Ioffe Physico-Technical Institute

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Rippling, crumpling, and folding in disordered free-standing graphene

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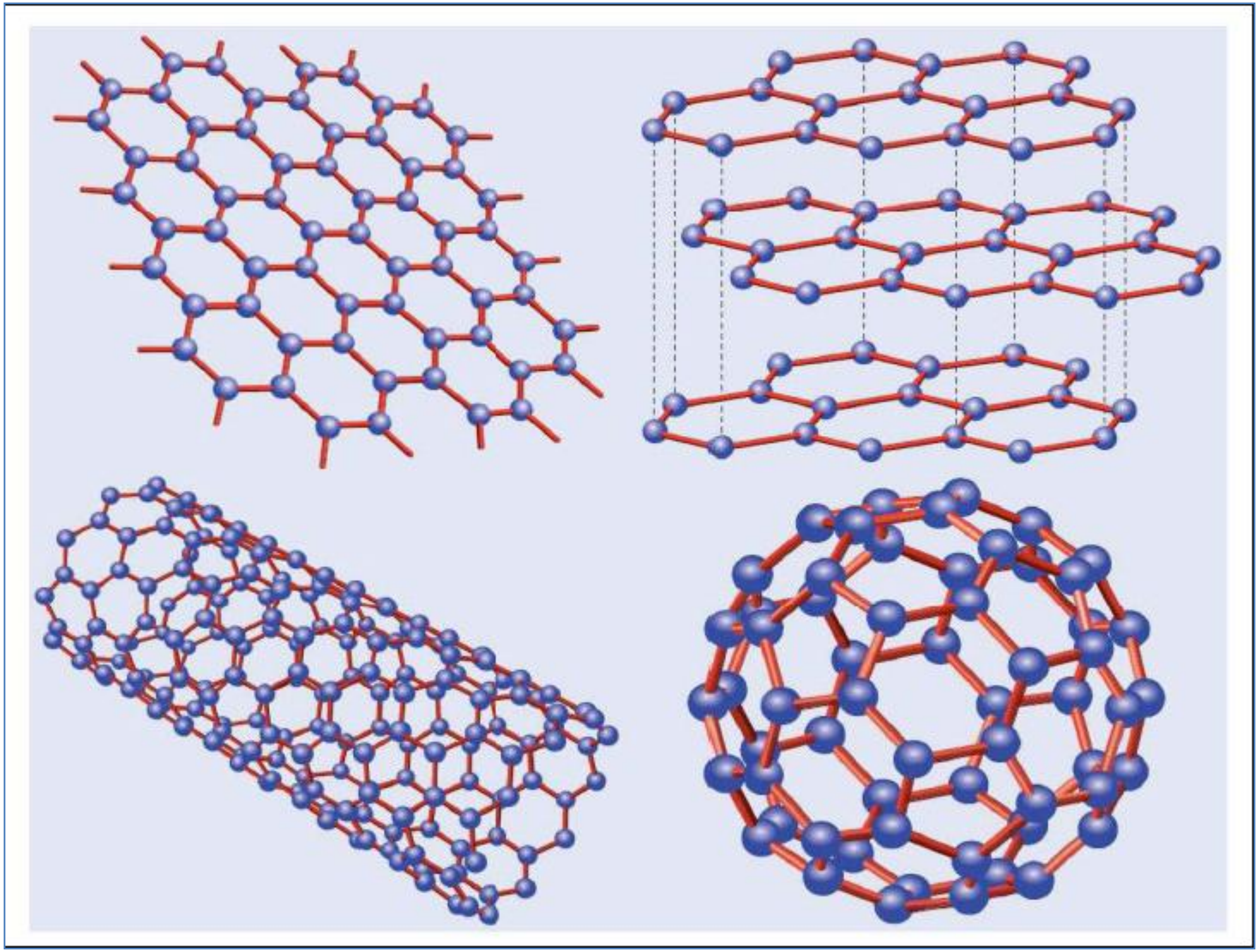
Outline

- *Introduction.* Graphene as elastic membrane. Flexural phonons
- *Formation of flat phase at low temperatures.* Mean field approximation
- *Beyond mean field.* Softening of membrane due to thermal fluctuations and disorder. Anharmonicity–induced increase of bending rigidity
- *Crumpling transition.* Competition between anharmonicity and fluctuations
- *Geometry of the membrane.* Fractal behavior in the near-critical region
- *Effect of disorder on crumpling transition in graphene*. Non-monotonous scaling of bending rigidity. Static, frozen-out fluctuations – ripples.

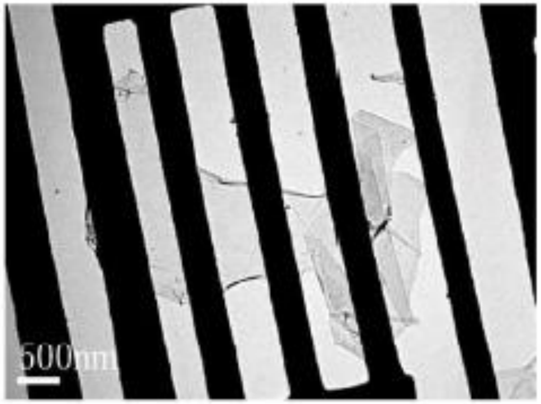
Graphene: monoatomic layer of carbon



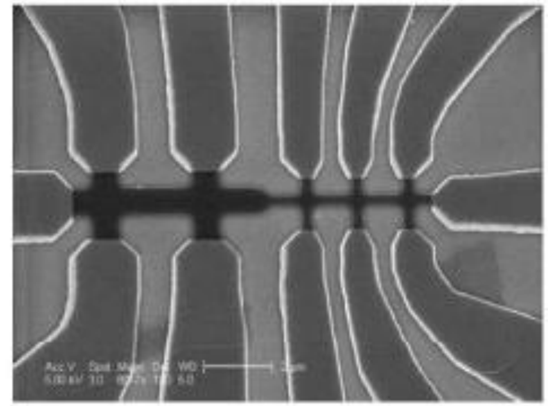
First isolated and explored: Manchester (Geim, Novoselov, et al., 2004)

Nobel Prize 2010 (Andre Geim & Konstantin Novoselov)

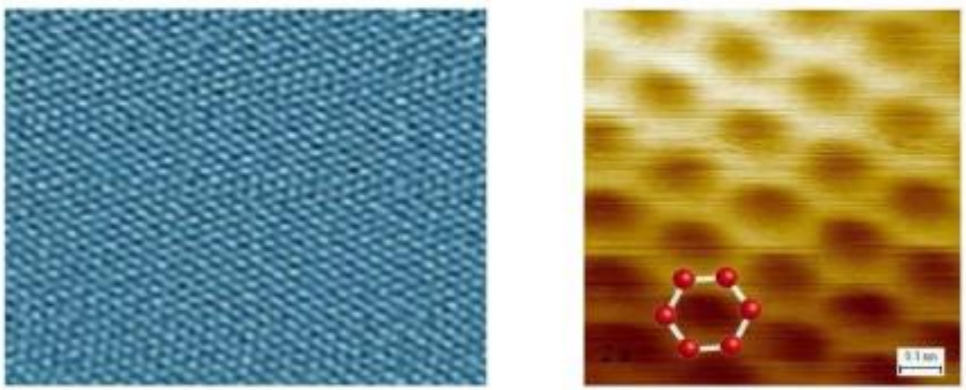
Graphene samples



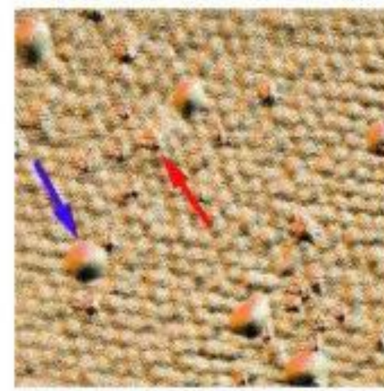
Suspended sample



Hall bar



Micro-mechanical cleavage

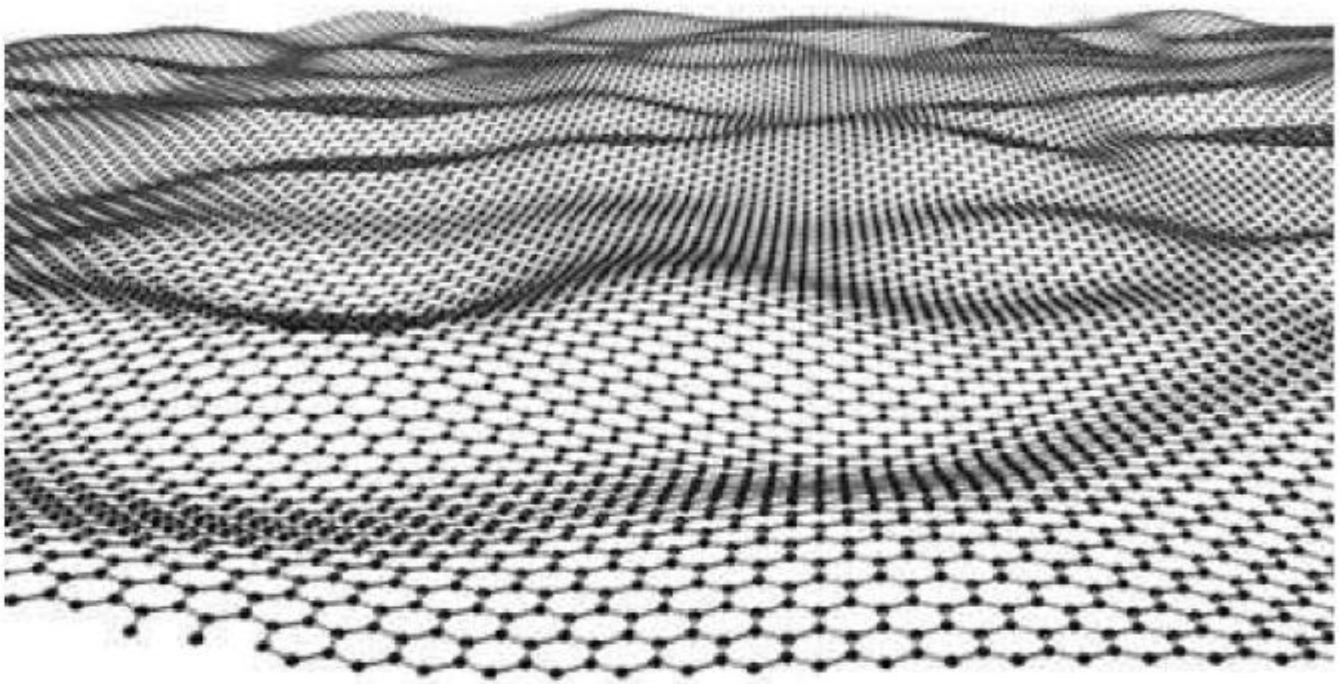


Epitaxial growth

carrier mobility: up to ~20,000 cm^2/V·s at 300K; ~200,000 cm^2/V·s at 4K

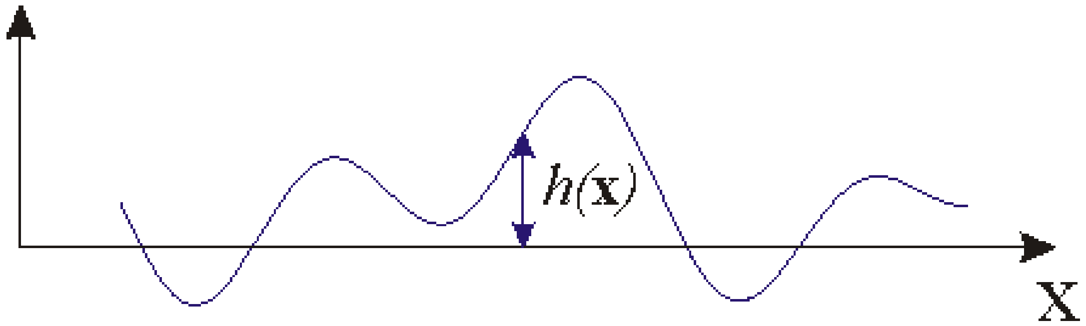
Suspended graphene:

dynamical out-of-plane deformations (flexural phonons)
+ static frozen-out deformations (ripples)



Meyer, Geim, Katsnelson, Novoselov, Booth, Roth, Nature'07

Flexural phonons (FP)



$$E = \frac{1}{2}\int d\mathbf{x}\left[\rho\dot{h}^2 + \varkappa(\Delta h)^2\right]$$

$\varkappa \simeq 1$ eV.

$$h(\mathbf{r}) = \sum_{\mathbf{q}}\sqrt{\frac{\hbar}{2\rho\omega_{\mathbf{q}}S}}(b_{\mathbf{q}} + b^{\dagger}_{-\mathbf{q}})e^{i\mathbf{qr}}$$

out-of-plane flexural mode

$$\omega_q = Dq^2$$

soft dispersion of FP

$$D = \sqrt{\varkappa/\rho}$$

Thermal fluctuations

$$b_{\mathbf{q}} = \sqrt{N_{\mathbf{q}}} e^{-i\varphi_{\mathbf{q}}}$$

$$N_{\mathbf{q}} \approx \sqrt{T/\hbar\omega_{\mathbf{q}}} \gg 1$$

$$h(\mathbf{r}) = \sum_{\mathbf{q}} \sqrt{\frac{2T}{\varkappa q^4 S}} \cos(\mathbf{qr} + \varphi_{\mathbf{q}})$$

$$G(\mathbf{q}) = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = \frac{T}{\varkappa q^4}$$

correlation function of FP

$$\sqrt{\langle h^2(\mathbf{r}) \rangle} \propto \sqrt{\frac{T}{\varkappa} \int \frac{d^2\mathbf{q}}{q^4}} \propto \sqrt{\frac{T}{\varkappa}} L$$

for graphene at room temperature: $\sqrt{T/\varkappa} \approx 0.2$

Thermal fluctuations with small q are huge !!!!!

Quasielastic scattering by FP

$$V_{e,ph} = V + V_{\mathbf{A}} = g_1 u_{ii} + g_2 \boldsymbol{\sigma} \mathbf{A}$$

$$A_x = 2u_{xy}, \quad A_y = u_{xx} - u_{yy}$$

$$V = g_1 (\nabla h)^2 / 2$$

FP contribution to the deformation potential

$g_1 \simeq 30$ eV deformation coupling constant,

$g_2 \simeq 1.5$ eV coupling to gauge field

$$V(\mathbf{r}) = \frac{g_1 T}{\varkappa S} \sum_{\mathbf{q}_1, \mathbf{q}_2} \frac{\mathbf{q}_1 \mathbf{q}_2}{q_1^2 q_2^2} \sin(\mathbf{q}_1 \mathbf{r} + \varphi_{\mathbf{q}_1}) \sin(\mathbf{q}_2 \mathbf{r} + \varphi_{\mathbf{q}_2})$$

Theory:

Golden rule calculation

$$\sigma_{\rm ph} = \frac{e^2}{\hbar} \frac{\pi^2 N}{24 g^2 \ln(q_T L)} \approx 10^{-3} \frac{e^2}{h}$$

theory yields unrealistic (too small) values of conductivity in the Dirac point!!!

Experiment:

$$\sigma_{\rm ph} \sim 10 \div 50 \frac{e^2}{h}$$

K. Bolotin *et al* PRL (2008)

$$g = \frac{g_1}{\sqrt{32\varkappa}} \simeq 5.3$$

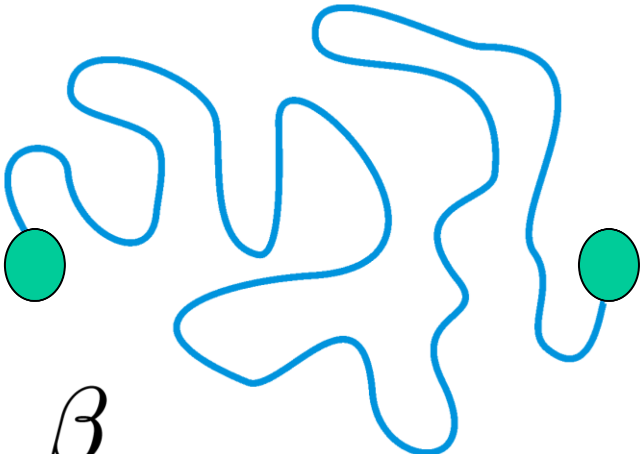
dimensionless coupling constant

$N = 4$ spin×valleys,

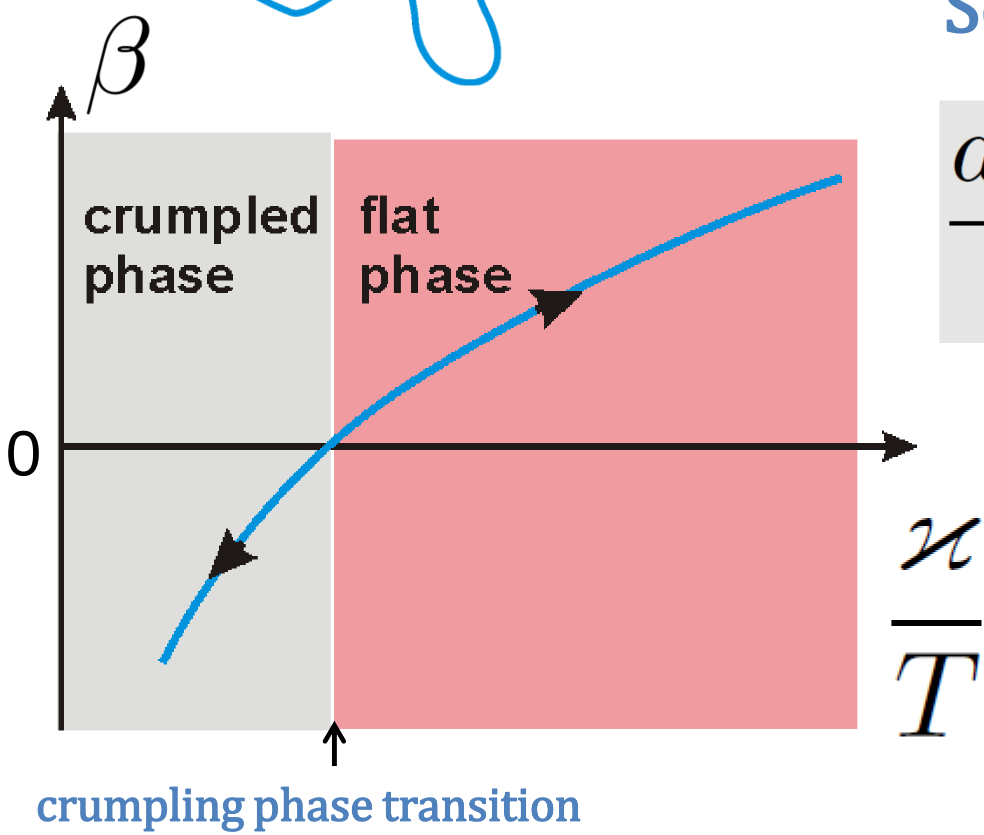
$q_T = T/\hbar v$

Crumpling transition of membrane: key parameter $\varkappa/T$

Crumpled phase: $\varkappa/T \to 0$



Flat phase: $\varkappa/T \to \infty$



Scaling of bending rigidity

$$\frac{d\ln(\varkappa/T)}{d\ln L} = \beta(\varkappa/T)$$

D. Nelson, T. Piran, S. Weinberg *Statistical Mechanics of Membranes and Surfaces* (1989).

Physics behind: anharmonic coupling with in-plane modes

For graphene $\varkappa/T \approx 30$ even for T=300 K ➔ flat phase

Bending rigidity increases with increasing the system size (or decreasing the wave vector):

$$\varkappa \propto L^{\eta} \propto \frac{1}{q^{\eta}}$$

critical behavior of bending rigidity

η - critical exponent ($\approx$ 0.7)

F.David and E. Guitter, Europhys. Lett. (1988);
P. Le Doussal , L. Radzihovsky, PRL (1992)

$$\frac{h}{L} \sim \frac{1}{L^{\eta/2}}$$

➔ deformation potential is suppressed ➔

$$\sigma_{\rm ph} \sim 10\frac{e^2}{h}$$

agrees with experiment

(Dirac point, T=300 K)

in the thermodynamic limit fluctuations are suppressed

Gornyi, Kachorovskii, Mirlin RRB (2012)

Theory of crumpling transition

$$F = \int d^D x \left\{ \frac{\varkappa_0}{2} (\partial_\alpha \partial_\alpha \mathbf{R})^2 - \frac{t}{2} (\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R}) + u (\partial_\alpha \mathbf{R} \partial_\beta \mathbf{R})^2 + v (\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R})^2 \right\}$$

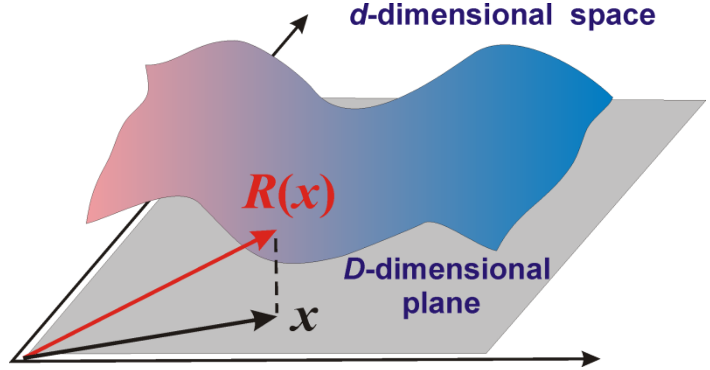
$$\alpha, \beta = 1, ..., D$$

Paczuski, Kardar, Nelson , PRL,1988

$\mathbf{R}(\mathbf{x})$ is d-dimensional vector

$\mathbf{x}$ is D-dimensional vector

For physical membranes d=3, D=2



Mean field $\rightarrow$ $\mathbf{R} = \xi \mathbf{x}$ $\rightarrow$ $F = -\xi^2 t + 2\xi^4 (u + Dv)$

$\partial F / \partial \xi = 0$ $\rightarrow$

$$\xi^2 = \begin{cases} \dfrac{t}{4(u+Dv)}, & \text{for} \quad t > 0 \\ 0, & \text{for} \quad t < 0 \end{cases}$$

flat phase

crumpled phase

$t \propto T_c - T$ $\rightarrow$ $\xi^2 \propto T_c - T$

Flat phase ($T < T_c$, $\xi > 0$)

$$\mathbf{R} = \xi \mathbf{r} \qquad \mathbf{r} = \mathbf{x} + \underbrace{\mathbf{u} + \mathbf{h}}_{\text{in-plane and out-of-plane fluctuations}}$$

Elastic energy

$$F = \int d^D x \left\{ \frac{\varkappa}{2} (\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

strong anharmonicity

$$u_{\alpha\beta} = \frac{1}{2} (\partial_\alpha \mathbf{r} \partial_\beta \mathbf{r} - \delta_{\alpha\beta}) \approx \frac{1}{2} (\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha \mathbf{h} \partial_\beta \mathbf{h})$$

strain tensor

$$\varkappa = \varkappa_0 \xi^2, \;\; \mu = 4u\xi^4, \;\; \lambda = 8v\xi^4$$

$$\mu, \lambda \propto (T_c - T)^2, \quad \kappa \propto T_c - T$$

Elastic constants turn to zero in the transition point

Disorder

Clean membrane

$$F = \int d^D\mathbf{x} \left\{ \frac{\varkappa}{2}(\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

Random curvature

$$F = \int d^D\mathbf{x} \left\{ \frac{\varkappa}{2}[\Delta \mathbf{h} + \boldsymbol{\beta}(\boldsymbol{x})]^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

random vector

$$P(\boldsymbol{\beta}) = Z_{\boldsymbol{\beta}}^{-1} \exp\left(-\frac{1}{2b} \int \beta^2(\mathbf{x}) d^D\mathbf{x} \right)$$

In-plane disorder

$$F = \int d^D\mathbf{x} \left\{ \frac{\varkappa}{2}(\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} [u_{ii} + c(\mathbf{x})]^2 \right\}$$

$$P(c) = Z_c^{-1} \exp\left(-\frac{1}{2\sigma} \int c^2(\mathbf{x}) d^D\mathbf{x} \right)$$

Beyond mean field: Fluctuations

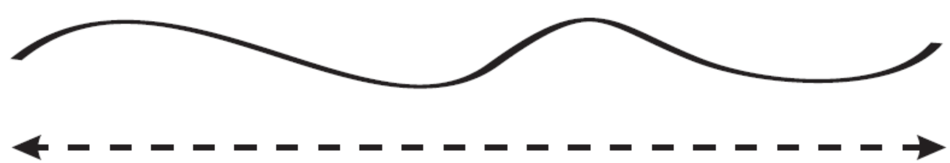
$$\mathbf{r} = \xi \mathbf{x} + \mathbf{u} + \mathbf{h}$$

Mean field: $\xi = 1$

$$\tilde{\mathbf{u}} = \xi \mathbf{u}$$

$$F_0 = \frac{DL^D(\mu + \lambda D/2)}{4}\left[(\xi^2-1)^2 + \underbrace{\frac{2(\xi^2-1)}{D}\int \frac{d^D\mathbf{x}}{L^D}\,\partial_\alpha \mathbf{h}\partial_\alpha \mathbf{h}}_{\text{coupling between stretching and fluctuations}}\right] + F(\tilde{\mathbf{u}},\mathbf{h})$$

Physics behind: transverse fluctuations lead to decrease of membrane size in x-direction



$$R = \xi_L L$$

minimization of energy $\Rightarrow$

$$\xi^2 = 1 - \frac{1}{D}\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle$$

harmonic approximation $\Rightarrow$

$$\langle \partial_\alpha \mathbf{h} \partial_\alpha \mathbf{h} \rangle = d_c \left(\frac{T}{\varkappa} + b \right) \int \frac{d^D \mathbf{q}}{(2\pi)^D} \frac{1}{q^2}$$

$$d_c = d - D$$

Logarithmic divergence for D=2 $\rightarrow$ renormalization group (RG)

$$\frac{d\xi^2}{d\Lambda} = -\frac{d_c}{4\pi}\left(\frac{T}{\varkappa} + b \right), \qquad D = 2$$

thermal fluctuations

disorder

$$\Lambda = \ln(L/a)$$

L – system size

a - ultraviolet cutoff

$\xi \rightarrow 0$, for certain value of L $\Rightarrow$ Flat phase is destroyed both by thermal fluctuations and by disorder

Renormalization of bending rigidity (clean case)

David, Guitter, Europhys. Lett. (1988), Le Doussal, Radzihovsky, PRL (1992)

$$F = \int d^D x \left\{ \frac{\varkappa}{2} (\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

Correlation function of transverse modes:

$$G_{ij} = \langle h_i(\mathbf{q}) h_j(-\mathbf{q}) \rangle = \frac{\int h_i(\mathbf{q}) h_j(-\mathbf{q}) e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h} d\mathbf{u}\}}{\int e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h} d\mathbf{u}\}} = \delta_{ij} G(q)$$

$$G^0_{\mathbf{q}} = \frac{T}{\varkappa q^4}$$

Interaction between in-plane and out-of-plane modes is neglected

However, such interaction dramatically change the small q behavior of $G(q)$ due to strong anharmonicity

Anomalous scaling of bending rigidity

Integrate out the in-plane modes

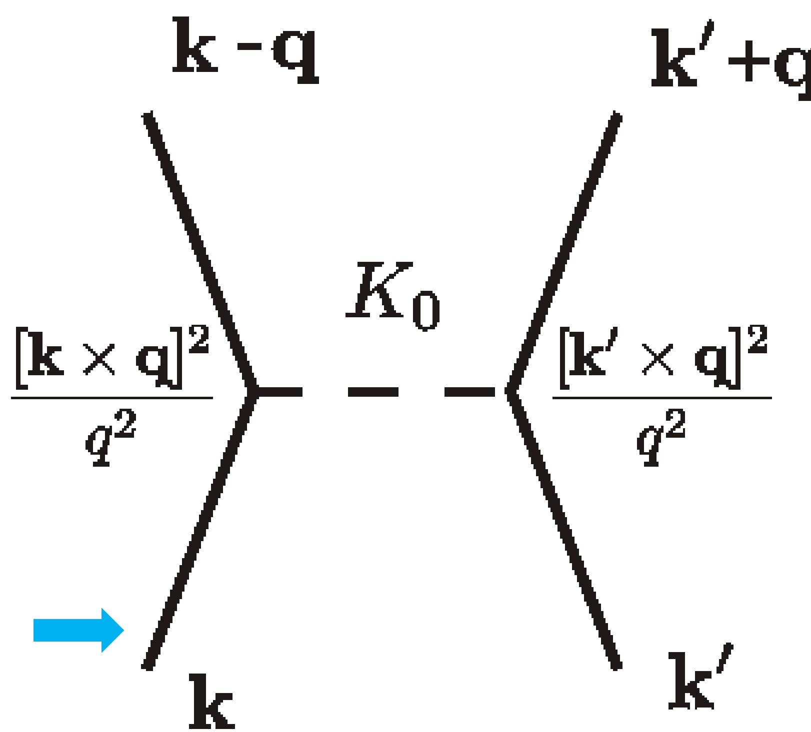
$$F(\mathbf{h}) = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\Big[\varkappa q^4 \mathbf{h}_{\mathbf{q}}\mathbf{h}_{-\mathbf{q}} + \frac{1}{4d_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} \times \underline{R(\mathbf{k},\mathbf{k}',\mathbf{q})(\mathbf{h}_{-\mathbf{k}}\mathbf{h}_{\mathbf{k}+\mathbf{q}})(\mathbf{h}_{\mathbf{k}'}\mathbf{h}_{-\mathbf{q}-\mathbf{k}'})\Big]}$$

Interaction between out-of-plane modes: $d_c = d - D$

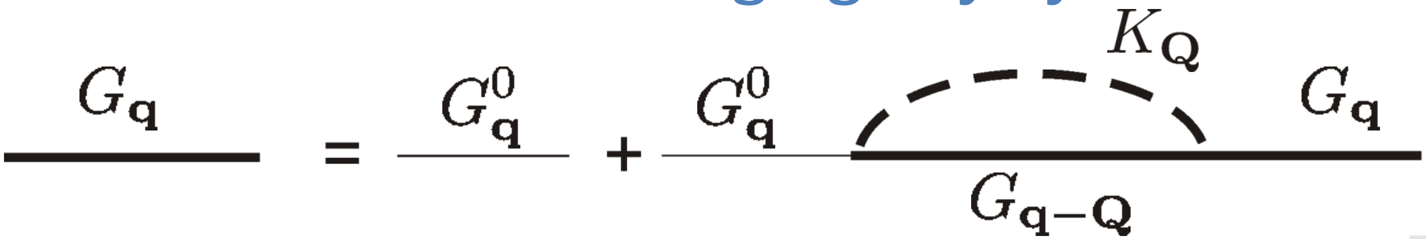
$$R(\mathbf{k},\mathbf{k}',\mathbf{q}) = K_0 \frac{[\mathbf{k}\times\mathbf{q}]^2}{q^2}\frac{[\mathbf{k}'\times\mathbf{q}]^2}{q^2}$$

$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

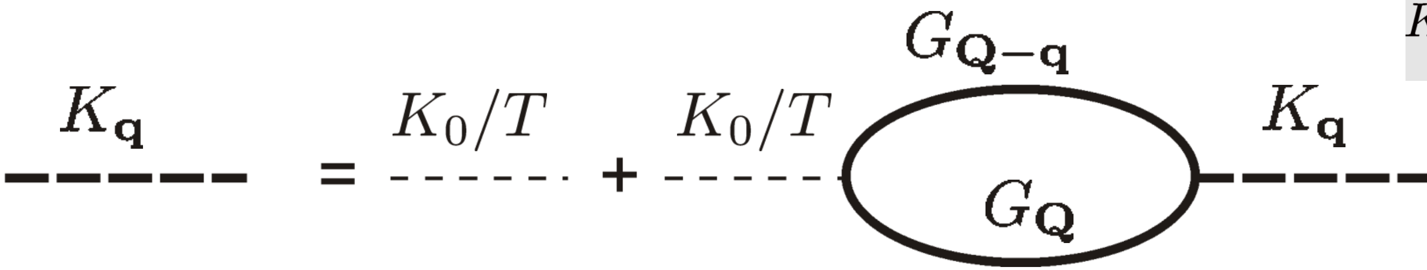
$$G^0_{\mathbf{k}} = \frac{T}{\varkappa k^4}$$



Renormalization of bending rigidity by screened interaction



$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$



$$G_{\mathbf{q}} = \frac{T}{\varkappa q^4 + \Sigma_{\mathbf{q}}}$$

$$\Sigma_{\mathbf{q}} = \frac{2T}{d_c} \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{Q^4} K_{\mathbf{Q}} G_{\mathbf{q}-\mathbf{Q}}$$ self-energy

Interaction is screened:

$$K_{\mathbf{q}} = \frac{(K_0/T)}{1+(K_0/T)\Pi_{\mathbf{q}}}$$

$$\Pi_{\mathbf{q}} = \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{q^4} G_{\mathbf{Q}-\mathbf{q}} G_{\mathbf{Q}}$$ polarization operator

$$\Pi^0_{\mathbf{q}} = \frac{3}{16\pi}\left(\frac{T}{\varkappa}\right)^2 \frac{1}{q^2} \to \infty, \qquad \text{for } q \to 0$$

$\Rightarrow$

$$K_{\mathbf{q}} \approx \frac{1}{\Pi^0_{\mathbf{q}}} = \frac{16\pi}{3}\left(\frac{\varkappa}{T}\right)^2 q^2$$

bare coupling drops out !

Universal scaling ($q < q^*$)

$$q_* = \sqrt{\frac{K_0 T}{\varkappa^2}}$$

ultraviolet cutoff (Ginzburg scale)

$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

$$\Sigma_{\mathbf{q}} \approx \varkappa q^4 \frac{2}{d_c} \ln\left(\frac{q_*}{q}\right), \qquad \text{for } q \ll q^*$$

$$\delta\varkappa = \varkappa \frac{2}{d_c} \ln\left(\frac{q_*}{q}\right)$$

$$G_{\mathbf{q}} = \frac{T}{\varkappa q^4 + \Sigma_{\mathbf{q}}}$$

$$\frac{d\varkappa}{d\Lambda} = \eta\varkappa$$

anharmonicity-induced increase of bending rigidity

$$\Lambda = \ln(q^*/q)$$

$$G_q \propto \frac{1}{q^{4-\eta}}$$

$$\eta \simeq \frac{2}{d_c} \quad \text{for } D = 2 \text{ and } d_c \gg 1$$

David, Guitter, Europhys. Lett. (1988)

$$\eta = 0.821 \quad \text{for } D=2,\ d=3$$

self consistent screening approximation

Le Doussal, Radzihovsky, PRL (1992)

Crumpling transition (clean case)

$$\frac{d\varkappa}{d\Lambda} = \frac{2}{d_c}\varkappa$$

$$\frac{d\xi^2}{d\Lambda} = -\frac{d_c T}{4\pi\varkappa}$$

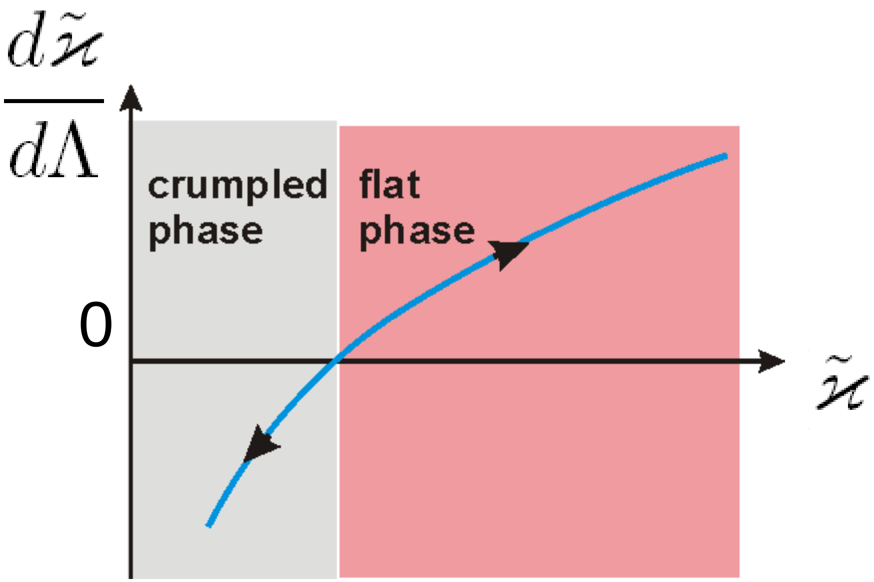
rescaling of bending rigidity:

$$\tilde{\varkappa} = \varkappa\xi^2$$

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \eta\left(\tilde{\varkappa} - \varkappa_{\rm cr}\right)$$

$$\frac{d\xi^2}{d\Lambda} = -\eta\xi^2\frac{\varkappa_{\rm cr}}{\tilde{\varkappa}}$$

$$\eta = 2/d_c$$



$$\varkappa_{\rm cr} = \frac{d_c^2\ T}{8\pi}$$

unstable fixed point

agrees with David, Guitter, Europhys. Lett. (1988),

$$\xi_\infty^2 = \frac{\varkappa_0 - \varkappa_{cr}}{\varkappa_0}$$

For $\varkappa_0 > \varkappa_{\rm cr}$, membrane remains in the flat phase in the course of renormalization

Lower critical dimension for crumpling transition

$$D \neq 2$$

$$\tilde{\varkappa} = \varkappa \xi^2 q^{2-D}$$

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \eta \left[\tilde{\varkappa}(1+\epsilon_2) - \varkappa_{\text{cr}} \right]$$

$$\frac{d\xi^2}{d\Lambda} = -\eta \xi^2 \frac{\varkappa_{\text{cr}}}{\tilde{\varkappa}}$$

$$\eta = 2/d_c$$

$$1 + \epsilon_2 > 0 \;\Rightarrow\; D > D_{\text{cr}}$$

$$\epsilon_2 = \frac{D-2}{\eta}$$

$$D_{\text{cr}} = 2 - \frac{2}{d_c}$$

Aronovitz, Golubovic, Lubensky, J.Phys. France (1989)

Disordered case (random curvature)

Replicas: $\mathbf{h} \to \mathbf{h}^n \;\; n = 1, \ldots, N$

$$F^{\mathrm{rep}} = \sum_{n=1}^{n=N} \frac{\varkappa}{2} \int (dk) k^4 |\mathbf{h}^n_{\mathbf{k}} + \boldsymbol{\beta}_{\mathbf{k}}|^2$$

$$= \frac{1}{4d_c} \sum_{n=1}^{n=N} \int (dk dk' dq) R_{\mathbf{q}}(\mathbf{k}, \mathbf{k}') \left(\mathbf{h}^n_{\mathbf{k}+\mathbf{q}} \mathbf{h}^n_{-\mathbf{k}}\right) \left(\mathbf{h}^n_{-\mathbf{k}'-\mathbf{q}} \mathbf{h}^n_{\mathbf{k}'}\right)$$

interaction

$$\langle \exp(-F_{\mathrm{rep}}/T) \rangle_{\boldsymbol{\beta}} \longrightarrow \exp(-F_{\mathrm{eff}}/T)$$

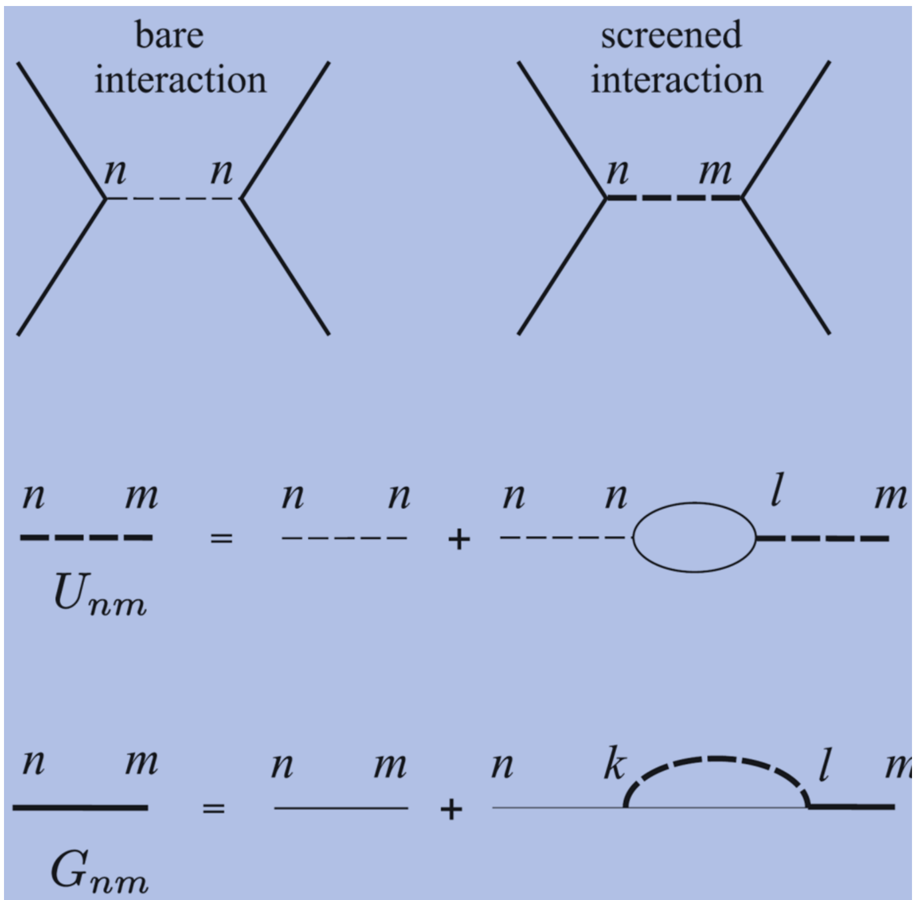
$$P(\boldsymbol{\beta}) = Z_{\boldsymbol{\beta}}^{-1} \exp\left(-\frac{1}{2b} \int \beta^2(\mathbf{x}) d^D \mathbf{x}\right)$$

$$F_{\mathrm{eff}} = \sum_{n,m} \frac{\varkappa^{nm}}{2} \int (dk) k^4 \mathbf{h}^n_{\mathbf{k}} \mathbf{h}^m_{-\mathbf{k}} + F_{\mathrm{int}}$$

Effective bending rigidity $\rightarrow$ b-dependent matrix in the replica space

$$\hat{\varkappa} = \varkappa - \frac{b\varkappa^2}{T}\hat{J}$$

$$\hat{J}\,:\; J^{nm} = 1$$



$$\hat{G}^0_{\mathbf{k}} = \frac{T\hat{\varkappa}^{-1}}{k^4} = \frac{T}{\varkappa k^4}\left(1 + f\hat{J}\right)$$

$$\hat{\Pi}_{\mathbf{q}} = A_D \frac{T^2}{\varkappa^2 q^{4-D}}\left(1 + 2f + f^2\hat{J}\right)$$

$$f = \frac{b\varkappa}{T}$$

dimensionless disorder strength

$$\hat{U} = \frac{D\hat{\Pi}_{\mathbf{q}}^{-1}}{2(D+1)}$$

$$\Sigma^{nm}_{\mathbf{k}} = \frac{2T}{d_c}\int (dq)\, k_\perp^4\, U^{nm}_{\mathbf{q}}\, G^{0,nm}_{\mathbf{k}-\mathbf{q}}$$

In-plane disorder

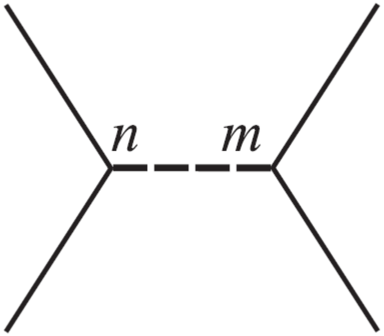
$$F = \frac{\varkappa}{2}\int (dk) k^4 |\mathbf{h}_\mathbf{k}|^2 + \frac{K_0}{4d_c}\int (dq) \left| \int (dk) \frac{(\mathbf{k}\times\mathbf{q})^2}{q^2} \mathbf{h}_{\mathbf{k}+\mathbf{q}} \mathbf{h}_{-\mathbf{k}} + c_\mathbf{q} \right|^2$$

$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

$$P[c(\mathbf{x})] \propto \exp\left(-\frac{1}{2\sigma}\int d\mathbf{x} c^2(\mathbf{x})\right)$$

Replicate and average over disorder

$$U^0_{nm} = K_0 \delta_{nm} - K_0^2 \sigma \hat{J}$$



Screening

$$\hat{U} = (1 + \hat{U}_0 \Pi)^{-1} \hat{U}_0 = (1 + \Pi^{-1} \hat{U}_0^{-1})^{-1} \Pi^{-1}$$

$$\hat{U} \to \Pi^{-1}, \quad \text{for } q \ll q^*$$

In-plane disorder is irrelevant

Disordered case (random curvature)

RG equations

$$\frac{df}{d\Lambda} = -\eta \frac{f(1+3f)}{(1+2f)^2}$$

renormalization of disorder

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \eta \left[\tilde{\varkappa} \frac{(1+3f+f^2)}{(1+2f)^2} - \varkappa_{\mathrm{cr}}(1+f) \right]$$

$$\frac{d\xi^2}{d\Lambda} = -\eta \frac{\xi^2(1+f)\varkappa_{\mathrm{cr}}}{\tilde{\varkappa}}$$

$$f = \frac{b\varkappa}{T}$$

dimensionless disorder

$$\tilde{\varkappa} = \varkappa \xi^2$$

rescaled rigidity

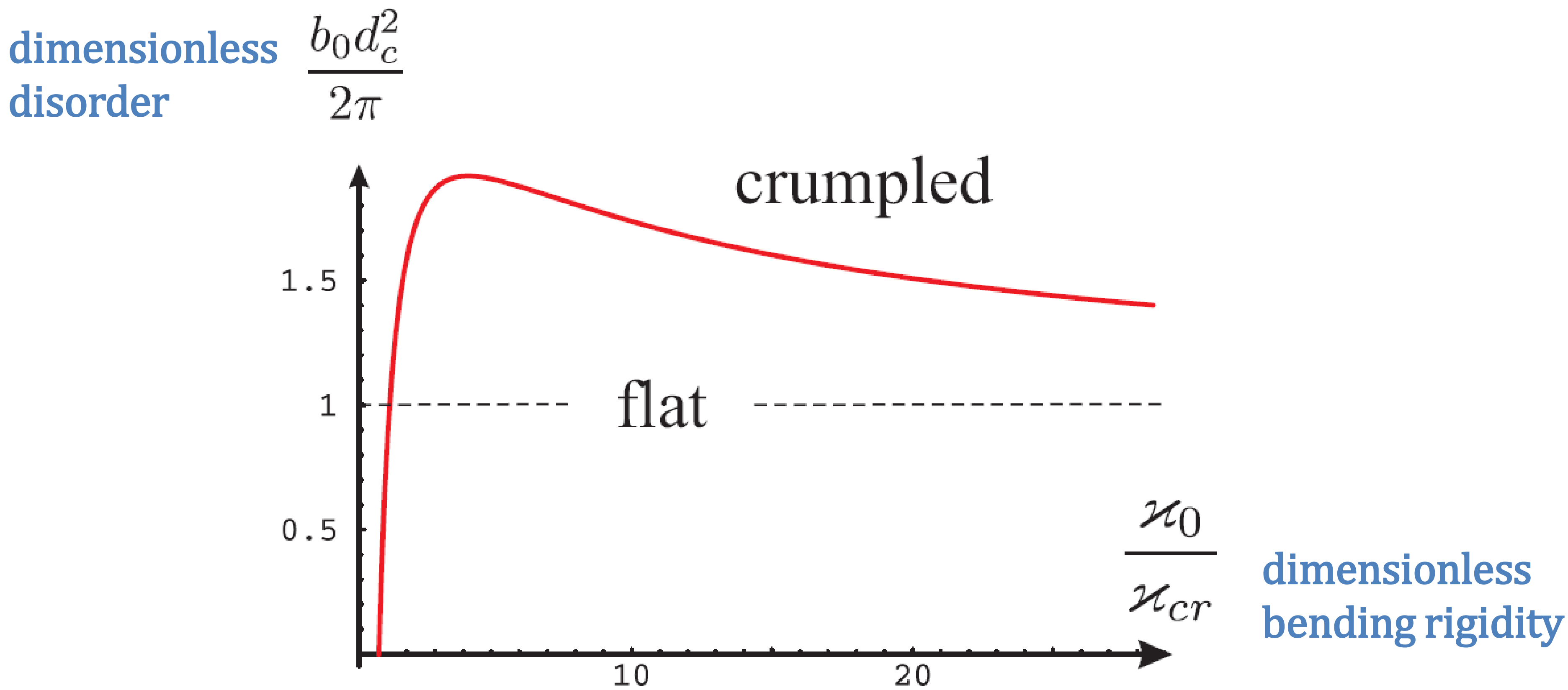
$$\eta \simeq \frac{2}{d_c}$$

critical index for D=2

$$\varkappa_{\mathrm{cr}} = \frac{d_{\mathrm{c}}^2\, T}{8\pi}$$

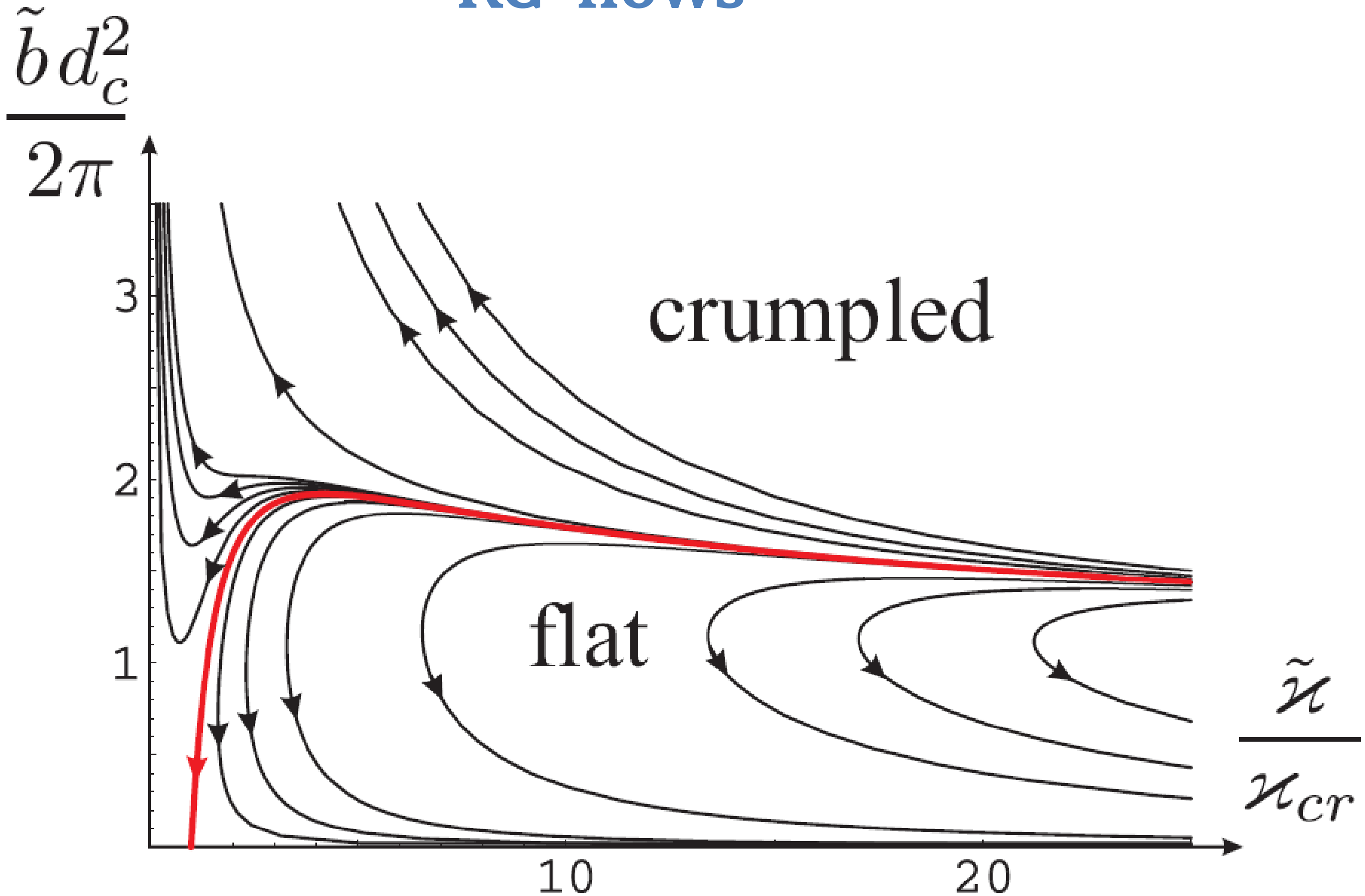
fixed point for clean membrane

Phase diagram of crumpling transition in disordered membrane



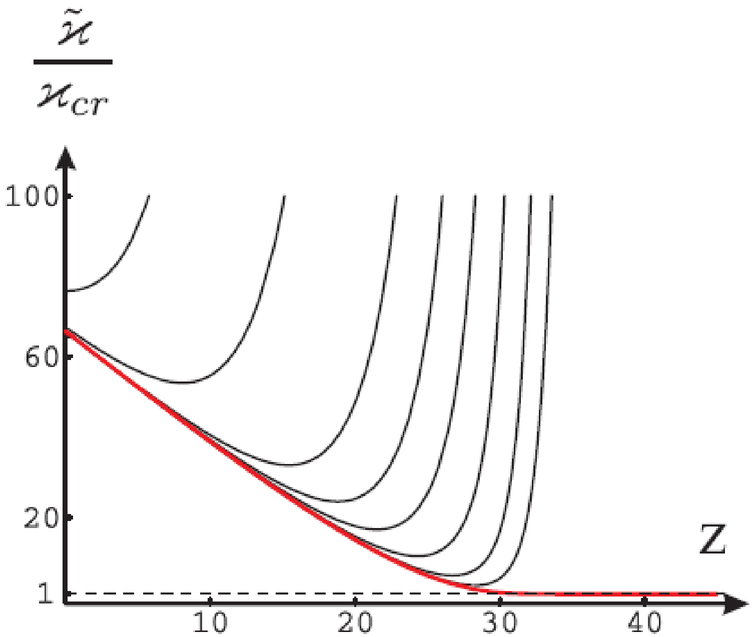
Critical bending rigidity becomes disorder dependent

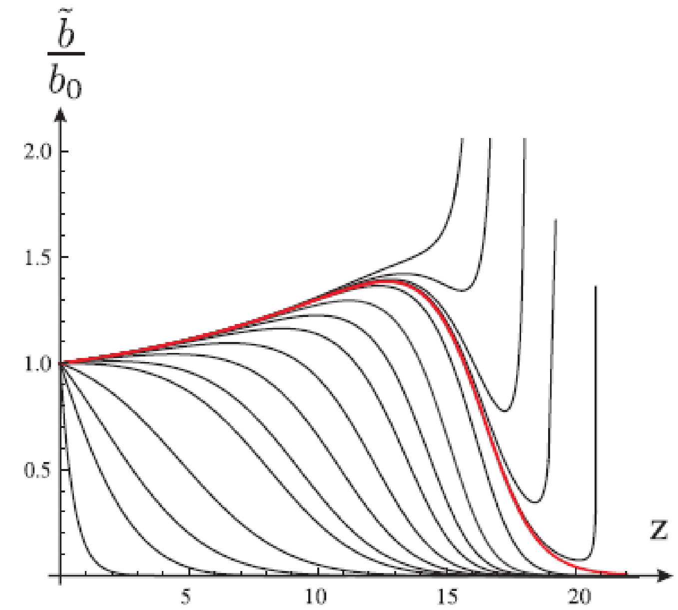
RG flows



Non-monotonous scaling of bending rigidity and disorder

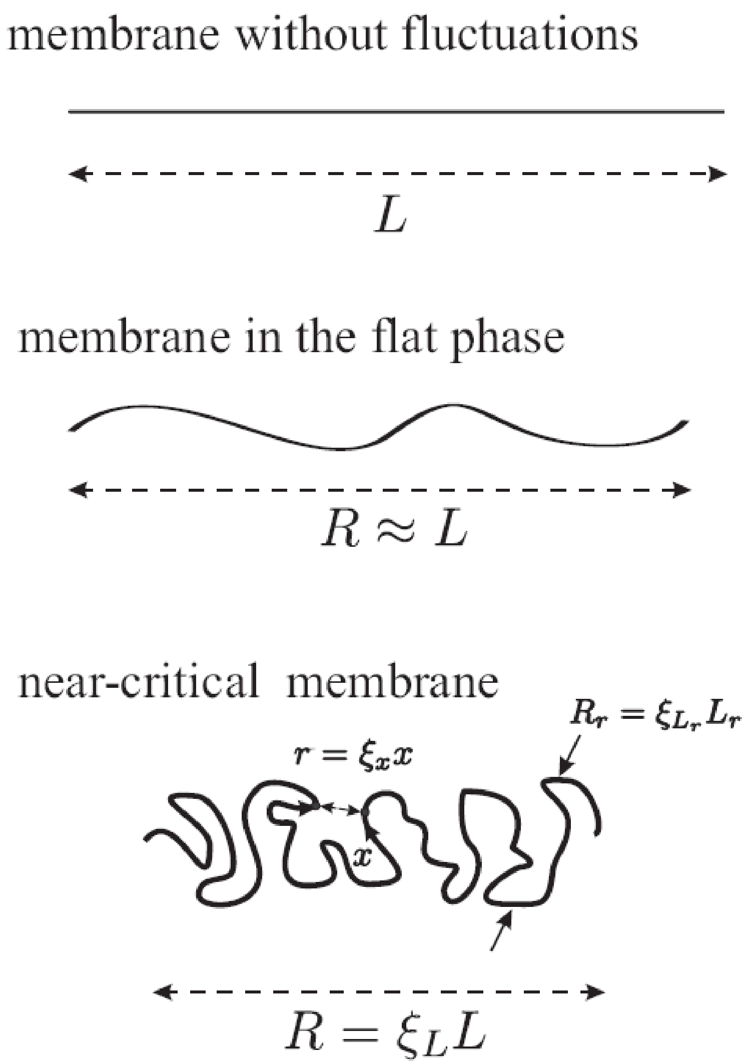
Scaling of bending rigidity and disorder in the flat phase





$$z = \eta \ln (L\, q^*)$$

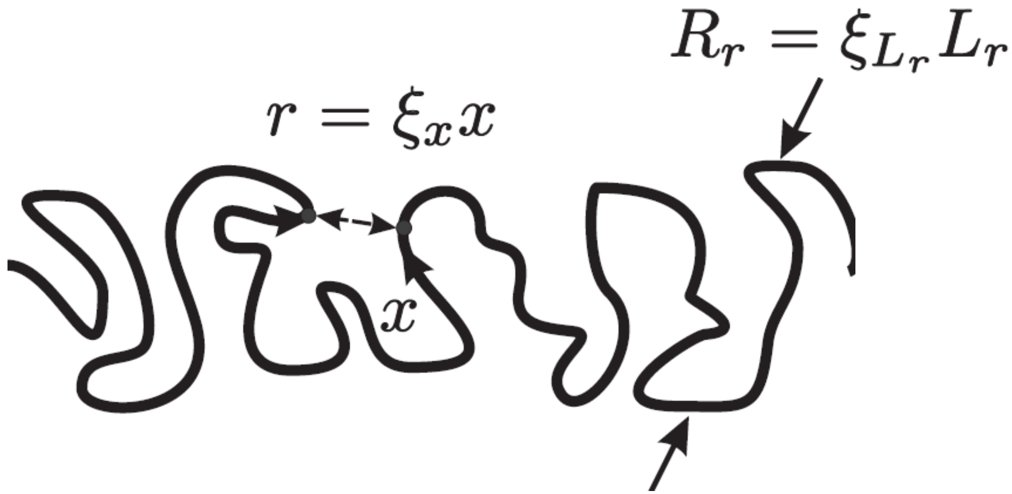
Geometry of the membrane



small fluctuations:
dynamical (flexural phonons)
+ static (ripples)

fractal behavior

Multiple folding of membrane in the near-critical region



$$\langle |\mathbf{r}_1 - \mathbf{r}_2|^2 \rangle \sim \xi^2_{|\mathbf{x}_1 - \mathbf{x}_2|} (\mathbf{x}_1 - \mathbf{x}_2)^2$$

$$\frac{\xi_L}{\xi_{L'}} \sim \frac{1}{2}$$

folding of membrane

Clean membrane at criticality: Fractal dimension

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \eta\left(\tilde{\varkappa} - \varkappa_{\rm cr}\right)$$

$$\frac{d\xi^2}{d\Lambda} = -\eta\xi^2\frac{\varkappa_{\rm cr}}{\tilde{\varkappa}}$$

Critical point

$$\tilde{\varkappa} = \varkappa_{\rm cr}$$

$$\xi \propto \frac{1}{L^{\eta/2}}$$

$$\eta = 2/d_c, \;\; \Lambda = \ln(L/L^*)$$

Size of the membrane in the embedding space:

$$R = L\xi_L \propto L^{1-\eta/2}$$

$$R^{D_H} \propto L^D$$

$$D_H = \frac{2}{1-\eta/2}$$

fractal (Hausdorff) dimension

Near-critical membrane (clean case)

$$\xi^2 = \delta + (1-\delta)\left(\frac{L^*}{L}\right)^{\eta}$$

small deviation from the critical point

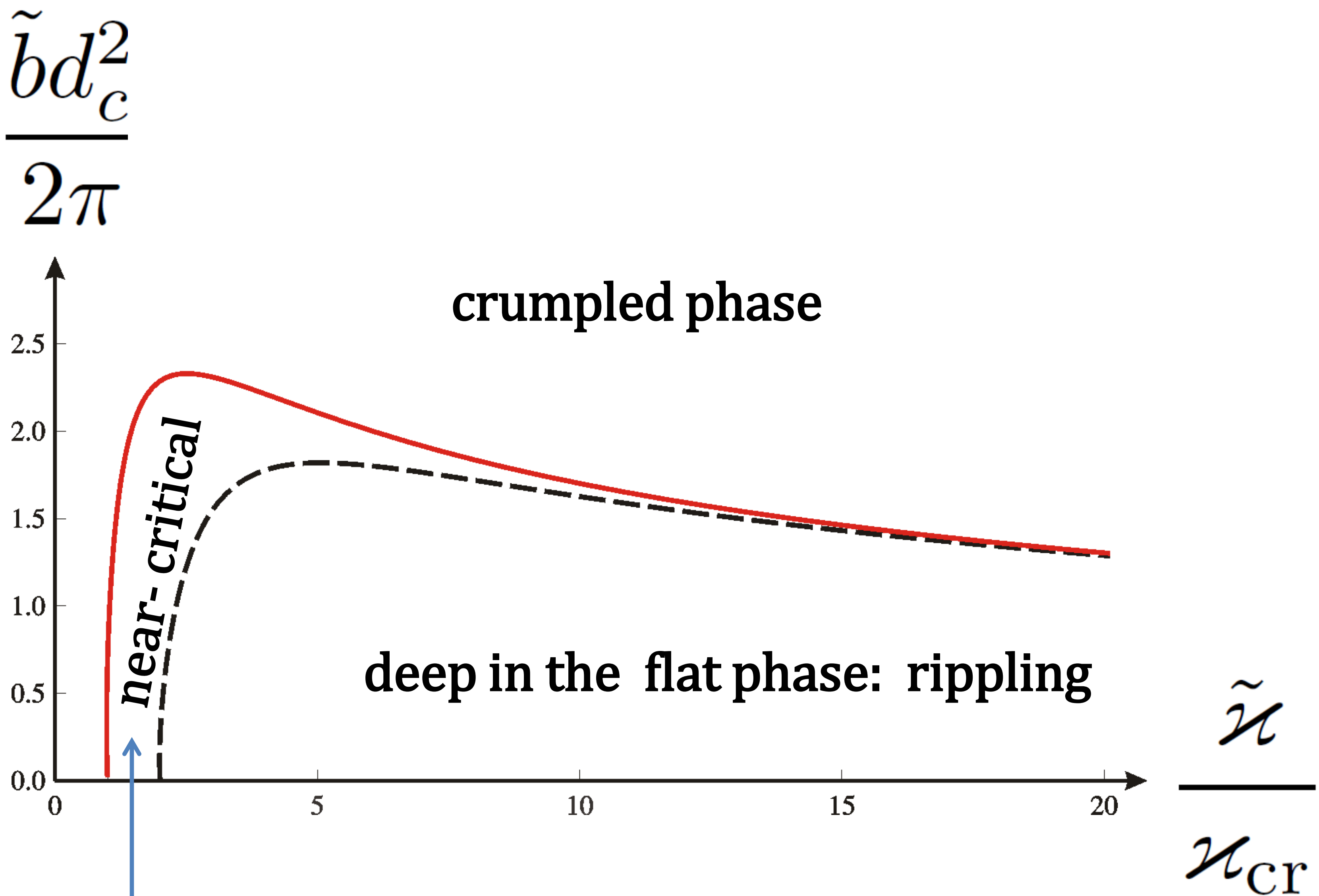
$$\delta = \frac{\varkappa_0 - \varkappa_{\rm cr}}{\varkappa_0} \ll 1$$

Fractal behavior: $L_1 \ll L \ll L_2$

$$L_1 \sim L^* e^{1/\eta}, \qquad L_2 \sim L^* \left(\frac{1}{\delta}\right)^{1/\eta}$$

$L \sim L_2$ ➡ $\xi \sim \xi_\infty \sim \sqrt{\delta} \ll 1$ multiple folding

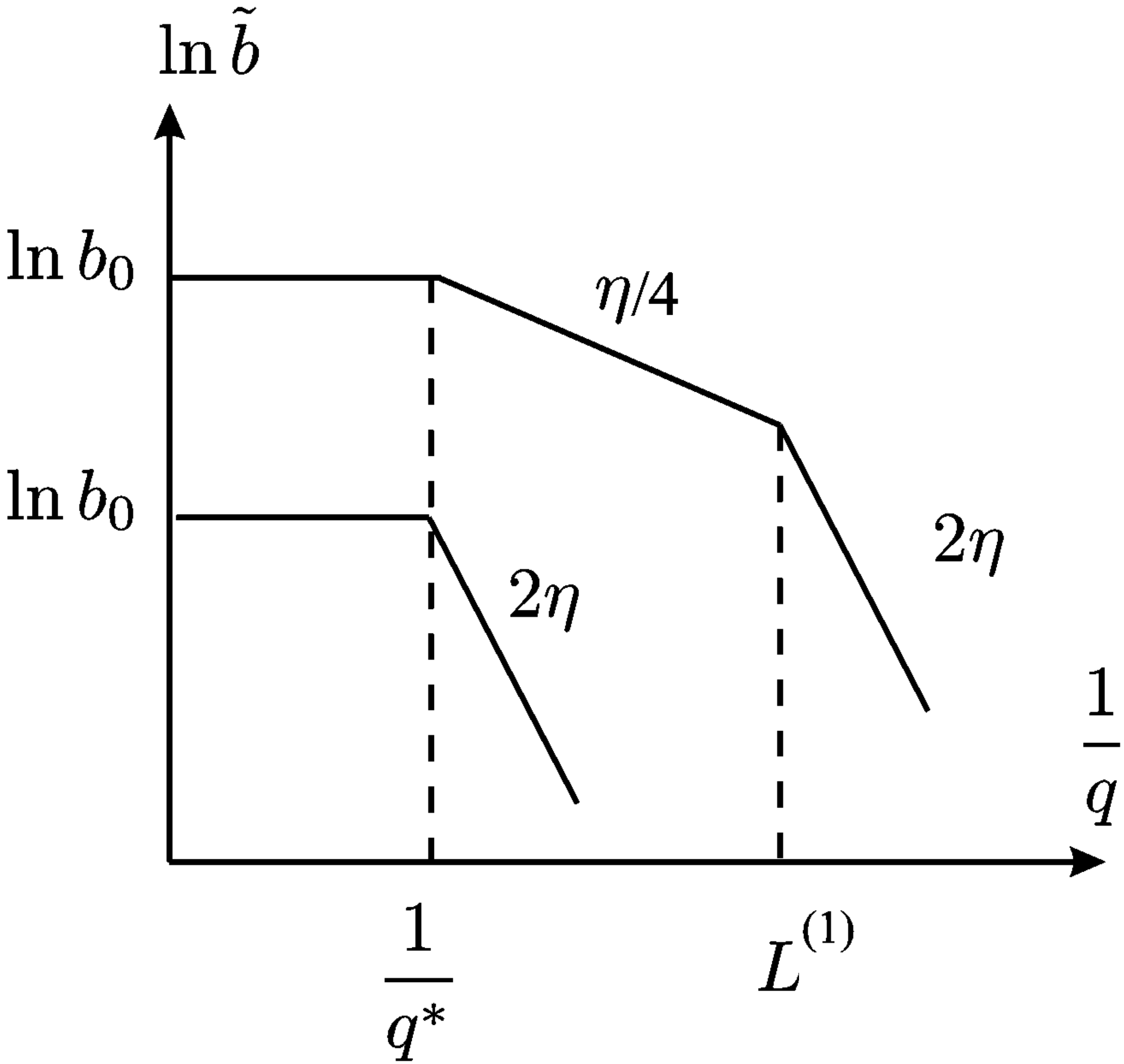
$L \gg L_2$ ➡ membrane flattens



fractal behavior: $L < L_2 \sim L^* \left(\frac{1}{\delta}\right)^{1/\eta} \exp\left(\frac{f_0}{3\eta}\right)$

disorder-induced factor

Scaling of the disorder deep in the flat phase



Ripples: Static frozen-out deformations

Disorder generates new correlation functions:

conventional correlation function (dynamical+static)

$$\overline{\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle} \propto \frac{1}{q^{4-\eta}} \Longrightarrow h_{\rm rms} \propto L^{1-\eta/2}$$

flat phase

disorder-induced correlation function (static)

$$\overline{\langle h_{\mathbf{q}} \rangle \langle h_{-\mathbf{q}} \rangle} \propto \frac{1}{q^{4-2\eta}} \Longrightarrow h'_{\rm rms} \propto L^{1-\eta}$$

Spatial size and amplitude of ripples

$$H(\mathbf{r}) = \overline{\nabla_{\mathbf{r}}\langle\mathbf{h}(0)\rangle\nabla_{\mathbf{r}}\langle\mathbf{h}(\mathbf{r}/\xi)\rangle}$$
$$= \int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{\tilde{b}_q}{q^2}e^{i\mathbf{q}\mathbf{r}}$$

$$H(r) \sim \frac{b_0}{2\pi}\begin{cases} \ln\left(\frac{1}{q^* r}\right), & \text{for } a < r < 1/q^*, \\ \left(\frac{1}{q^* r}\right)^{2\eta}, & \text{for } 1/q^* < r, \end{cases}$$

Both spatial size and amplitude of ripples decrease with temperature:

$$L_r \simeq \frac{2\pi}{q^*} \propto \frac{1}{\sqrt{T}}$$

$$H(0) = \frac{b_0}{2\pi}\ln\left(\frac{1}{q^* a}\right) = \frac{b_0}{4\pi}\ln\left(\frac{T^*}{T}\right)$$

Agrees with experiment:

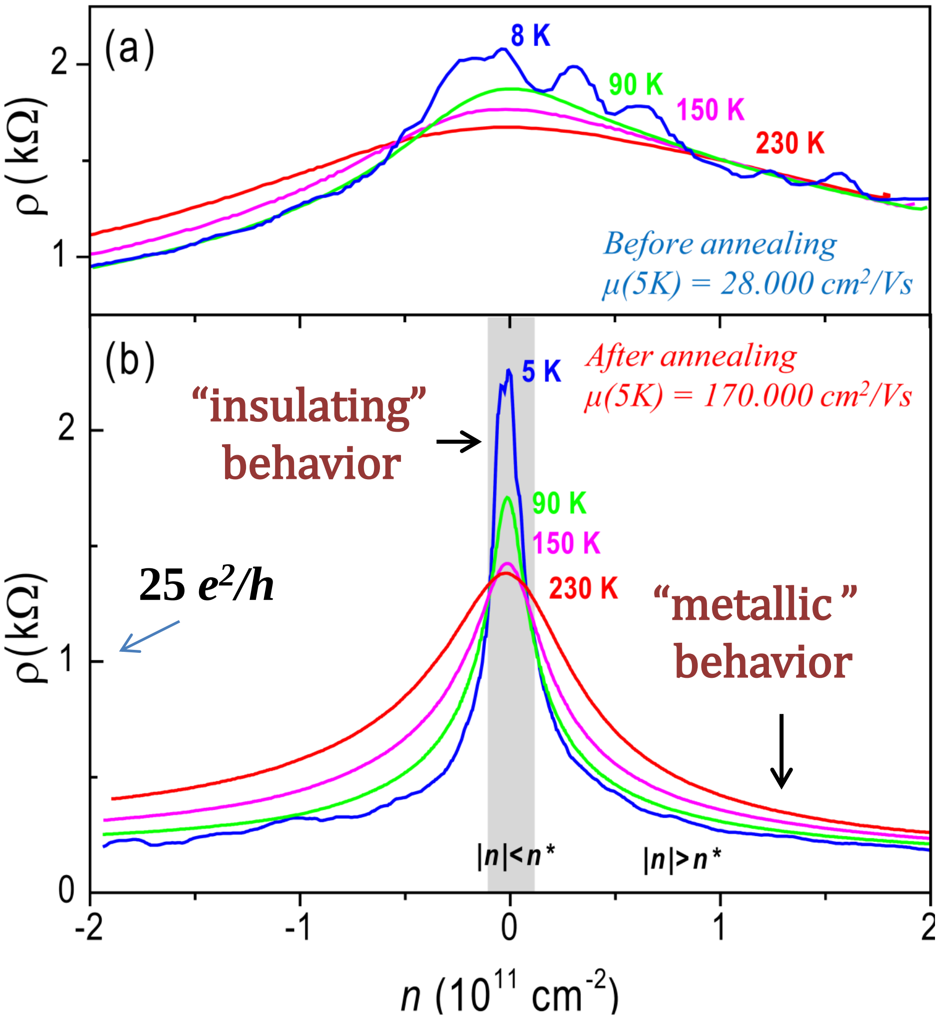
D. A. Kirilenko,
A. T. Dideykin,
G. V. Tendeloo,
PRB (2011)

Main results

- Anharmonicity crucially effects elastic and transport properties of graphene
- Bending rigidity and disorder show non-monotonous scaling
- Membrane demonstrates fractal behavior in the near-critical region
- Amplitude and size of ripples in disordered graphene decrease with temperature

Effect of anharmonicity on transport properties: Comparison with experiment

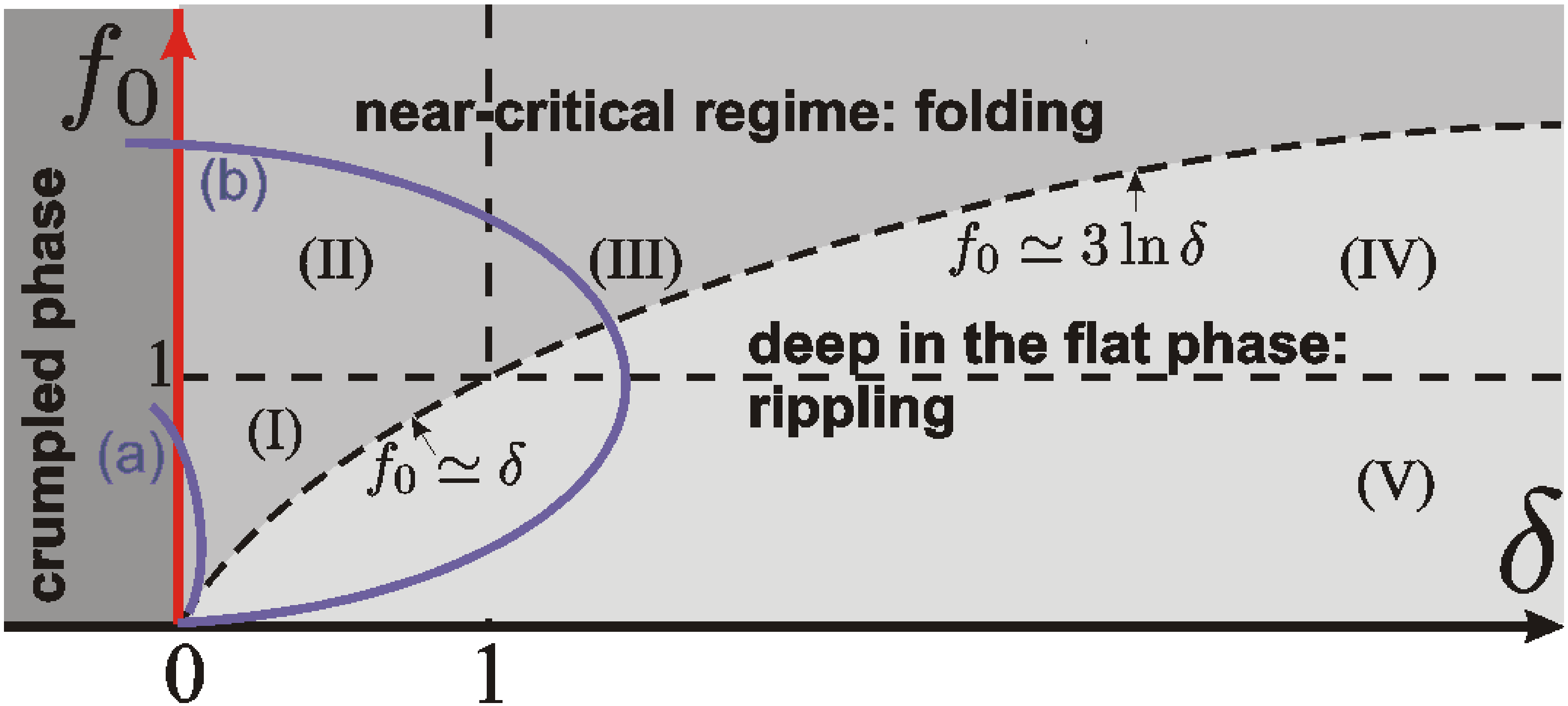
K. Bolotin, K.Sikes, J.Hone, H.Stormer, P.Kim, PRL (2008)



Suspended graphene (experiment):

"metallic" ↔ "insulating" T-dependence

Phase diagram of disordered membrane in the (δ, f_0) *plane.*



Critical curve (shown in red) separates flat and crumpled phases. Clean case corresponds to horizontal axis ($f_0 = 0$).

Regions (I), (II), (III) $\rightarrow$ near-critical regime within the flat phase. $\rightarrow$ membrane shows critical (fractal) folding at intermediate scales before flattening at larger scales.

Regions (IV) and (V) $\rightarrow$ rippled membrane deep in the flat phase.

Blue curves $\rightarrow$ fixed values of bare bending rigidity. Bare disorder increases along these curves from the bottom to the top $(a)\quad (\varkappa_0 - \varkappa_{\rm cr})/\varkappa_{\rm cr} \ll 1, \quad (b) \quad (\varkappa_0 - \varkappa_{\rm cr})/\varkappa_{\rm cr} \gg 1$

Suspended graphene (theory) : “metallic”↔“insulating ” T-dependence

Realistic samples: disorder + Coulomb + phonons

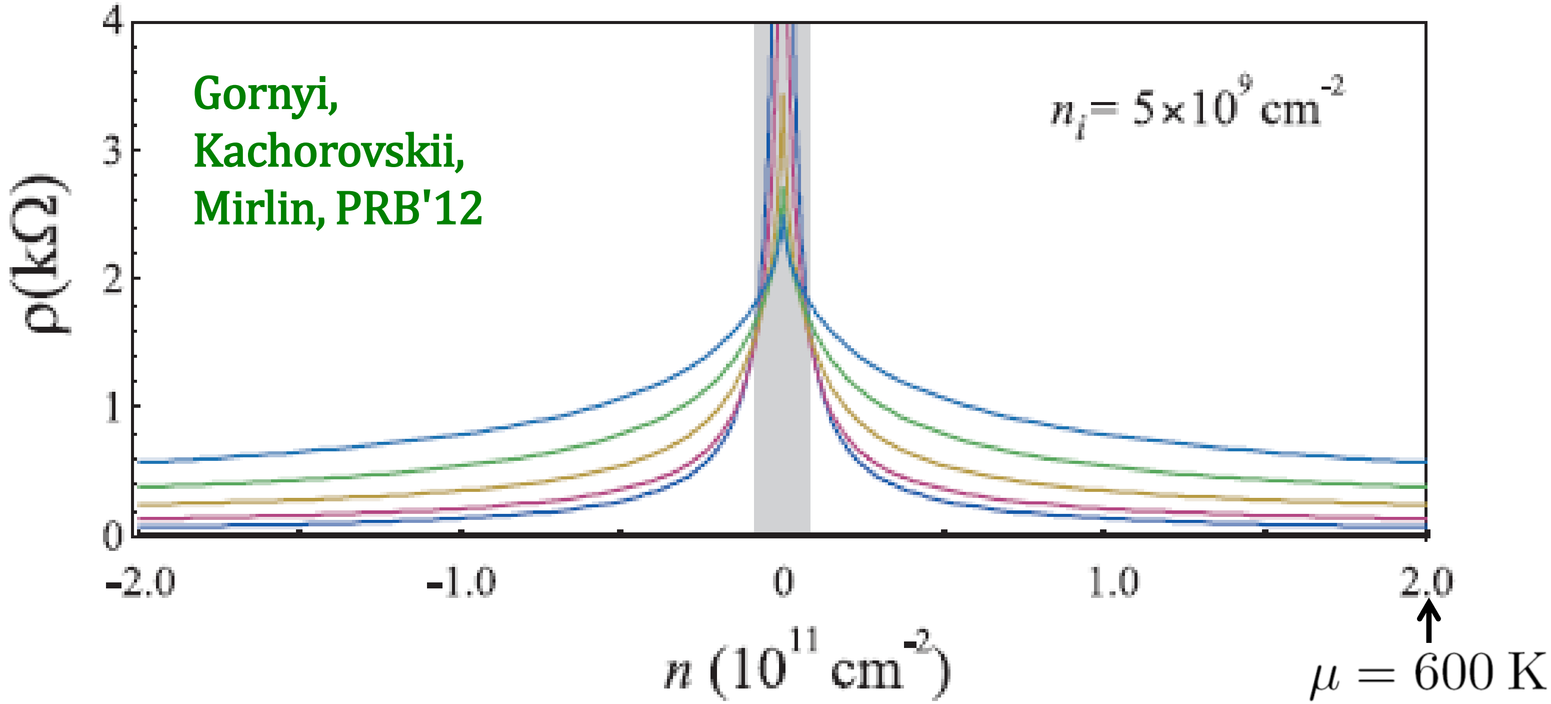


FIG. Resistivity as a function of electron concentration at $n_i = 5\times10^9\ \mathrm{cm}^{-2}$ and different temperatures ($T/1\mathrm{K} = 5,\ 40,\ 90,\ 150,\ 230$) increasing from the bottom to the top at large n. Within the grey area temperature dependence is “insulating”, while outside this region it is “metallic”.

Graphene as elastic membrane

Elastic energy

$$E = \frac{1}{2}\int d\mathbf{r}\left[\rho(\dot{\mathbf{u}}^2 + \dot{h}^2) + \varkappa(\Delta h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

$\mathbf{u}(\mathbf{r}), h(\mathbf{r})$ are in-plane and out-of-plane distortions

$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Strain tensor

$\rho \simeq 7.6 \cdot 10^{-7}\mathrm{kg/m^2}$	mass density of graphene
$\lambda \simeq 3\mathrm{eV}/\mathrm{\AA}^2 \quad \mu \simeq 3\mathrm{eV}/\mathrm{\AA}^2$	elastic constants
$\varkappa \approx 1\mathrm{eV}$	bending rigidity

$\omega_{\parallel\mathbf{q}} = s_\parallel q \;, \quad \omega_{\perp\mathbf{q}} = s_\perp q$ in-plane phonons

$$s_\parallel = [(2\mu+\lambda)/\rho]^{1/2} \simeq 2\cdot 10^6\,\mathrm{cm/s}, \quad s_\perp = (\mu/\rho)^{1/2} \simeq 1.3\cdot 10^6\,\mathrm{cm/s}$$