Normal modes of inhomogeneous Josephson junction chains

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Thanks to the collaborators:

Theory

F. Hekking
M. Taguchi
V.-D. Nguyen

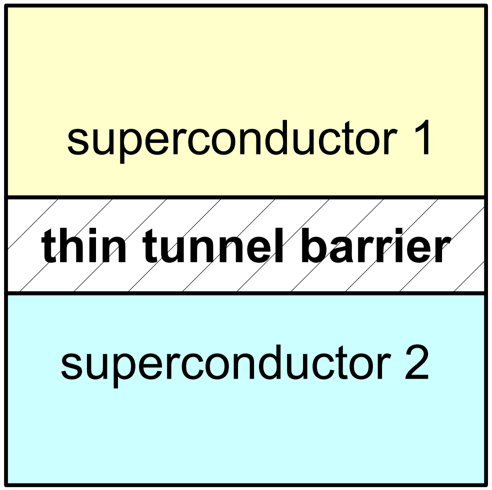
Experiment

W. Guichard
O. Buisson
N. Roch
Yu. Krupko

Outline

- JJ chains and their normal modes
- Disorder in JJ chains and localization of normal modes
- Effect of spatial inhomogeneity on Josephson energy renormalization
- Spatial inhomogeneity for construction of a superinductance?

Josephson junction

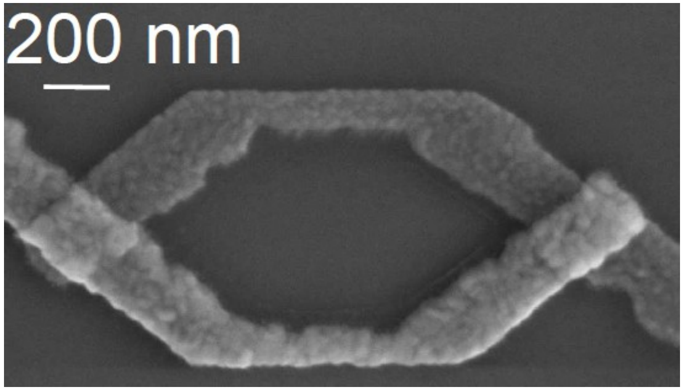


Josephson current: $I_{1\to 2} = I^c \sin(\phi_1 - \phi_2)$

critical current
(characterizes
the structure
of the contact)
nanoamperes

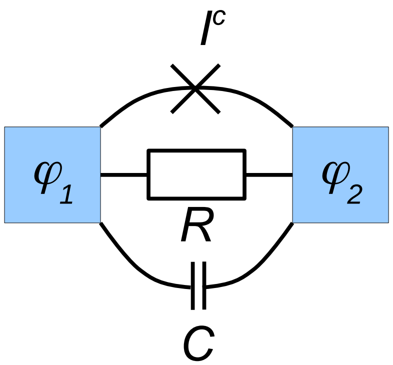
superconducting
phases
(characterize the state
of each island)

- Superconducting QUantum Interference Devices
  (the most sensitive magnetometers)
- Qbits
- metrology (frequency → voltage)



SQUID (image from W. Guichard)

Josephson junction: RCSJ model



The total current between the islands

$$I_{1\to 2} = I^c \sin(\phi_1 - \phi_2) + \frac{V_1 - V_2}{R} + C\,\frac{d(V_1 - V_2)}{dt}$$

Josephson (Cooper pairs) | normal (quasiparticles) | displacement (capacitance)

Neglected in this talk

$$\frac{d\phi_n}{dt} = -2eV_n$$ gauge invariance in the superconductor

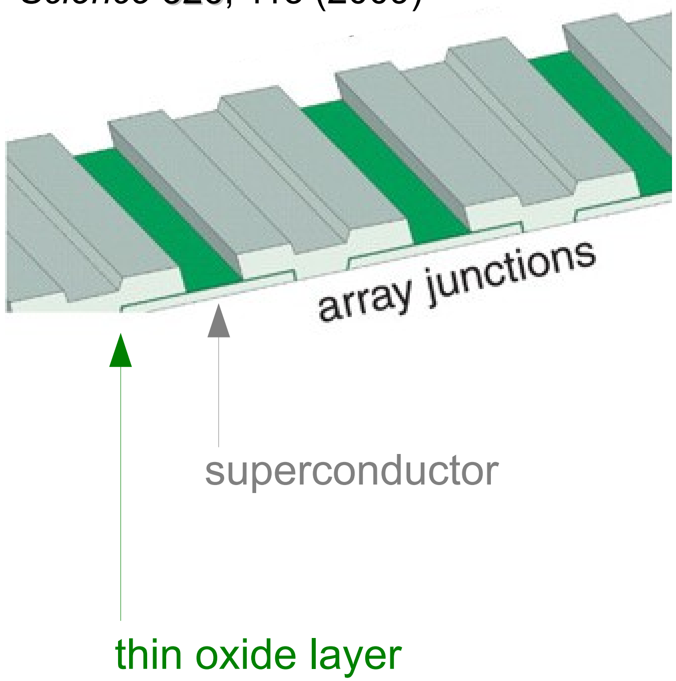
Classical regime:

inverse time of transfer of one Cooper pair $$E_J \equiv \frac{I^c}{2e} \gg \frac{(2e)^2}{2C}$$ charging energy due to capacitance between the islands

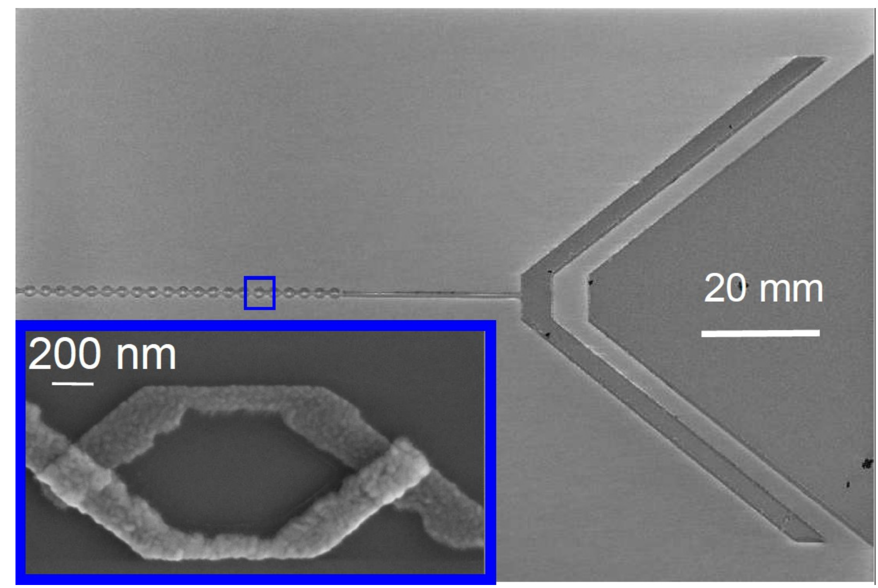
Otherwise: strong Coulomb blockade
(charge discreteness matters)

Josephson junction chains

V. Manucharyan *et al.*,
Science 326, 113 (2009)

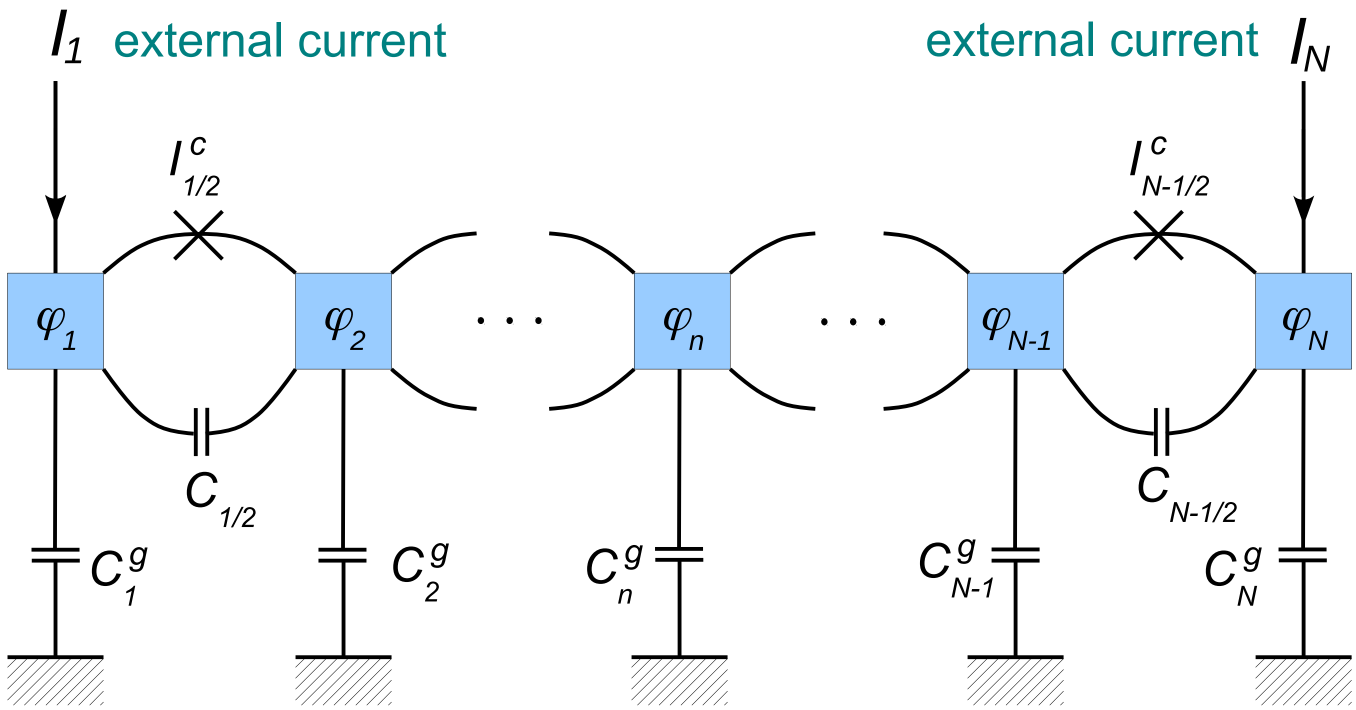


SQUID chain (image from W. Guichard):



- large impedance with little dissipation Maslyuk *et al.*, PRL **109**, 137002 (2012)
- control over quantum coherence (phase slips) Pop *et al.*, Nature Phys. **6**, 589 (2010)

Josephson junction chains



a small capacitance between the islands and the ground

$$I_{n-1\to n} - I_{n\to n+1} = C^g_n \frac{dV_n}{dt}$$

nonlinear "wave" equation

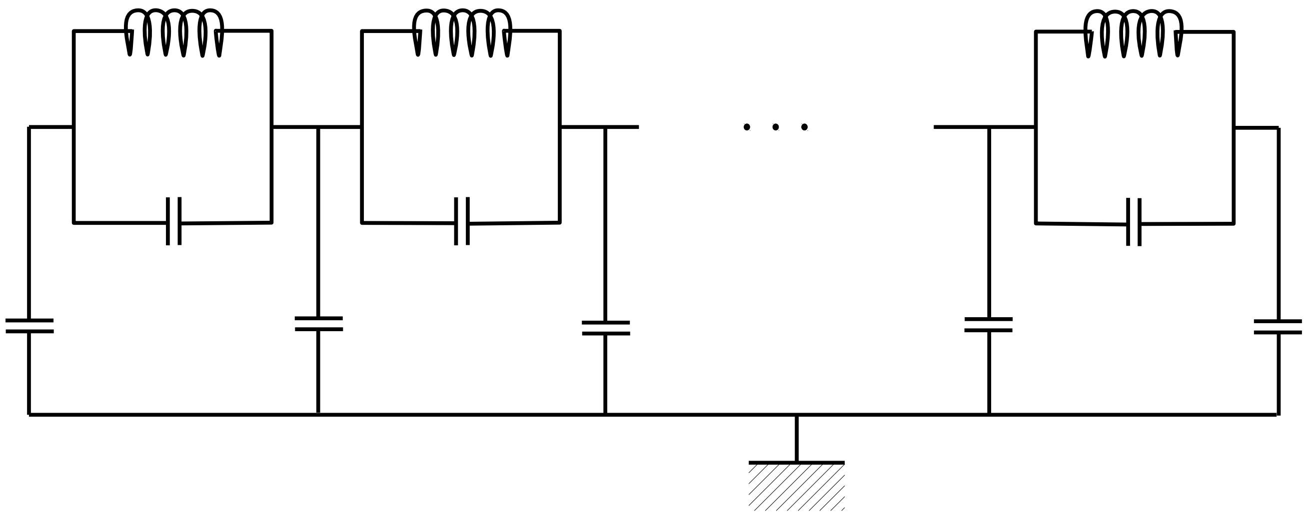
Josephson + displacement

because of $I^c \sin(\phi_n - \phi_{n+1})$

Small oscillations of the phase

$$\sin(\phi_n - \phi_{n+1}) \to \phi_n - \phi_{n+1}$$

Josephson current ~ inductance



$$Y_{n-1/2}(V_n - V_{n-1}) + Y_{n+1/2}(V_n - V_{n+1}) - i\omega C_n^g V_n = 0$$ linear “wave” equation

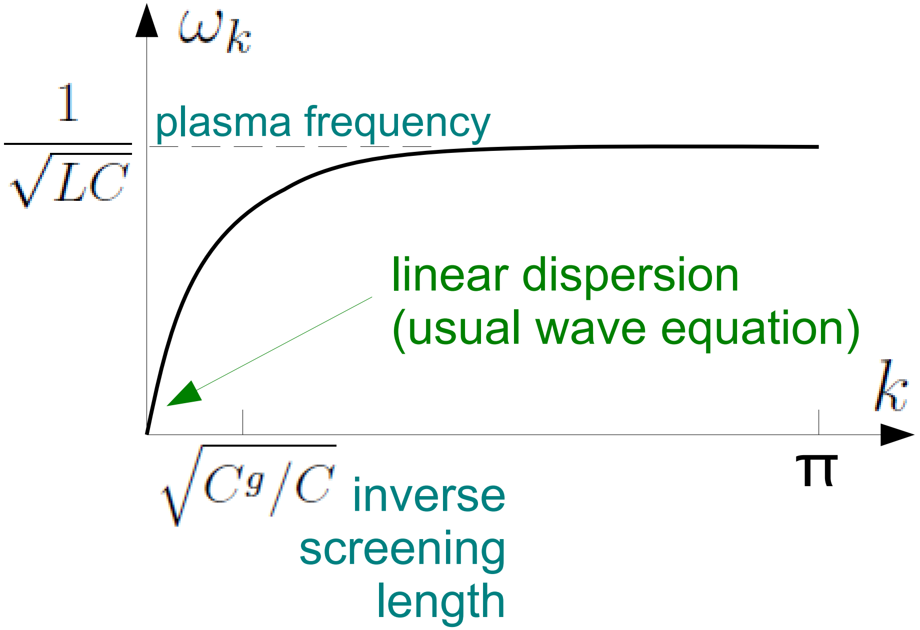
$$Y_{n+1/2}(\omega) = -\frac{1}{i\omega L_{n+1/2}} - i\omega C_{n+1/2} \cdot$$ complex admittance of the junction

Quadratic eigenvalue problem

Spatially homogeneous chains

Infinitely long chain: $V_n \propto e^{ikn}$

$$\omega_k = \frac{1}{\sqrt{LC}}\sqrt{\frac{4\sin^2(k/2)}{4\sin^2(k/2) + C^g/C}}$$



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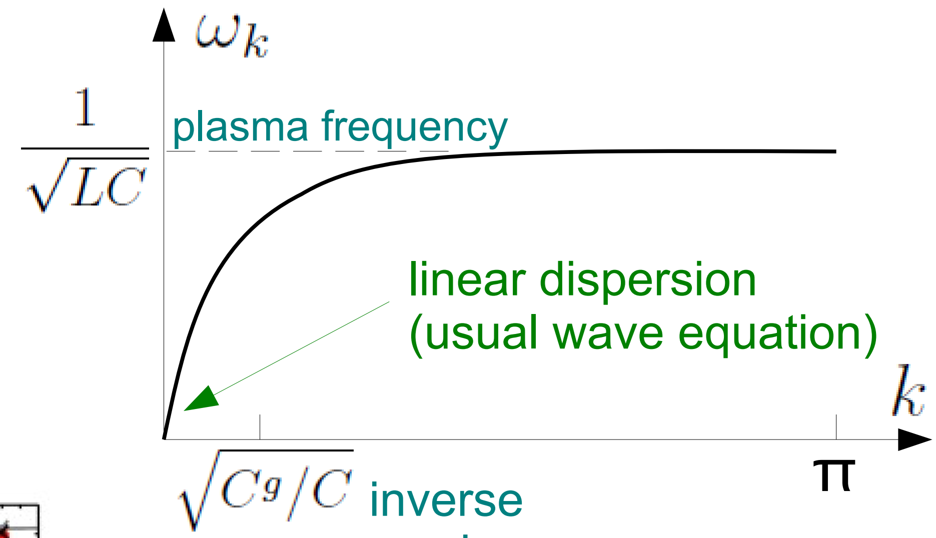
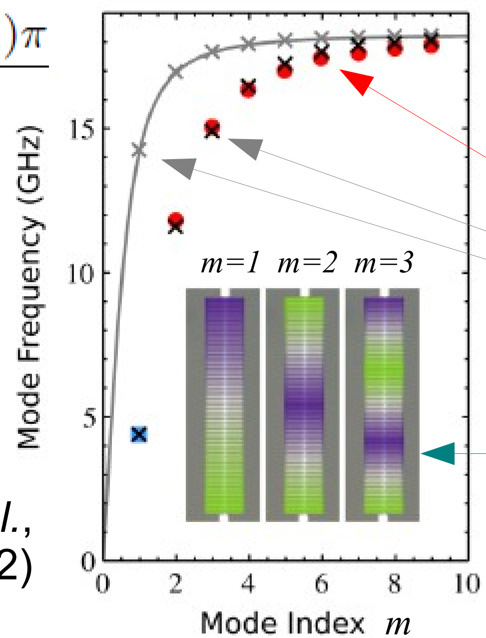


Finite length, N junctions:

$$k = 0, \frac{\pi}{N}, \frac{2\pi}{N}, \ldots, \frac{(N-1)\pi}{N}$$

80 junctions:



experimental frequencies

theoretical calculation $C^g/C \sim 10^{-3}$

mode profiles

Masluk *et al.*,
PRL **109**, 137002 (2012)

Disordered chains

Critical current, junction capacitance $\propto$ **junction area** – the main source of disorder

$$L_{n+1/2} = \frac{L}{1+\zeta_n}, \quad C_{n+1/2} = C(1+\zeta_n), \quad \left\langle \zeta_n^2 \right\rangle = \sigma_S^2 \ll 1$$

weak relative fluctuations of the junction areas

$$C_n^g = C^g(1+\eta_n), \quad \left\langle \eta_n^2 \right\rangle = \sigma_g^2$$

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Long chains: the inverse localization length from the DMPK equation

$$\frac{1}{\xi} = \frac{\sigma_S^2 + \sigma_g^2}{2} \tan^2 \frac{k}{2}$$

at $k \to 0$ goes as k^2 (standard behavior for Goldstone modes)
at $k \to \pi$ diverges

Short chains $N \ll \xi$: random perturbative shifts of the discrete frequencies

$$\left\langle \delta\omega_k^2 \right\rangle = \frac{3/8}{LC} \frac{\sigma_S^2 + \sigma_g^2}{N} \frac{(C^g/C)^2\, 4\sin^2(k/2)}{[4\sin^2(k/2) + C^g/C]^3}$$

motional narrowing

Basko & Hekking, PRB **88**, 094507 (2013)

Spatial inhomogeneity

can be introduced on demand

(lithographic patterning)

Can it be used for something?

(engineering the environment)

Renormalization of the Josephson energy

weak link
("black sheep" junction)

$E_J^{bs} \ll E_J$ rest of the junctions

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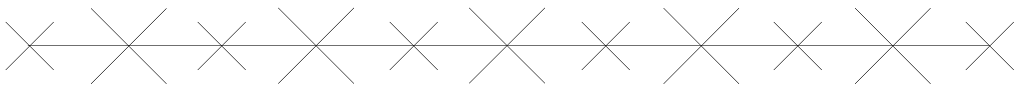
Transfer of a Cooper pair through the "black sheep" junction

Fast redistribution of phases on the rest of the junctions
(displacement of normal mode oscillators)

Debye-Waller factor $E_J^{bs} = E_{J0}^{bs} e^{-W}$

Hekking & Glazman,
PRB **55**, 6551 (1997)

Renormalization in a modulated JJ chain

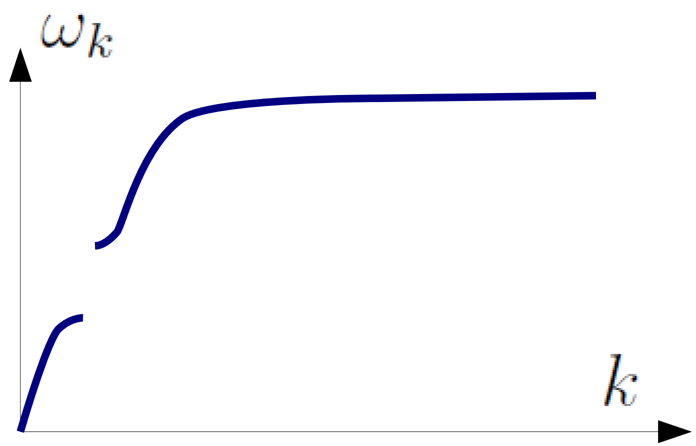


$$E_{J,n} = E_J\left(1 + t\cos\frac{2\pi n}{a}\right)$$

modulation depth << 1

modulation period > $\sqrt{C/C_g}$

Mode dispersion in the modulated chain



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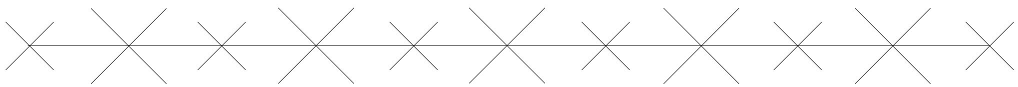

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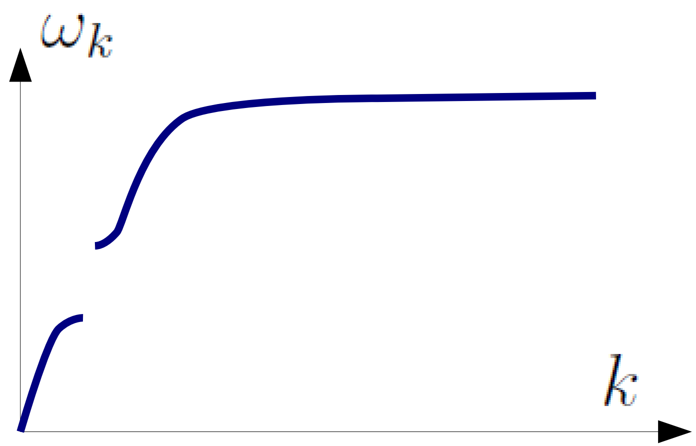

screening length

$$\frac{e^{-W}}{e^{-W_0}} = \left(\frac{\sqrt{C/C_g}}{a/\pi}\right)^{t/(2g)}$$

without modulation

$$g \equiv \sqrt{\frac{C_g}{L}}\frac{\pi}{(2e)^2} > 1$$

inverse dimensionless impedance of the chain

The most important effect of the modulation
is not
the gap in the frequency spectrum,
but
the change in the normal mode wave functions

Taguchi, Basko & Hekking
arXiv:1505.00385
PRB (in press)

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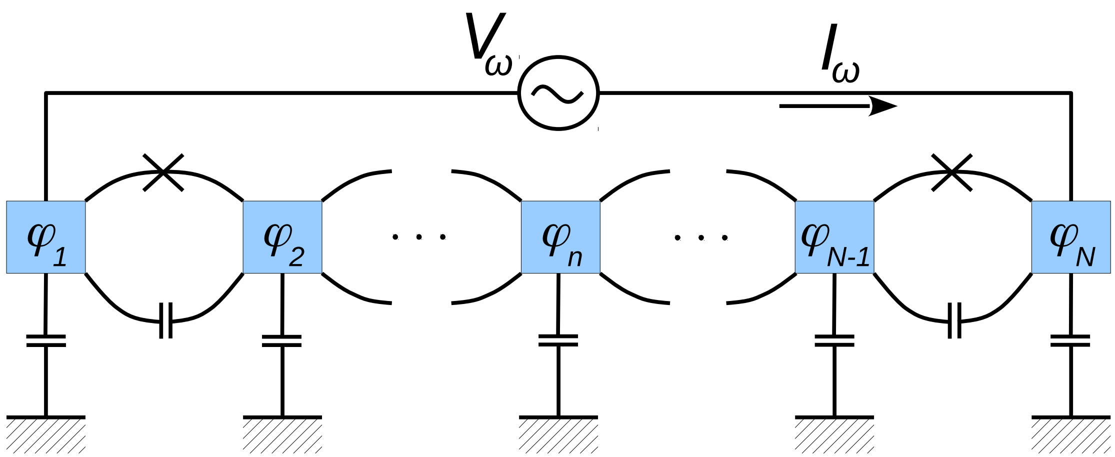
Superinductance

Highly inductive environments
without dissipation

- to suppress charge fluctuations in Josephson qubits
- to build a current standard

Exploit the kinetic inductance
of superconducting JJ chains

Masluk *et al*., PRL **109**, 137002 (2012)
Bell *et al*., PRL **109**, 137003 (2012)



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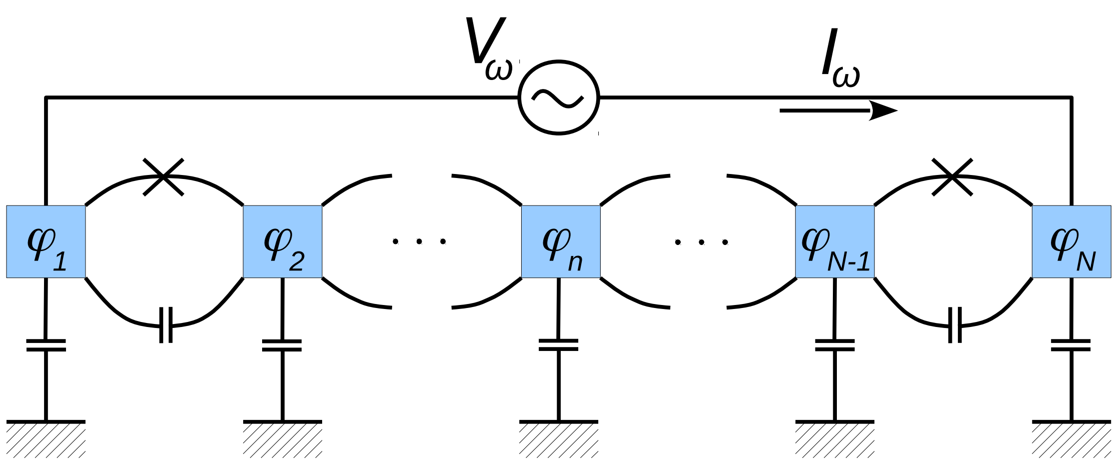
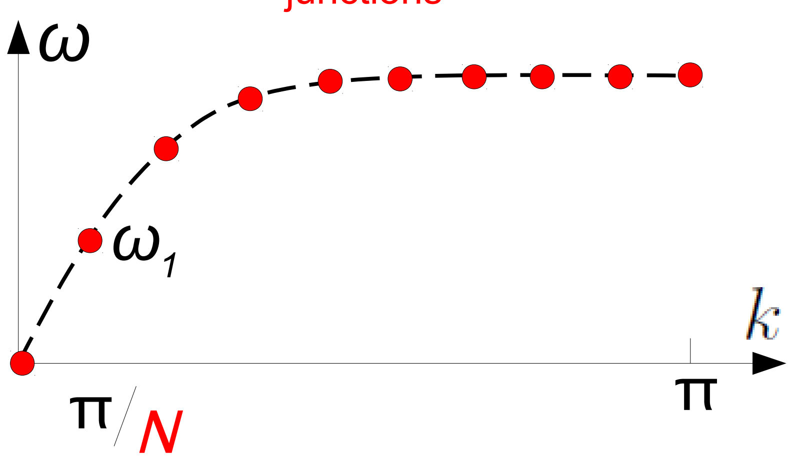


Inductive response at $\omega \ll \omega_1$:

$$Z(\omega) \approx -\frac{1}{i\omega N L}$$

single junction inductance (L)

number of junctions (N)



Competition

More junctions → The total inductance increases (~ N)

BUT

More junctions → The bandwidth shrinks (ω_1 decreases)

Is it possible to improve the bandwidth by modulating the chain?

(N-1 degrees of freedom to optimize!)

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More junctions → The bandwidth shrinks (ω_1 decreases)

Is it possible to improve the bandwidth
by modulating the chain?

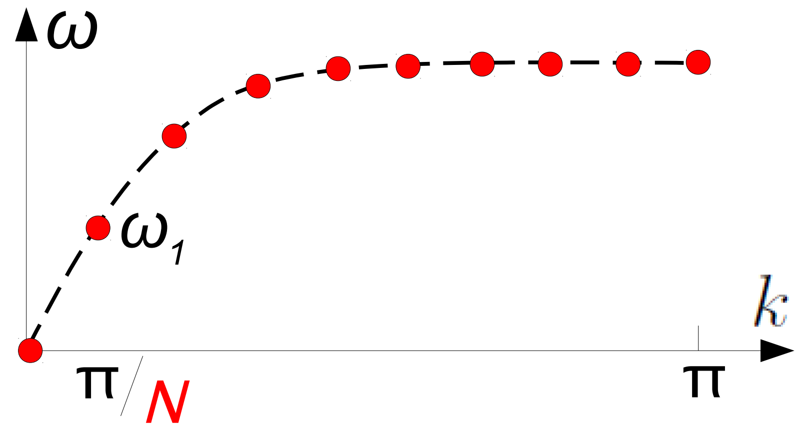
(N-1 degrees of freedom to optimize!)

No

Minimize the ground capacitance

Increase the slope

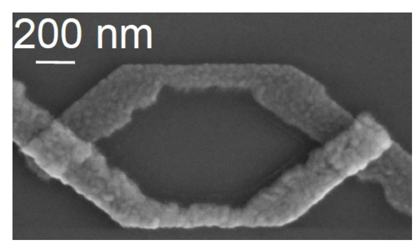
C^g as small as possible



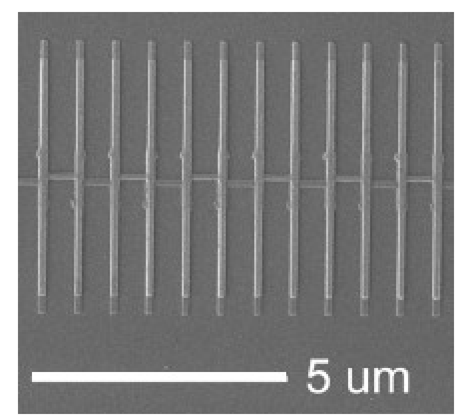
$$C^g \propto \text{island area}$$

$$C \propto \text{junction area}$$

$$L \propto \frac{1}{\text{junction area}}$$



not good



good

All island area should be involved in the junctions

Masluk *et al.*,
PRL **109**, 137002 (2012)

Optimize the junction areas

Islands 1, ... , N

Junction areas $A_{3/2}$, ... , $A_{N-1/2}$

$\Rightarrow$

junction inductance $L_i \propto 1/A_i$

junction capacitance $C_i \propto A_i$

ground capacitance $C_n^g \propto A_{n+1/2} + A_{n-1/2}$

$$\left(\frac{1}{L_{n-1/2}} - \omega^2 C_{n-1/2}\right)(\phi_n - \phi_{n-1}) + \left(\frac{1}{L_{n+1/2}} - \omega^2 C_{n+1/2}\right)(\phi_n - \phi_{n+1}) = \omega^2 C_n^g \phi_n$$

Homogeneous rescaling of all areas does not change the frequencies

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Optimization for a homogeneous chain:

1. Make the areas as small as possible
   $A = A_{\min}$ (**cannot be made too small**)

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Freedom left for modulation:

1. Vary N

2. Vary $A_i > A_{\text{min}}$

The optimum is reached in the corner

Conclusions

1. Normal modes of a JJ chain
   with a random inhomogeneity are localized

2. Spatial modulation affects renormalization
   of the Josephson energy of a weak link

3. But it is of no help for constructing a superinductance

Basko & Hekking, PRB **88**, 094507 (2013)
Taguchi, Basko & Hekking, arXiv:1505.00385, PRB (in press)

Thank you for your attention!