Superconductor-insulator transitions: phase diagram and magnetoresistance

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in collaboration with

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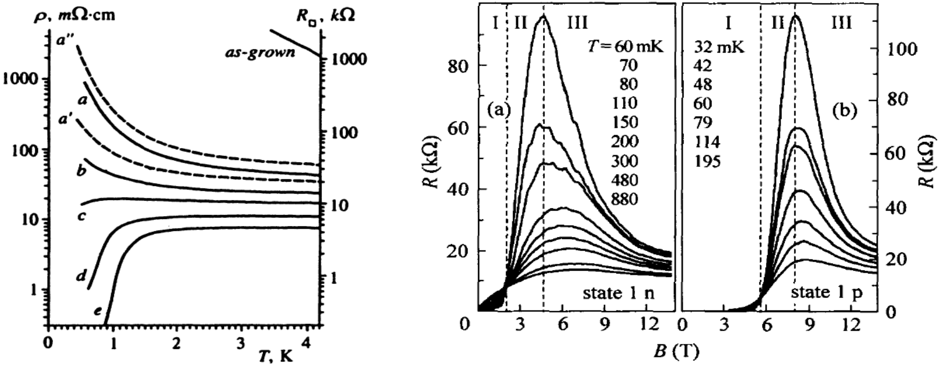
Motivation: experiments on SIT in homogeneously disordered materials - I

- ◦ Mo-Ge films (thickness $b = 15 \div 1000$ Å) [Graybeal, Beasley (1984)]
- ◦ Bi and Pb layers on Ge ($b = 4 - 75$ Å) [Strongin et al. (1971); Haviland et al. (1989)]
- ◦ ultrathin Be films ($b = 4 - 15$ Å) [Bielejec et al. (2001)]
- ◦ thin TiN films [Baturina et al. (2007)]
- ◦ Li_xZrNCl powders [Kasahara et al. (2009)]
- ◦ In-O films ($b = 100 - 2000$ Å) [Shahar, Ovadyahu (1992); Gantmakher et al. (1996-2000);] [Sambandamurthy et al.(2004); Sacépé et al. (2011)]

- $LaAlO_3/SrTiO_3$ interface [Caviglia et al. (2008), Gariglio et al. (2009)]
- δ-doped Nb:$SrTiO_3$ films [Kim et al. (2012)]
- monolayer MoS_2 [Ye et al. (2012, 2014); Taniguchi et al. (2012)]

see review by V. Gantmakher and V. Dolgopolov, 2010

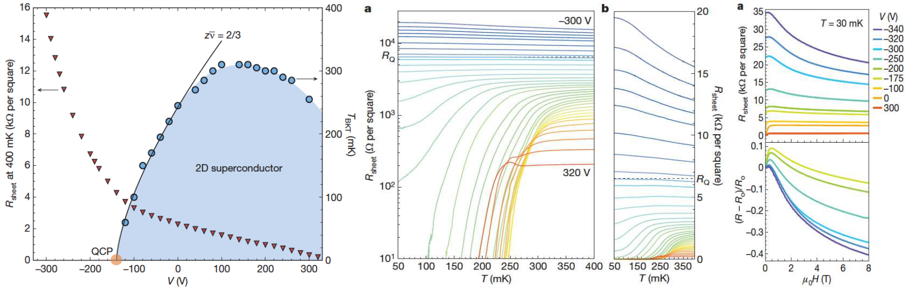
Motivation: experiments on SIT in homogeneously disordered materials - II



[Gantmakher et al. (1996, 1998)]

amorphous In-O film ($b \approx 200$ Å): resistance vs temperature (left), perpendicular (middle) and parallel (right) magnetic field

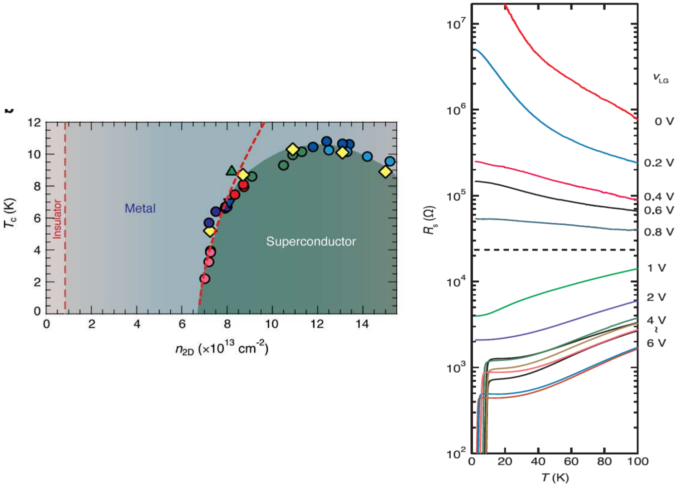
Motivation: experiments on SIT in homogeneously disordered materials - III



[Caviglia et al. (2008)]

SIT in $LaAlO_3/SrTiO_3$: phase diagram (left), resistance vs T (middle), and resistance vs $H_\perp$ (right)

Motivation: experiments on SIT in homogeneously disordered materials - IV



[Ye et al. (2012,2014)]

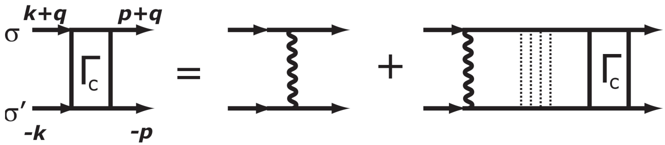
SIT in MoS_2: phase diagram (left) and resistance vs T (right)

"Anderson theorem":

- nonmagnetic impurities do not affect s-wave superconductors

  [A.A. Abrikosov, L.P. Gorkov (1958); P.W. Anderson (1959)]

- Cooper instability is the same for clean and diffusive electrons:

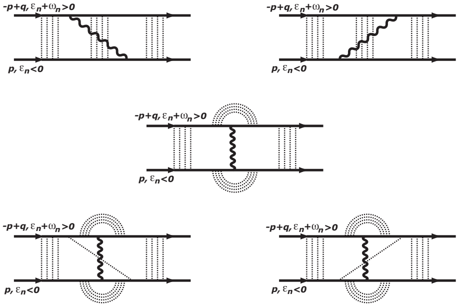


- mean free path l does not enter expression for T_c

N.B.: in the presence of spin-orbit coupling nonmagnetic impurities can affect T_c [see Samokhin (2012)]

Motivation: the effect of impurities - II

- renormalization of cooperon by interaction (first order corrections)



- perturbative correction to T_c due to Coulomb interaction:

$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\square \left(\ln \frac{1}{T_c^{BCS}\tau} \right)^3 < 0$$

[Ovchinnikov (1973); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982)]

Coulomb repulsion and disorder do suppress T_c: "Anderson theorem" is not the theorem suppression of T_c:

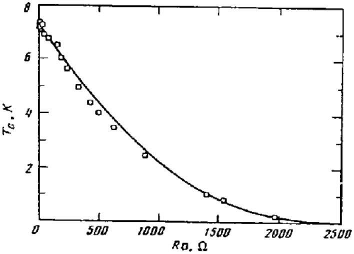
- perturbation theory:

$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\square \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^3 < 0$$

[Ovchinnikov (1973); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982)]

- renormalization group:

$$T_c = 0 \text{ at } R_\square \sim \frac{h}{e^2}\left(\ln \frac{1}{T_c^{BCS}\tau}\right)^{-2}$$



[Finkelstein (1987)]

competition between localization and attraction:

- BCS model in the basis of electron states for a given disorder: superconductivity survives in the localized phase as long as

$$T_c^{BCS} \propto \exp(-2/\lambda) \gtrsim \delta_\xi \propto \xi^{-d}$$

where ξ stands for the localization length, d is dimensionality

[Bulaevskii, Sadovskii (1984); Ma, Lee (1985); Kapitulnik, Kotliar (1985)]

- enhancement of T_c near Anderson transition

$$T_c \propto \lambda^{d/|\Delta_2|}$$

where $\Delta_2 < 0$ is multifractal exponent for inverse participation ratio

[Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007); I.S.B., Gornyi, Mirlin (2012)]

without Coulomb repulsion intermediate disorder enhances T_c

Question to answer

How resistance and magnetoresistance are described near the superconductor-metal/insulator transition within RG approach?

$H = H_0 + H_{\text{dis}} + H_{\text{int}}$:

- free electrons in d-dimensions

$$H_0 = \int d^d\boldsymbol{r}\,\overline{\psi}_\sigma(\boldsymbol{r})\left[-\frac{\nabla^2}{2m}\right]\psi_\sigma(\boldsymbol{r})$$

where $\sigma = \pm 1$ is spin projection

- scattering off white-noise random potential

$$H_{\text{dis}} = \int d^d\boldsymbol{r}\,\overline{\psi}_\sigma(\boldsymbol{r})V(\boldsymbol{r})\psi_\sigma(\boldsymbol{r}), \qquad \langle V(\boldsymbol{r})V(0)\rangle = \frac{1}{2\pi\nu\tau}\delta(\boldsymbol{r})$$

where ν denotes the thermodynamics density of states

- electron-electron interaction

$$H_{\text{int}} = \frac{1}{2}\int d^d\boldsymbol{r}_1 d^d\boldsymbol{r}_2\, U(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)\,\overline{\psi}_\sigma(\boldsymbol{r}_1)\psi_\sigma(\boldsymbol{r}_1)\overline{\psi}_{\sigma'}(\boldsymbol{r}_2)\psi_{\sigma'}(\boldsymbol{r}_2)$$

- Coulomb repulsion with BCS-type attraction ($\lambda > 0$):

$$U(\boldsymbol{R}) = \frac{e^2}{\epsilon R} - \frac{\lambda}{\nu}\delta(\boldsymbol{R})$$

- short-ranged repulsion with BCS-type attraction ($\lambda > 0$):

$$U(\boldsymbol{R}) = u_0 \left[1 + \frac{R^2}{a^2}\right]^{-\alpha/2} - \frac{\lambda}{\nu}\delta(\boldsymbol{R}), \qquad \alpha > d, \qquad u_0 > 0$$

- assumptions

$$\mu \gg \tau^{-1} \gg T$$

where

μ – chemical potential
τ – transport mean-free time
T – temperature

small momentum transfers:

- particle-hole channel:

$$H_{\rm int}^{\rm p-h} = \frac{1}{2\nu} \int\limits_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \sum_{j=0}^{3} F_j(q) m_j(\boldsymbol{q}) m_j(-\boldsymbol{q})$$

where $l = v_F \tau$ denotes mean-free path, $m_j(\boldsymbol{q}) = \int_{\boldsymbol{k}} \bar{\psi}_\sigma(\boldsymbol{k}+\boldsymbol{q}) s_j^{\sigma\sigma'} \psi_{\sigma'}(\boldsymbol{k})$ and

$$F_0(q) = F_s, \qquad F_{1,2,3}(q) = F_t$$

- particle-particle channel:

$$H_{\rm int}^{\rm p-p} = -\frac{F_c}{\nu} \int\limits_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \int \frac{d^d \boldsymbol{k}_1 d^d \boldsymbol{k}_2}{(2\pi)^{2d}} \bar{\psi}_\sigma(\boldsymbol{k}_1) \bar{\psi}_{-\sigma}(-\boldsymbol{k}_1+\boldsymbol{q}) \psi_{-\sigma}(\boldsymbol{k}_2+\boldsymbol{q}) \psi_\sigma(-\boldsymbol{k}_2)$$

estimates for interaction parameters in $d = 2$:

$$F_s = \nu U(q) + F_t \qquad \text{singlet (p-h) channel}$$

$$F_t = -\frac{\nu}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}\,U_{scr}\left(2k_F\sin\frac{\theta}{2}\right) \qquad \text{triplet (p-h) channel}$$

$$F_c = \frac{F_t}{2} - \frac{\nu}{4}\int_0^{2\pi}\frac{d\theta}{2\pi}\,U_{scr}\left(2k_F\left|\cos\frac{\theta}{2}\right|\right) = F_t \qquad \text{singlet (p-p) channel}$$

where $U_{scr}(q) = U(q)/[1+\nu U(q)]$ stands for the statically screened interaction

BCS attraction only ($\lambda \ll 1$): $\quad -F_s = F_t = F_c = \lambda/2$

Coulomb interaction only ($\varkappa/k_F \ll 1$): $F_s \to \infty, \quad F_t = F_c \approx -\frac{\varkappa}{4\pi k_F}\ln\frac{4k_F}{\varkappa}$
where inverse static screening length $\varkappa = 2\pi e^2\nu$

nonlinear sigma-model (NLSM) action:

$$\mathcal{S} = -\frac{g}{32}\,\mathrm{Tr}(\nabla Q)^2 + 4\pi T Z_\omega\,\mathrm{Tr}\,\eta Q - \frac{\pi T}{4}\sum_{\alpha,n,r,j}\int_{\boldsymbol{r}} \Gamma_{rj}\,\mathrm{tr}\Big[t_{rj} J^\alpha_{n,r} Q\Big]\,\mathrm{tr}\Big[t_{rj}\big(J^\alpha_{n,r}\big)^T Q\Big]$$

[Finkelstein(1983)]

where the matrix field Q (Matsubara, replica, spin and particle-hole spaces) obeys

$$Q^2(\boldsymbol{r}) = 1, \qquad \mathrm{tr}\,Q(\boldsymbol{r}) = 0, \qquad Q(\boldsymbol{r}) = C^T Q^T(\boldsymbol{r}) C,$$

g – conductivity in units e^2/h,
Z_ω – Finkelstein's parameter
Γ_{rj} – interaction parameters:

$$\Gamma = \begin{pmatrix} \Gamma_s & \Gamma_t & \Gamma_t & \Gamma_t \\ \Gamma_c & 0 & 0 & 0 \\ \Gamma_c & 0 & 0 & 0 \\ \Gamma_s & \Gamma_t & \Gamma_t & \Gamma_t \end{pmatrix}$$

$SU(4)$ generators in spin and particle-hole spaces (τ_r and s_j are Pauli matrices)

$$t_{rj} = \tau_r \otimes s_j, \quad r, j = 0, 1, 2, 3$$

matrices involved:

$$\Lambda^{\alpha\beta}_{nm} = \mathrm{sgn}\, n\, \delta_{nm}\delta^{\alpha\beta} t_{00}, \quad \eta^{\alpha\beta}_{nm} = n\,\delta_{nm}\delta^{\alpha\beta} t_{00}$$
$$J^\alpha_{n,0} = J^\alpha_{n,3} = I^\alpha_{n,-}, \quad J^\alpha_{n,1} = J^\alpha_{n,2} = I^\alpha_{n,+}$$
$$(I^\gamma_{k,\pm})^{\alpha\beta}_{nm} = \delta_{n\pm m,k}\delta^{\alpha\beta}\delta^{\alpha\gamma} t_{00}, \qquad C = i t_{12}$$

initial values of interaction parameters:

- convenient dimensionless interaction parameters: $\gamma_{s,t,c} = \Gamma_{s,t,c}/z$
- initial values (at the energy scale $\min\{\omega_D, \tau^{-1}\}$):

$$\gamma_{s0} = -\frac{F_s}{1+F_s}, \qquad \gamma_{t0} = -\frac{F_t}{1+F_t},$$

$$\gamma_{c0} = -\frac{F_c}{1 - F_c \ln \max\{1, \omega_D \tau\}} = -\frac{1}{\ln \frac{\min\{\omega_D, \tau^{-1}\}}{T_c^{BCS}}}$$

where $T_c^{BCS} = \omega_D \exp(-1/F_c)$

- BCS attraction only ($\lambda \ll 1$, $\omega_D \tau \ll 1$): $\gamma_{c0} = \gamma_{t0} = -\gamma_{s0} = -\lambda/2$
- Coulomb interaction only ($\varkappa/k_F \ll 1$):

$$\gamma_{s0} = -1, \quad \gamma_{t0} = \gamma_{c0} \approx \frac{\varkappa}{4\pi k_F} \ln \frac{4k_F}{\varkappa}$$

- RG eqs in the lowest order in $t = 2/(\pi g)$, but exact in $\gamma_{s,t,c}$:

$$\frac{dt}{d\ln L} = t^2\Big[\overbrace{\frac{n-1}{2}}^{WL/WAL} + \overbrace{f(\gamma_s) + nf(\gamma_t)}^{AA} \quad \overbrace{-\gamma_c}^{DOS}\Big]$$

$$\frac{d\gamma_s}{d\ln L} = -\frac{t}{2}(1+\gamma_s)\Big[\gamma_s + n\gamma_t + 2\gamma_c(1+2\gamma_c)\Big]$$

$$\frac{d\gamma_t}{d\ln L} = -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - (n-2)\gamma_t - 2\gamma_c(1+2\gamma_t - 2\gamma_c)\Big)$$

$$\frac{d\gamma_c}{d\ln L} = \overbrace{-2\gamma_c^2}^{BCS} - \frac{t}{2}\Big[(1+\gamma_c)(\gamma_s - n\gamma_t) - 2\gamma_c\big(\gamma_c - 2\gamma_c^2 - n\gamma_t + n\ln(1+\gamma_t)\big)\Big]$$

$$\frac{d\ln Z_\omega}{d\ln L} = \frac{t}{2}\Big[\gamma_s + n\gamma_t + 2\gamma_c(1+2\gamma_c)\Big]$$

where $f(x) = 1 - (1 + 1/x)\ln(1+x)$

- the number of triplet diffusons:
  - $n = 3$ – $SU(2)$ spin-rotational symmetry preserved
  - $n = 1$ – spin-rotational symmetry is broken down to $U(1)$
  - $n = 0$ – spin-rotation symmetry is fully broken

$$
\begin{aligned}
\frac{dt}{d\ln L} &= t^2\Big[\overbrace{\frac{n-1}{2}}^{WL/WAL} + \overbrace{f(\gamma_s)+nf(\gamma_t)}^{AA} \quad \overbrace{-\gamma_c}^{DOS}\Big] \\
\frac{d\gamma_s}{d\ln L} &= -\frac{t}{2}(1+\gamma_s)\Big[\gamma_s + n\gamma_t + 2\gamma_c(1+2\gamma_c)\Big] \\
\frac{d\gamma_t}{d\ln L} &= -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - (n-2)\gamma_t - 2\gamma_c(1+2\gamma_t-2\gamma_c)\Big) \\
\frac{d\gamma_c}{d\ln L} &= \overbrace{-2\gamma_c^2}^{BCS} - \frac{t}{2}\Big[(1+\gamma_c)(\gamma_s - n\gamma_t) - 2\gamma_c\Big(\gamma_c - 2\gamma_c^2 - n\gamma_t + n\ln(1+\gamma_t)\Big)\Big] \\
\frac{d\ln Z_\omega}{d\ln L} &= \frac{t}{2}\Big[\gamma_s + n\gamma_t + 2\gamma_c(1+2\gamma_c)\Big]
\end{aligned}
$$

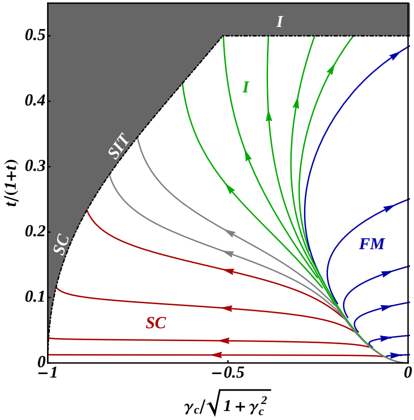
- lowest order in γ_c: Finkelstein (1984, 1985); Castellani, Di Castro, Forgacs, Sorella (1984); Ma, Fradkin (1986)
- all orders in γ_c but problem with renormalization in the Coulomb case [$\ln(1+\gamma_s)$ term]: Belitz, Kirkpatrick (1994); Dell'Anna (2013)
- our RG eqs.: smooth limit for the Coulomb case [no $\ln(1+\gamma_s)$ term]

RG eqs for Coulomb interaction, $\gamma_s = -1$

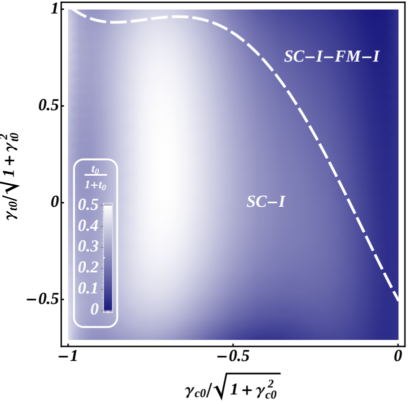
$$\frac{dt}{d\ln L} = t^2\Big[2 + 3f(\gamma_t) - \gamma_c\Big]$$
$$\frac{d\gamma_t}{d\ln L} = \frac{t}{2}(1+\gamma_t)\Big(1 + \gamma_t + 2\gamma_c(1 + 2\gamma_t - 2\gamma_c)\Big)$$
$$\frac{d\gamma_c}{d\ln L} = -2\gamma_c^2 + \frac{t}{2}\Big[(1+\gamma_c)(1+3\gamma_t) + 2\gamma_c\big(\gamma_c - 2\gamma_c^2 - 3\gamma_t + 3\ln(1+\gamma_t)\big)\Big]$$

- marginal line of fixed points at $t = \gamma_c = 0$ (clean FL)
- line of fixed points at $t = 0$ and $\gamma_c = -\infty$ (clean SC)
- line of fixed points at $\gamma_t = \infty$ and $\gamma_c = 1$ (FM)
- insulating phase with $t = \infty$
- towards SC phase RG eqs are valid upto scale L_X: $t(L_X) \sim 1/|\gamma_c(L_X)| \ll 1$

Coulomb interaction, $\gamma_s = -1$

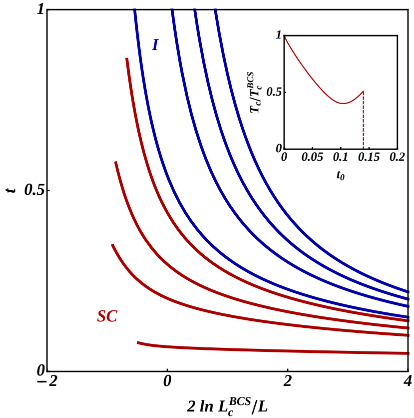


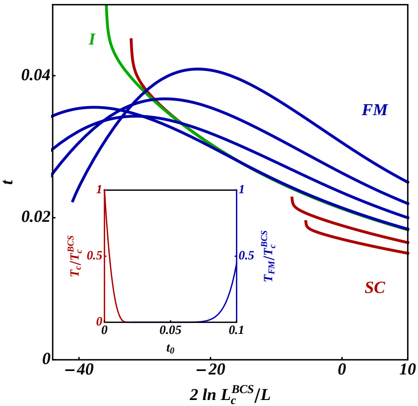
sketch of RG flow ($\gamma_{t0} = 0.2$)



phase diagram

Coulomb interaction, $\gamma_s = -1$





t vs L $(\gamma_{c0} = -0.25,\ \gamma_{t0} = 0.01,\ t_0 = 0.05 \div 0.22)$ t vs L $(\gamma_{c0} = -0.1,\ \gamma_{t0} = 0.4,\ t_0 = 0.01 \div 0.03)$

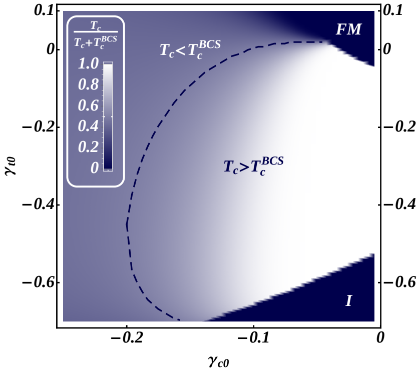
$$T_c \sim L_X^{-2},\ T_c^{BCS} \sim (L_c^{BCS})^{-2} \sim \exp(2/\gamma_{c0})$$

RG eqs for the short-ranged interaction, $\gamma_s > -1$

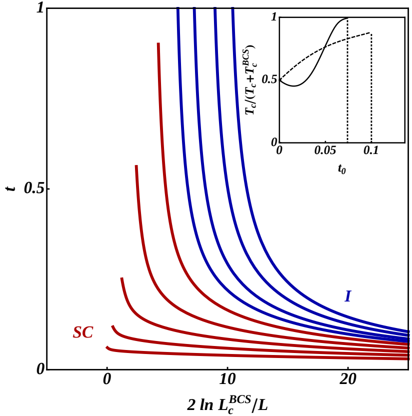
$$\frac{dt}{d\ln L} = t^2\Big[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c\Big]$$
$$\frac{d\gamma_s}{d\ln L} = -\frac{t}{2}(1+\gamma_s)\Big[\gamma_s + 3\gamma_t + 2\gamma_c(1+2\gamma_c)\Big]$$
$$\frac{d\gamma_t}{d\ln L} = -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - \gamma_t - 2\gamma_c(1+2\gamma_t-2\gamma_c)\Big)$$
$$\frac{d\gamma_c}{d\ln L} = -2\gamma_c^2 - \frac{t}{2}\Big[(1+\gamma_c)(\gamma_s-3\gamma_t) - 2\gamma_c\Big(\gamma_c - 2\gamma_c^2 - 3\gamma_t + 3\ln(1+\gamma_t)\Big)\Big]$$

- marginal line of fixed points at $t = \gamma_c = 0$ (clean FL)
- line of fixed points at $t = 0$ and $\gamma_c = -\infty$ (clean SC)
- line of fixed points at $\gamma_t = \infty$, $\gamma_s = -1$ and $\gamma_c = 1$ (FM)
- insulating phase with $t = \infty$
- towards SC phase RG eqs are valid upto scale L_X: $t(L_X) \sim 1/|\gamma_c(L_X)| \ll 1$

short-ranged interaction, $\gamma_s > -1$



T_c ($\gamma_{s0} = -0.05,\ t_0 = 0.06$)



t vs L ($\gamma_{c0} = -0.04,\ \gamma_{t0} = 0.005,$
$\gamma_{s0} = -0.05,\ t_0 = 0.01 \div 0.1$)

enhancement of T_c due to interplay of short-ranged interaction and disorder in 2D

[I.S.B., Gornyi, Mirlin (2012)]

- weak short-ranged interaction, $|\gamma_{s0}|, \gamma_{t0} \lesssim |\gamma_{c0}| \ll t_0 \ll 1$:

$$\frac{dt}{d\ln L} \approx t^2, \quad \frac{d\gamma_s}{dy} \approx -\frac{t}{2}\big(\gamma_s + 3\gamma_t + 2\gamma_c\big),$$

$$\frac{d\gamma_t}{dy} \approx -\frac{t}{2}\big(\gamma_s - \gamma_t - 2\gamma_c\big), \quad \frac{d\gamma_c}{d\ln L} \approx -2\gamma_c^2 - \frac{t}{2}\gamma_s$$

- three-step RG:
  - (i) from $t = t_0$ to $t \approx t_0/2$: approaching BCS-like line $\gamma_c = -\gamma_s = \gamma_t = \gamma$
  - (ii) increase of $|\gamma|$ from $|\gamma_{c0}|$ to $|\gamma_*| \sim t_0^2/|\gamma_{c0}|$:

$$\frac{d|\gamma|}{d\ln L} = 2t|\gamma| \quad \Longrightarrow \quad |\gamma| \sim \frac{|\gamma_{c0}|t^2}{t_0^2} \quad \Longrightarrow \quad |\gamma_*| \sim t_* \sim t_0^2/|\gamma_{c0}|$$

  - (iii) superconducting instability with initial attraction γ_*

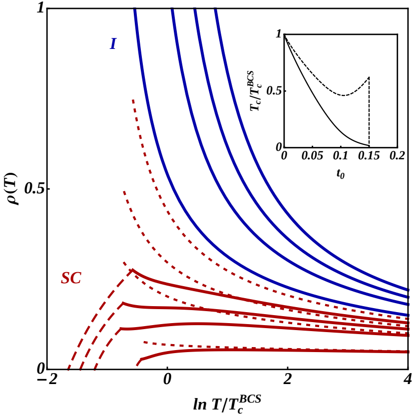
$$\frac{d\gamma}{d\ln L} = -2\gamma^2 \quad \Longrightarrow \quad \ln\frac{1}{T_c\tau} \sim \frac{2}{t_0}\left(1 - \frac{t_0}{t_*}\right) \quad \Longrightarrow \quad T_c \gg T_c^{BCS}$$

- conductance $1/\rho(T)$ is obtained from the Kubo formula
- $t(L_T)$ is determined by action renormalization from $1/\tau$ down to T
- $\rho(T) \neq t(L_T)$ because of fluctuation corrections due to 'real processes' $(|\omega_n|, Dq^2 \lesssim T)$
- at $|\gamma_c(L_T)| \gg 1$ the most important one is anomalous Maki-Thompson correction:

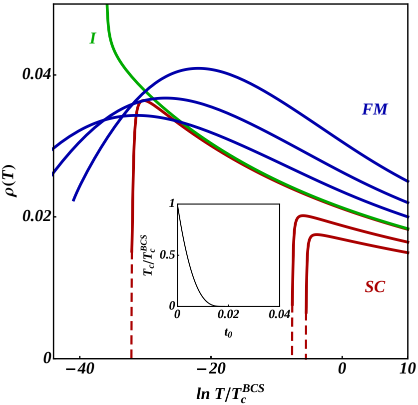
$$\frac{1}{\rho(T)} = \frac{1}{t(L_T)} - \frac{\pi^2}{4}\gamma_c(L_T)\ln\frac{L_\phi}{L_T} \qquad T_X < T\,(L_T < L_X)$$

N.B.: $t(L_T)$ can be significantly different from t_0 due to renormalization between $1/\tau$ and T contrary to Glatz, Varlamov, Vinokur (2011); Tikhonov, Schwiete, Finkelstein (2012)

Coulomb interaction, $\gamma_s = -1$

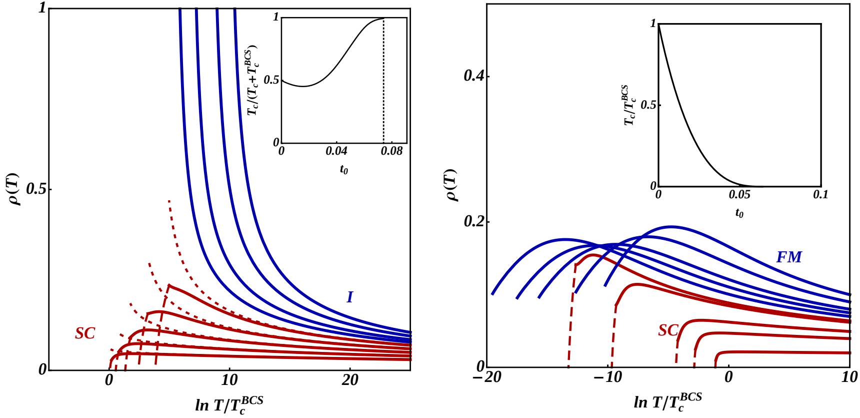


ρ vs L $(\gamma_{c0} = -0.25,\ \gamma_{t0} = 0.01,\ t_0 = 0.05 \div 0.22)$



ρ vs L $(\gamma_{c0} = -0.1,\ \gamma_{t0} = 0.4,\ t_0 = 0.01 \div 0.03)$

short-ranged interaction, $\gamma_s > -1$



ρ vs L ($\gamma_{c0} = -0.04,\ \gamma_{t0} = 0.005,$
$\gamma_{s0} = -0.05,\ t_0 = 0.01 \div 0.1$)

ρ vs L ($\gamma_{c0} = -0.1,\ \gamma_{t0} = 0.2,$
$\gamma_{s0} = -0.05,\ t_0 = 0.02 \div 0.1$)

- perpendicular field $H_\perp$ suppresses cooperons at scales

$$L \gg l_{H_\perp} = \sqrt{\frac{\phi_0}{H_\perp}}$$

- parallel field H suppresses copperons at scales

$$L \gg l_Z = \frac{l_{H_{||}}}{\sqrt{2\pi g_L (1+\gamma_{t0}) t_0}}$$

- RG equations at $L \gg l_{HZ} = \min\{l_H, l_Z\}$

$$\frac{dt}{d\ln L} = t^2\Big[\frac{1}{2} - \frac{1+\gamma_s}{\gamma_s}\ln(1+\gamma_s)\Big], \qquad \frac{d\gamma_s}{d\ln L} = -\frac{t}{2}(1+\gamma_s)\gamma_s$$

[Finkelstein (1983, 1984); Castellani, Di Castro, Forgacs, Sorella (1984)]

N.B.: smooth dependence of T_c on H as well as dependence of initial parameters γ_{c0}, γ_{s0} and t_0 on H are not taken into account

- RG at lengthscales $L \ll l_H$

$$\frac{dt}{d\ln L} = t^2\Big[1 + f(\gamma_s) + 3f(\gamma_t) - \gamma_c\Big]$$
$$\frac{d\gamma_s}{d\ln L} = -\frac{t}{2}(1+\gamma_s)\Big[\gamma_s + 3\gamma_t + 2\gamma_c(1+2\gamma_c)\Big]$$
$$\frac{d\gamma_t}{d\ln L} = -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - \gamma_t - 2\gamma_c(1+2\gamma_t-2\gamma_c)\Big)$$
$$\frac{d\gamma_c}{d\ln L} = -2\gamma_c^2 - \frac{t}{2}\Big[(1+\gamma_c)(\gamma_s - 3\gamma_t) - 2\gamma_c\big(\gamma_c - 2\gamma_c^2 - 3\gamma_t + 3\ln(1+\gamma_t)\big)\Big]$$

- RG at lengthscales $L \gg l_H$

$$\frac{dt}{d\ln L} = t^2[f(\gamma_s) + 3f(\gamma_t)], \quad \frac{d\gamma_s}{d\ln L} = -\frac{t}{2}(1+\gamma_s)(\gamma_s+3\gamma_t), \quad \frac{d\gamma_t}{d\ln L} = -\frac{t}{2}(1+\gamma_t)(\gamma_s-\gamma_t)$$

- resistance for $|\gamma_c(l_H)| \gg 1$

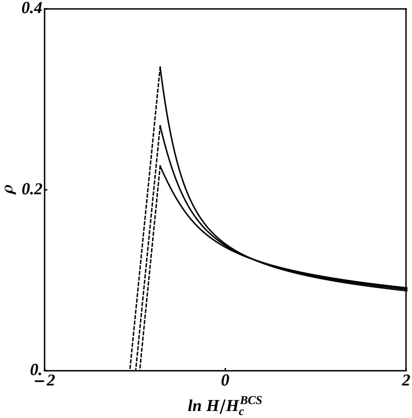
$$\rho^{-1}(T,H) = t^{-1}(l_H) - \ln\frac{L_T}{l_H} - \frac{2}{3}\ln|\gamma_c(l_H)|$$

[Galitski, Larkin (2001)]

N.B.: $t(l_H)$ can be significantly different from t_0 due to renormalization between l and l_H

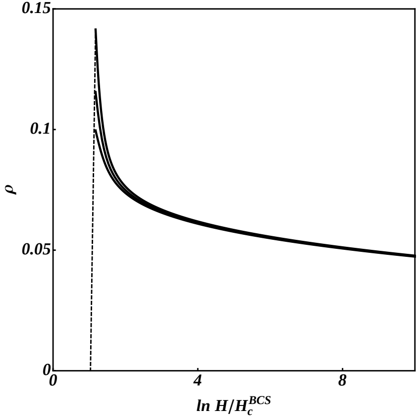
contrary to Glatz, Varlamov, Vinokur (2011); Tikhonov, Schwiete, Finkelstein (2012)

Results: magnetoresistance in perpendicular H - II



ρ vs H ($\gamma_{c0} = -0.45$, $\gamma_{t0} = 1$, $\gamma_{s0} = -1$, $t_0 = 0.1$)

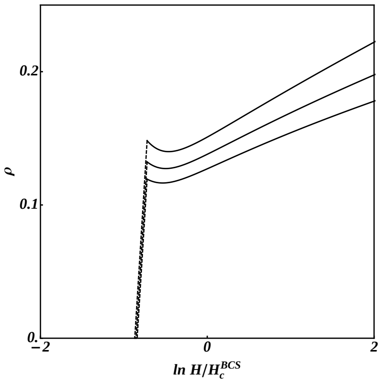
$T = T_c/2, T_c/4, T_c/8$



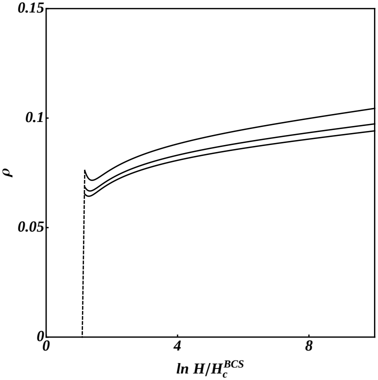
ρ vs H ($\gamma_{c0} = -0.1$, $\gamma_{t0} = -0.1$, $\gamma_{s0} = 0.1$, $t_0 = 0.05$)

$T = T_c/2, T_c/4, T_c/16$

Results: magnetoresistance in parallel H - I



ρ vs H ($\gamma_{c0} = -0.45,\ \gamma_{t0} = 1,$
$\gamma_{s0} = -1,\ t_0 = 0.1$)
$T = T_c/2, T_c/4, T_c/8$



ρ vs H ($\gamma_{c0} = -0.1,\ \gamma_{t0} = -0.1,$
$\gamma_{s0} = 0.1,\ t_0 = 0.05$)
$T = T_c/2, T_c/4, T_c/16$

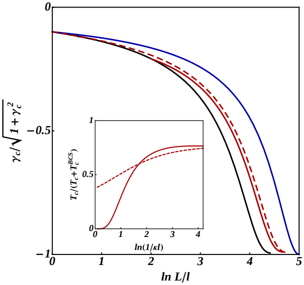
N.B.: $\rho^{-1} = t^{-1}(L_T) + (4/\pi) \ln |\gamma_c(l_H)|$

[Khodas, Levchenko, Catelani (2012)]

- superconducting transition in 2D is of Berezinsky-Kosterlitz-Thouless type
- our (mean-field) T_c is the upper estimate for T_c^{BKT}
- description of BKT transition within NLSM (transformation of fermionic to bosonic mechanism) [in progress]

- if static screening length $\varkappa^{-1} \ll l$, effective singlet-channel interaction is universal: $\gamma_s = -1$ (long-ranged), i.e., suppression of T_c

- if static screening length $l \ll L_c \ll \varkappa^{-1}$, effective singlet-channel interaction is short-ranged at relevant scales, i.e. enhancement of T_c is possible



$$\gamma_{t0} = -\gamma_{s0} = \gamma_{c0} = -0.1,$$
$$t_0 = 0.05,\ \ln 1/(\varkappa l) = 2$$

Conclusions

- one-loop RG eqs exact in the Cooper-channel interaction are derived
- one-loop RG eqs are valid upto $|\gamma_c| \sim 1/t \gg 1$
- phase diagram within one-loop RG eqs is explored
- enhancement of T_c in the case of short-ranged interaction is found in wide range of parameters
- SIT is controlled by an unstable fixed point at $R \sim h/e^2$ as in the 'bosonic' mechanism
- RG eqs and fluctuation corrections results in nonmonotonous magnetoresistance at $T < T_c$