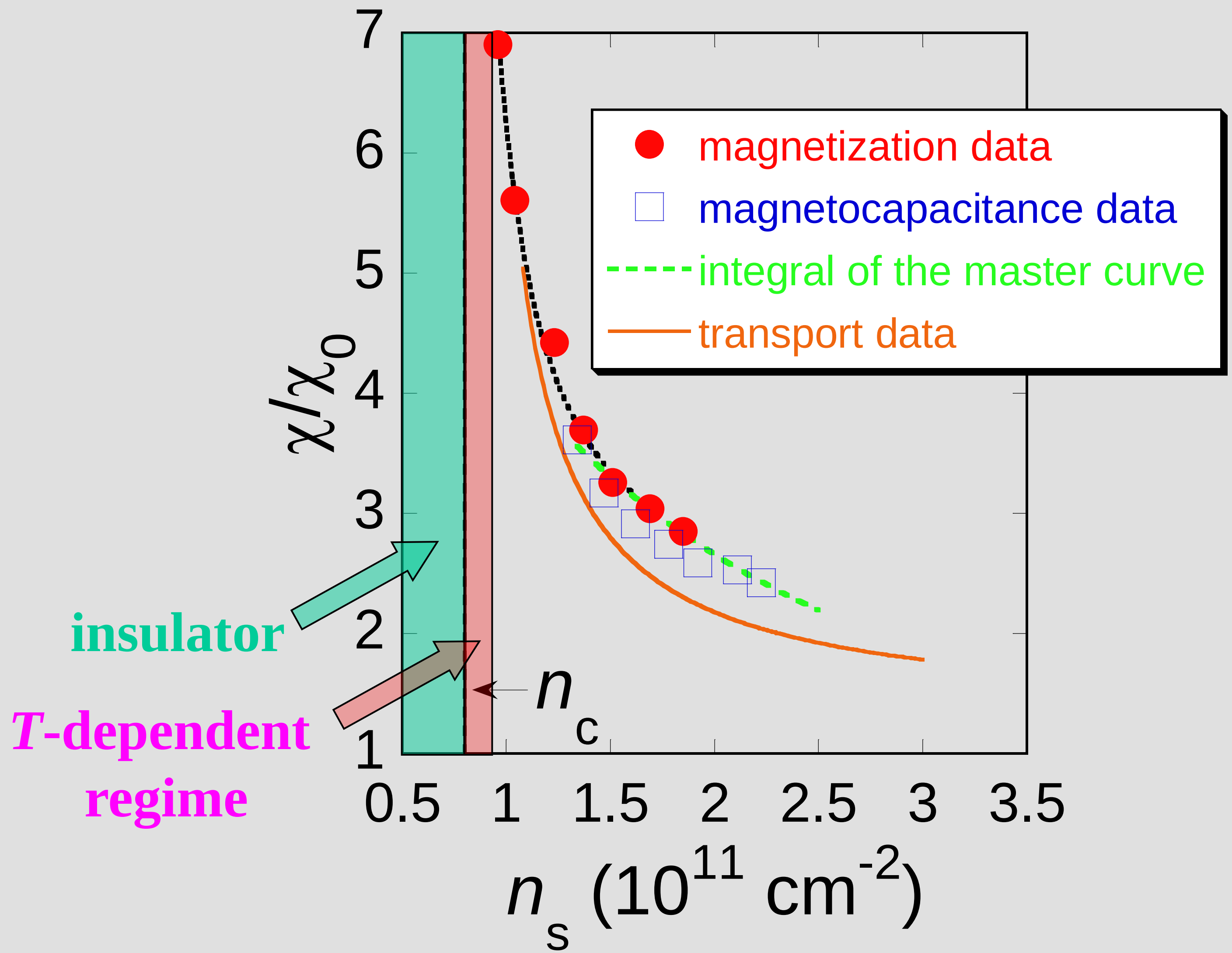
Integral of the previous slide gives $M\ (n_s)$:

complete spin polarization
at n_s=1.5x10^{11} cm^{-2}



Spin susceptibility exhibits critical behavior near the sample-independent critical density n_χ : $\chi \propto n_s/(n_s - n_\chi)$

Critical behavior of a thermodynamic parameter suggests a phase transition!



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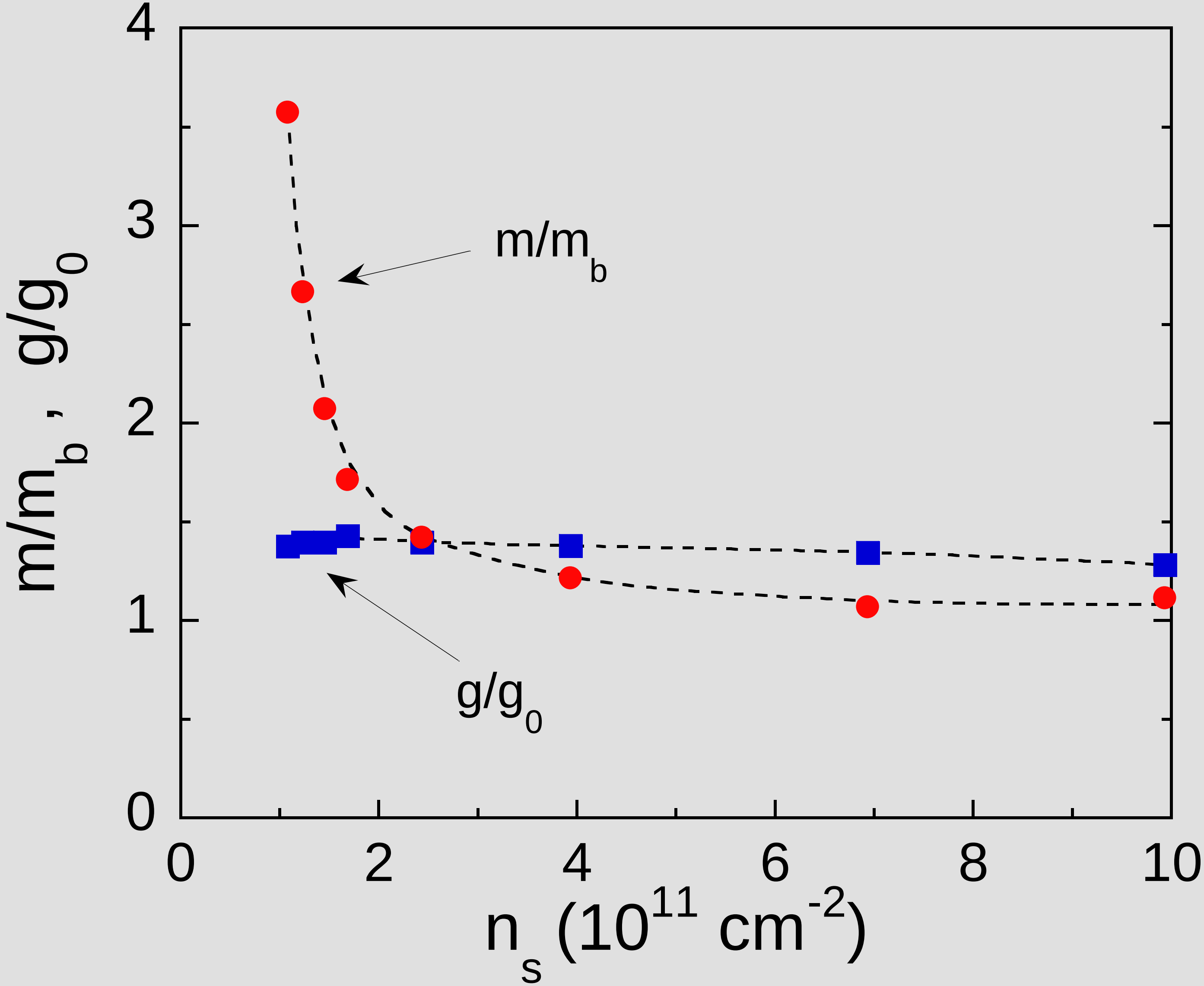
Effective mass *vs*. g-factor
(from the analysis of the transport data in spirit of Zala, Narozhny, and Aleiner, *PRB* 2001) :

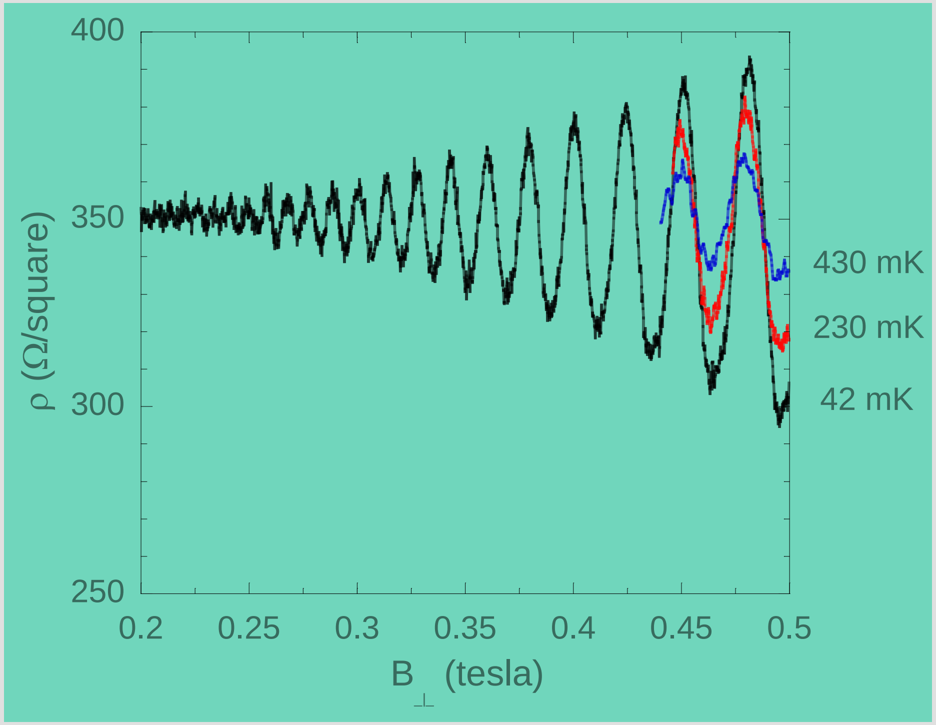


Shashkin, S.V.K., Dolgopolov, and Klapwijk, *PRB* (2002)

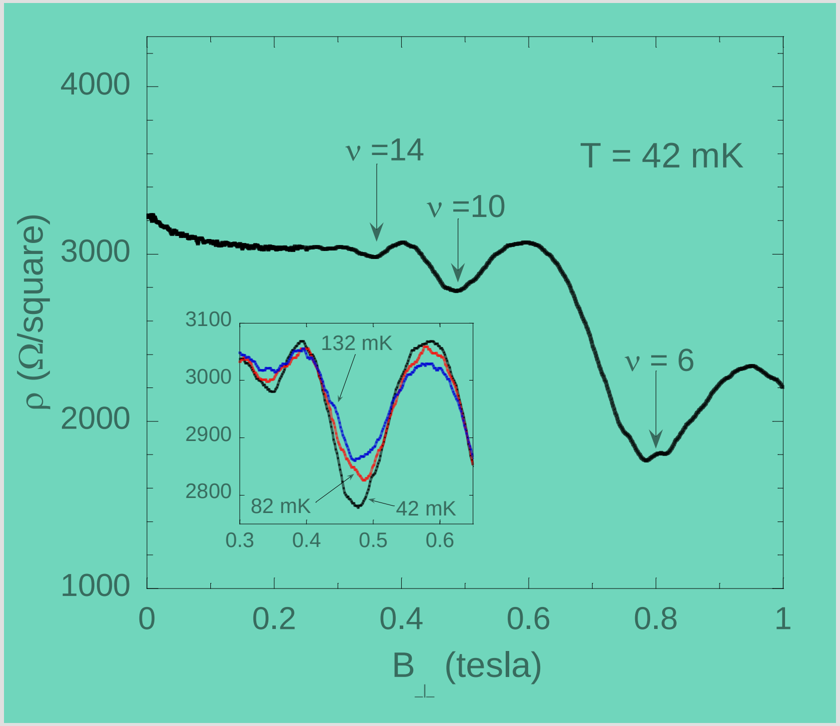
Another way to measure m*:

amplitude of the weak-field Shubnikov-de Haas oscillations vs. temperature

high density

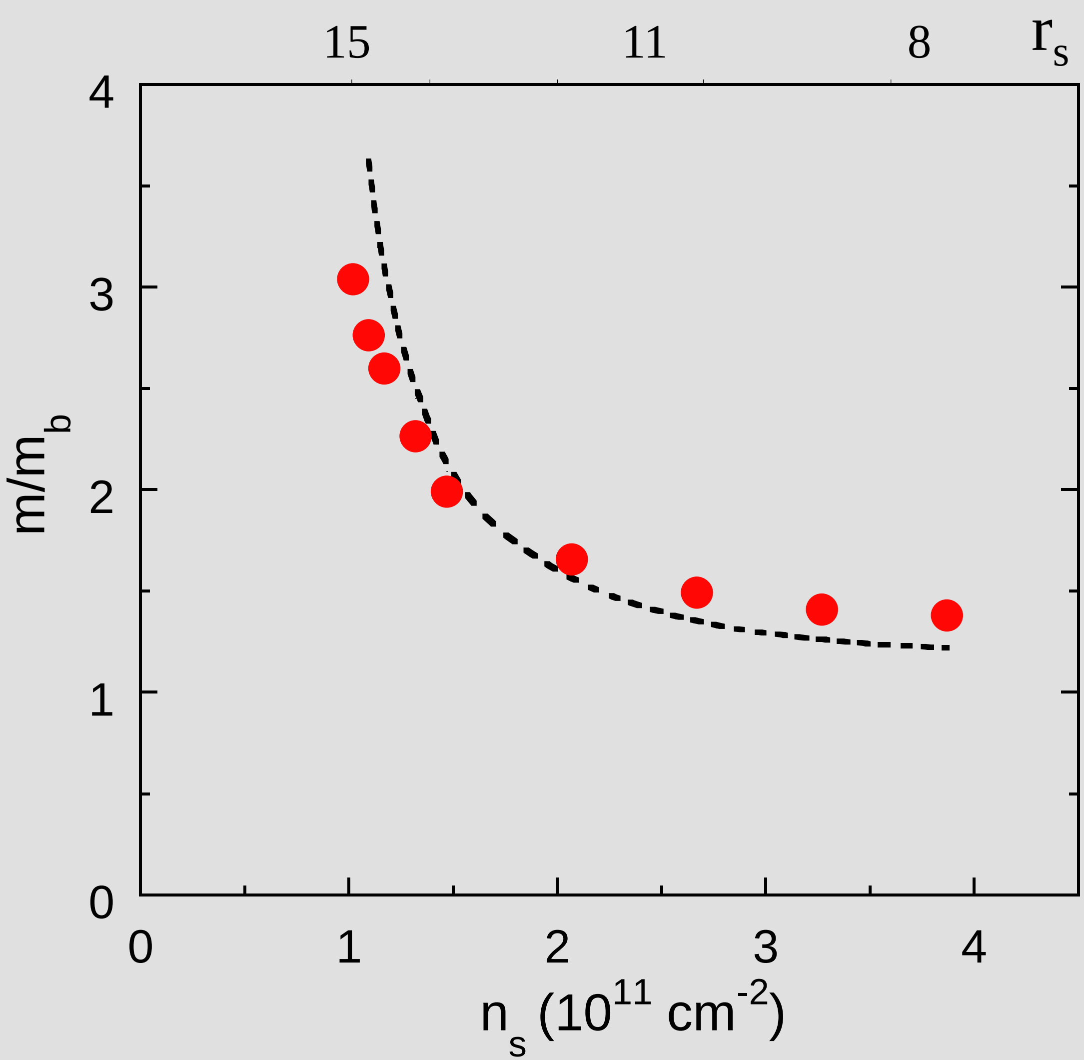


low density



(Rahimi, Anissimova, Sakr, S.V.K., and Klapwijk, *PRL* 2003)

Comparison of the effective masses determined by two independent experimental methods:



(Shashkin, Rahimi, Anissimova, S.V.K., Dolgopolov, and Klapwijk, *PRL* 2003)

Yet another way to measure the effective mass: Thermopower

In the low-temperature metallic regime, the diffusion thermopower of strongly interacting 2D electrons is given by the relation

$$S = -\alpha \frac{2\pi k_B^2 m T}{3e\hbar^2 n_s}$$

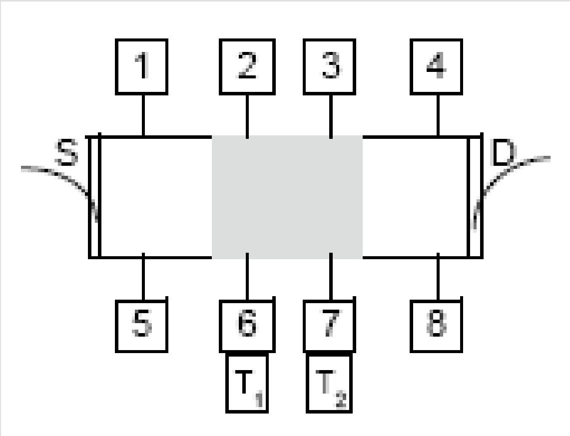
(Dolgopolov and Gold, 2011)

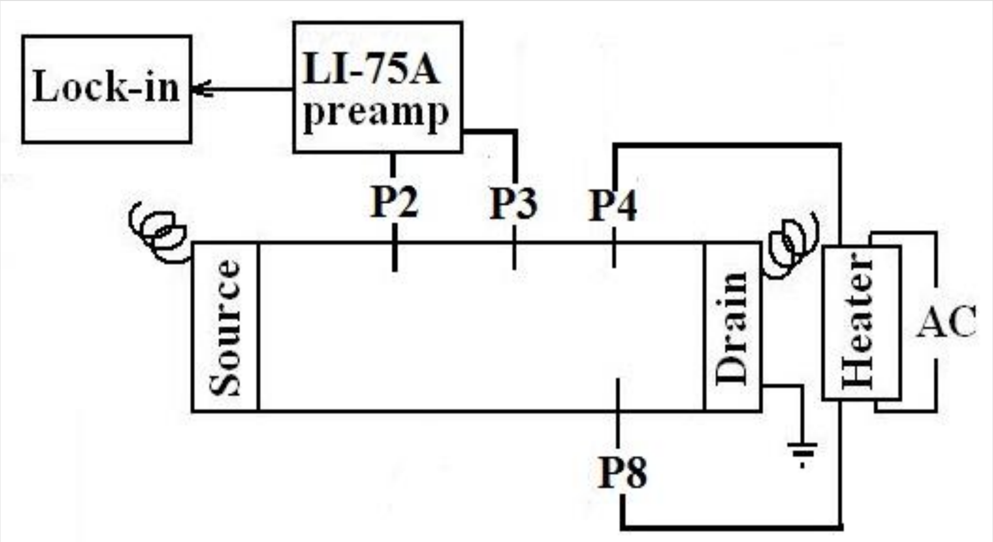
Thermopower : $S = -\Delta V / (\Delta T)$

$$S = S^d + S^g = \alpha T + \beta T^s$$

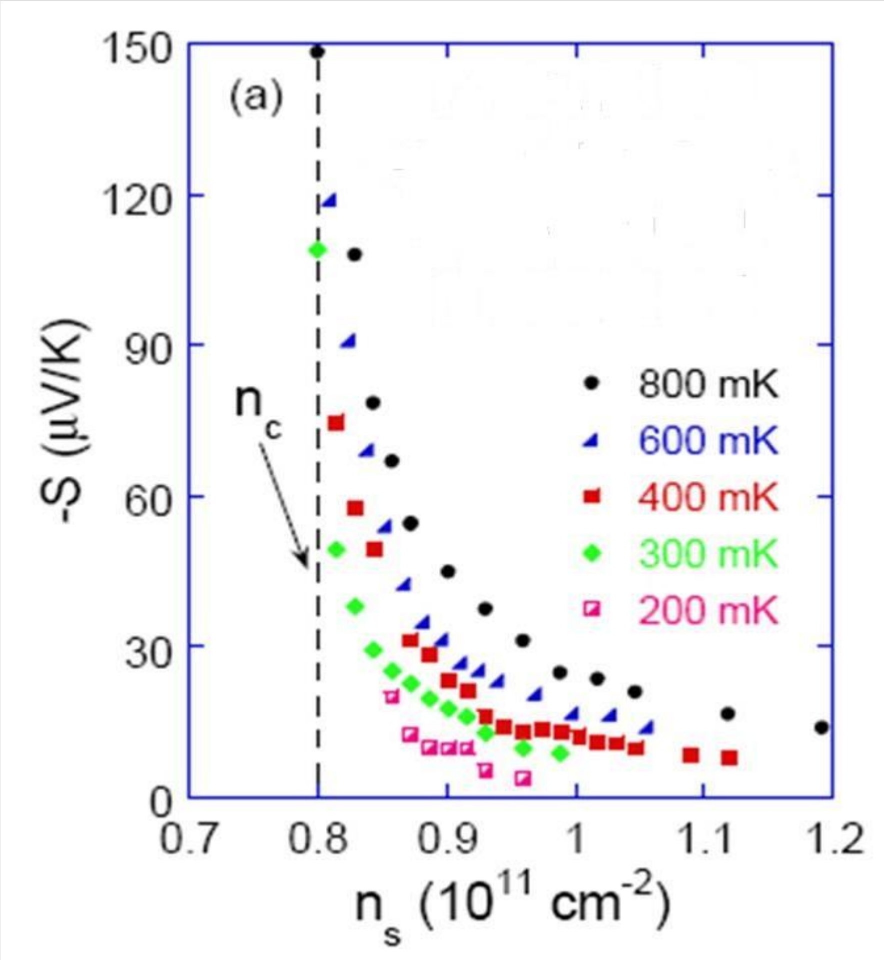
ΔV : heat either end of the sample, measure the induced voltage difference in the shaded region

ΔT : use two thermometers to determine the temperature gradient

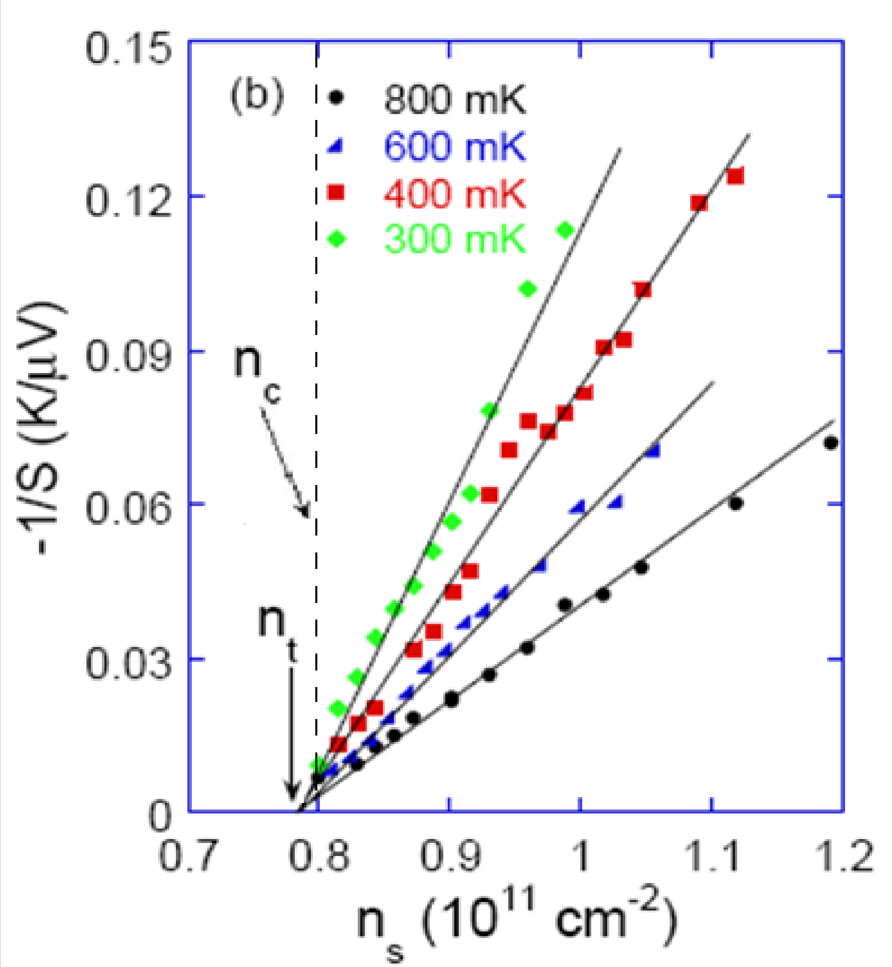




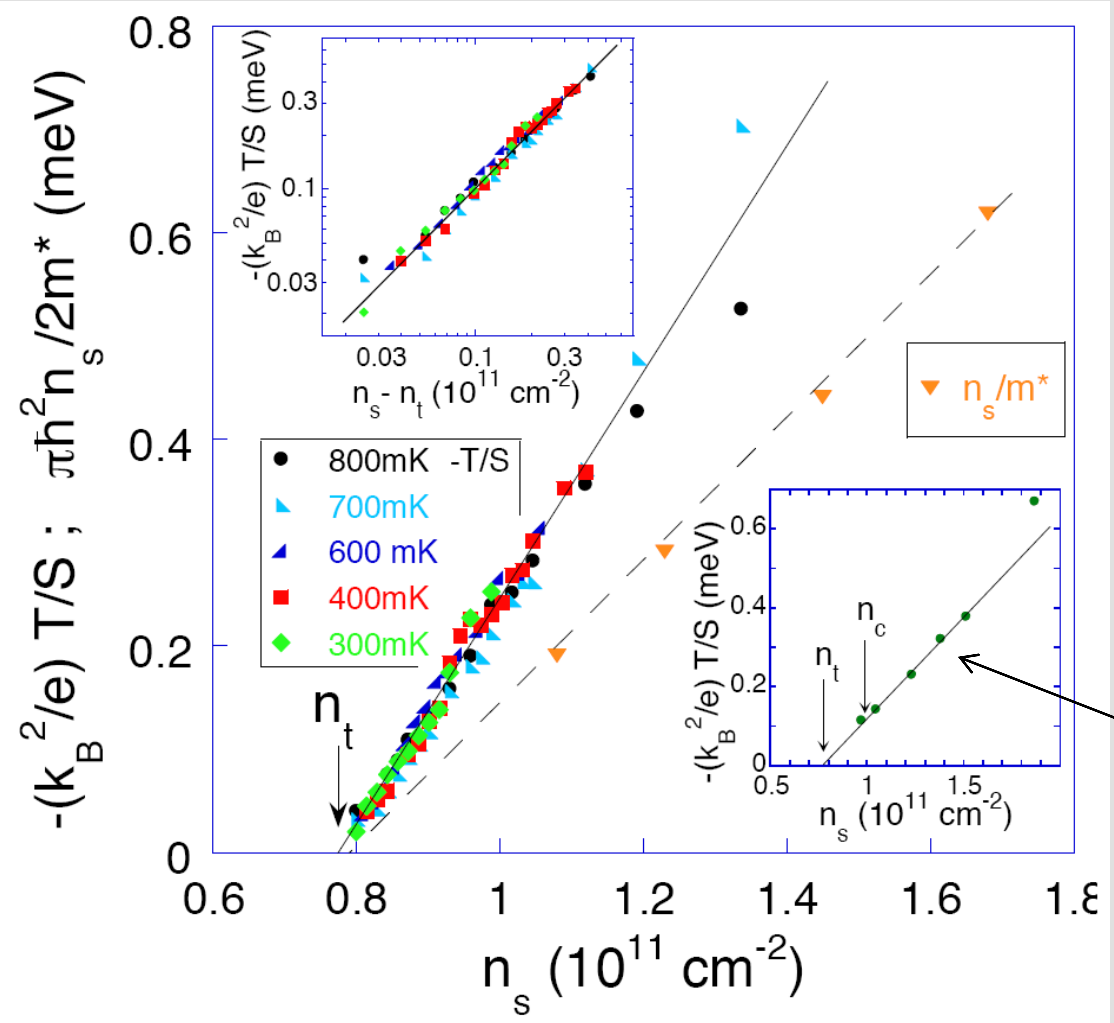
Divergence of thermopower



1/S tends to vanish at n_t



Critical behavior of thermopower



$(-T/S) \propto (n_s - n_t)^x$
where $x = 1.0 \pm 0.1$

$n_t = (7.8 \pm 0.1) \times 10^{10}\ \mathrm{cm}^{-2}$

and is independent of the level of the disorder

Data of Fletcher, Pudalov et al.

Since $S/T \propto m/n_s$, divergence of the thermopower indicates a divergence of the effective mass:

$$m \propto n_s/(n_s - n_t)$$

We observe the increase of the effective mass up to $m \cong 25 m_b \cong 5 m_e$!!

A divergence of the effective mass has been predicted…

i. using Gutzwiller's theory (Dolgopolov, *JETP Lett*. 2002)

ii. solving an extended Hubbard model using dynamical mean-field theory (Pankov and Dobrosavljevic, *PRB* 2008)

iii. from a renormalization group analysis for multi-valley 2D systems (Punnoose and Finkelstein, *Science* 2005)

iv. by Monte-Carlo simulations (Marchi *et al*., *PRB* 2009; Fleury and Waintal, *PRB* 2010)

v. using an analogy with He^3 near the onset of Wigner crystallization (Spivak and Kivelson, *PRB* 2004)

Transport properties of the insulator

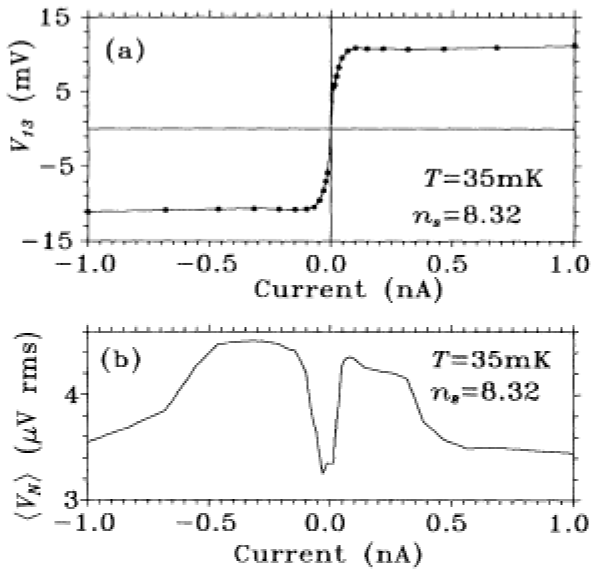
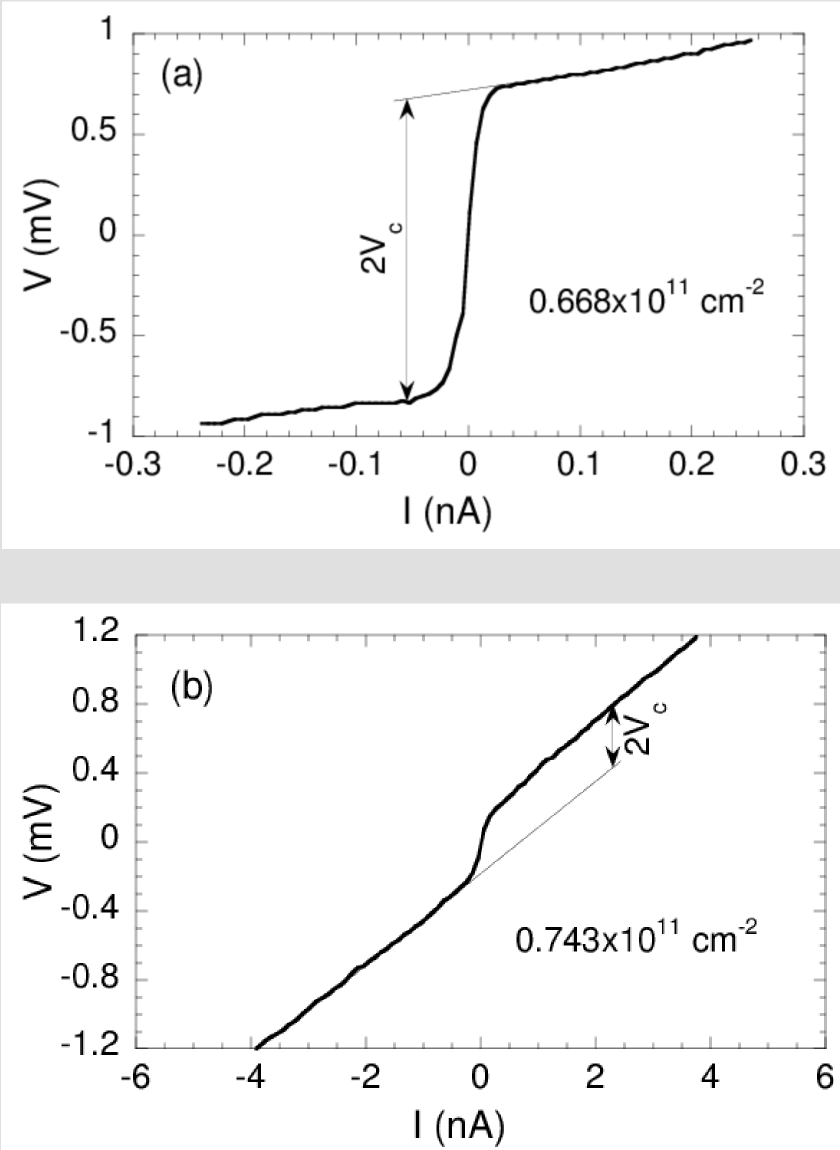


FIG. 2. (a) Typical threshold *I*-*V* characteristic. (b) Broad-band noise voltage measured between probes 1 and 3 as a function of source-drain current. n_s is in units of 10^{10} cm^{-2}.

V. M. Pudalov et al, PRL 1993



If the insulating state were due to a single-particle localization, the electric field needed to destroy it would be of order (the most conservative estimate)

$E_{th} \sim W_b / le \sim$ $10^3 - 10^4$ V/m

However, in experiment
$E_{th} =$ 1 – 10 V/m !

De-pinning of a pinned Wigner solid?

SUMMARY:

- Competition between electron-electron interactions and disorder leads to the existence of the metal-insulator transition in two dimensions. The metallic state is stabilized by the electron-electron interactions. Disorder-interactions flow diagram of the metal-insulator transition reveals a quantum critical point.

- In the clean (ballistic) regime, spin susceptibility critically grows upon approaching to some sample-independent critical point, n_χ, pointing to the existence of a phase transition.

- The dramatic increase of the spin susceptibility is due to the divergence of the effective mass rather than that of the g-factor and, therefore, is not related to the Stoner instability. It may be a precursor phase or a direct transition to the long sought-after Wigner solid.

- However, the existing data, although consistent with the formation of the Wigner solid, are not enough to reliably confirm its existence.