Low-energy decoherence threshold in strongly disordered spin models

M. V. Feigel'man and L. B. Ioffe

Content:

Random-field XY spin-1/2 model and its physical origin

$$H_0 = -J \sum_{(ij)} (s_i^+ s_j^- + s_i^- s_j^+) - W \sum_i \xi_i s_i^z$$

Low-energy effective Hamiltonian for strongly disordered superconductors with a pseudo-gap (near SIT)

M.Feigelman, L.Ioffe, V.Kravtsov, E.Yuzbashyan,
Phys Rev Lett.**98**, 027001, 2007
M.Feigelman, L.Ioffe, V.Kravtsov, E.Cuevas,
Ann.Phys. **325**, 1390 (2010)

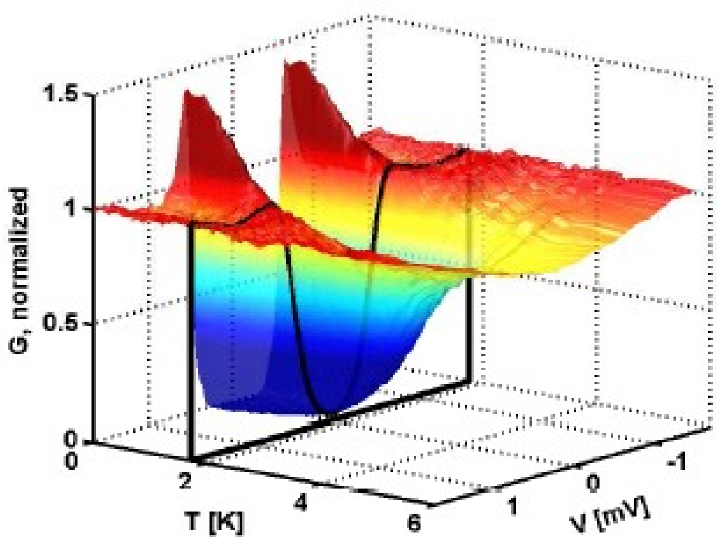
B. Sacépé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia,
D. Shahar, M. Feigel'man, and L. Ioffe,
Nature Physics **7**, 239 (2011)

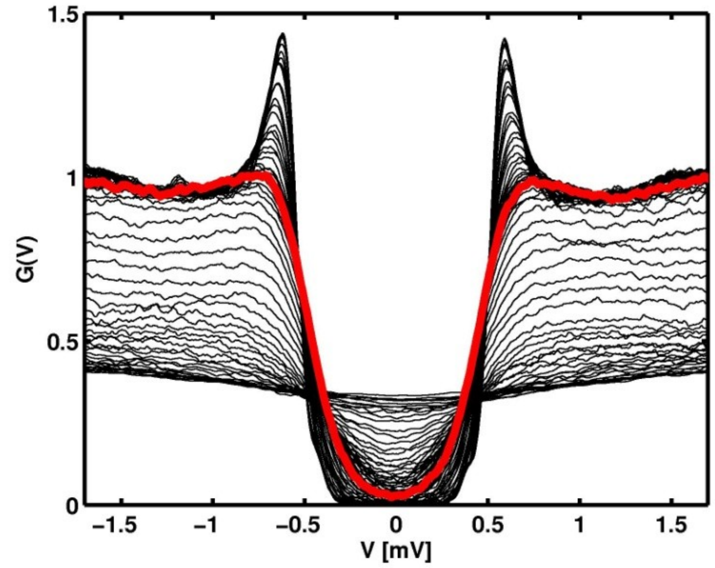
T. Dubouchet, B. Sacépé, C. Chapelier, et al, to be published (2015)

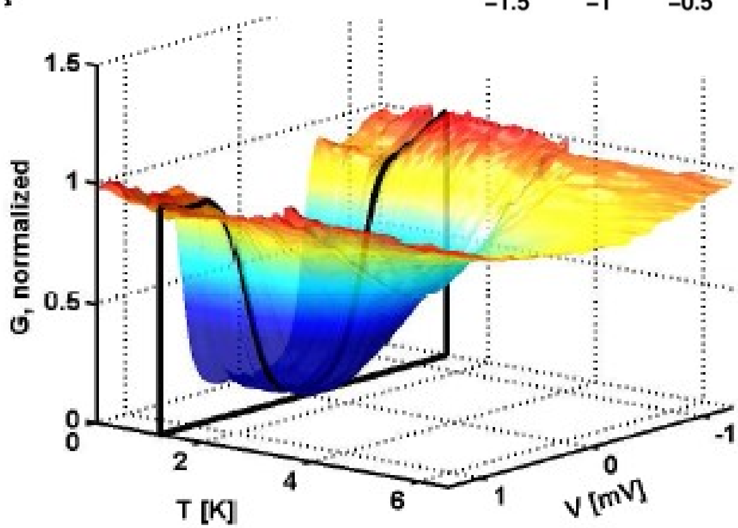
Local tunneling conductance

Spectral signature of localized Cooper pairs in disordered superconductors.

Benjamin Sacépé,[1,*] Thomas Dubouchet,[1] Claude Chapelier,[1] Marc Sanquer,[1] Maoz Ovadia,[2] Dan Shahar,[2] Mikhail Feigel'man,[3] and Lev Ioffe[4,3]

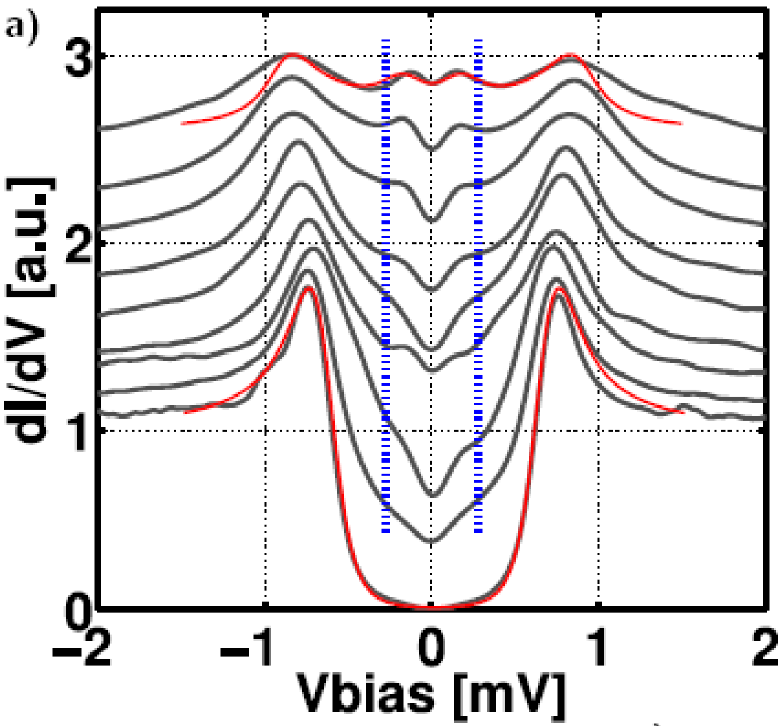






Andreev point-contact spectroscopy

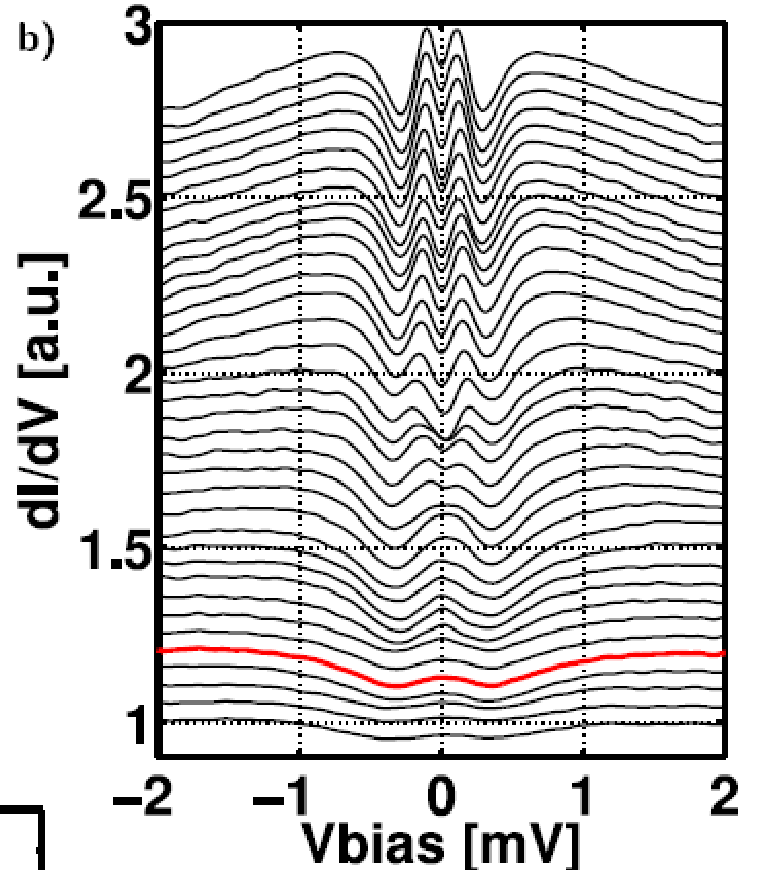
T. Dubouchet,[1,*] C. Chapelier,[1] M. Sanquer,[1] B. Sacépé,[2,3] Maoz Ovadia,[3] and Dan Shahar[3] (to be published)

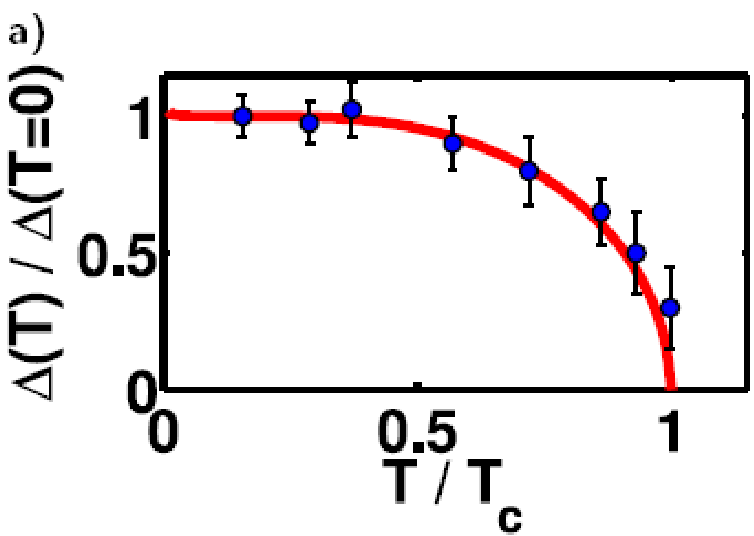


$2eV_1 = 2\Delta$

$eV_2 = \Delta + \Delta_P$

T.Dubouchet, thesis, Grenoble (11 Oct. 2010)





Theory: Cooper pairing of electrons in localized eigenstates

T_c versus Pseudogap

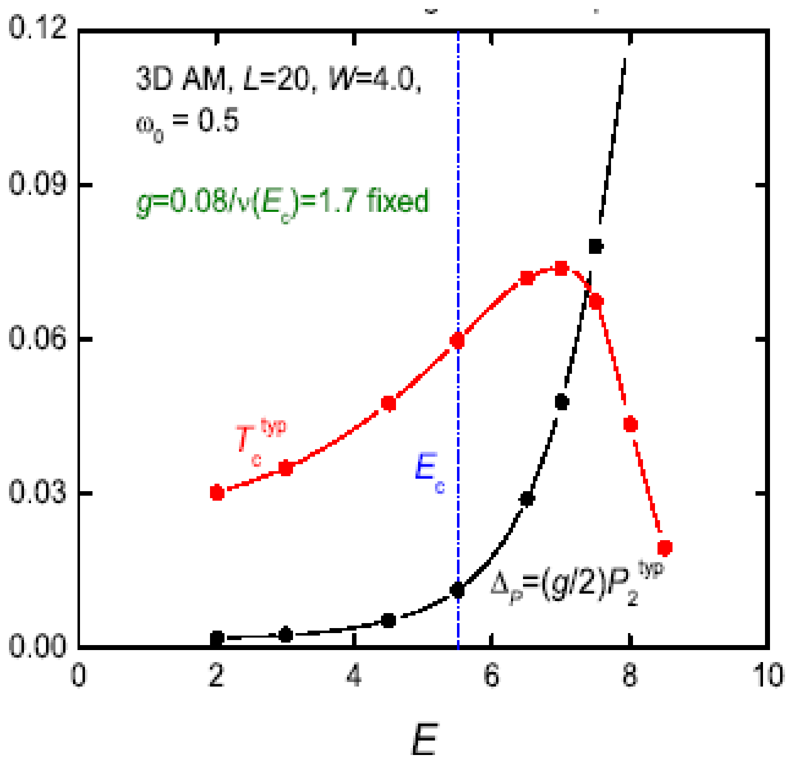


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

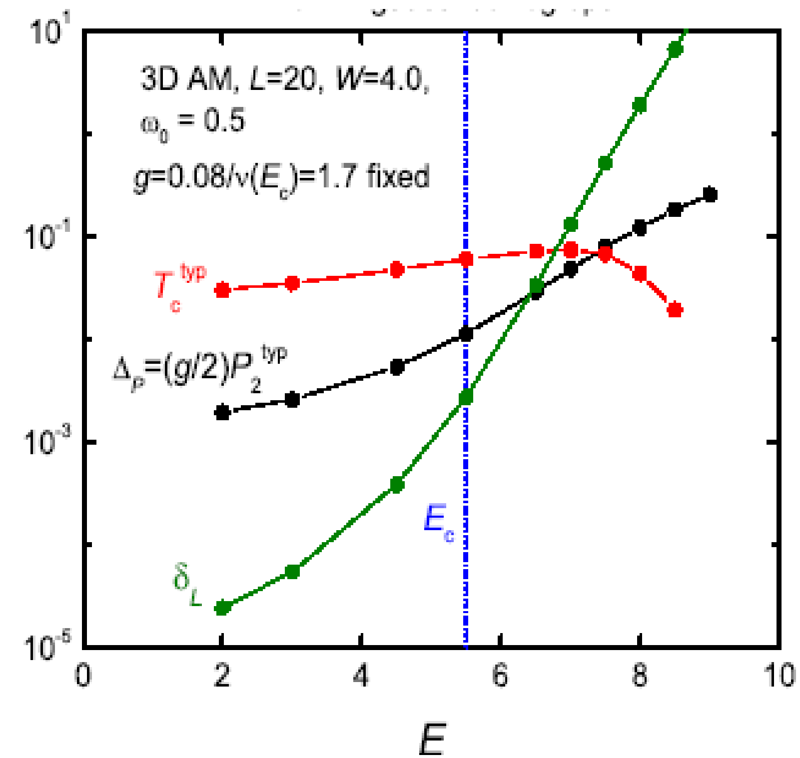


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

Transition exists even at $\delta_L \gg T_{c0}$ and $\Delta_P \gg T_c$

Single-electron states suppressed by pseudogap

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector: all states which are 2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g \sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu} \left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\overline{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

D.o.S ↑ ↖ total volume

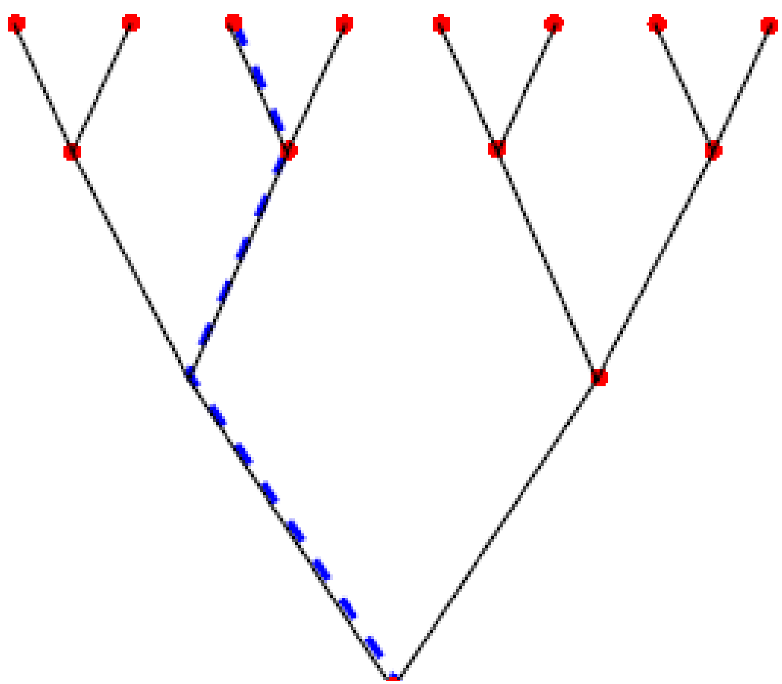
"Pseudospin" approximation

Similar Hamiltonian appears in the study of JJ arrays
If off-set charge disorder is taken into account
- ***see talk by Lev Ioffe***

Bethe lattice approximation and quantum phase transition

Phys.Rev. B **82,** 184534 (2010**)**
M. Feigelman, L.Ioffe, M. Mezard

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$



$M_{ij} = g/K$

Mean field approximation
($K \to \infty$) :

$H_j = \xi_j \sigma_j^z - \sigma_j^x (g/K) \Sigma_k \langle \sigma_k^x \rangle$

$B_j = (g/K) \Sigma_k \langle \sigma_k^x \rangle$

$$1 = g \int d\xi \, p(\xi) \, \frac{\tanh(\beta\sqrt{\xi^2 + B^2})}{\sqrt{\xi^2 + B^2}} \; .$$

$$\frac{1}{g} = \frac{1}{2}\int_{-1}^{1} d\xi \frac{\tanh(\xi/T_c)}{\xi} \quad \Longrightarrow \quad T_{c0} = \frac{4e^{\mathrm{C}}}{\pi}\exp(-1/g)$$

Nontrivial distribution function for O.P.

General recursion:

$$B_j = \frac{g}{K}\sum_{k=1}^{K}\frac{B_k}{\sqrt{B_k^2+\xi_k^2}}\tanh\beta\sqrt{B_k^2+\xi_k^2}\ .$$

Linear recursion ($T=T_c$)

$$B_i = (g/K)\sum_k (B_k/\xi_k)\tanh(\beta\xi_k)\ ,$$

$$P(B) = \frac{B_0^m}{B^{1+m}}$$

Laplace transform satisfies the equation:

$$\mathcal{P}(s) = \left[\int_0^1 d\xi\ \mathcal{P}\left(s\frac{g}{K}\frac{\tanh\beta\xi}{\xi}\right)\right]^K$$

Diverging 1st moment

Solution in the RSB phase: $\mathcal{P}(s) = 1 - As^x$ with $x<1$

T=0

$$\frac{K}{2}\int_{-1}^{1} d\xi\left|\frac{g}{K\xi}\right|^x = 1$$

$$\int_{-1}^{1}\frac{d\xi}{|\xi|^x}\ln\frac{g}{K|\xi|} = 0$$

$$\Rightarrow$$

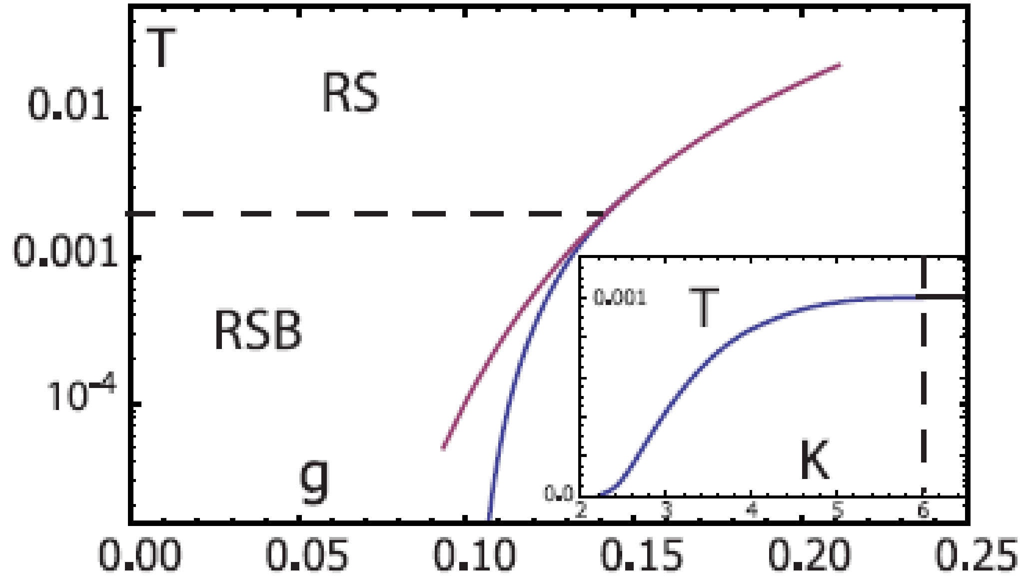
$$g_c e^{1/(eg_c)} = K$$

$$m = 1 - eg_c$$

Vicinity of the Quantum Critical Point

$$T_c(K) = \vartheta(y_c)\left(\frac{K}{K_c} - 1\right)^{1/y_c}$$

$y_c = eg.$ **<< 1**



Order parameter near transition point:

typical value near the T=0 critical point

$$B_0 \simeq e^{-1/(eg_c)} \exp\left[-\frac{C}{(g/g_c)^m - 1}\right]$$

No long-range order at $g < g_c$;

What is the nature of excitations ?

Criteria for localization in the interacting system

- 1. Level statistics (Poisson vs WD statistics of the full system spectrum)
- 2. Do local excitations decay completely ?
- 3. Does the external noise propagate into the interior of the system ?
- .... (many others)

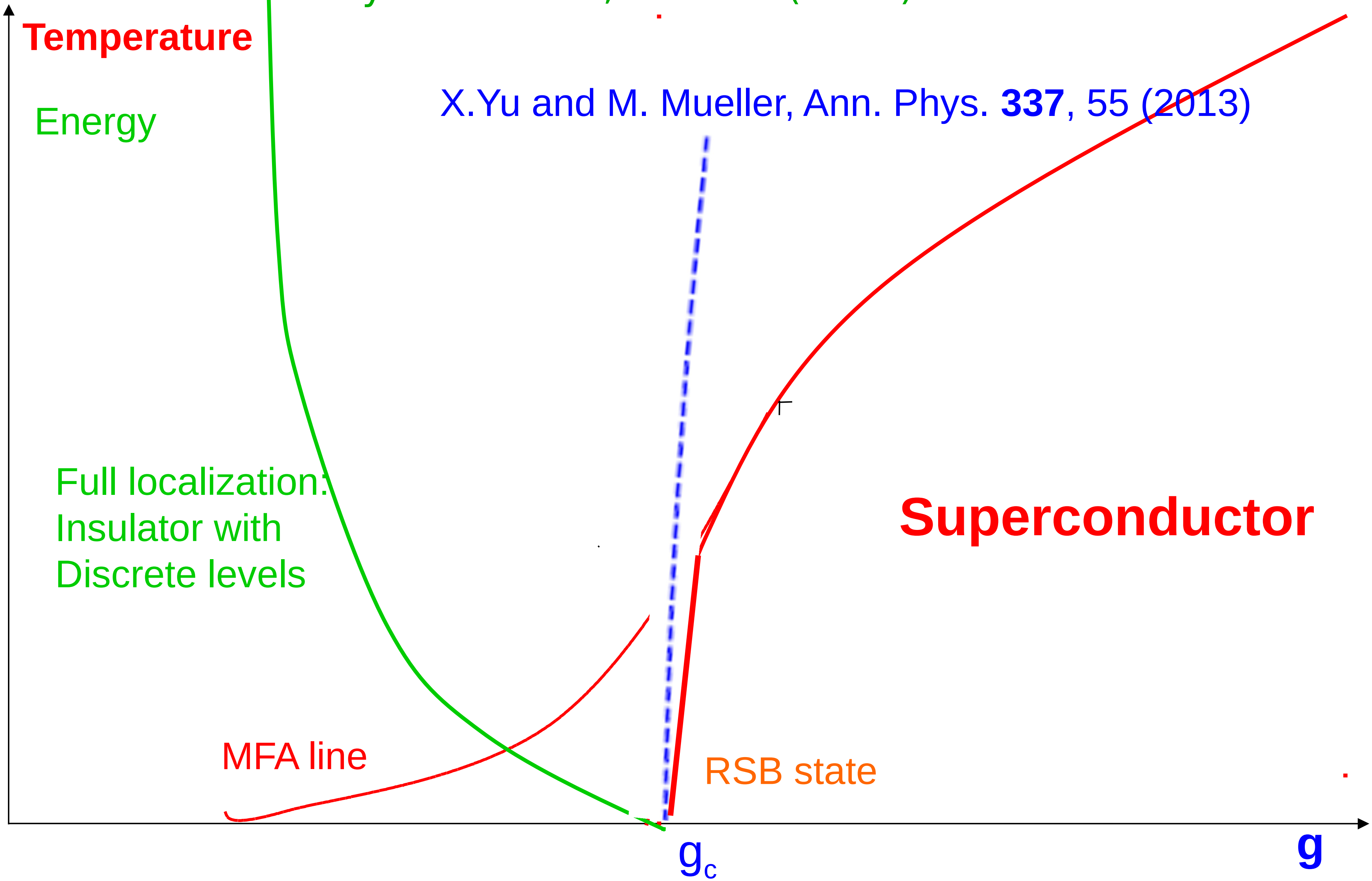
What determines a boundary of the coherent (noiseless) state ?

Extensive energy (i.e. temperature) v/s intensive (excitation) energy ?

Full phase diagram: previous results

M.Feigelman, L.Ioffe and M.Mezard,
Phys. Rev.B **82**, 184534 (2010)

X.Yu and M. Mueller, Ann. Phys. **337**, 55 (2013)



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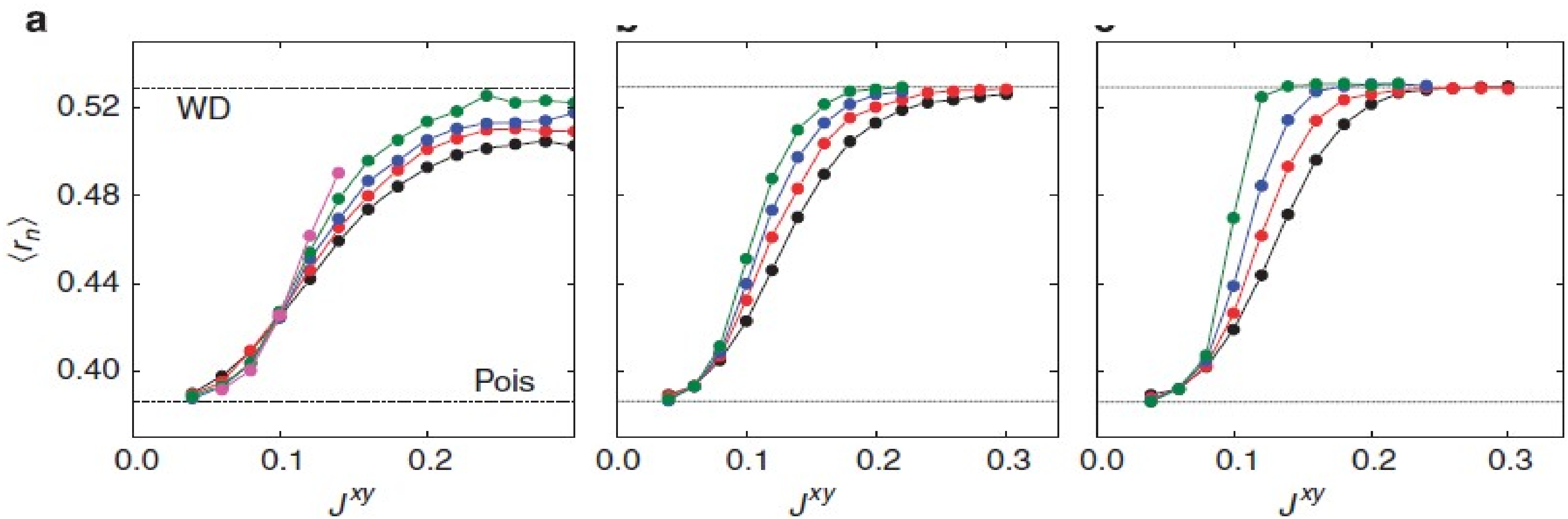
Level statistics of disordered spin-1/2 systems and materials with localized Cooper pairs

Emilio Cuevas[1], Mikhail Feigel'man[2,3], Lev Ioffe[4,5] & Marc Mezard[6]

$$0 < r_n = \min(\delta_n, \delta_{n-1}) / \max(\delta_n, \delta_{n-1}) < 1$$



refers to the sector with $S_z^{tot}=0$ for the model (1) with $J^{zz}=0$, defined on a $Z=3$ random graph with bandwidth $W=1$. Panel **a** shows the statistics of the low-energy excitations in the energy interval (E_{gs}, $E_{gs}+1.5$). Data points are shown for system sizes $N=14$ (black dots), $N=16$ (red), $N=18$ (blue), $N=2$ (green) and $N=22$ (violet). The critical value of the coupling $J_c^{xy}=0.095\pm0.003$ is determined via a crossing point analysis. Panel **b** shows similar result for intermediate excitation energies, ($E_{gs}+1.5$, $E_{gs}+2.5$), leading to the critical point $J_c^{xy}=0.066\pm0.002$. Panel **c** corresponds to high energies, close to t centre of the many-body spectrum, with the critical point $J_c^{xy}=0.061\pm0.002$. Each data point represents the average over $N_r=2{,}000$, 200, 100 and 60 disorder realizations for $N_S=14$, 16, 18 and 20, respectively. A large (exponential) increase in the number of states implies that larger samples require

Phase diagram (for J^{zz}=0, T=0)

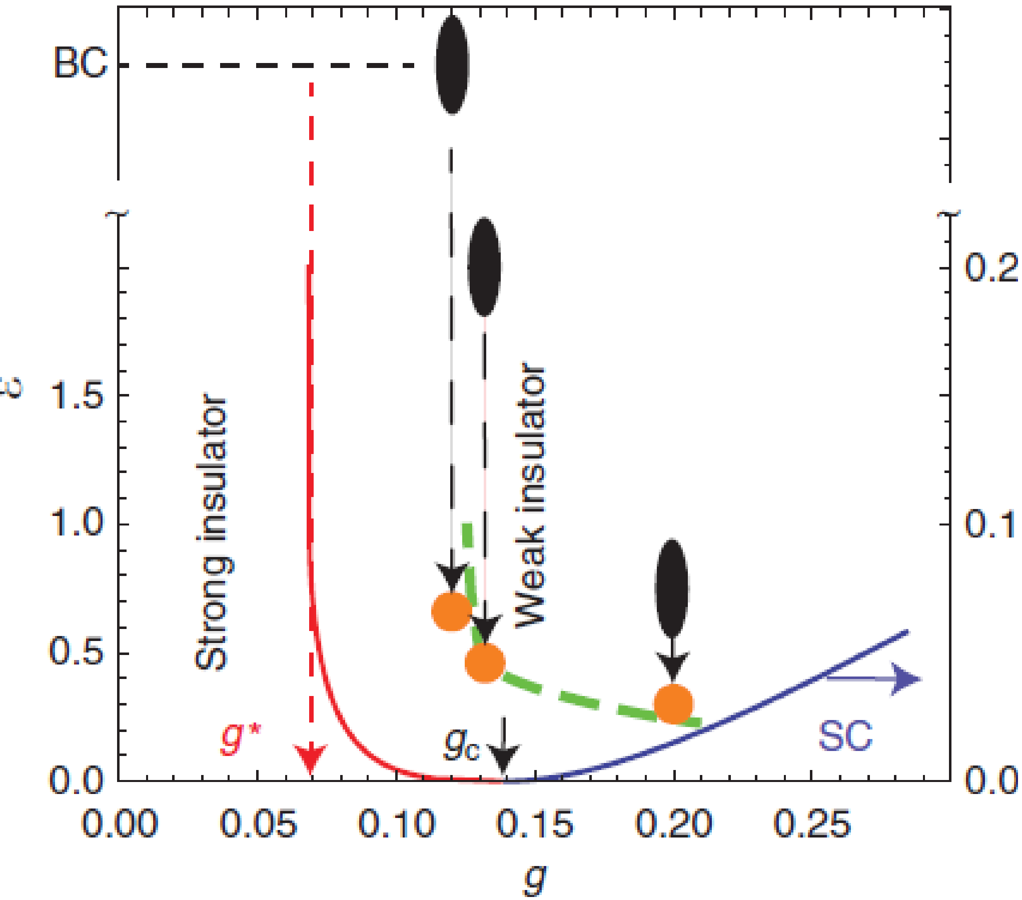


Figure 3 | Phase diagram and finite-size effects. Phase diagram for the model (1) with $J^{zz} = 0$ as a function of the interaction constant g. The full lines show the predictions of the analytical study of the model (1) for the critical temperature (right vertical axis) and the threshold energy, ϵ, (left axis) of spin-flip excitations in infinite random graphs with $Z = 3$ neighbours. The vertical ovals show the values of the critical coupling constant that correspond to a transition between different types of spectra for different energies E in finite random graphs of small size ($N = 16$–20) as determined by direct numerical simulations. The uppermost oval shows the transition at the many-body band centre (corresponding to $E \gg 1$) that sets a lower bound for the critical $g(E)$. The thick dashed line shows the position of the spectral threshold for single-spin excitations with energy ϵ adjusted by finite-size effects, as explained in the main text and in the Methods section. The small circles show the typical energy of the single-spin excitations, $\epsilon(E)$, that gives the main contribution to the many body excitations studied in direct numerical simulations. The good agreement between their position and expectations (dashed line) confirms the validity of the cavity method[5,6] that is used to obtain the results in infinite systems. The very small change in the critical value of the coupling constant between excitations at energy $E \approx 2.0$ and the centre of the many-body band implies that all excitations, at high and low energies, become localized when $g < g^*$.

The Question: delocalization transition at *intensive* energies ?

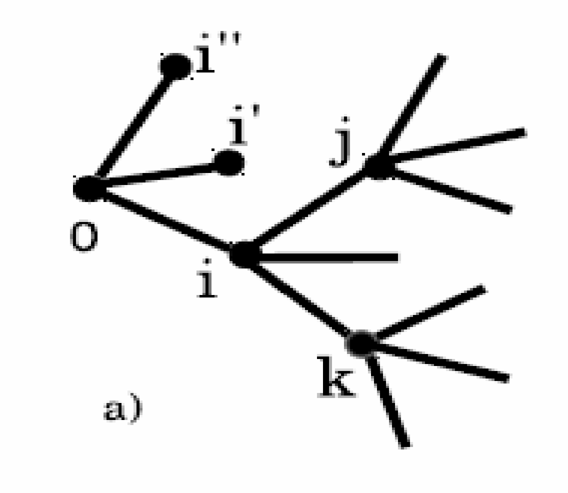
Qualitative reason for that: sharp growth of the total DoS with excitation energy

Bethe lattice recursions for noise power:

$$H = H_0 - \sum_i (s_i^+ \eta_i^-(t) + s_i^- \eta_i^+(t))$$

$$H_0 = -J \sum_{(ij)} (s_i^+ s_j^- + s_i^- s_j^+) - W \sum_i \xi_i s_i^z$$

$$\eta_0(t) = \sum_{i=1}^{i=K} J s_i^-(t) \qquad \eta_0^+(t) = \sum_{i=1}^{i=K} J s_i^+(t)$$



$$\frac{1}{2}\langle\{\eta_i^-(t), \eta_i^+(t')\}\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} D_i(\omega) e^{-i\omega(t-t')}$$

$$\epsilon_i = W\xi_i$$

$$p(\xi) = \frac{1}{2}\theta(1 - |\xi|)$$

$$C_i(\omega) = \frac{1}{2}\int dt e^{i\omega t}\langle\{s_i^+(t), s_i^-(0)\}\rangle$$

$$D_0(\omega) = J^2 \sum_{i=1}^{i=K} C_i(\omega)$$

$$J = \tilde{W} g/K$$

$$C_i^{(0)}(\omega) = \frac{D_i(\omega)}{(\epsilon_i - \omega)^2 + D_i^2(\omega)}$$

Single-spin approximation

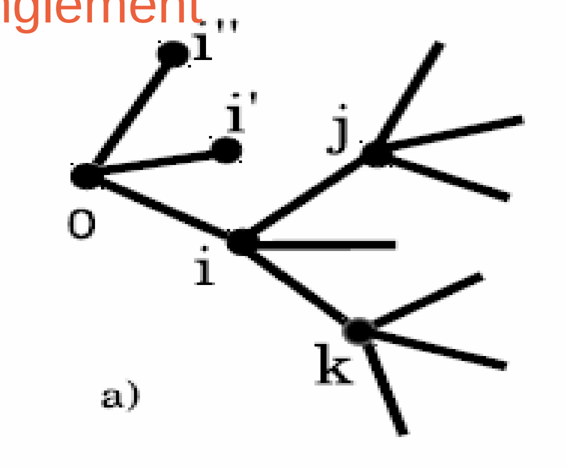
Yu & Mueller 2013

Bethe lattice recursions for noise power: many-body effects

Additional resonance terms due to 3-spin entanglement

$$\frac{D_0(\omega)}{J^2} = \sum_i \frac{D_i(\omega)}{(\epsilon_i - \omega)^2 + D_i^2(\omega)} + \sum_{ijk} \frac{\mathcal{A}_{ijk}^2 D_{ijk}(\omega)}{(E_{ijk}^{(3)} - \omega S_{ijk})^2}$$

$$S_{ijk} = -\mathrm{sgn}(\xi_i \xi_j \xi_k)$$



$$\langle \tilde{3} | s_i^- | \tilde{0} \rangle = \frac{2J^2}{(|\epsilon_k| + |\epsilon_j|)(|\epsilon_k| + |\epsilon_i|)} \equiv A_{k,ij}$$

$$\mathcal{A}_{ijk} = A_{i,jk} \quad \xi_j \xi_k > 0 \quad \xi_i \xi_j < 0$$
$$\mathcal{A}_{ijk} = A_{j,ik} \quad \xi_i \xi_k > 0 \quad \xi_j \xi_i < 0$$
$$\mathcal{A}_{ijk} = A_{k,ij} \quad \xi_i \xi_j > 0 \quad \xi_k \xi_i < 0$$

$$D_{ijk}(\omega) = D_i(\omega_{jk}) + D_j(\omega_{ik}) + D_k(\omega_{ij}) + \tilde{D}_{ijk}(\omega) \qquad \tilde{D}_{ijk}(\omega) = A_{i,jk}^2 D_i(\omega) \ll D_i(\omega)$$

$$\omega_{jk} = \omega - |\epsilon_j| - |\epsilon_k|, \text{ etc}$$

$$\mathcal{A}_{ijk}^2 \sim g^4/K^4$$

$$J = \tilde{W} g/K$$

Energy shifts with the recursion

3-spin entanglement and new resonances

example of $\xi_i > 0$, both $\xi_j, \xi_k < 0$

Ground state $|0>=|+,-,->$

1st excitation $|1>=|-,-,->$ $E_i^{(1)} = |\epsilon_i|$

3-spin-flip eigenstate $|3\rangle = |-,+,+\rangle$ energy $E_{ijk}^{(3)} = |\epsilon_i| + |\epsilon_j| + |\epsilon_k|$

Corrected eigenstates:

$$|\tilde{0}\rangle = |+,-,-> + \frac{J}{|\epsilon_i| + |\epsilon_j|}|-,+,-> + \frac{J}{|\epsilon_i| + |\epsilon_k|}|-,-,+>$$

$$|\tilde{3}\rangle = |-,+,+> - \frac{J}{|\epsilon_i| + |\epsilon_j|}|+,-,+> - \frac{J}{|\epsilon_i| + |\epsilon_k|}|+,+,->$$

New matrix elements:

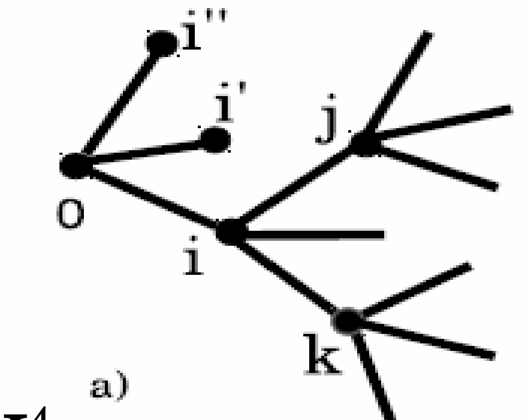
$$\langle\tilde{3}|s_i^+|\tilde{0}\rangle = -\frac{2J^2}{(|\epsilon_i| + |\epsilon_j|)(|\epsilon_i| + |\epsilon_k|)} \equiv -A_{i,jk}$$

matrix element for the case $\xi_i\xi_j > 0 > \xi_i\xi_k$

$$\langle\tilde{3}|s_i^-|\tilde{0}\rangle = \frac{2J^2}{(|\epsilon_k| + |\epsilon_j|)(|\epsilon_k| + |\epsilon_i|)} \equiv A_{k,ij}$$

Bethe lattice recursions for noise power: many-body effects

$$\frac{D_0(\omega)}{J^2} = \sum_i \frac{D_i(\omega)}{(\epsilon_i-\omega)^2 + D_i^2(\omega)} + \sum_{ijk} \frac{\mathcal{A}_{ijk}^2 D_{ijk}(\omega)}{(E_{ijk}^{(3)} - \omega S_{ijk})^2}$$



$$D_{ijk}(\omega) = D_i(\omega_{jk}) + D_j(\omega_{ik}) + D_k(\omega_{ij}) + \tilde{D}_{ijk}(\omega)$$

$\omega_{jk} = \omega - |\epsilon_j| - |\epsilon_k|$, etc

$\mathcal{A}_{ijk}^2 \sim g^4/K^4$ $\quad J = \tilde{W} g/K$

Recursions produce stationary *distribution functional*

$\mathcal{P}\{\tilde{D}(\omega)\}$ for *random functions* $D(\omega)$ which describes

correlations between $D(\omega)$ and $D(\omega')$ for $\omega \sim \omega'$.

Single-particle problem: random numbers Γ_i and distribution function $P(\Gamma)$

ω dumb parameter

Simplified recursions and low-energy decoherence line: "proof of the principle"

$$D_0(\omega) = J^2 \sum_{i=1}^{K} w_i(\omega) D_i(\omega)$$

Neglect energy shifts and look for the instability line

$$w_i(\omega) = \frac{1}{(E_i^{(1)} - \omega \mathrm{sgn}(\xi_i))^2} + \sum_{jk} \frac{\mathcal{A}_{ijk}^2}{(E_{ijk}^{(3)} - \omega S_{ijk})^2}$$

$$S_{ijk} = -\mathrm{sgn}(\xi_i \xi_j \xi_k)$$

"Directed polymer" approx
"Anderson upper limit"

solve two equations for J and $b < \frac{1}{2}$:

$$F_\omega(b) \equiv K J^{2b} \int_0^\infty P_\omega(w) w^b dw = 1; \quad \frac{\partial F_\omega(b)}{\partial b} = 0$$

$$p(\xi) = \tfrac{1}{2}\Theta(1 - |\xi|)$$

$\mathcal{A}_{ijk}^2$ is small in J^4

3-spin term is small unless very near resonance $0 < E_{ijk}^{(3)} \approx \omega S_{ijk}$

$$F_\omega(b) \equiv KJ^{2b}\int_0^\infty P_\omega(w)w^b dw = 1; \quad \frac{\partial F_\omega(b)}{\partial b} = 0$$

$$\int_0^\infty P_\omega(w)w^b dw = \frac{f_1(b,\omega)}{W^{2b}} + \frac{K^2}{2W^{2b}} f_2(b,\omega)$$

$$f_1(b,\omega) = \frac{1}{2}\int_{-1}^{1}\frac{d\xi}{[\xi - \frac{\omega}{W}]^{2b}} = \frac{1}{z} - \frac{1-z}{2}\frac{\omega^2}{W^2}$$

Yu & Mueller 2013

$$f_2(b,\omega) = \frac{3}{8}\int\int\int_0^1 \frac{d\xi_i \xi_j d\xi_k \; \mathcal{A}_{ijk}^{2b}}{[\xi + \xi_j + \xi_k - \frac{|\omega|}{W}]^{2b}}$$

3-spin resonances

$$f_2(b,\omega) = f_2(b,0) + \tilde{f}_2(b,\omega)$$

$$\tilde{f}_2(b,\omega) = \frac{3}{8}\left(\frac{2g^2}{K^2}\right)^{2b}\int_0^\infty dE Y(E)\left(\frac{1}{|E - \frac{|\omega|}{W}|^{2b}} - \frac{1}{E^{2b}}\right)$$

$$\tilde{f}_2(b,\omega) = \frac{3}{8}\left(\frac{2g^2}{K^2}\right)^{2b}\int_0^\infty dEY(E)\left(\frac{1}{|E-\frac{|\omega|}{W}|^{2b}} - \frac{1}{E^{2b}}\right)$$

where we introduce a distribution function

$$Y(E) = \int\int\int_0^1 \frac{dx_1 dx_2 dx_3\,\delta(E-x_1-x_2-x_3)}{[(x_1+x_2)(x_1+x_3)]^{2b}}$$

Calculation of $Y(E)$ at low $E \ll 1$ can be done analitically in the leading order over $1-2b = z \ll 1$:

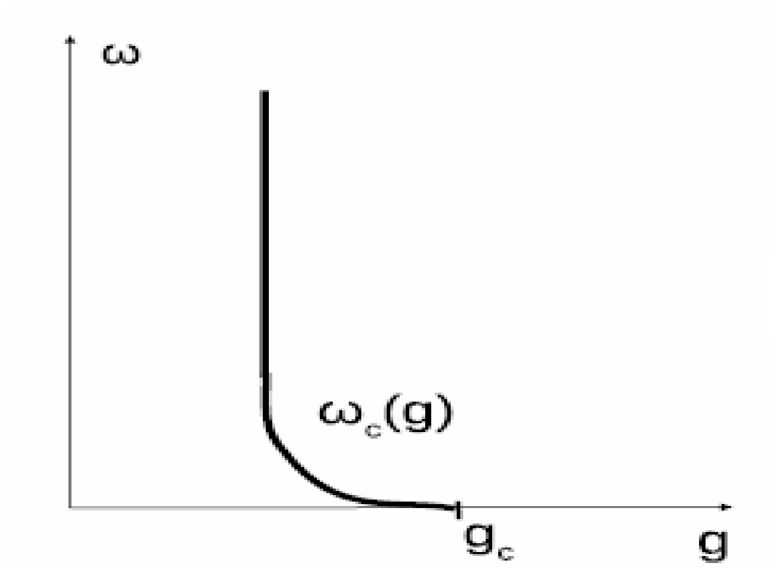
$Y(E) = Y_1 E^{2z}$ at $E \to 0$ $\quad Y_1(b) = \int_0^\infty dw e^{-w}\left[\int_0^\infty \frac{du\, e^{-u}}{(u+w)^{2b}}\right]^2,\ Y_1(1/2) \approx 1.64.$

$$1 = g\left(\frac{K}{g}\right)^z\left(\frac{1}{z} - \frac{1}{2}\frac{\omega^2}{W^2}\right) + \frac{g^3}{z}\left(\frac{K}{g}\right)^{3z}\frac{5Y_1}{8}\left|\frac{\omega}{W}\right|^{3z}$$

$$1 - \frac{g(\omega)}{g_c} = A(eg_c)^2\left(\frac{|\omega|}{W}\right)^{3eg_c} - \frac{eg_c}{2}\frac{\omega^2}{W^2}$$

$$\omega_c(g) = BW\left(1 - \frac{g}{g_c}\right)^{\frac{1}{3eg_c}}$$

Decoherence line



$$\omega_c(g) = BW\left(1 - \frac{g}{g_c}\right)^{\frac{1}{3eg_c}}$$

valid at $1 - g/g_c \ll (eg_c)^2$,

$$B \sim (eg_c)^{-\frac{2}{3eg_c}}$$

Actual critical energy $\omega'_c(g) > \omega_c(g)$

But it is of the same order of magnitude:

consider recursions at $\omega >> \omega_c(g)$

and proof their divergence

Major approximations employed

1. Neglect of the non-locality in the energy during the recursions

2. Forward path approximation
(similar to "upper limit" from AAT) → $2b < 1$

Conclusions

1. In a quantum disordered spin model decoherence at intensive energies occurs near the quantum critical point leading to the ground state with LRO

2. Therefore any $T>0$ leads to non-vanishing transport

3. Quantitative description of the decoherence line demands solution of the functional recursion relations