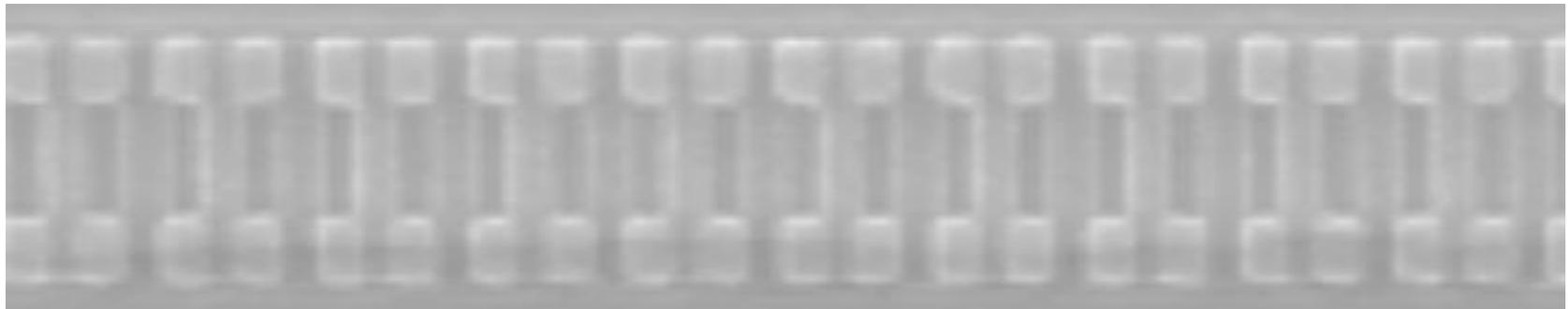


# Experiments with arrays of Josephson junctions near superconductor-insulator transition

Alexey Ustinov  
*Karlsruhe Institute of Technology, Germany*

Acknowledgements:

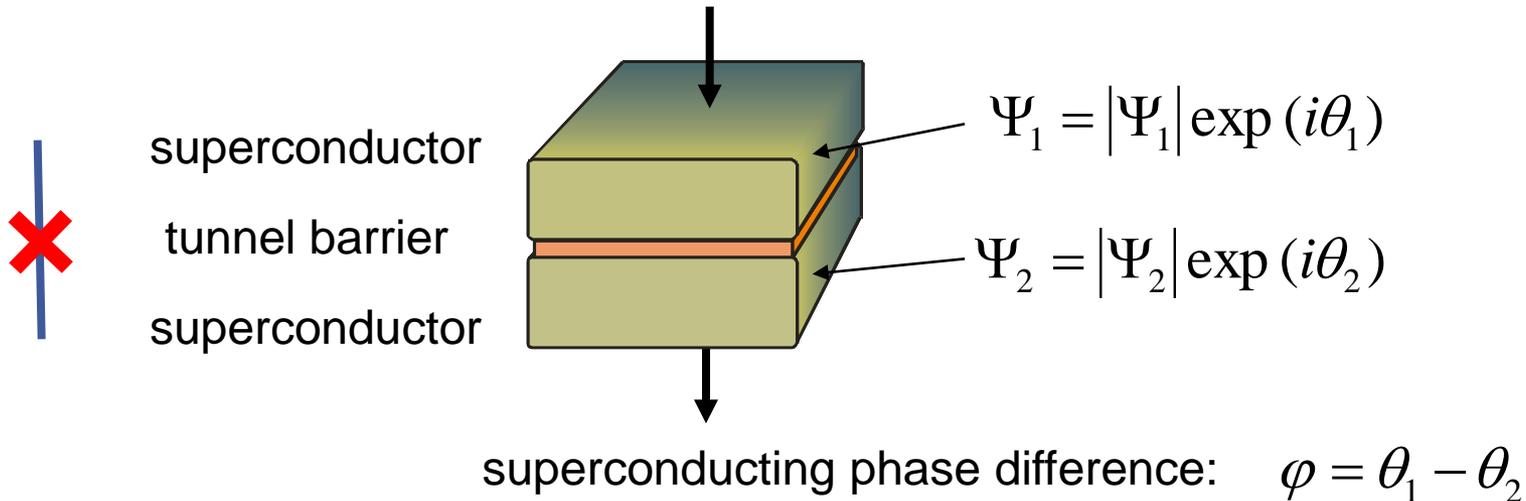
H. Rotzinger, R. Schäfer, A. Shnirman , N. Vogt, J. Zimmer



# Outline

- Josephson junctions and arrays
- Fabrication of Josephson junctions
- Measurement techniques
- Experimental limitations and typical problems
- Brief history of experiments with JJ arrays near S/I transition
  - 2D arrays
  - 1D/2D channels – shooting and localizing quantum vortices
  - 1D arrays – searching for ballistic Cooper pairs
- Our recent experiments with 1D arrays
  - tuning  $E_J/E_C$  , dual IV-curves
  - conductance  $\sim E_J^2$ , incoherent Cooper pair tunneling
  - thermal activation of charges
  - depinning of charges => **talk of A. Shnirman today**

# Josephson junction



Josephson relations

$$\left\{ \begin{array}{l} I_S = I_C \sin \varphi \\ V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \end{array} \right. \rightarrow \text{Josephson inductance}$$

$$L_J = V \frac{dI_S}{dt} = \frac{\Phi_0}{2\pi I_C \cos \varphi}$$

# Energy scales

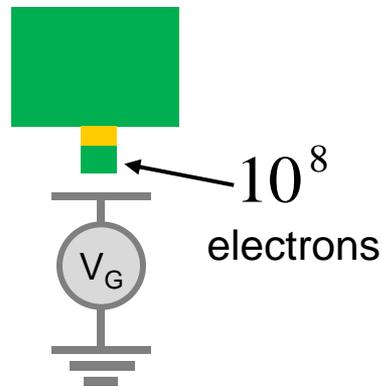
Uncertainty relation for a superconductor:  $\Delta n \cdot \Delta \varphi \geq 1$

Charging energy  $E_C = \frac{e^2}{2C_J}$

Josephson energy  $E_J = \frac{I_c \Phi_0}{2\pi}$

charge  
limit

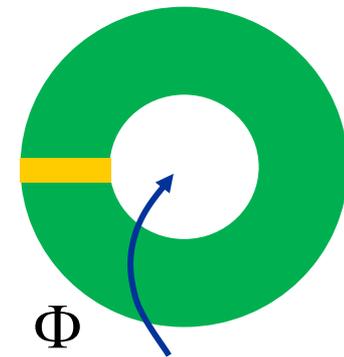
$$E_C \gg E_J$$



insulating state

flux  
limit

$$E_C \ll E_J$$



superconducting state

# Phase-charge duality in Josephson junction circuits

phase inertia



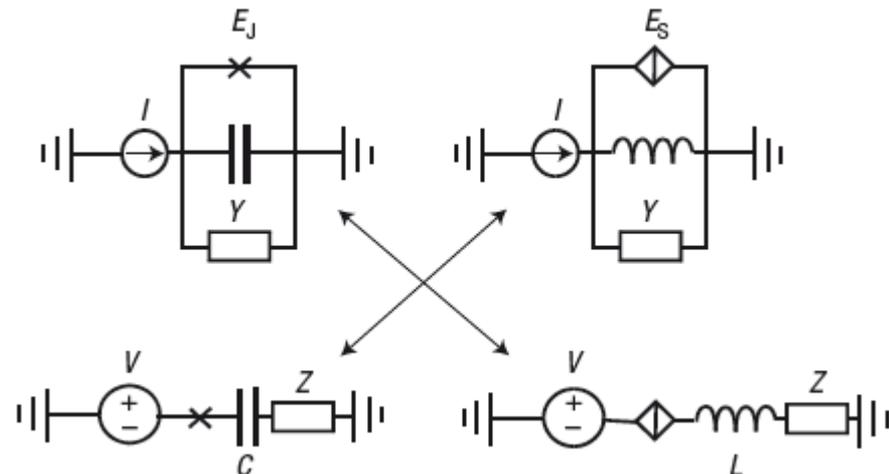
charge inertia



Review of the duality ideas for JJ arrays:

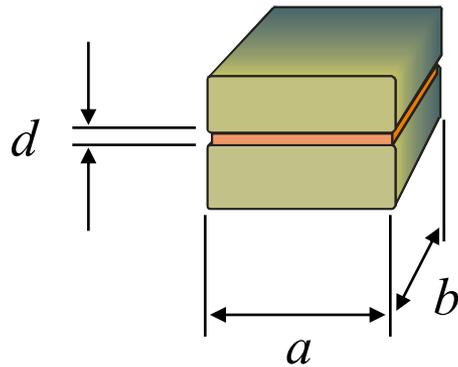
W. Guichard and F. W. J. Hekking, Phys. Rev. B **81**, 064508 (2010)

Duality between Josephson junction and phase slip junction



J. E. Mooij and Yu. V. Nazarov, Nature Phys. **2**, 169 (2006)

# Junction size and energy scales



tunnel barrier thickness  $d$

junction area  $A = ab$

Josephson energy  $E_J = \frac{I_c \Phi_0}{2\pi}$

charging energy  $E_C = \frac{e^2}{2C_J}$

$$I_c = \frac{\Phi_0 \Delta}{2R_n} \sim A \exp\left(-\frac{d}{d_0}\right) \quad C_J \sim \frac{A}{\epsilon d}$$

→  $\frac{E_J}{E_C} \sim A^2 d \exp\left(-\frac{d}{d_0}\right)$

Typically, achieving  $\frac{E_J}{E_C} < 1$  requires  $A < 0.01 \mu\text{m}^2$

# Simple estimate of conditions needed for observing quantum effects in a JJ

- A simple estimate for value of the junction resistance above which clear quantum effects become visible

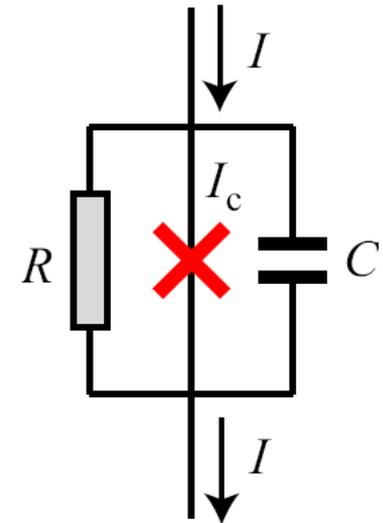
- The Heisenberg relation  $\Delta E \Delta \tau \geq \hbar$

- taking  $\Delta E \approx E_c = \frac{e^2}{2C}$  and  $\Delta \tau \approx RC$

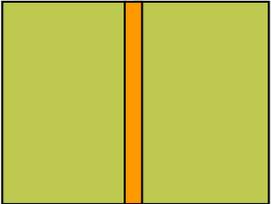
- the junction resistance should satisfy

$$R \geq R_Q = \frac{h}{4e^2}$$

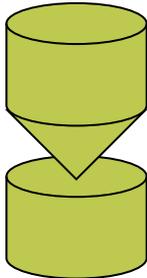
for quantum effects to be observable



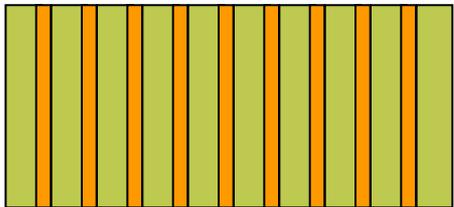
# Josephson junctions and weak links



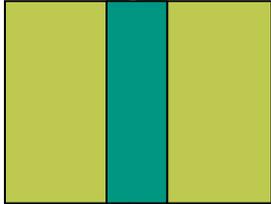
SIS  
tunnel junction



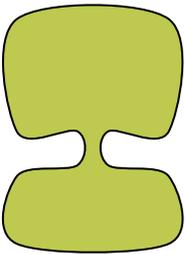
point contact



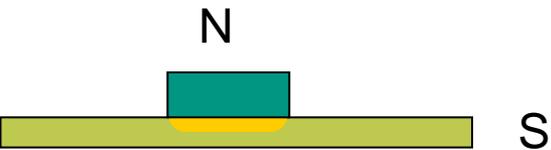
SISISISISISISISISIS  
intrinsic junctions  
(crystal)



SNS  
normal metal

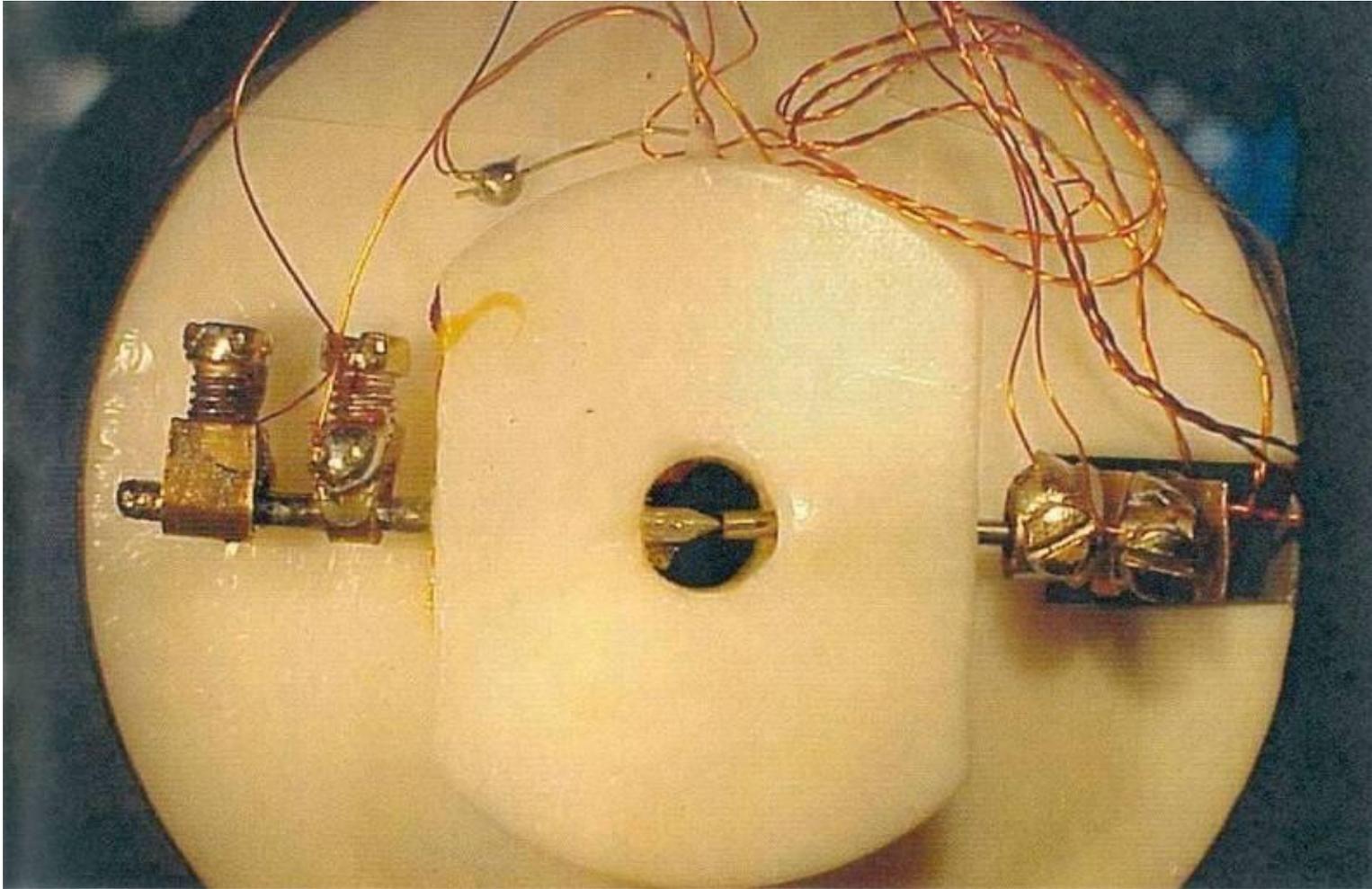


nano-bridge  
(constriction)

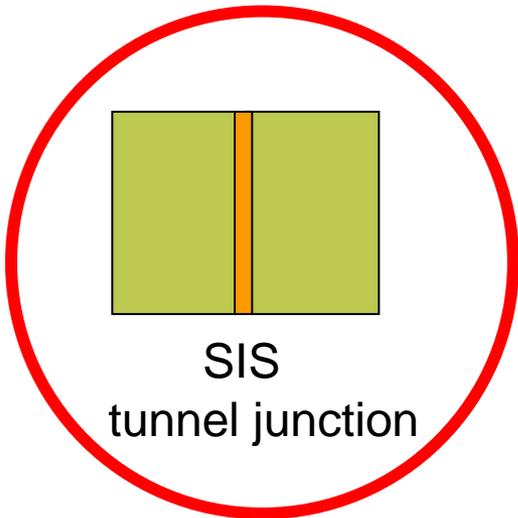


proximity link

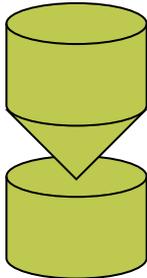
# The simplest way of making a Josephson junction



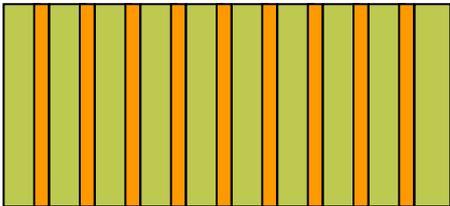
# Josephson junctions and weak links



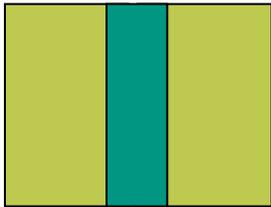
SIS  
tunnel junction



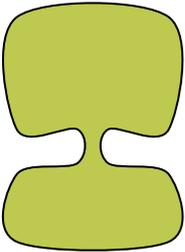
point contact



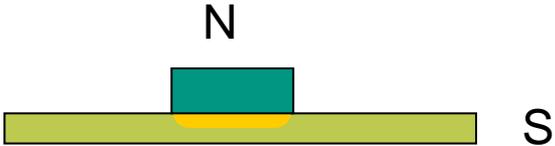
SISISISISISISISISIS  
intrinsic junctions  
(crystal)



SNS  
normal metal

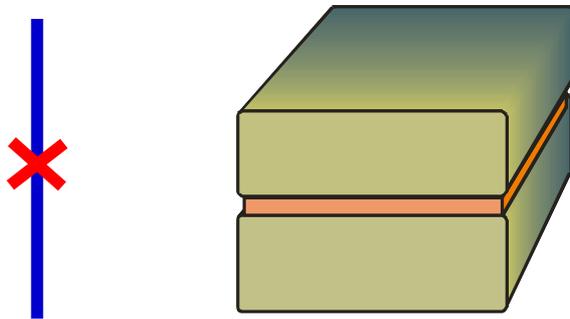


nano-bridge  
(constriction)



proximity link

# Fabrication of Josephson devices



1. circuit layout (CAD)
2. fabrication of photomasks
3. deposition of superconducting and insulating layers on a wafer
4. photo (or e-beam) lithography
5. dicing the wafer into chips

## ■ $Nb-AlO_x-Nb$

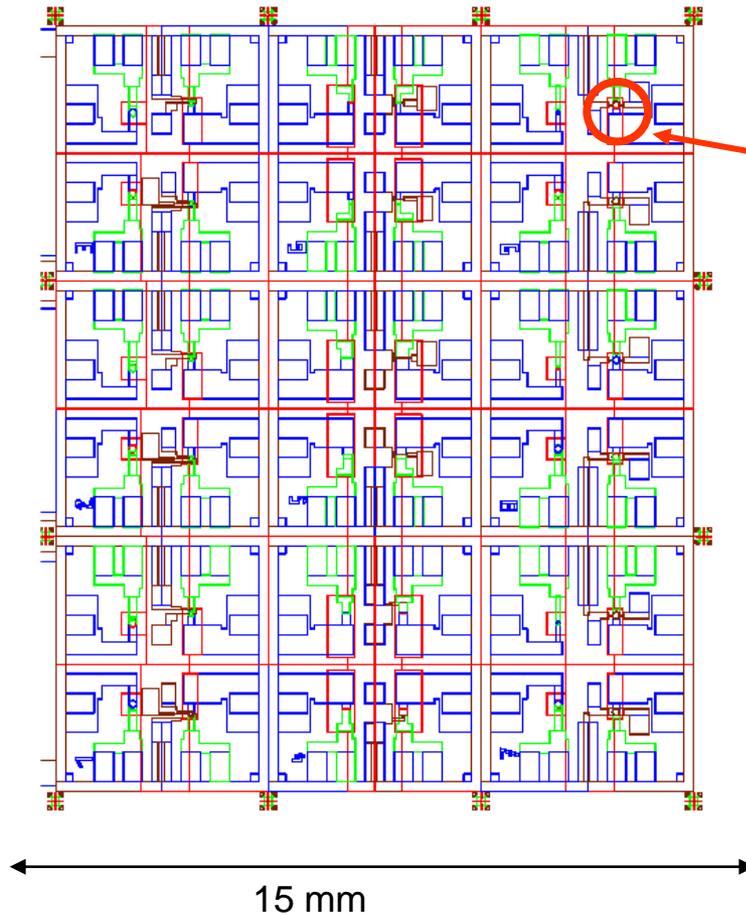
- $Nb$  sputtering
- $Al$  sputtering and oxidation
- $T_c = 9.2$  K
- $J_c$  from  $10^2$  to  $10^4$  A/cm<sup>2</sup>

## ■ $Al-AlO_x-Al$

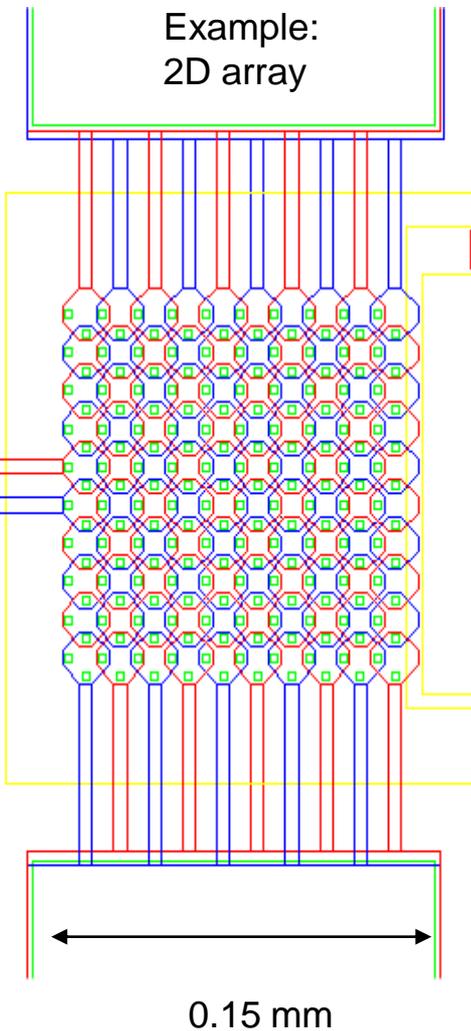
- $Al$  evaporation
- $Al$  oxidation
- $T_c = 1.2$  K
- $J_c$  from 1 to  $10^2$  A/cm<sup>2</sup>

# Circuit layout

CAD = computer aided design



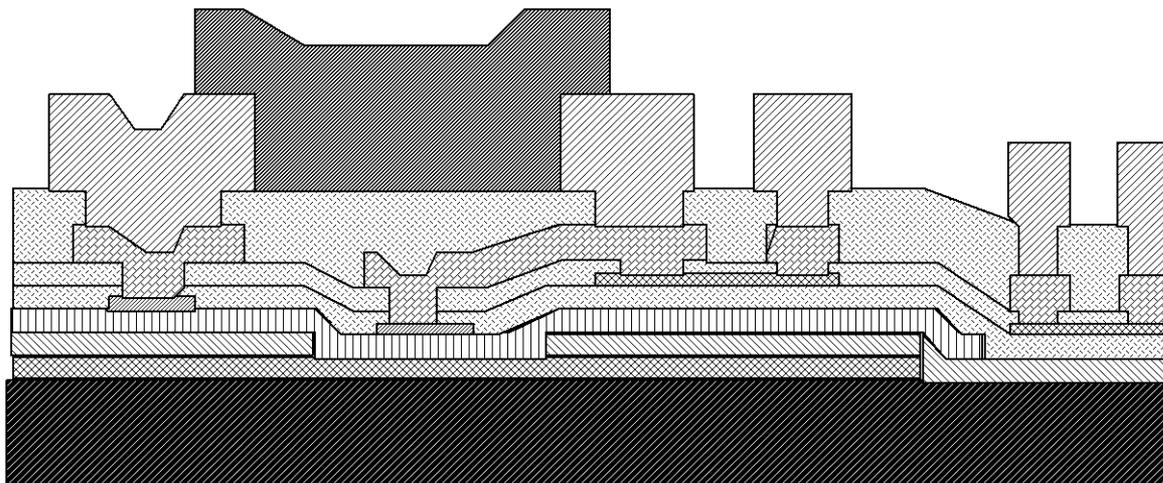
JJs



# Stacking thin-film layers: Example

design rules of Hypres Inc.

See: <http://www.hypres.com>



Si substrate

Insulator ( $SiO_2$ )

insulator

resistive layer ( $Mo$  or  $AuPd$ )

$Nb-AlO_x-Nb$

$Nb$

# Wafer processing

Photolithography can be used  
for JJ size down to  $1\ \mu\text{m}$

Clean room



mask alignment

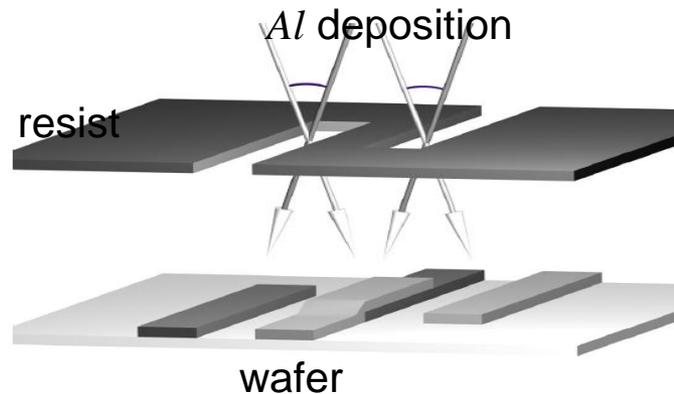
# Evaporation of superconducting thin films



PLASSYS electron beam evaporation unit with load lock. The unit is dedicated for fabrication of Josephson junctions and was installed at RQC lab at ISSP Chernogolovka in June 2014

# Shadow evaporation technique

Electron beam lithography can produce JJ size  $< 0.1 \mu\text{m}$



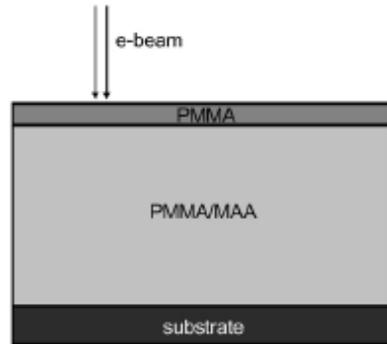
Electrom beam lithography



In a first step metal is evaporated from one angle, indicated by the dark arrows and dark structures on the substrate surface. The evaporation from another angle leads to an overlap of the features in the middle. (Picture by Mattias Urech)

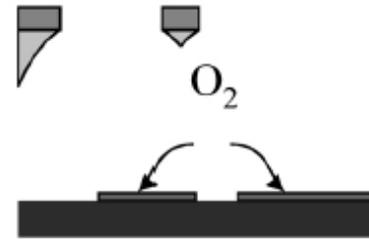
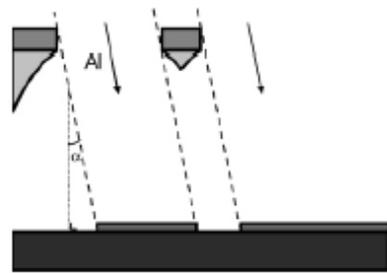
# Shadow evaporation of Al-AlO<sub>x</sub>-Al junctions

Pattern is written in the resist using an electron beam



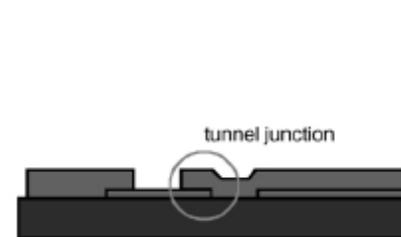
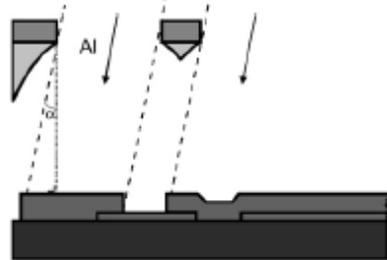
After the sample is developed, a suspended resist bridge is left

The bottom layer of aluminum is deposited under an angle  $+\alpha$



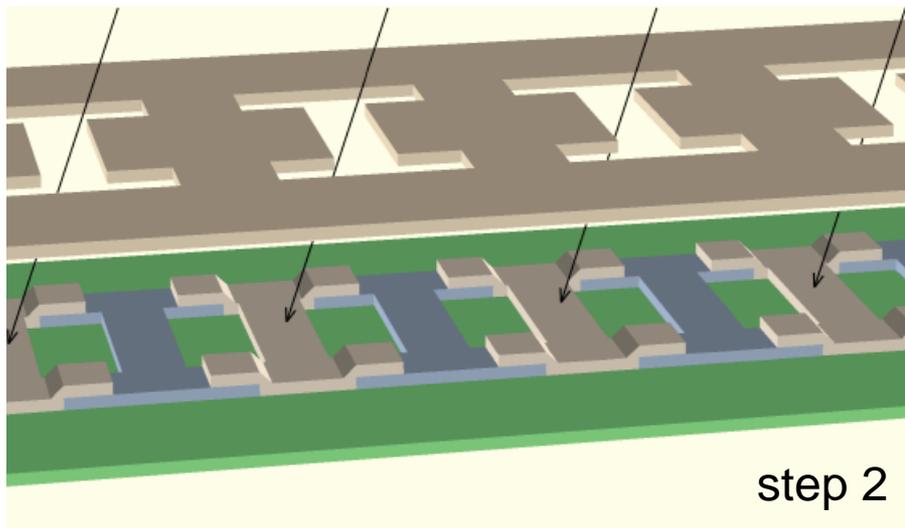
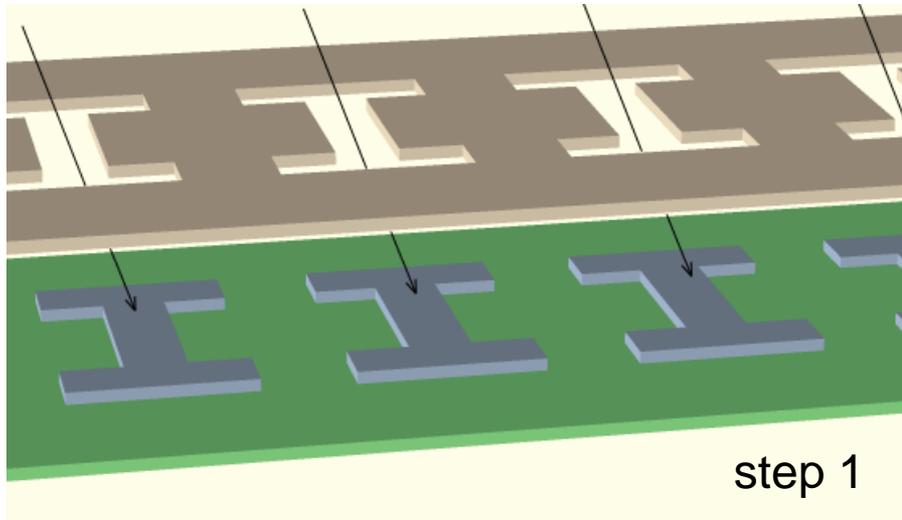
The sample is exposed to O<sub>2</sub> to form the oxide layer

The top layer of aluminum is deposited under an angle  $-\alpha$

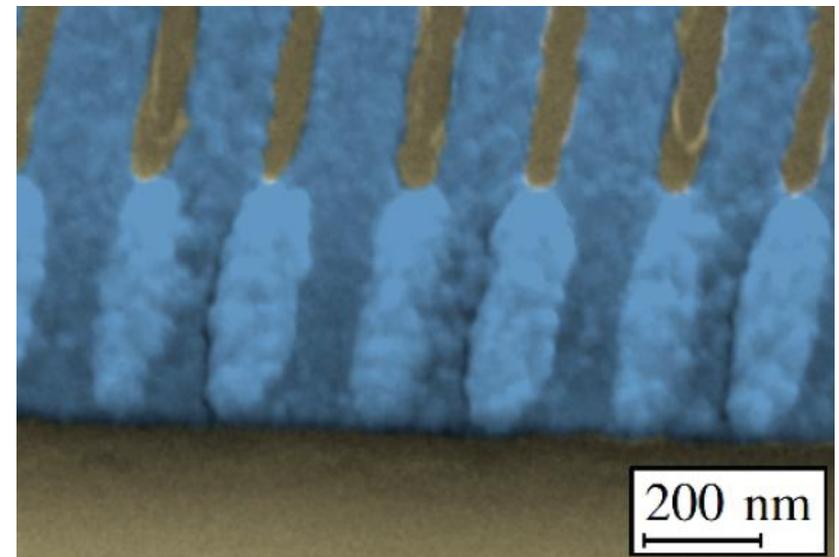


Lift-off in acetone removes the resist and a tunnel junction is left

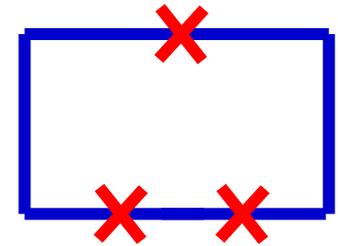
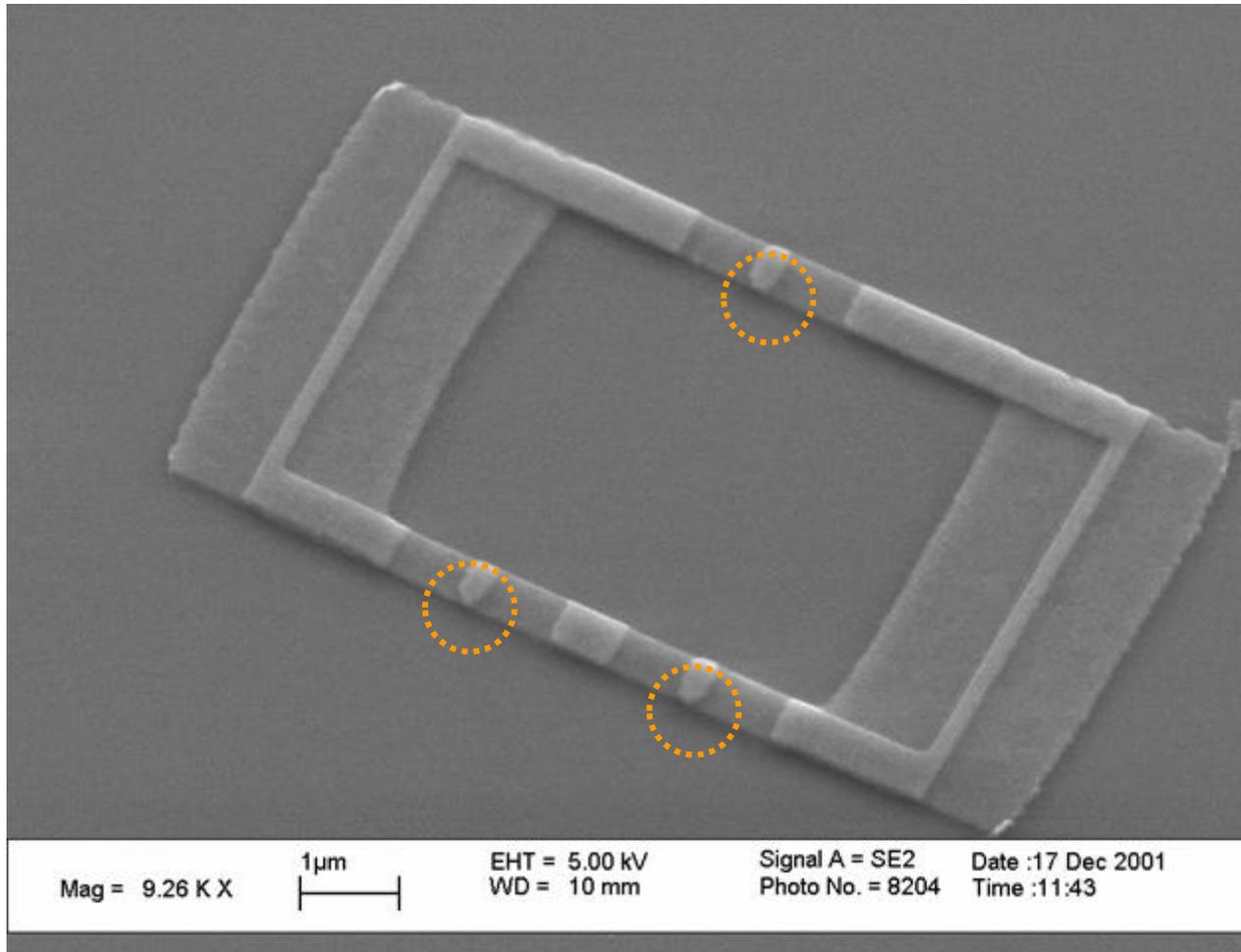
# Shadow evaporation of SQUID arrays



result

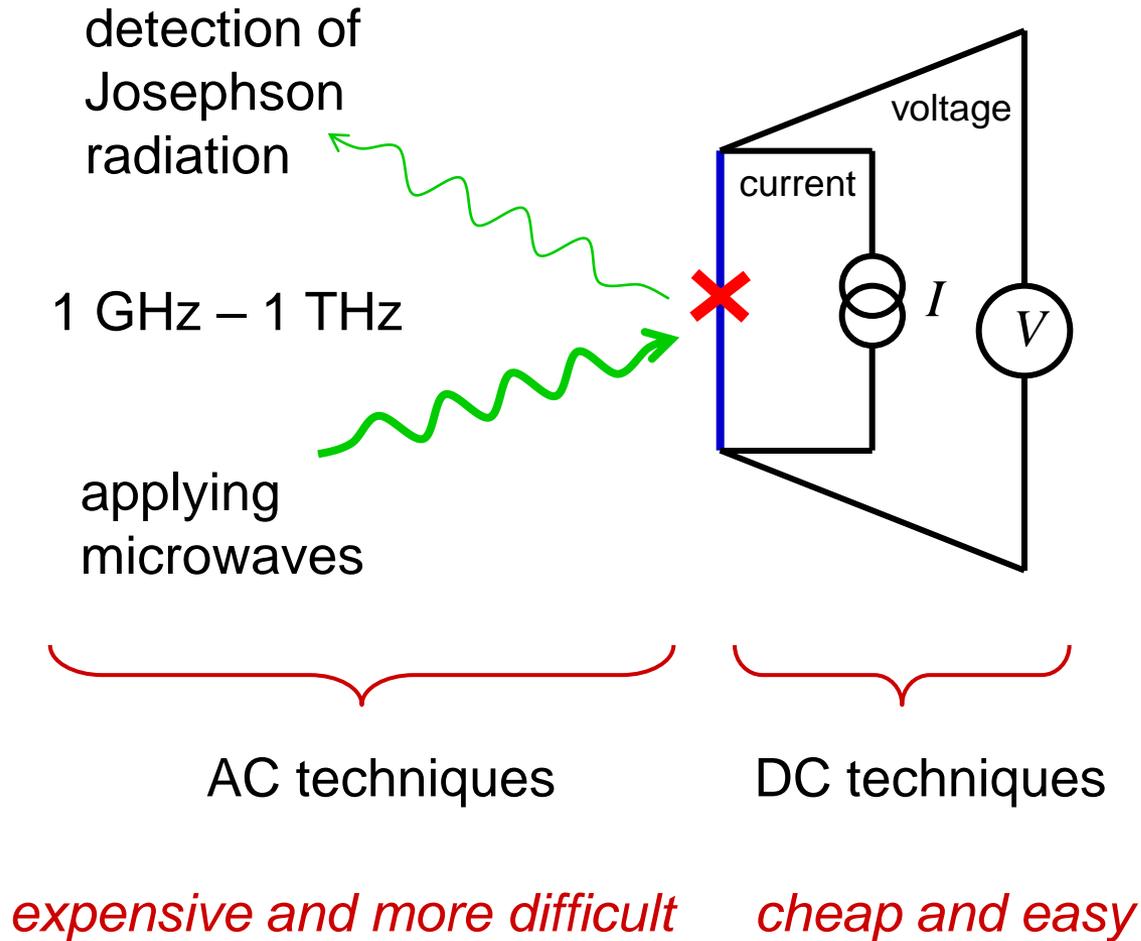


# Sub-micron Al-AlO<sub>x</sub>-Al JJs produced by electron beam lithography

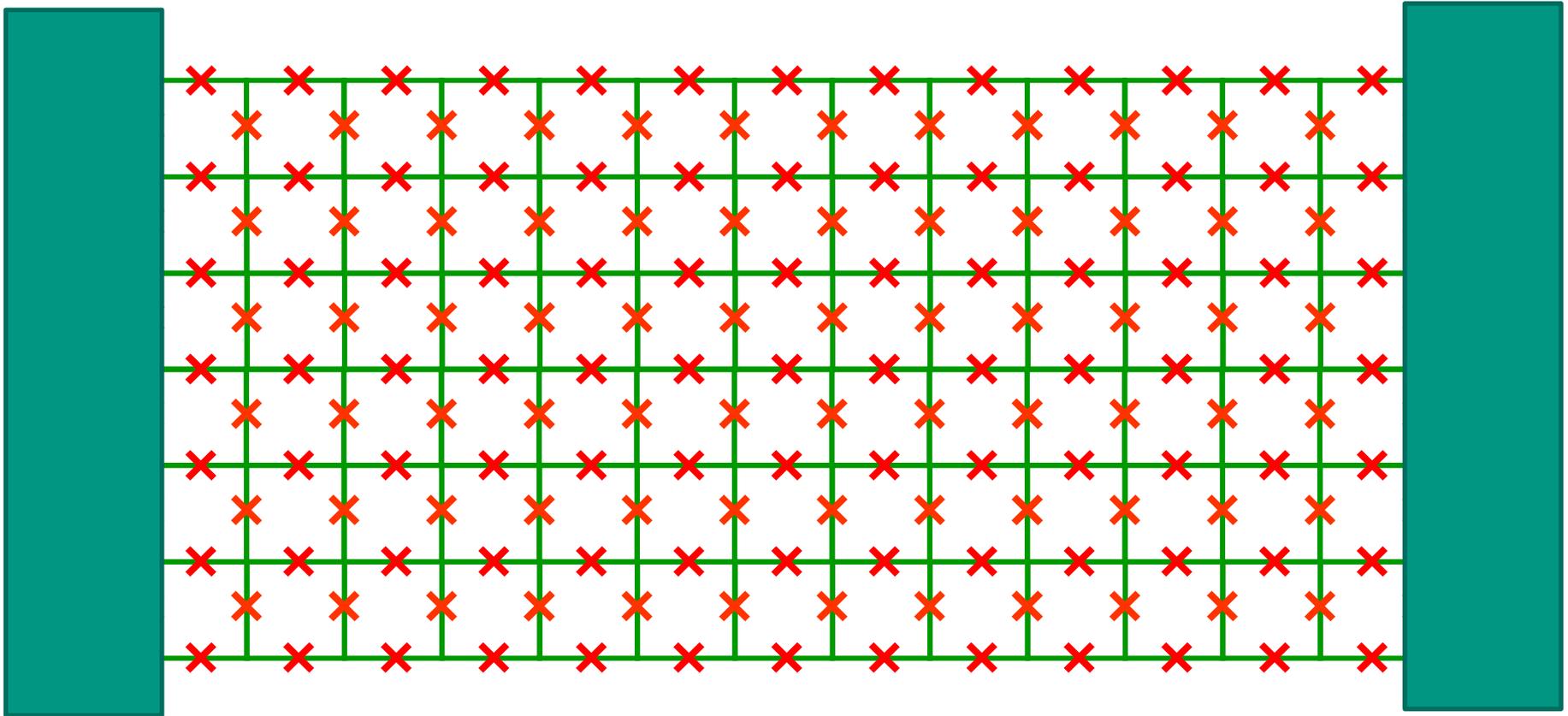


3-JJ flux qubit

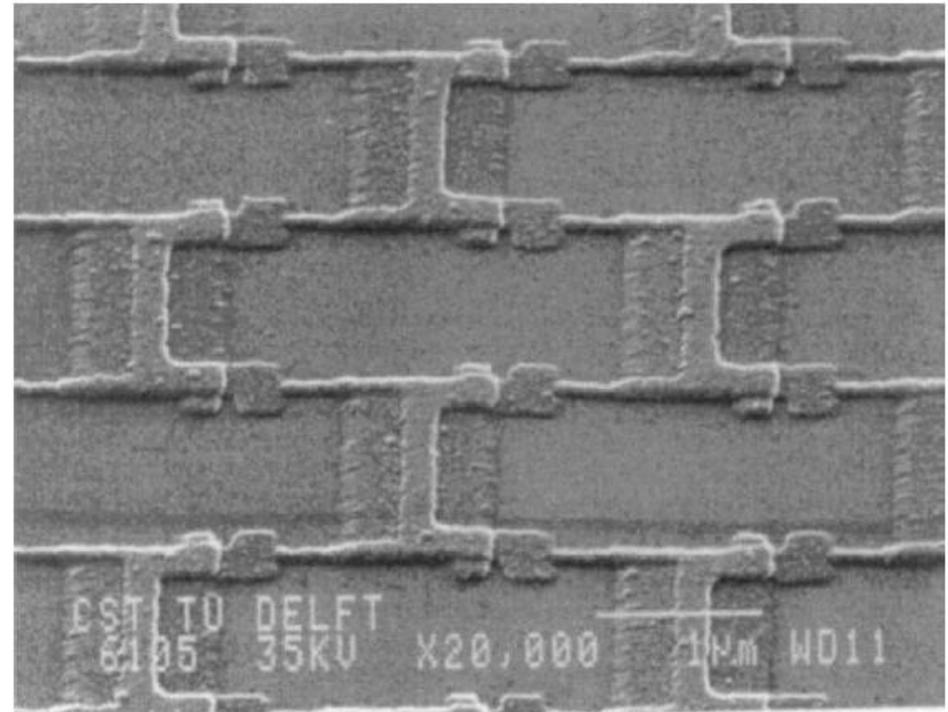
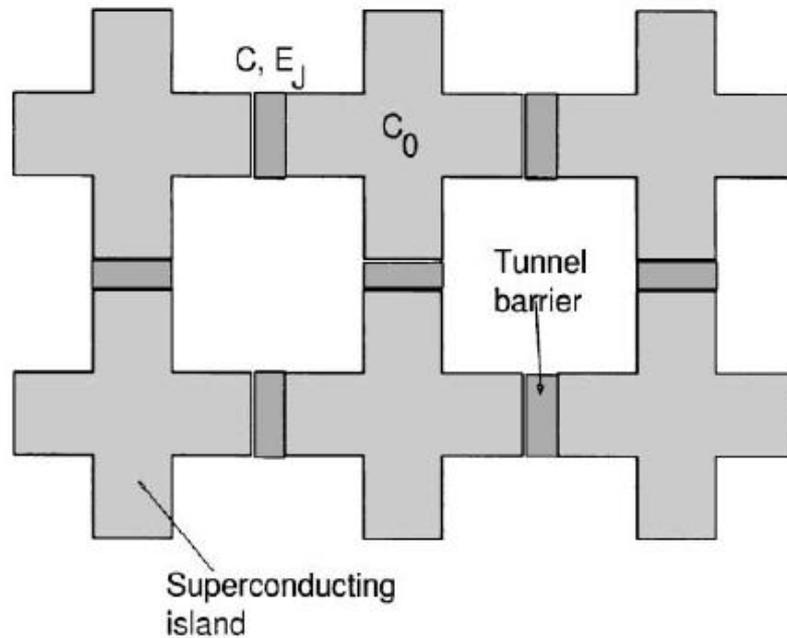
# Experiments with JJs



# Experiments with 2D arrays



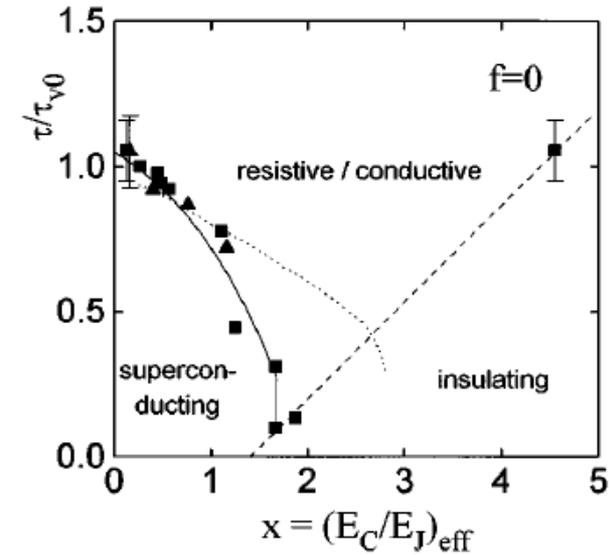
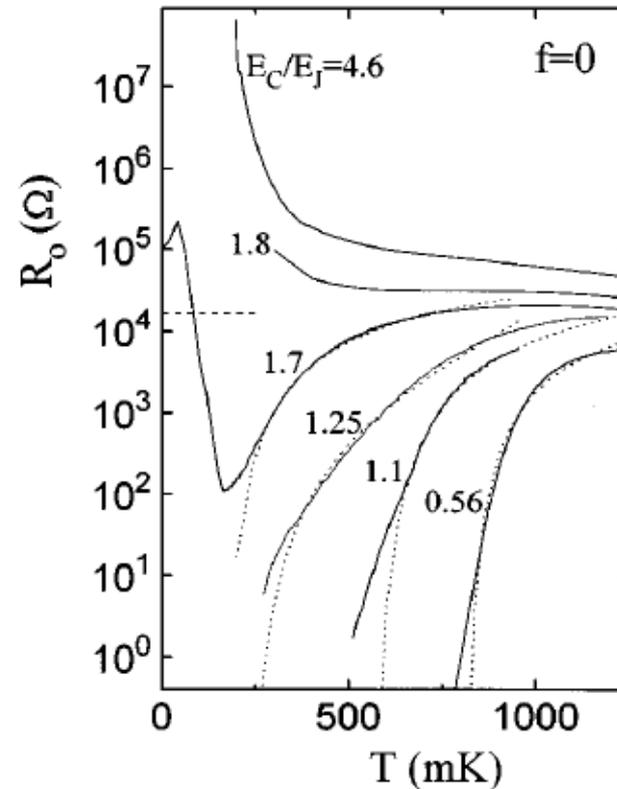
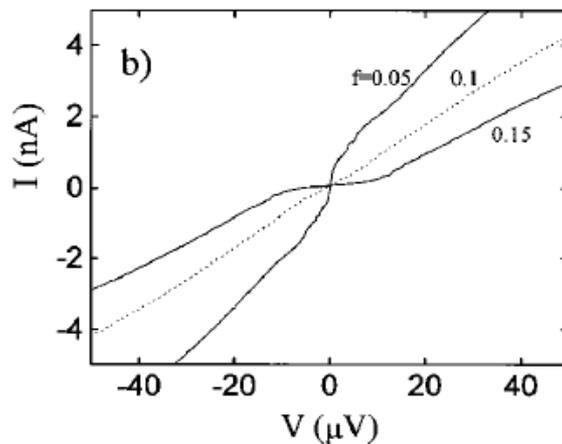
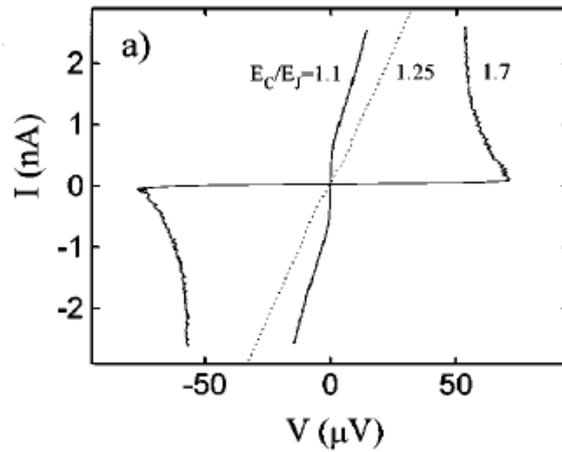
# Early works on 2D Josephson arrays



Ph.D. Thesis of L.J. Geerligs, Delft 1990

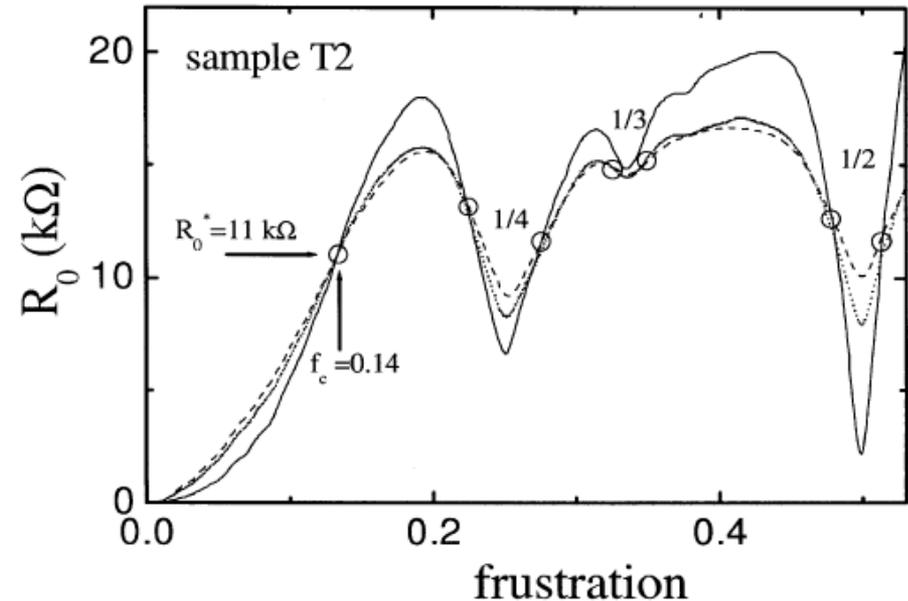
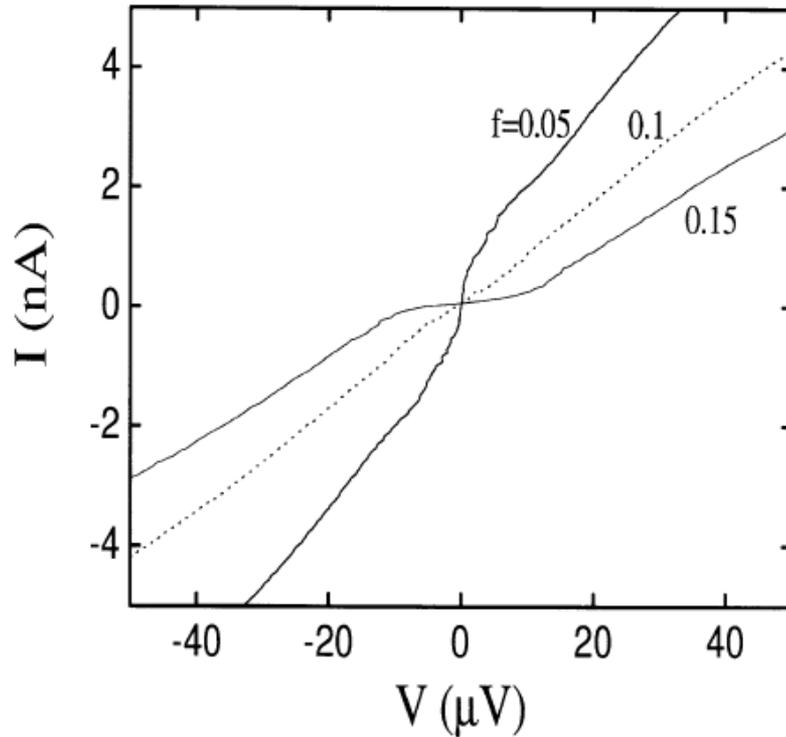
Review: R. Fazio and H. van der Zant, Phys. Rep. **355**, 235 (2001)

# Observation of S/I transition in 2D



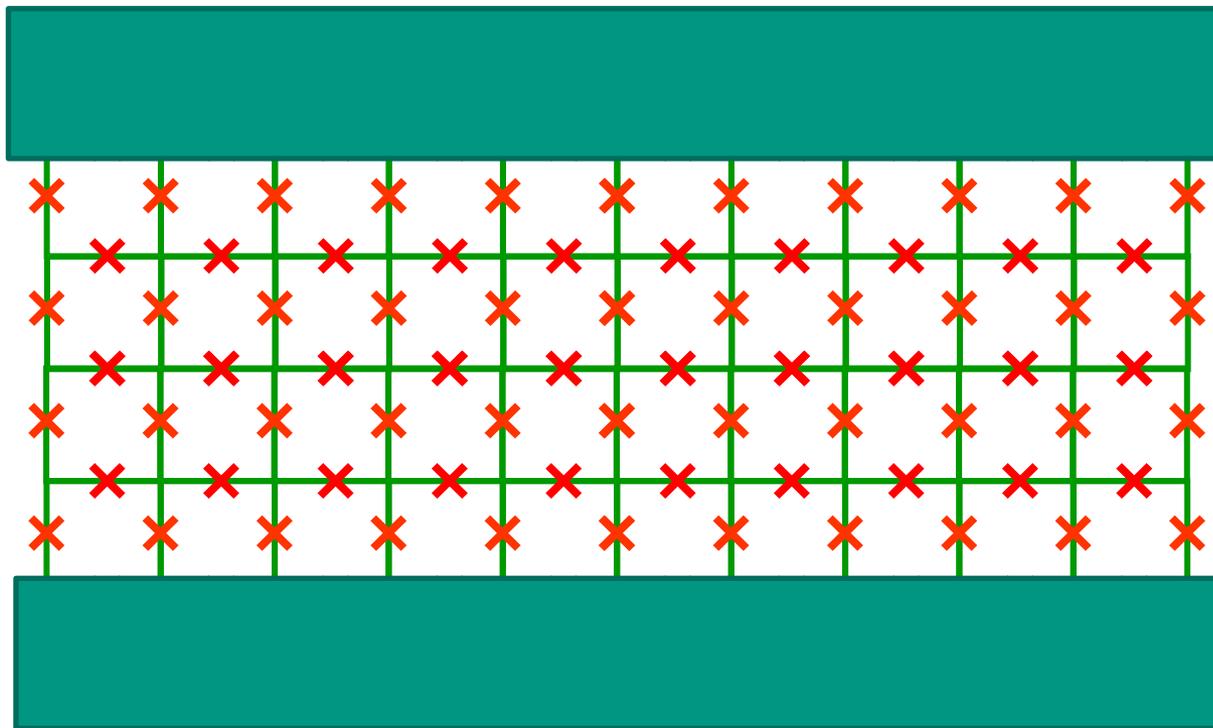
H.S.J. van der Zant, W.J. Elion, L.J. Geerligs, J.E. Mooij, Phys. Rev. B **54**, 10081 (1996)

# Field-induced S/I transition in 2D

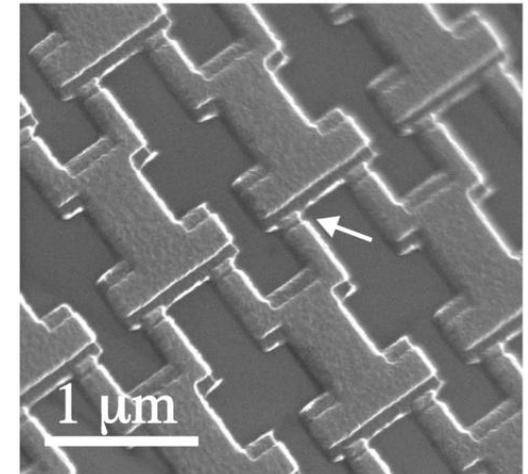
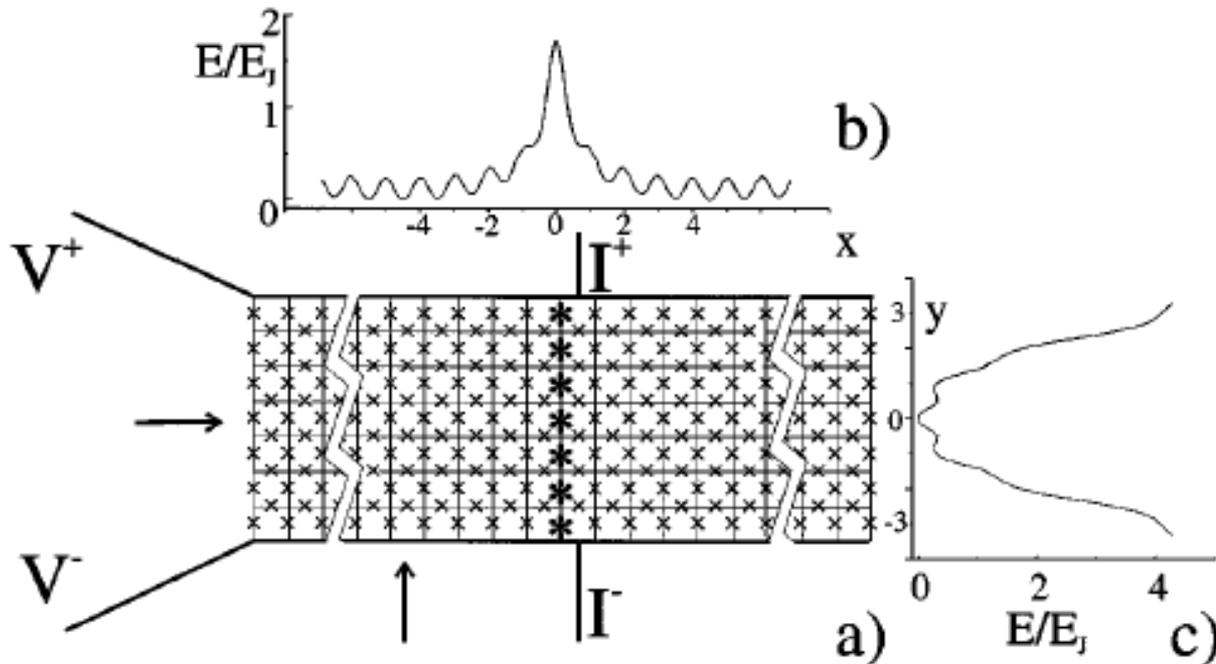


H.S.J. van der Zant, W.J. Elion, L.J. Geerligs, J.E. Mooij, Phys. Rev. B **54**, 10081 (1996)

# Experiments with 1D/2D channels



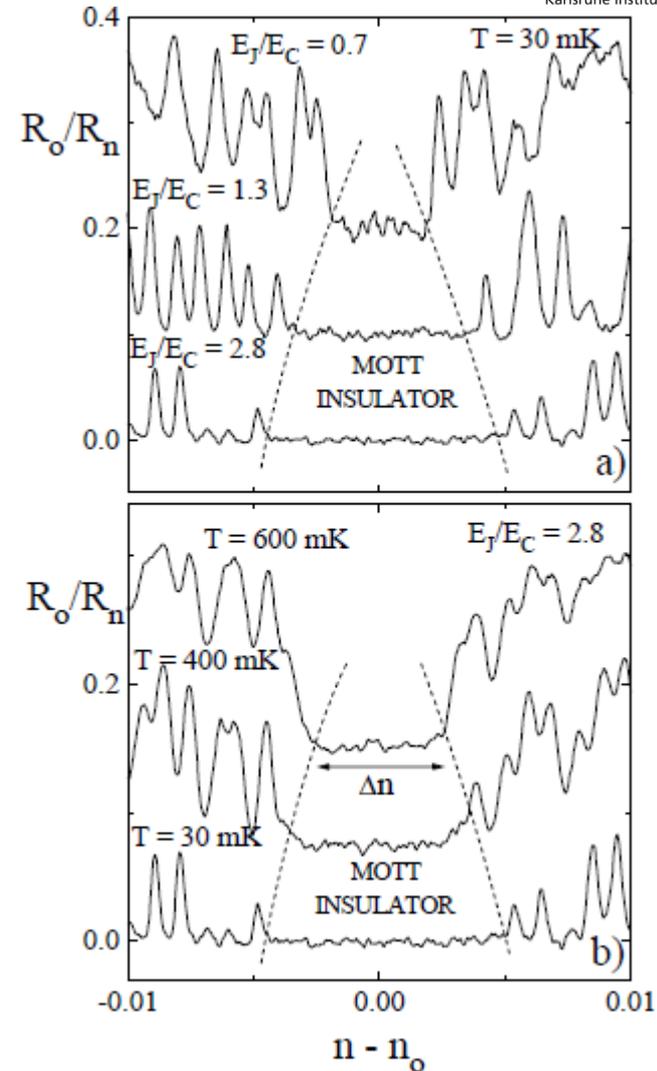
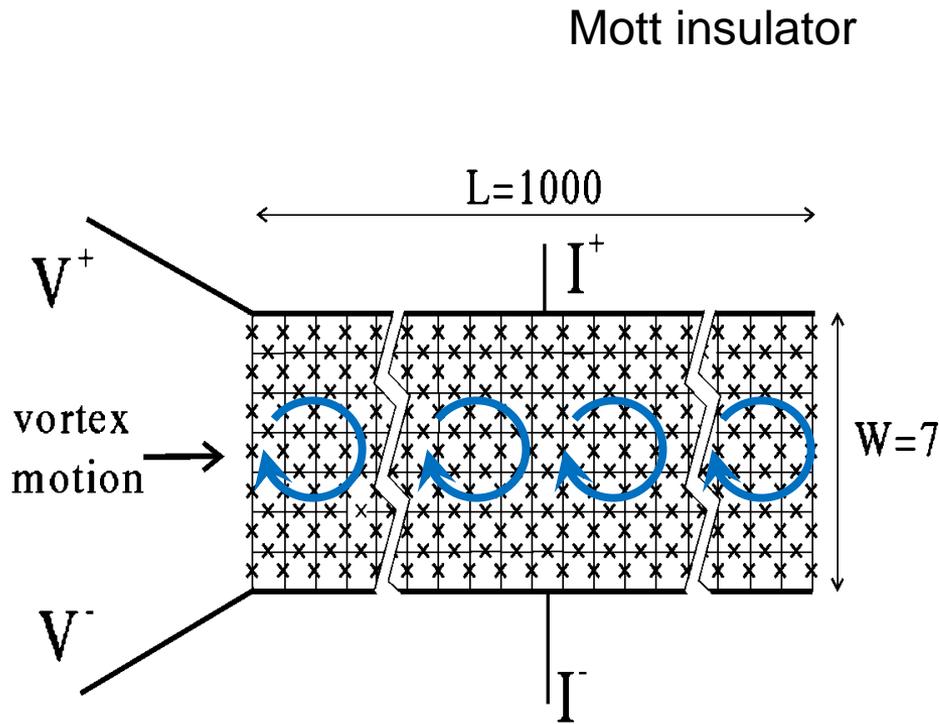
# Quantum vortices in 1D/2D channels



A. van Oudenaarden and J. E. Mooij, Phys. Rev. Lett. **76**, 4947 (1996)

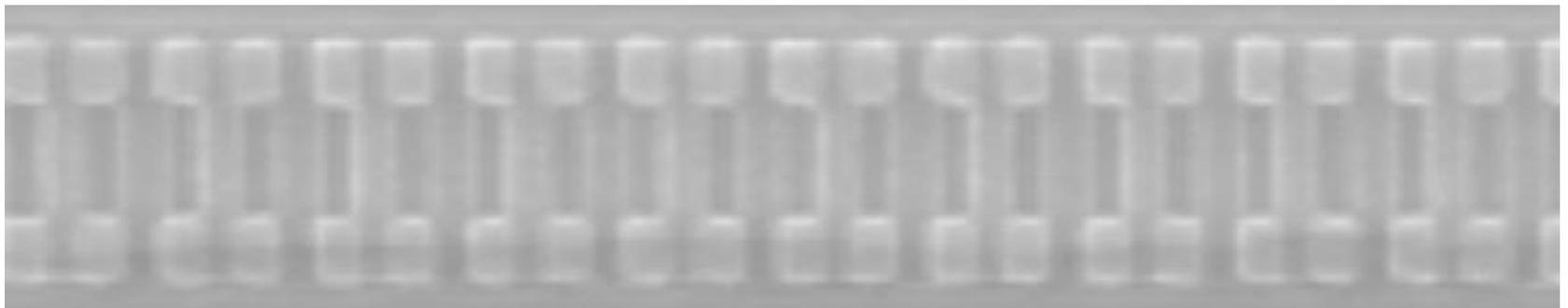
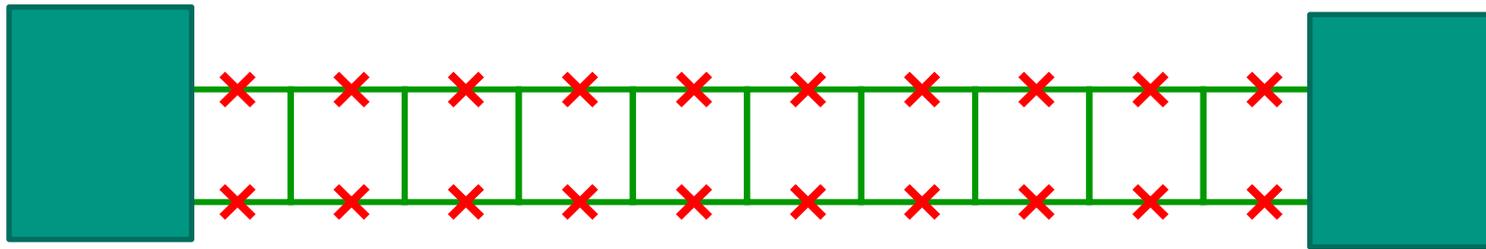
A. van Oudenaarden, S. J. K. Várdu, and J. E. Mooij, Phys. Rev. Lett. **77**, 4257 (1996)

# Quantum vortices in 1D/2D channels

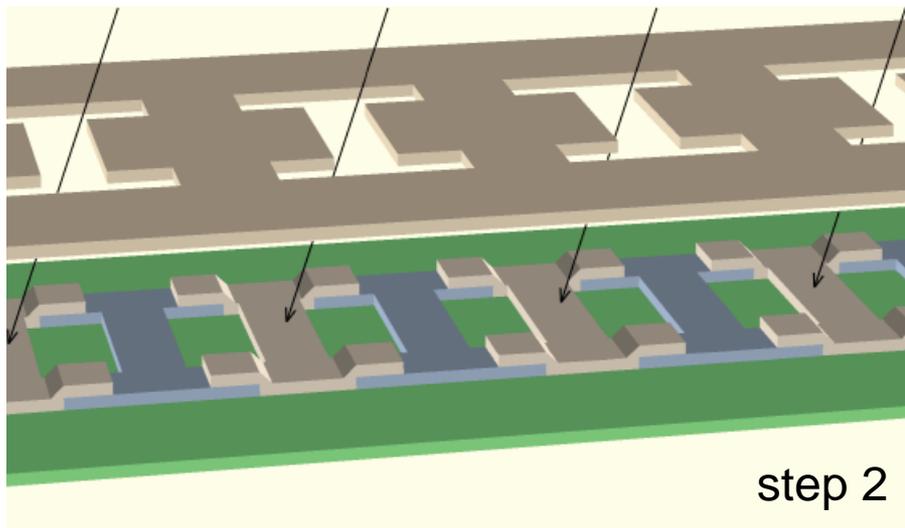
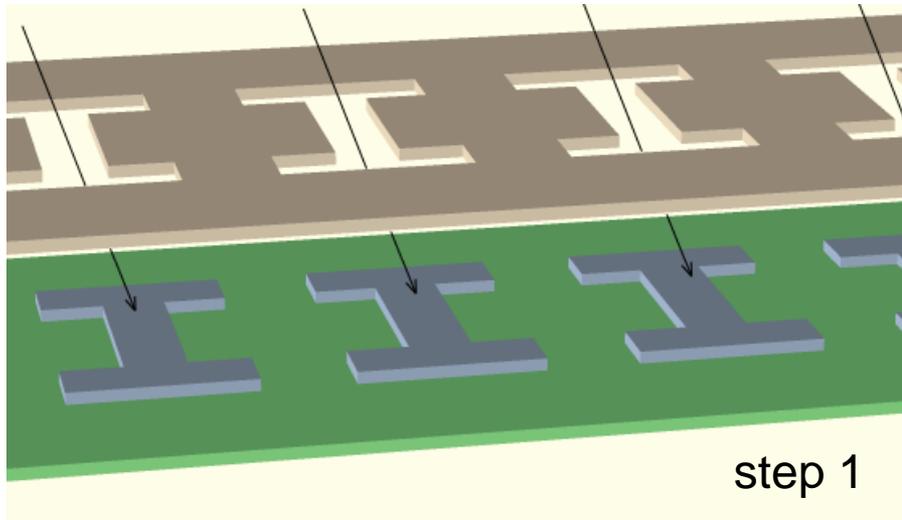


A. van Oudenaarden and J. E. Mooij, Phys. Rev. Lett. **76**, 4947 (1996)

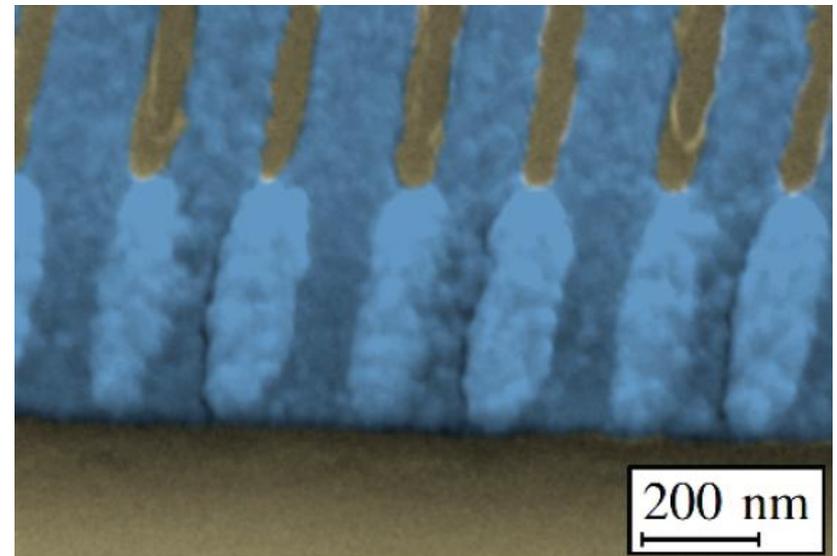
# Experiments with 1D arrays



# Shadow evaporation of SQUID arrays



result

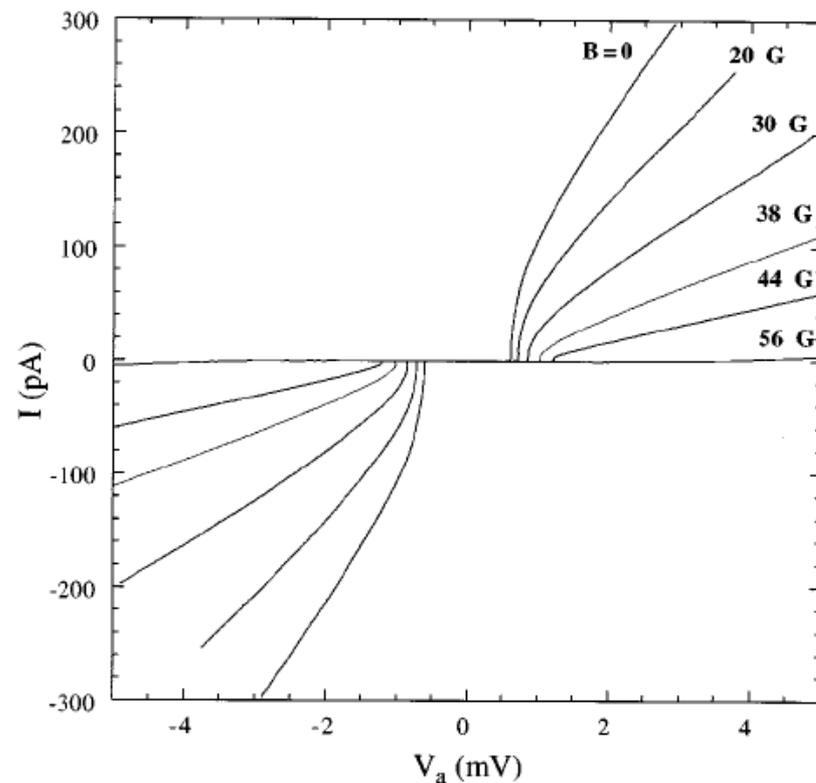
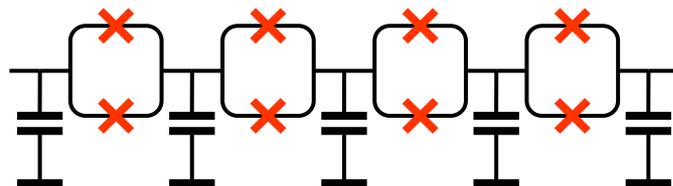
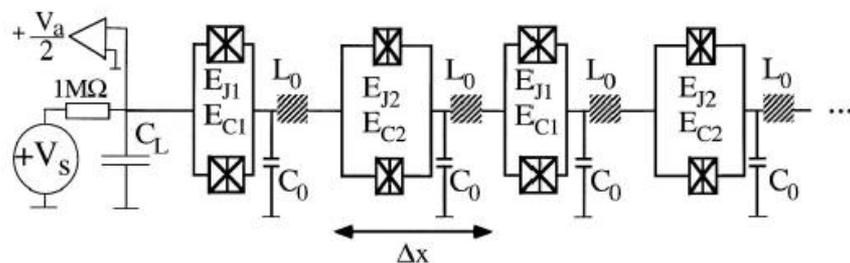
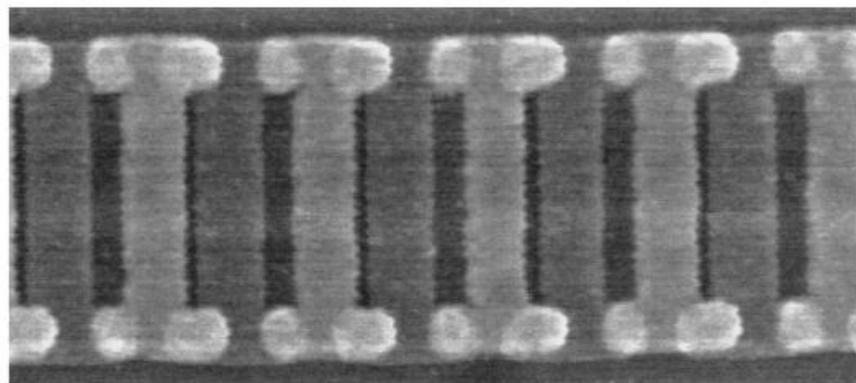


# Cooper-pair charge solitons: The electrodynamics of localized charge in a superconductor

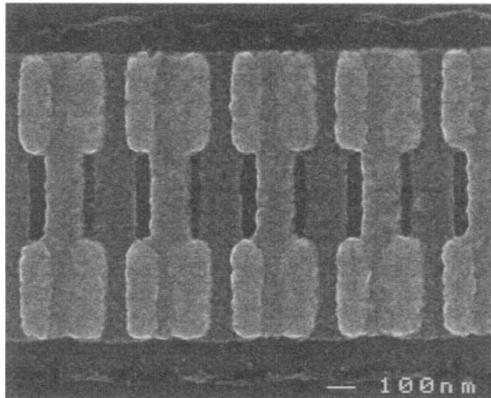
David B. Haviland and Per Delsing

*Department of Physics, Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden*

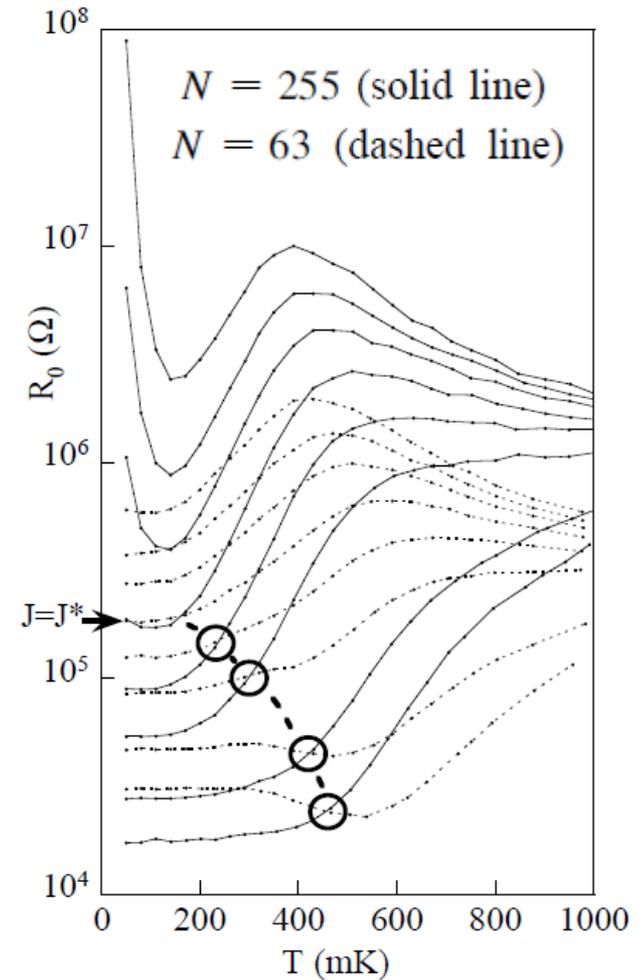
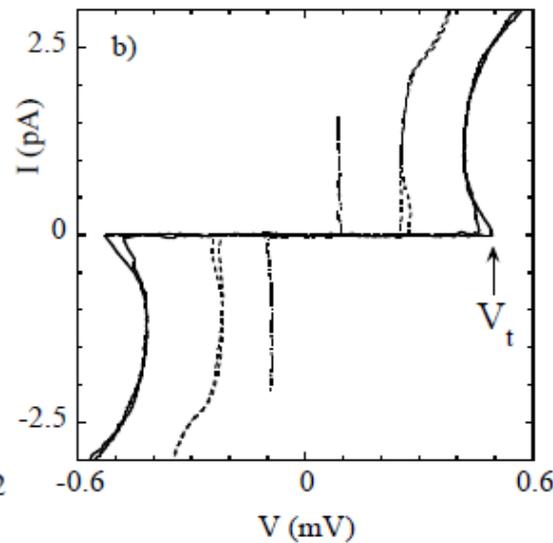
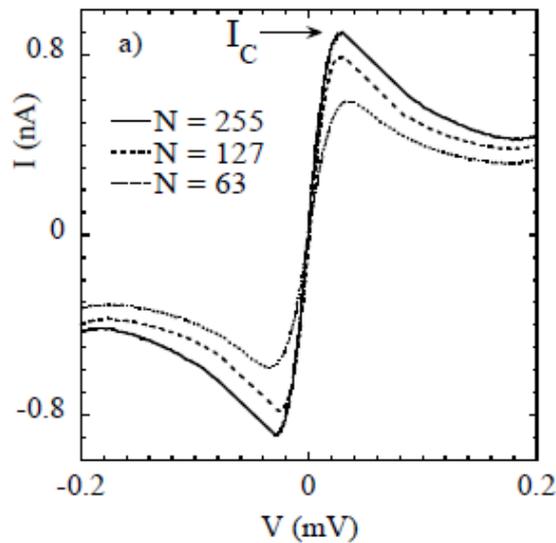
0.4  $\mu\text{m}$



# Observation of S/I transition in 1D



zero-bias resistance, extrapolated to  $T = 0$ , is independent of length at a critical magnetic field

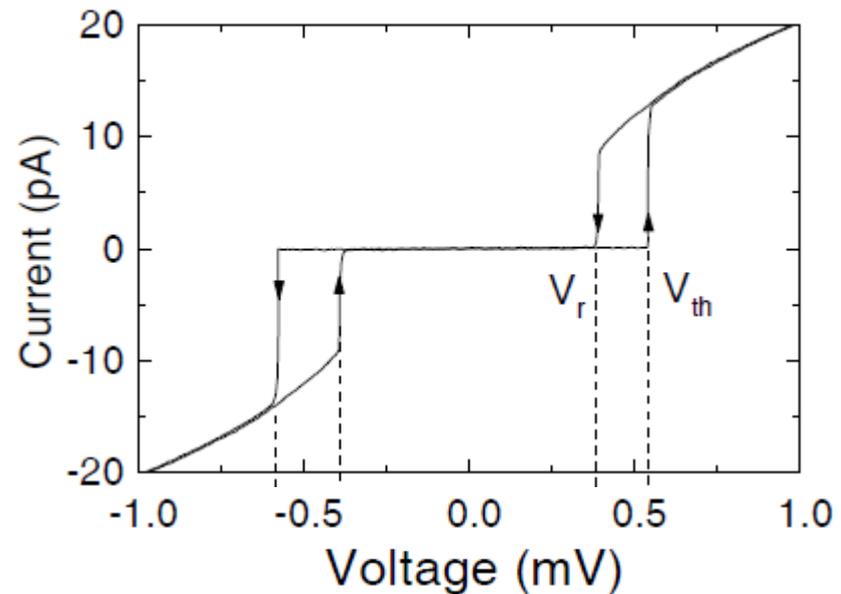
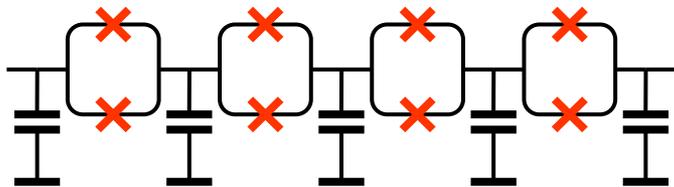
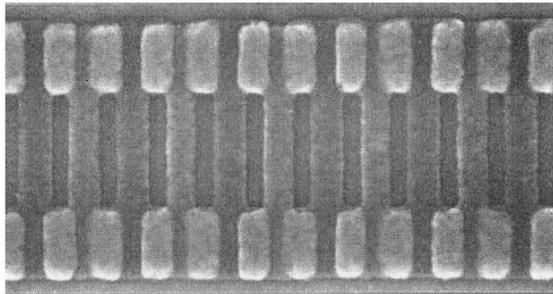


E. Chow, P. Delsing, and D. B. Haviland, Phys. Rev. Lett. **81**, 204 (1998)

# Kinetic Inductance and Coulomb Blockade in One Dimensional Josephson Junction Arrays

Peter Ågren, Karin Andersson and David B. Haviland

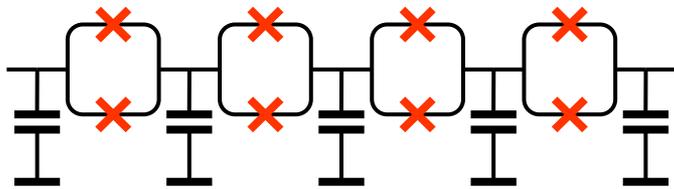
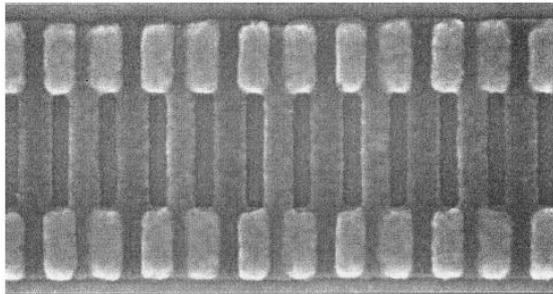
*Department of Physics, Royal Institute of Technology, Lindstedtsvägen 24, SE-100 44 Stockholm, Sweden*



# Kinetic Inductance and Coulomb Blockade in One Dimensional Josephson Junction Arrays

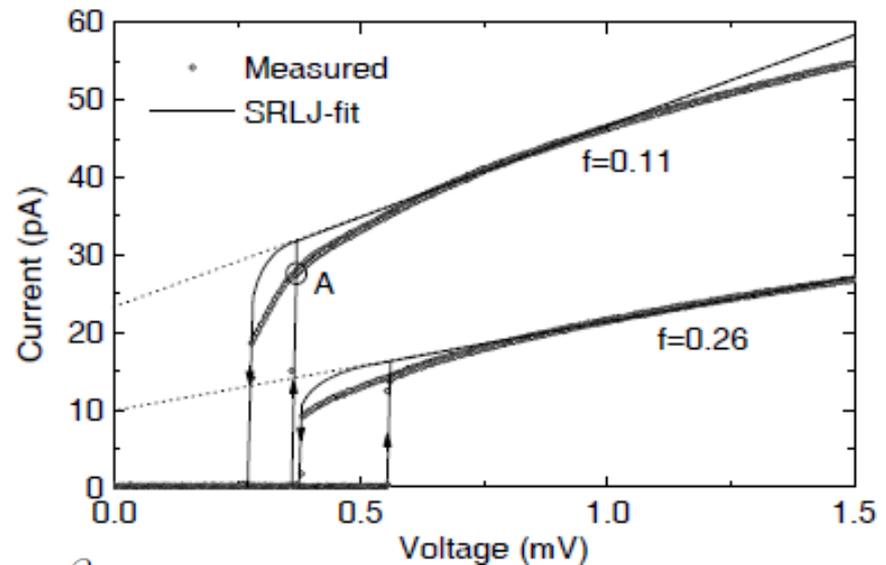
Peter Ågren, Karin Andersson and David B. Haviland

*Department of Physics, Royal Institute of Technology, Lindstedtsvägen 24, SE-100 44 Stockholm, Sweden*



damping parameter  $\beta$

The inductance per cell was then calculated from  $\beta$ ,  $L = \beta R^2 e / \pi V_C$ .



# Charge solitons in one-dimensional arrays of serially coupled Josephson junctions

Ziv Hermon

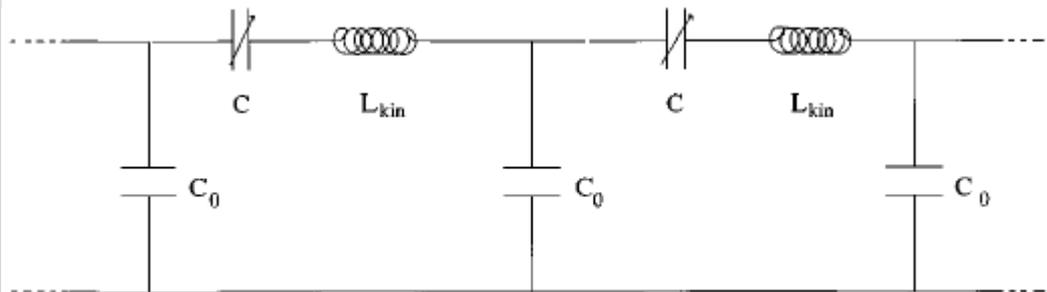
*Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*

Eshel Ben-Jacob

*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences,  
Tel-Aviv University, Ramat-Aviv, 69978 Tel-Aviv, Israel*

Gerd Schön

*Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*



assumption taken:

$$L_{\text{kin}} = \frac{m_e^* l_x}{e^*{}^2 n_s S} \gg L_J = \frac{1}{(2\pi)^2} \frac{\Phi_0^2}{E_J}$$

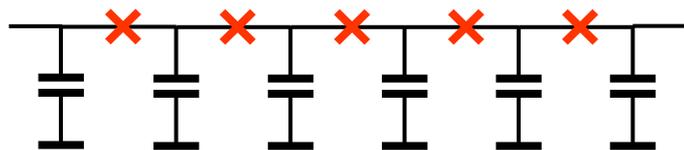
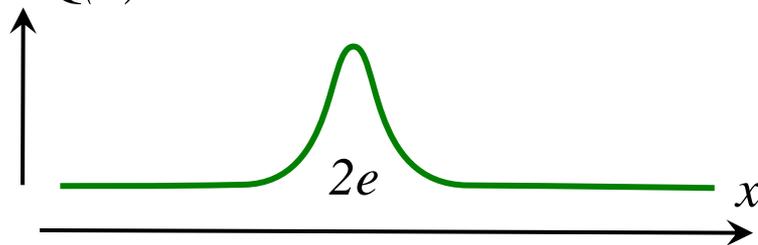
equation of motion for the charge:

$$\frac{1}{2\pi} 2e L_{\text{kin}} \ddot{q} - \frac{1}{2\pi} a^2 \frac{2e}{C_0} q_{xx} + \frac{1}{2\pi} V_C \sin q = 0$$

# Duality between charge and flux solitons

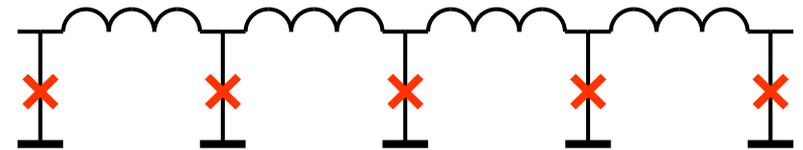
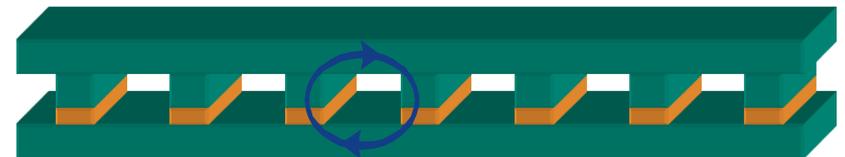
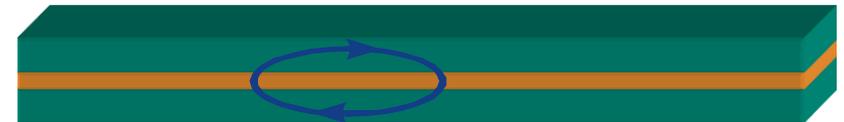
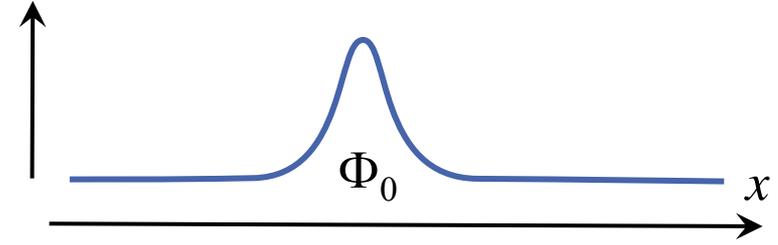
series array  $E_J \ll E_C$

$Q(x)$  electric charge



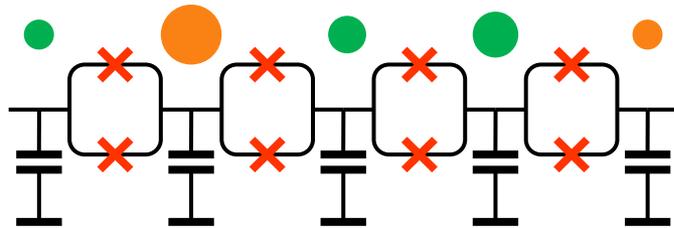
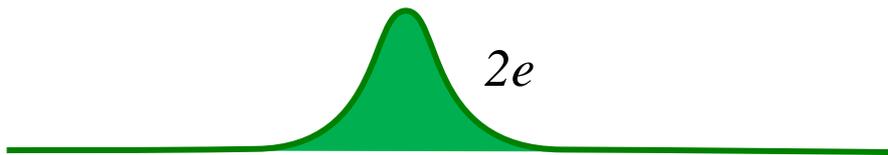
parallel array  $E_J \gg E_C$

$\Phi(x)$  magnetic flux



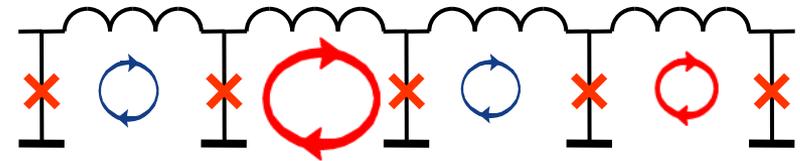
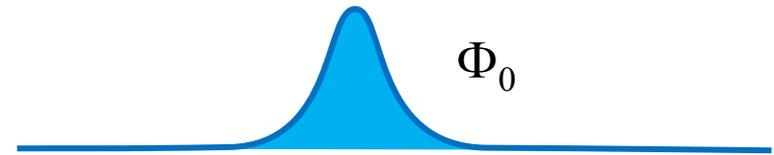
# Soliton pinning problem

pinned random charges



$$-e/2 < Q_i < e/2$$

pinned random fluxes



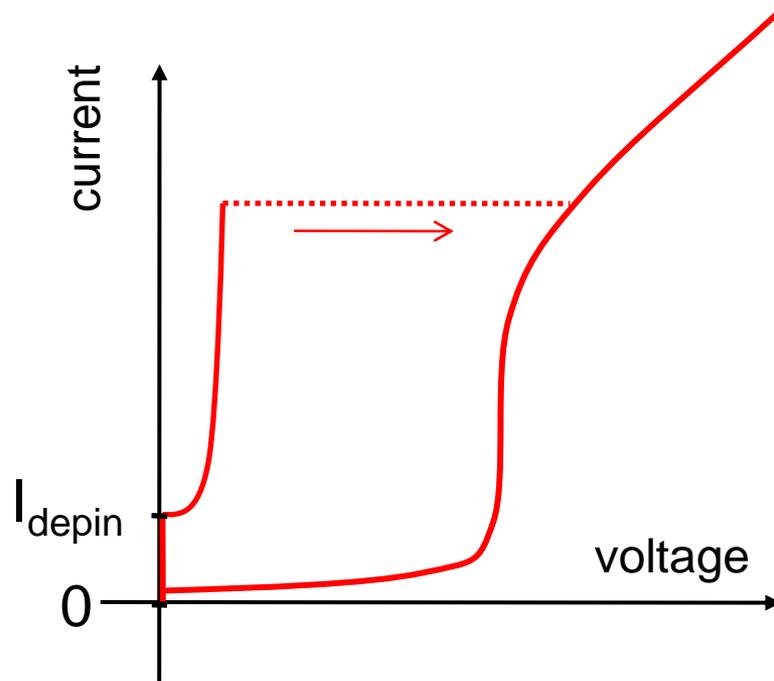
$$-\Phi_0/2 < \Phi_i < \Phi_0/2$$

For flux regime:

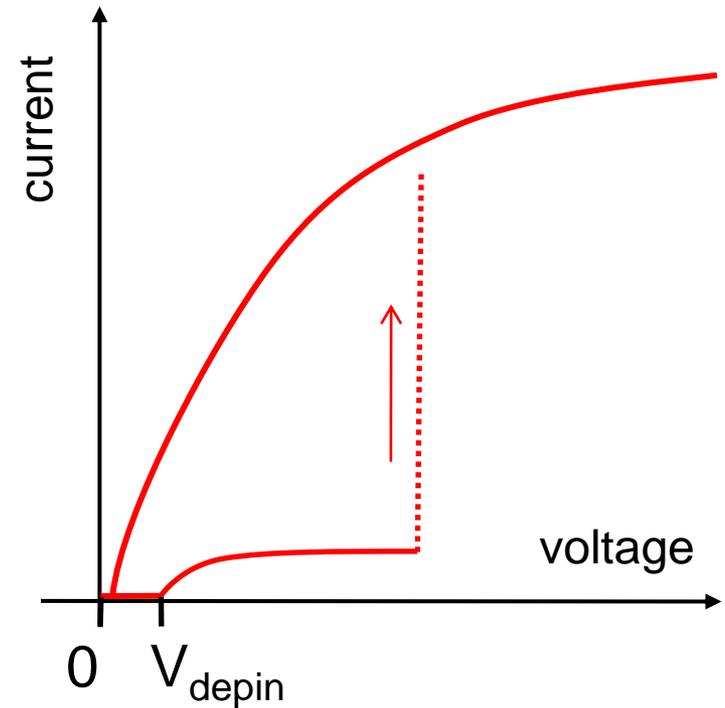
$$\frac{\partial^2 \varphi_i}{\partial t^2} + \alpha \frac{\partial \varphi_i}{\partial t} - \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{\alpha^2} = \sin \varphi_i - i + \Phi_i - \Phi_{i-1}$$

$\Phi_i$  – random fluxes

# Depinning of solitons

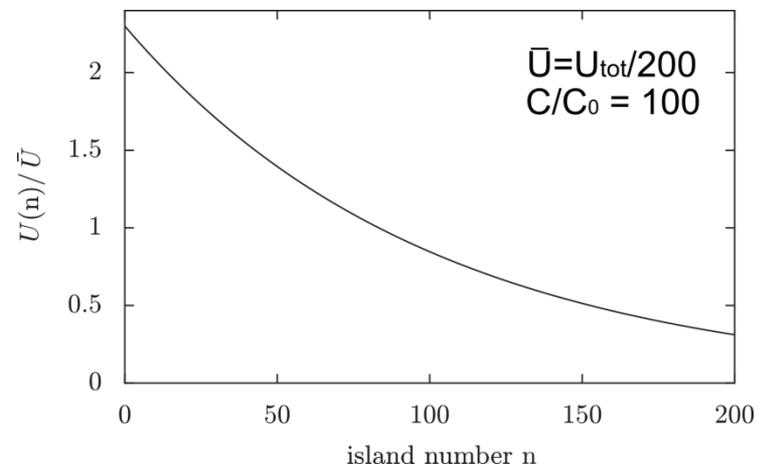
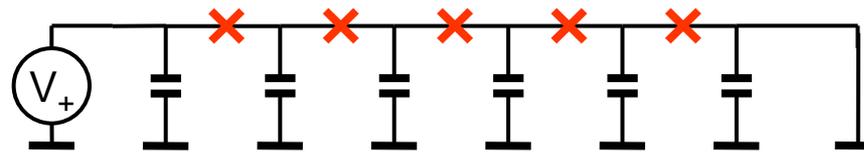


depinning of flux soliton

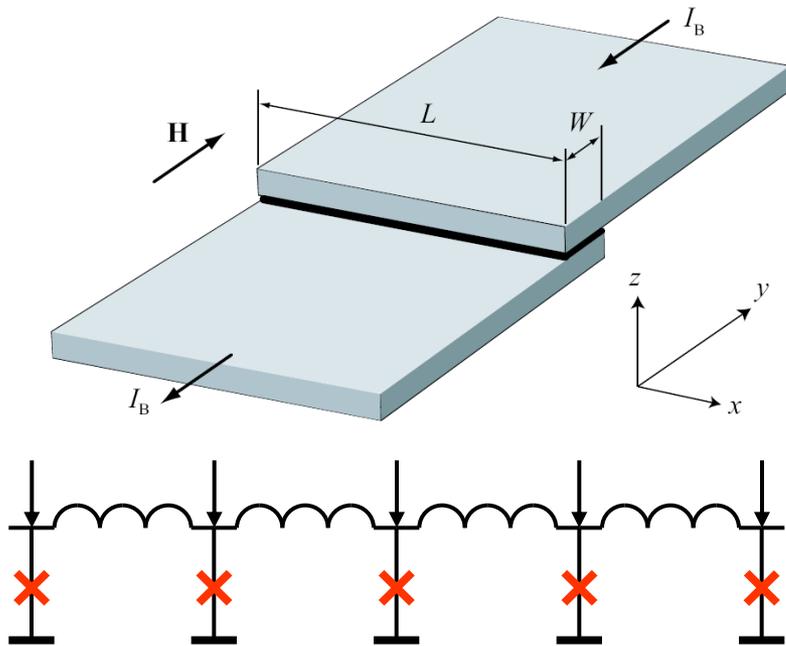
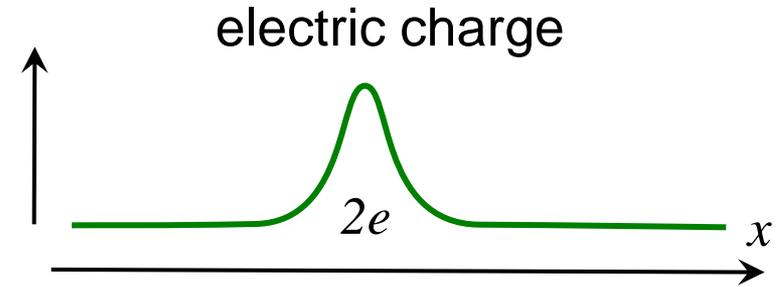
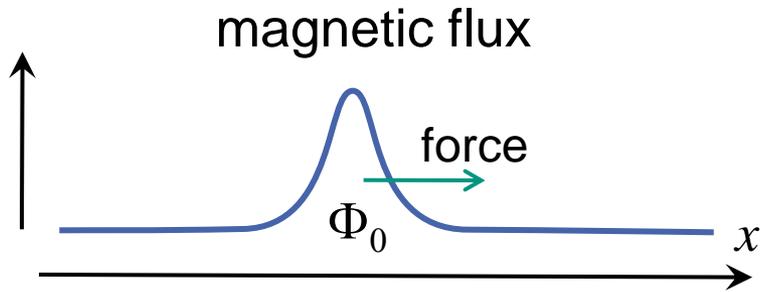


depinning of charge soliton

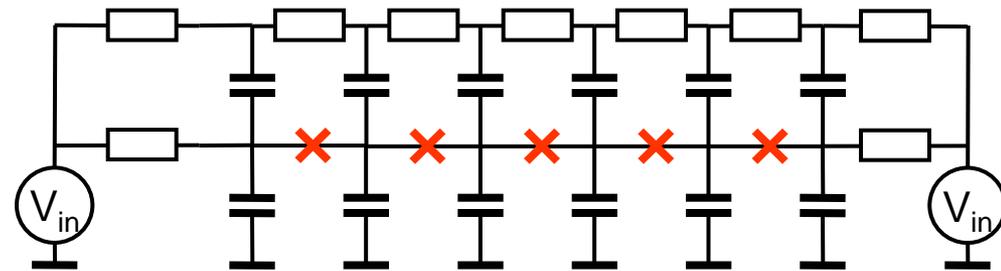
# Problem of non-uniform voltage bias



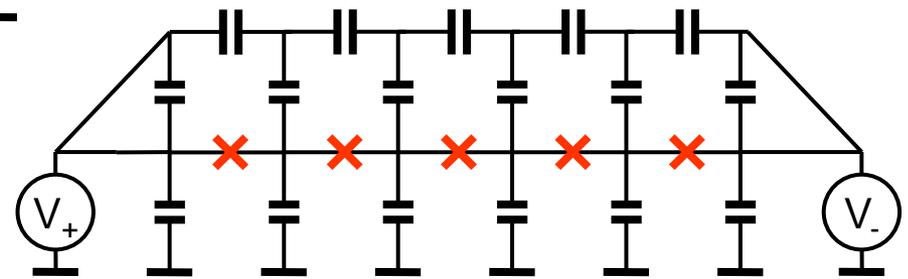
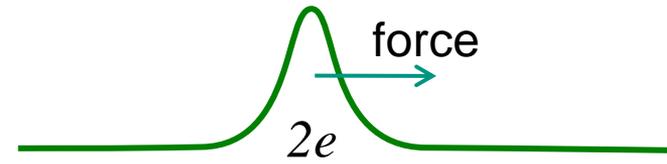
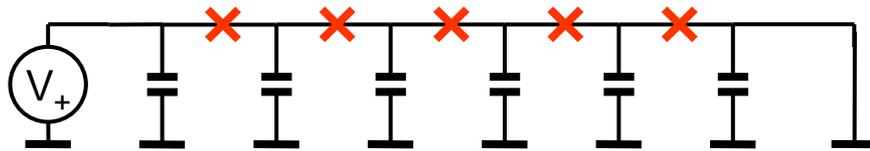
# Uniform forcing of solitons



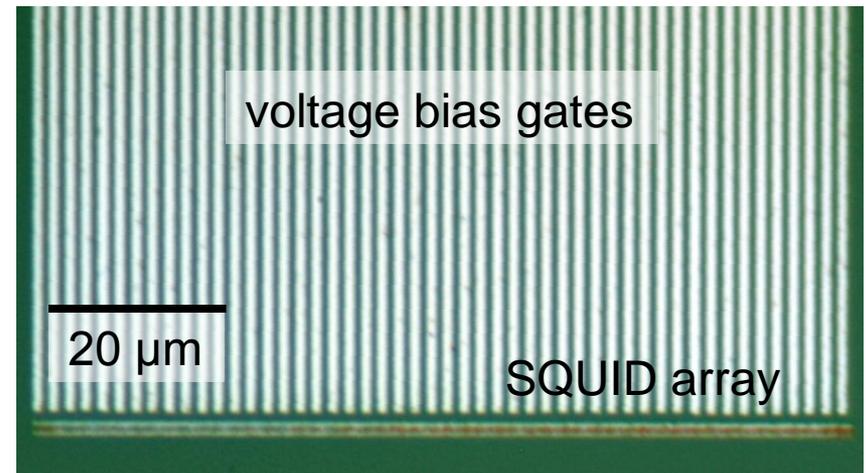
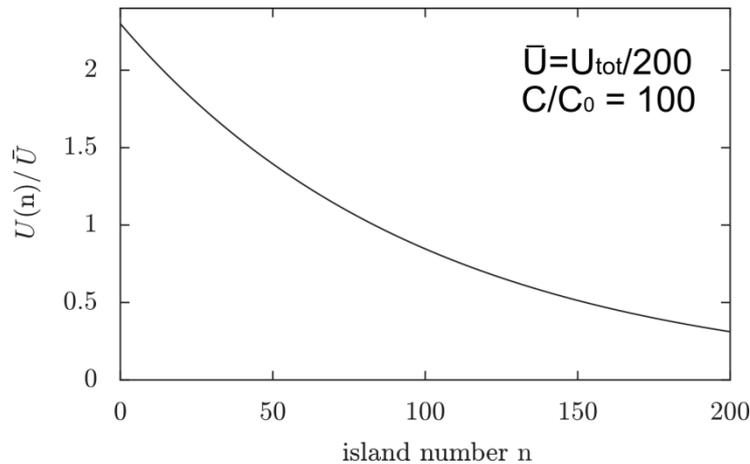
bias "comb" yielding uniform voltage bias



# Implementation of the uniform biasing for charge solitons



non-uniform voltage bias



# Experiments with JJ arrays in Karlsruhe

Josephson Energy:  $E_j = \Delta g / 8 = 1140 \text{ kB mK}$

Charging Energy:  $E_c = e^2 / 2C = 930 \text{ kB mK}$

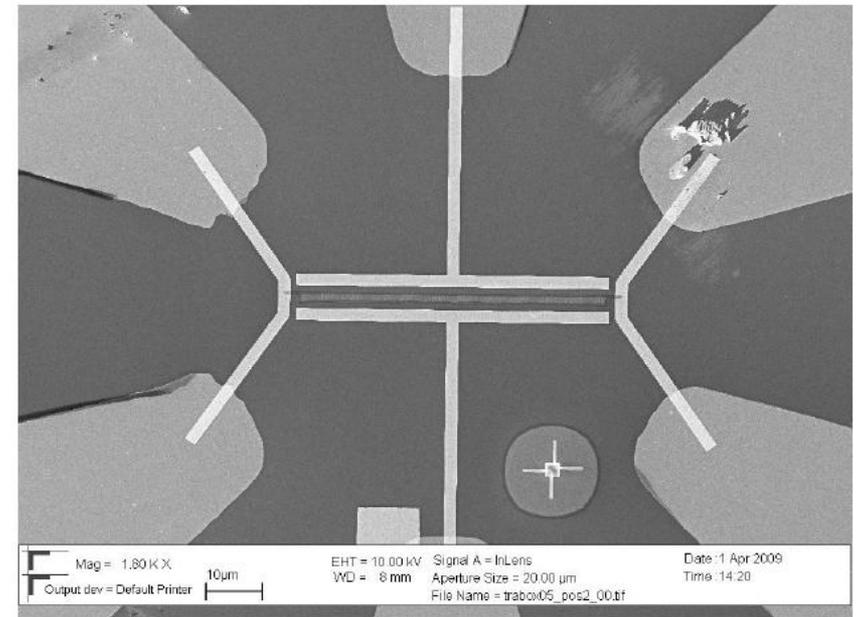
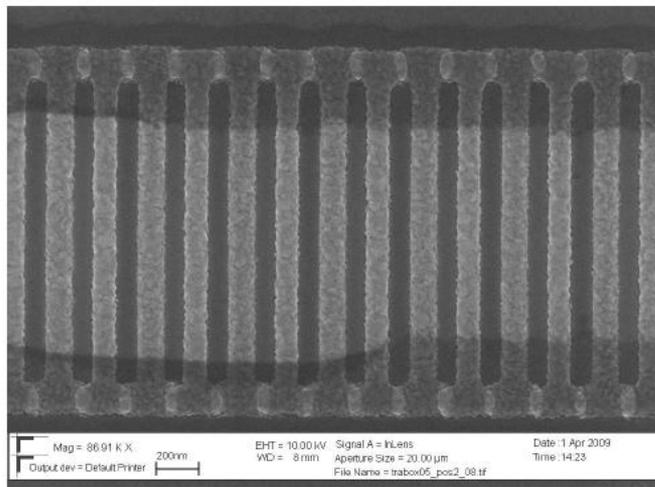
Capacitance to ground:  $C_0 = 5 \text{ aF} \dots 20 \text{ aF}$

Capacitance of single island:  $C = 1 \text{ fF}$

Size array length:  $L = 51 \mu\text{m}$

Array width:  $W = 1.6 \mu\text{m}$

Number of SQUID loops: 255

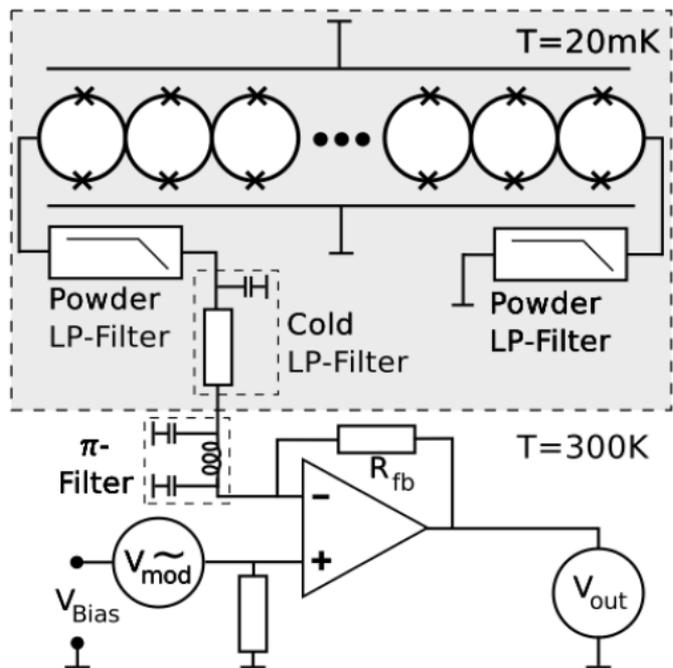


Loop size:  $1.44 \mu\text{m} \times 100 \text{ nm}$   
 $\rightarrow 14.3 \text{ mT per } \Phi_0$

R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

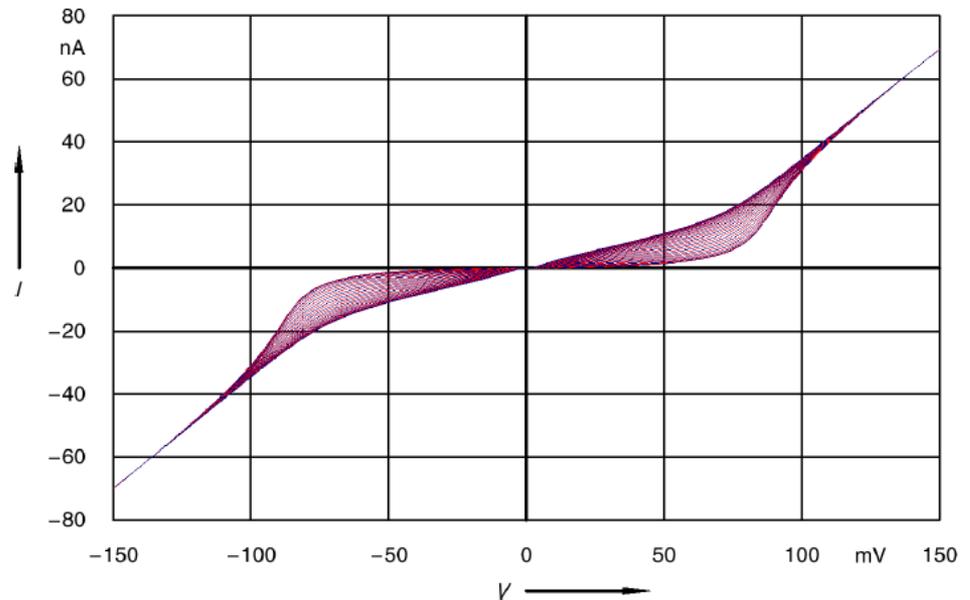
# Tunable Josephson energy $E_J$

Measurement setup



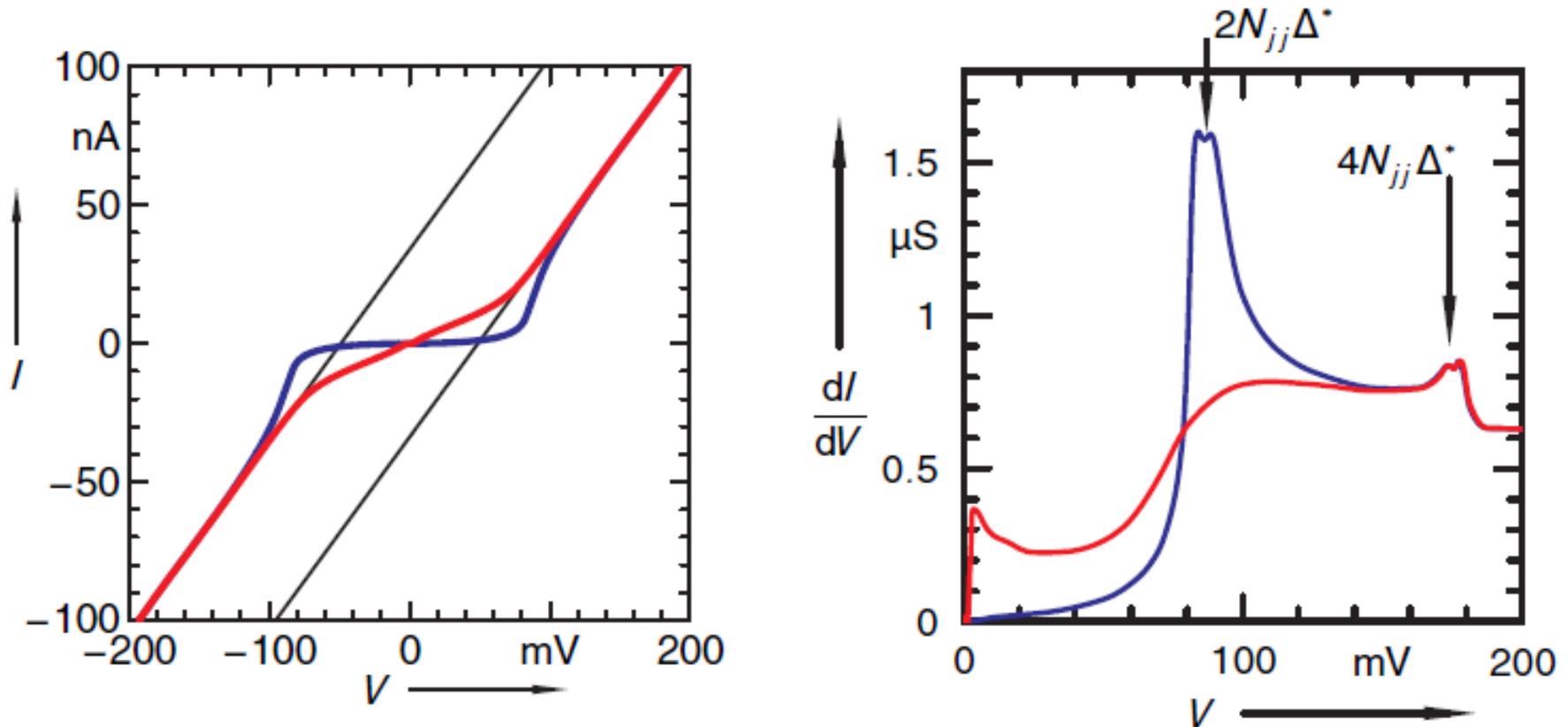
Measurements are done in voltage bias scheme.

Magnetic field dependence: I-V at large bias voltages



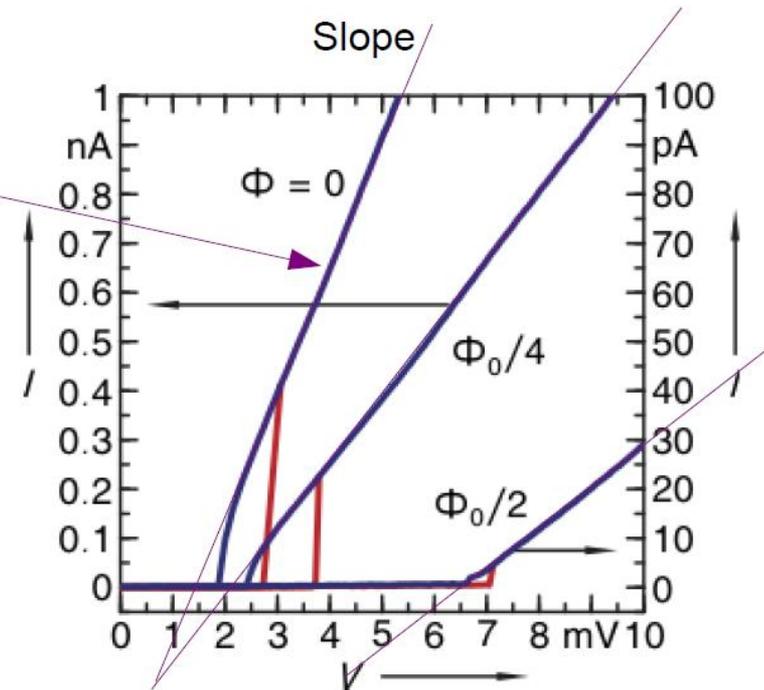
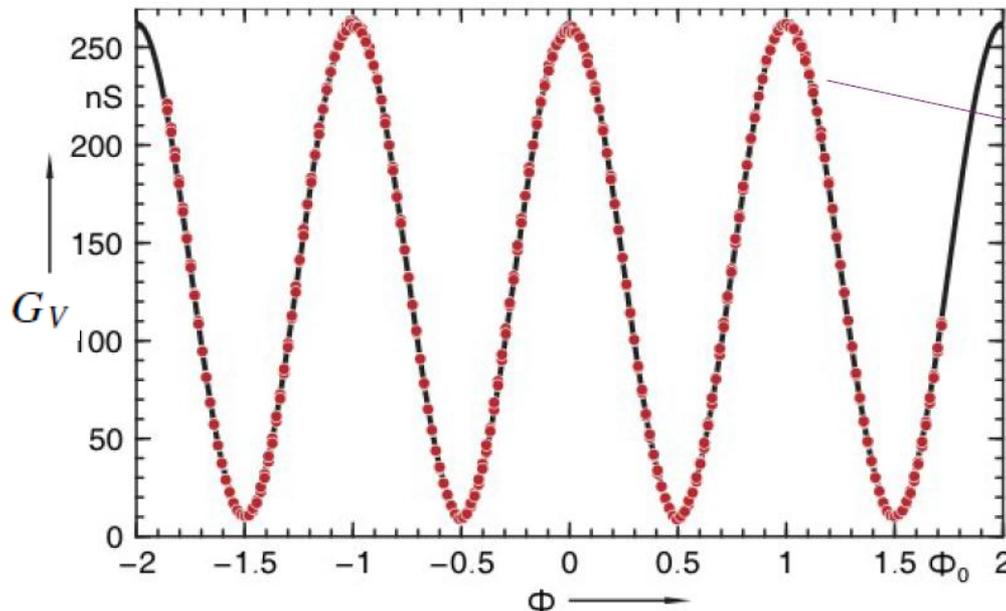
R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

# Large-scale I-V characteristics



R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

# Conductance of dissipative branch

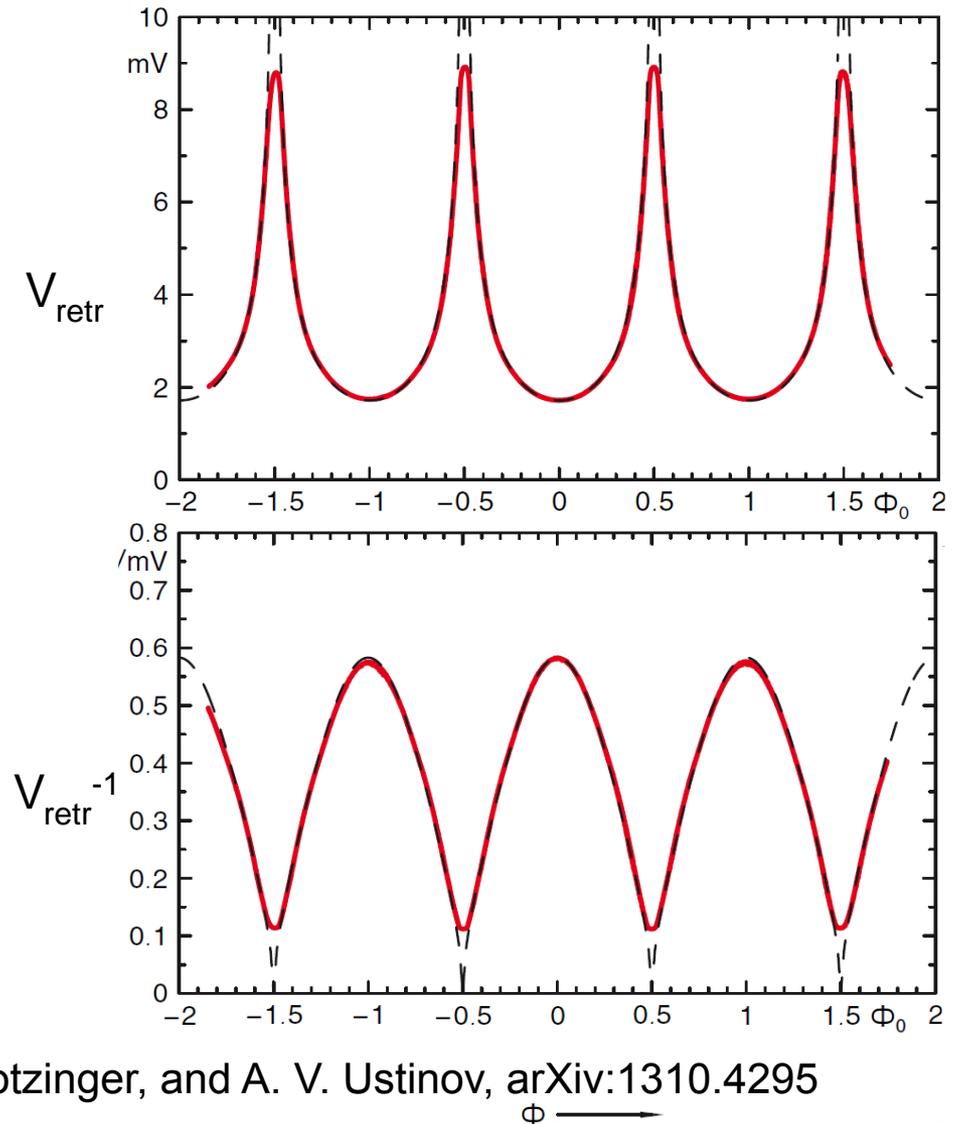
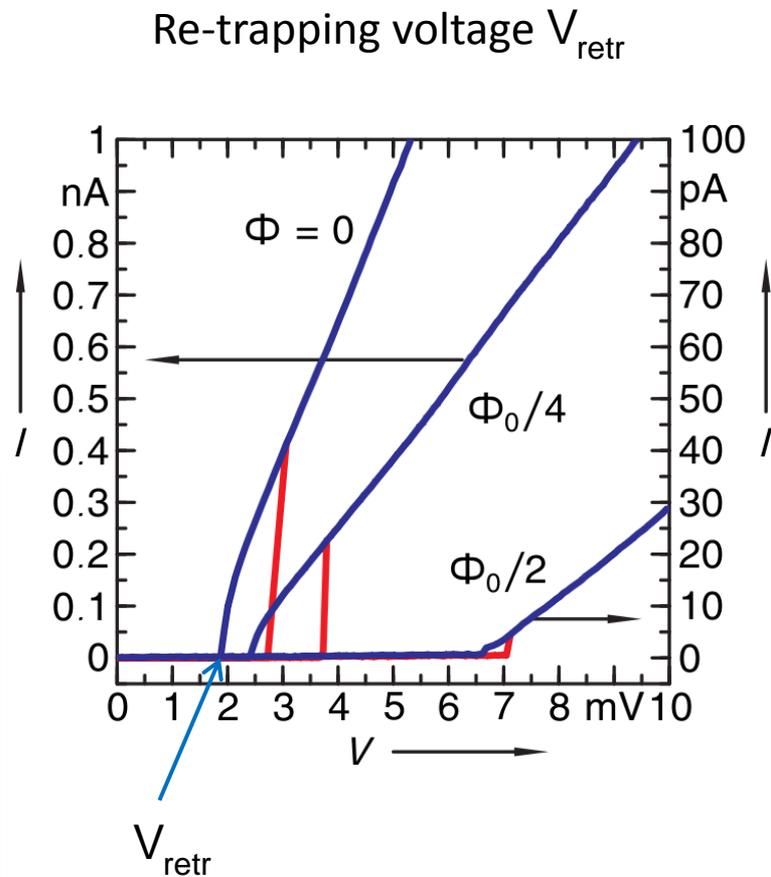


- $G_V \propto E_J^2$
- Conductance depends on the Josephson coupling of the islands

$$E_J(\Phi) = 2E_J \left| \cos \left( \frac{\pi \Phi}{\Phi_0} \right) \right|$$

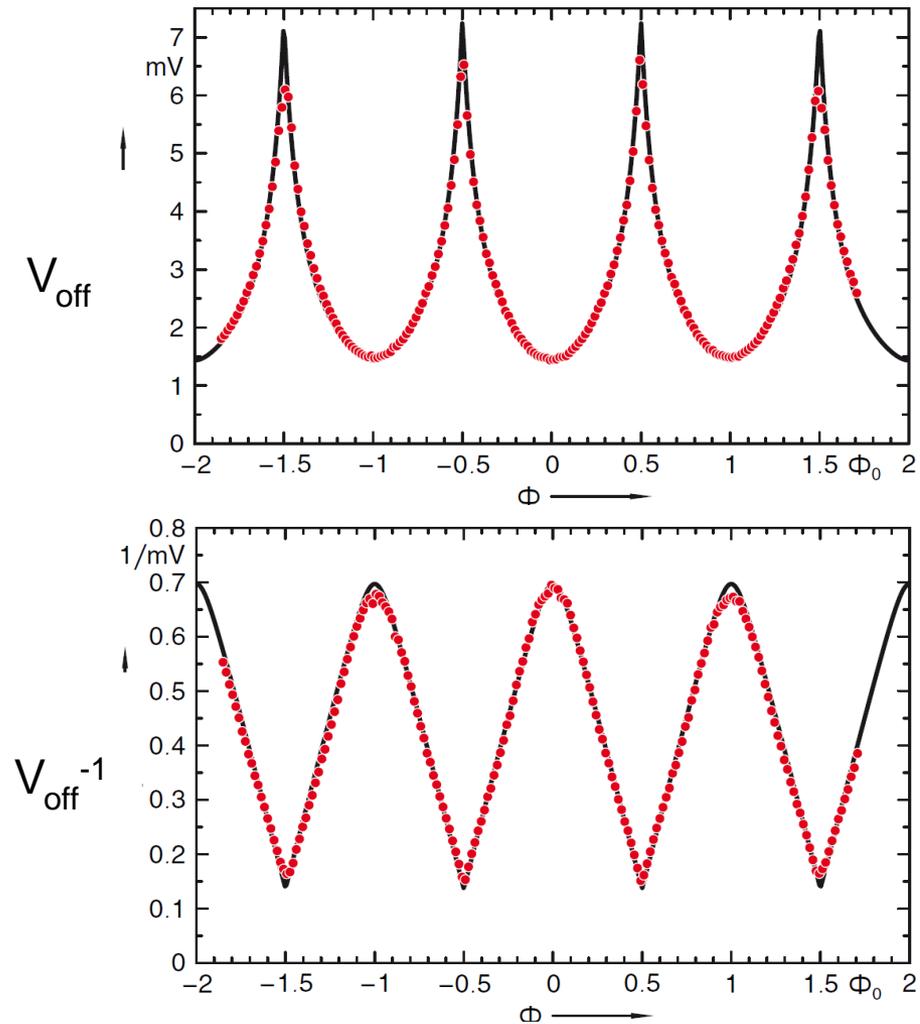
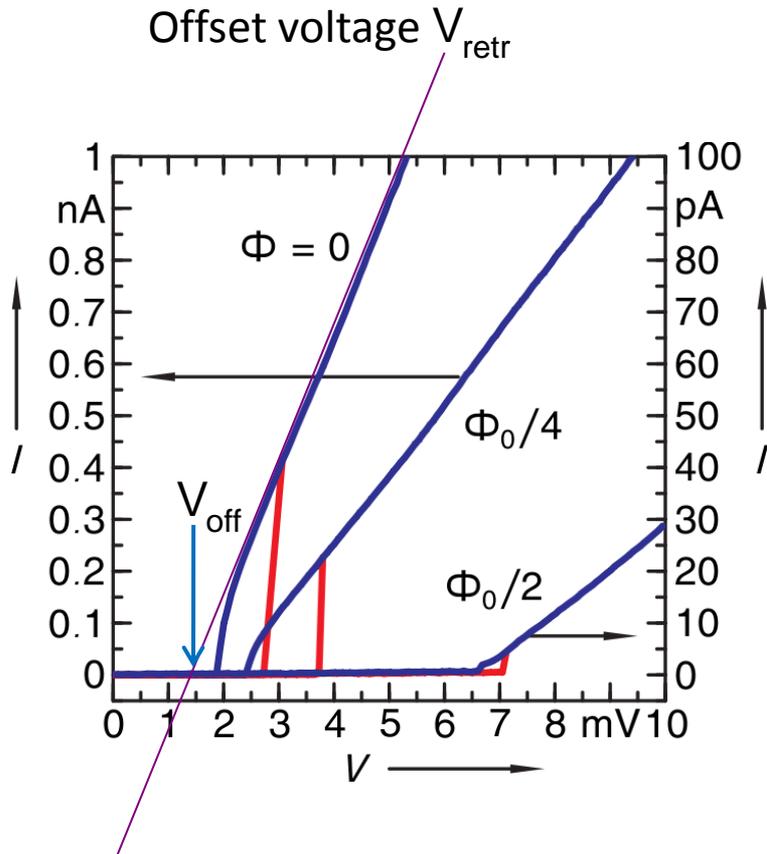
R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

# Re-trapping voltage change with flux



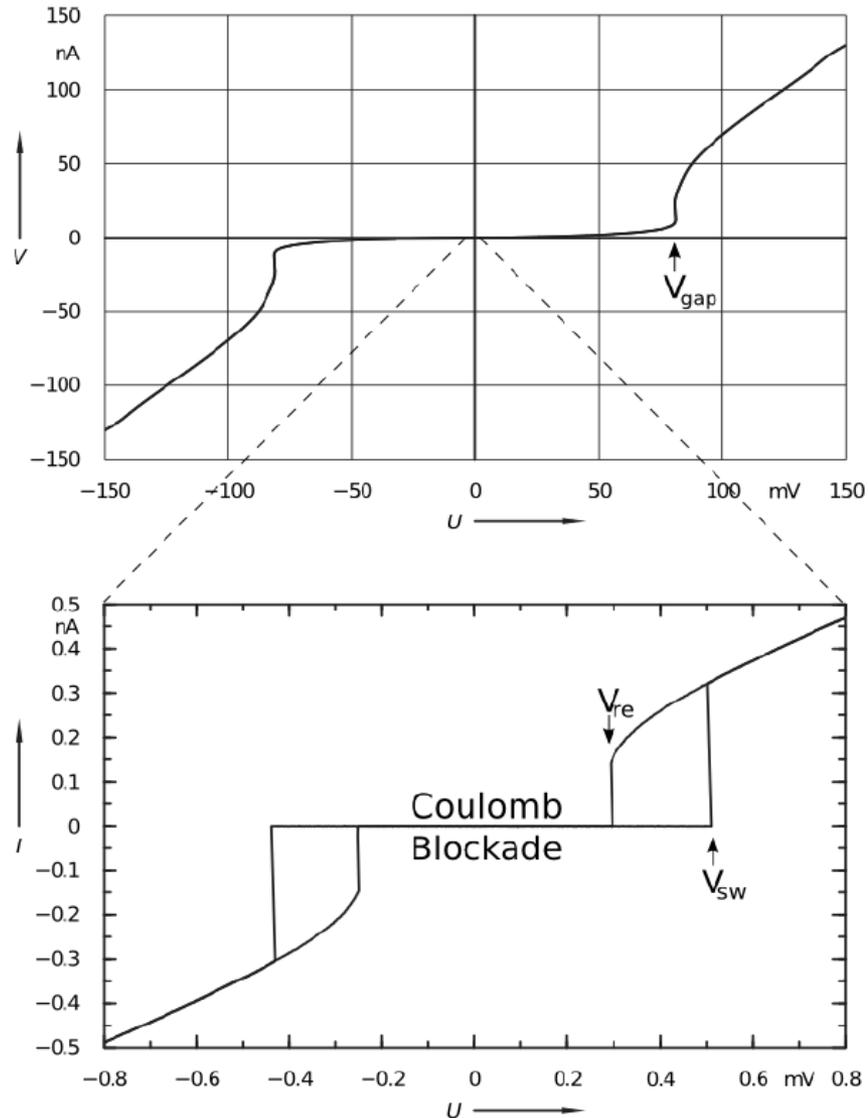
R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

# Offset voltage change with flux

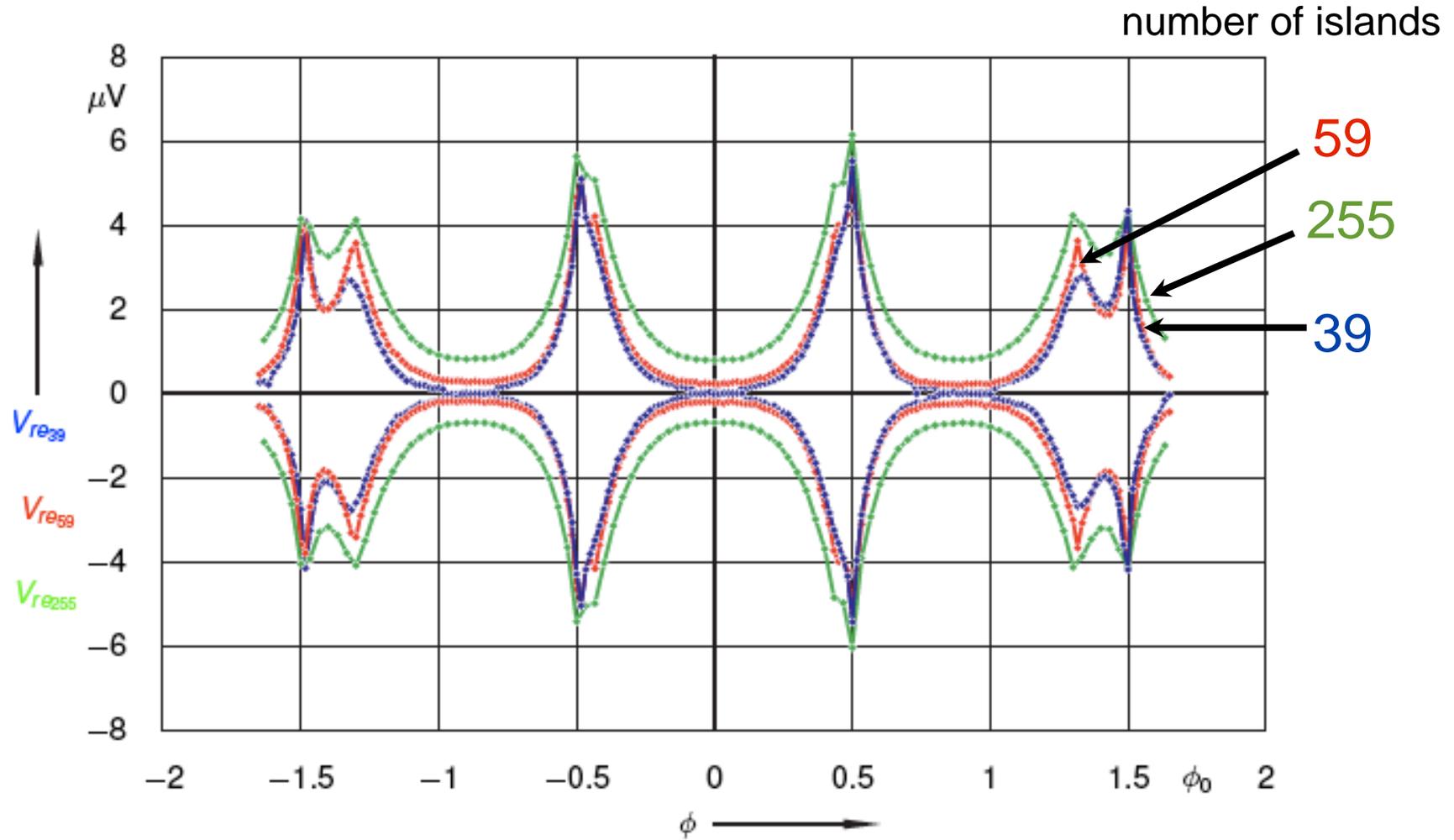


R. Schäfer, W. Cui, K. Grube, H. Rotzinger, and A. V. Ustinov, arXiv:1310.4295

# Switching and retrapping voltages

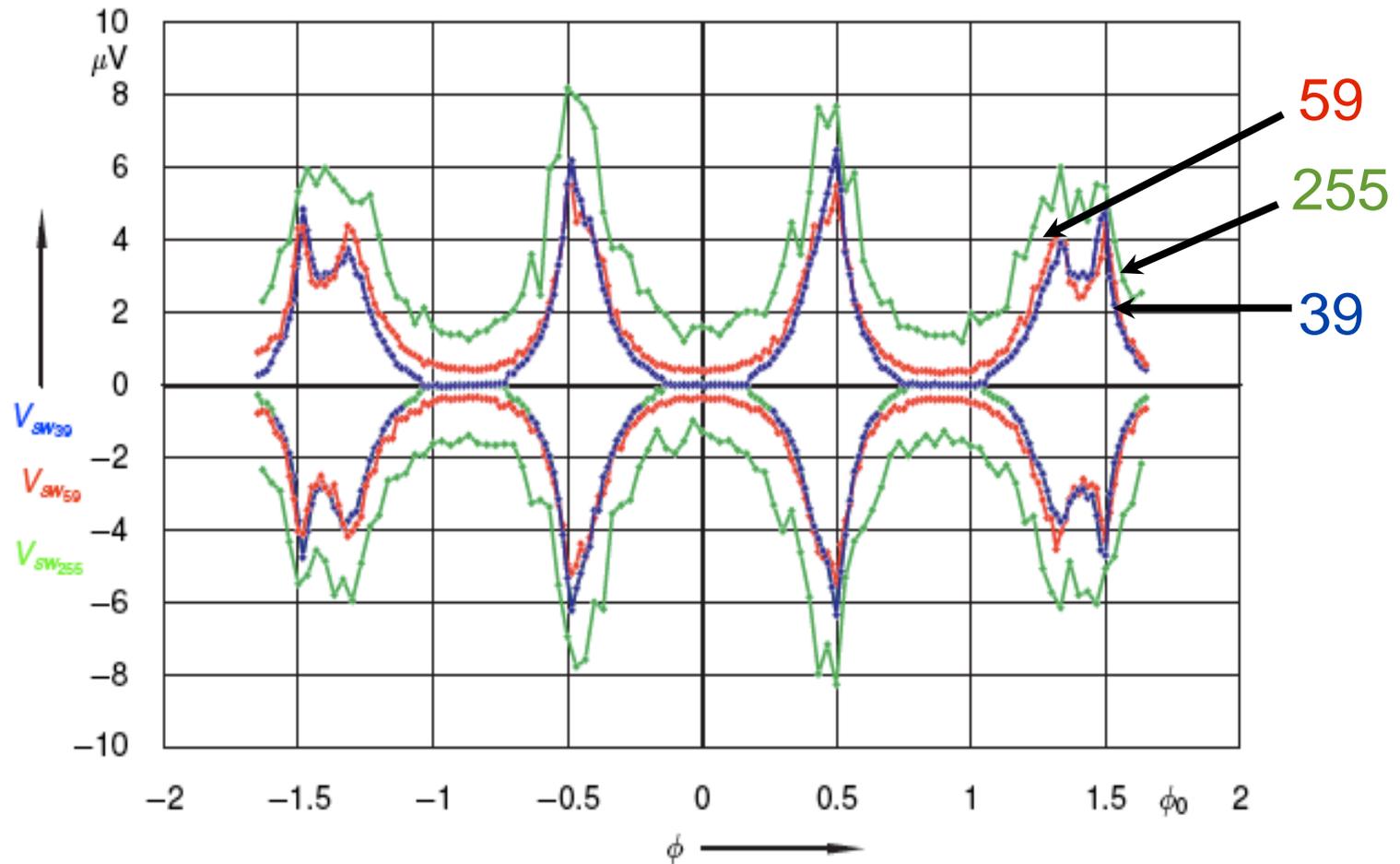


# Retrapping voltage normalized to the number of islands



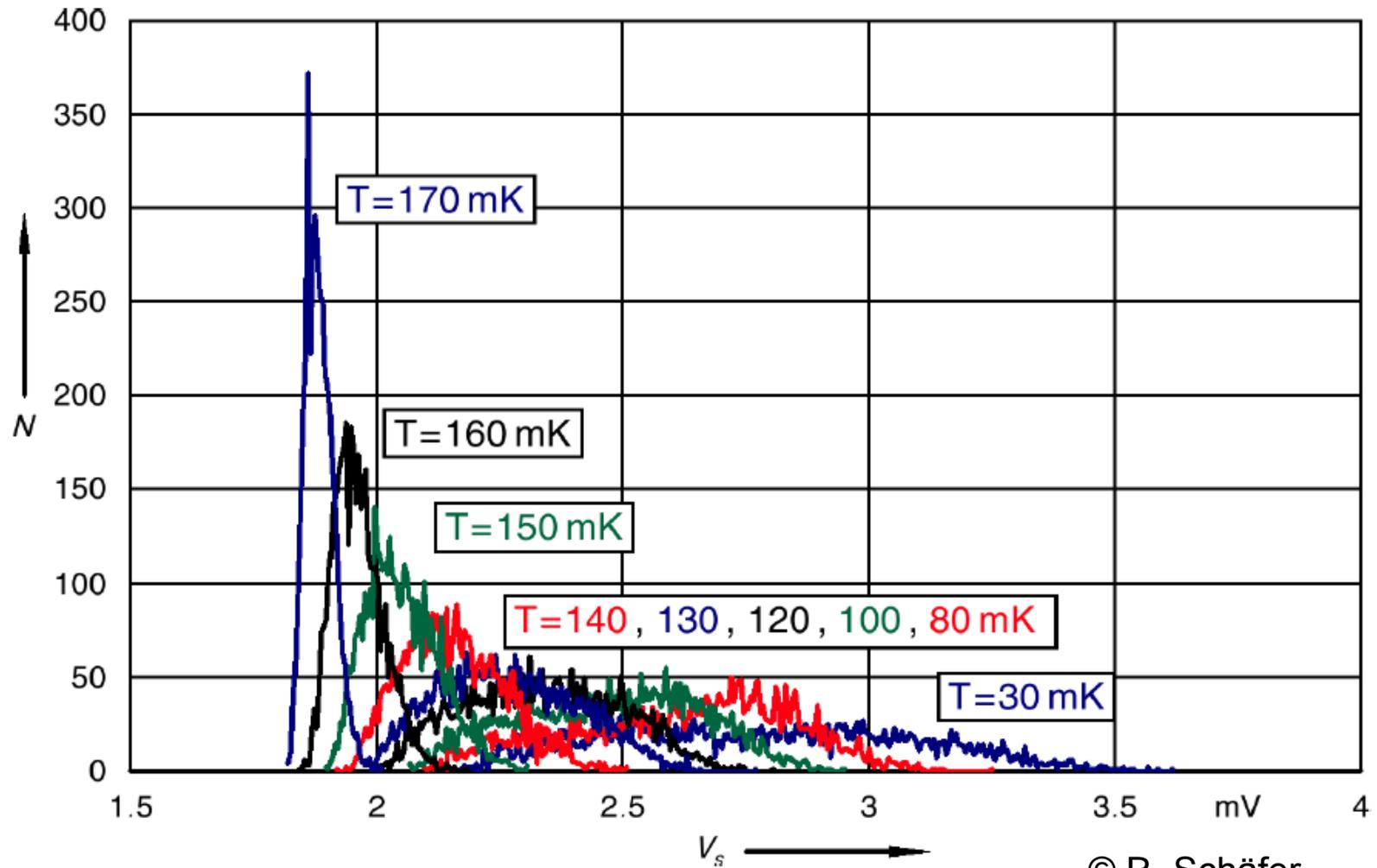
© A. Fiebig and R. Schäfer

# Switching voltage normalized to the number of islands



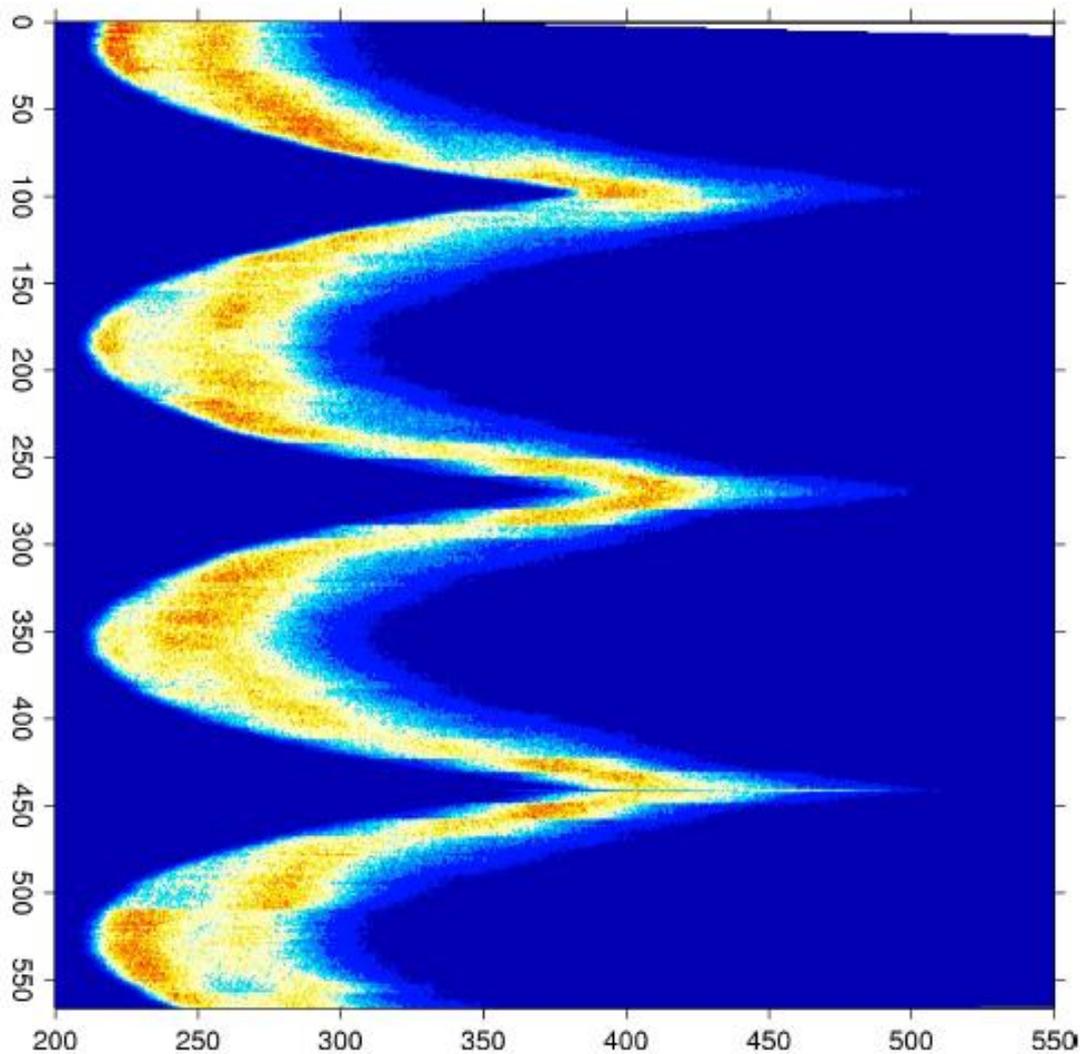
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# Switching voltage histograms



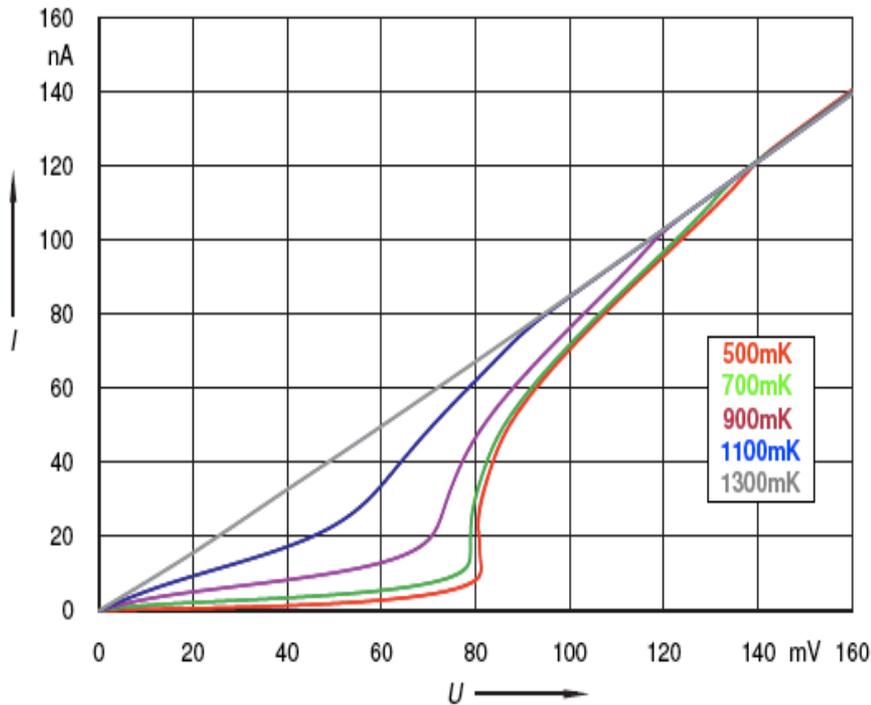
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# Switching voltage histograms

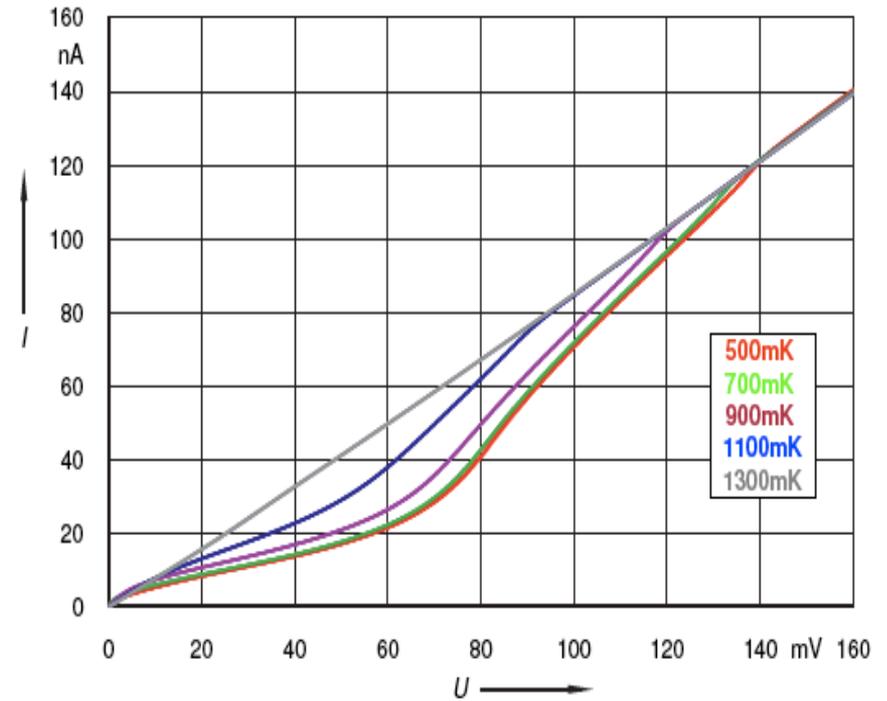


© R. Schäfer

# Thermally activated conductance



$$\Phi/\Phi_0 = 0.5$$

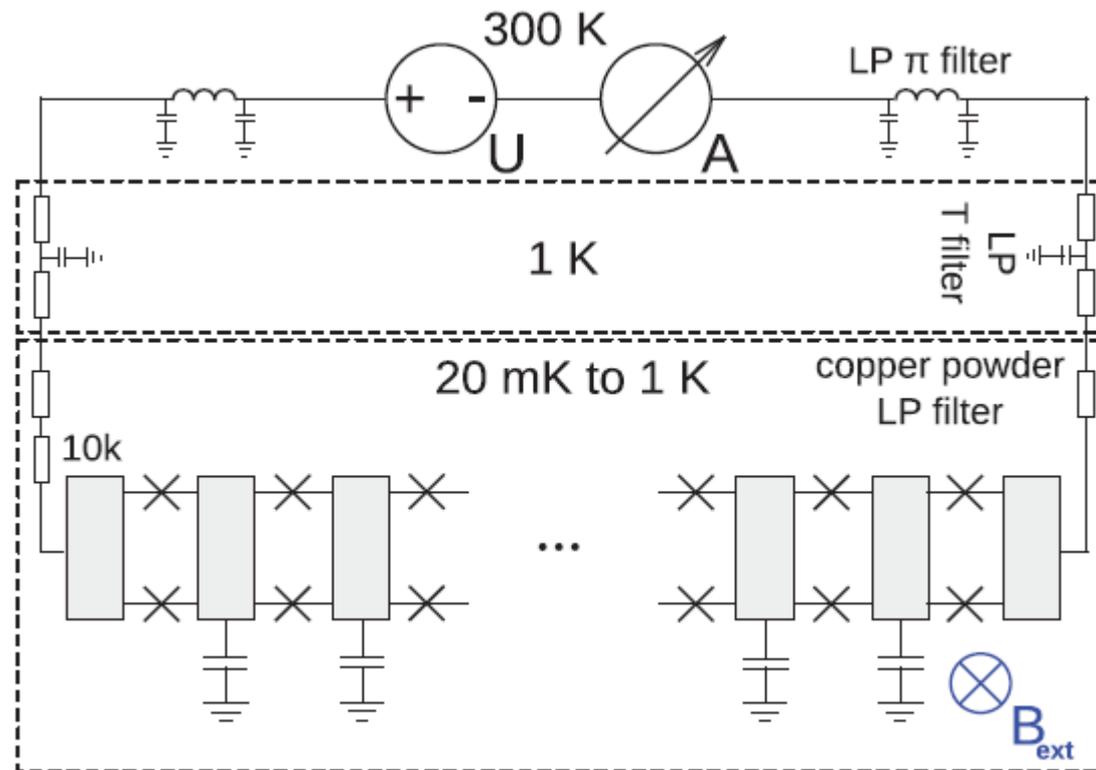


$$\Phi/\Phi_0 = 0$$

© R. Schäfer

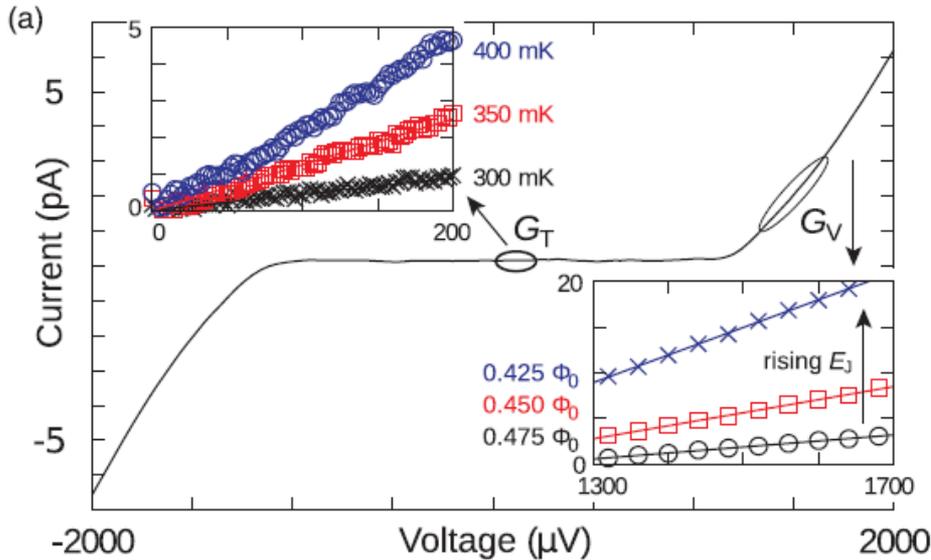
# Thermally activated conductance in arrays of small Josephson junctions

experimental setup



J. Zimmer, N. Vogt, A. Fiebig, S. V. Syzranov, A. Lukashenko, R. Schäfer, H. Rotzinger, A. Shnirman, M. Marthaler, and A. V. Ustinov, Phys. Rev. B **88**, 144506 (2013)

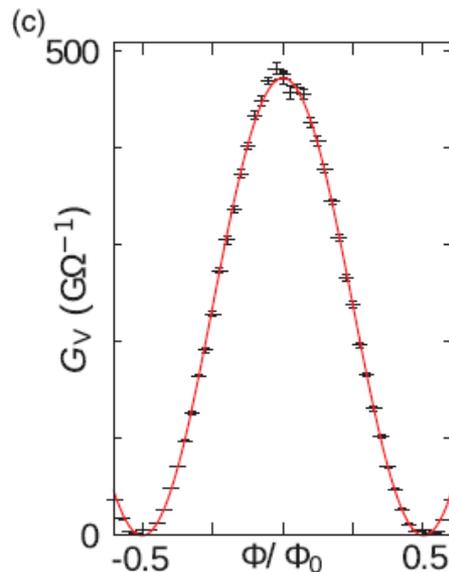
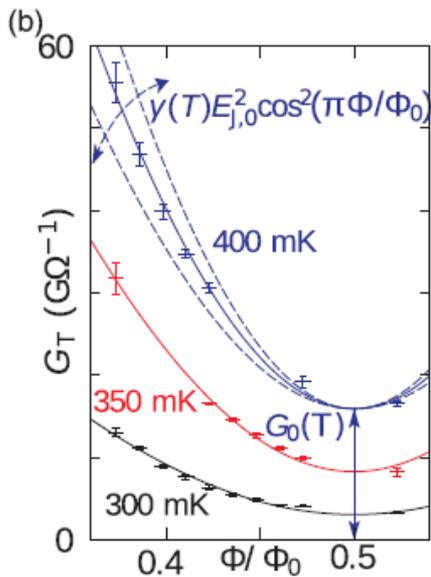
# I-V characteristic of the 255-JJ array



$G(T) \propto T^{-\alpha} \exp(-T_0/T)$   
 $\alpha$  depends on the nature of the bath

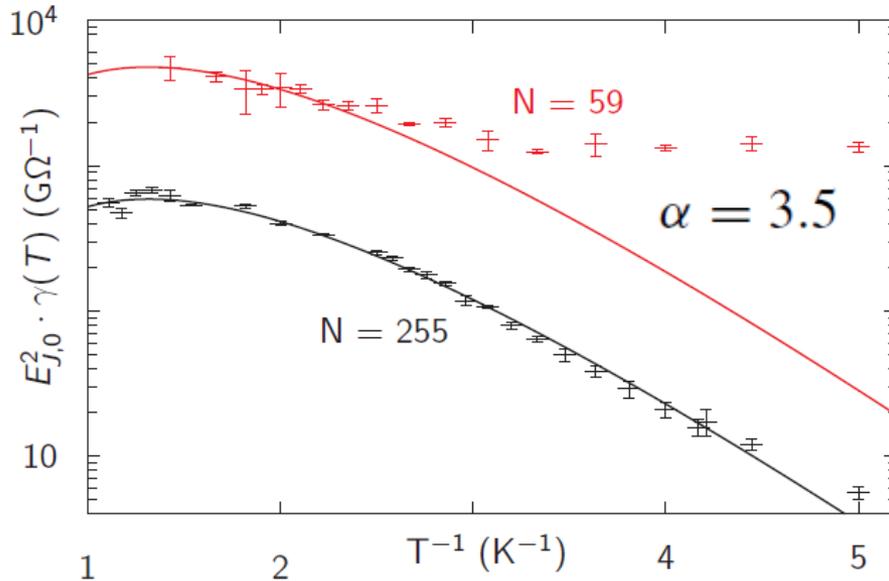
$$G_T(\Phi, T) = G_\Phi(T) + G_0(T)$$

$$G_\Phi(T) = (2E_J)^2 \cos^2\left(\frac{\pi\Phi}{\Phi_0}\right) \gamma(T)$$

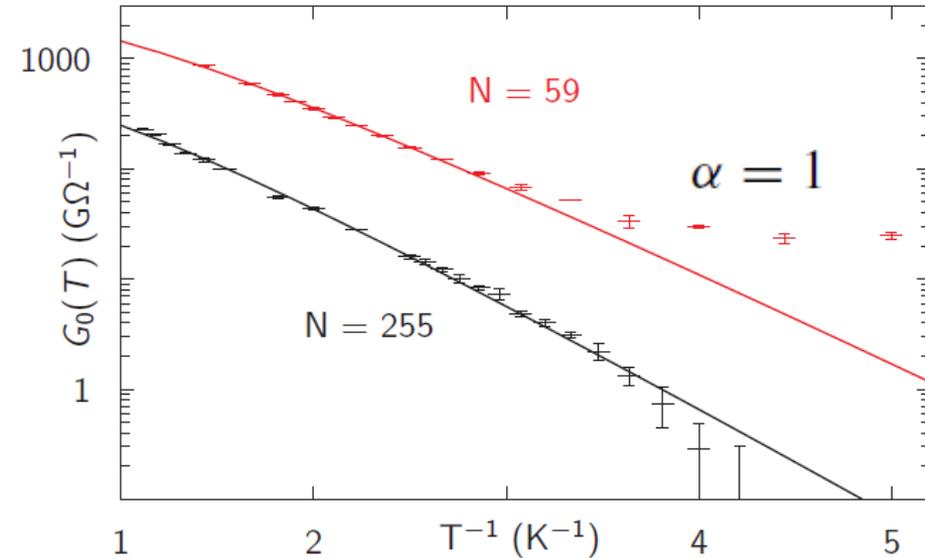


J. Zimmer, N. Vogt, A. Fiebig, S. V. Syzranov, A. Lukashenko, R. Schäfer, H. Rotzinger, A. Shnirman, M. Marthaler, and A. V. Ustinov, Phys. Rev. B **88**, 144506 (2013)

# Thermally activated conductance in arrays of small Josephson junctions



(a) Flux-dependent part  $\gamma(T)$  of  $G_T$



(b) Flux-independent part  $G_0(T)$  of  $G_T$

$$G(T) \propto T^{-\alpha} \exp(-T_0/T).$$

inelastic hopping of Cooper pairs  
between neighboring islands

J. Zimmer, N. Vogt, A. Fiebig, et al.,  
Phys. Rev. B **88**, 144506 (2013)

resulting from the fits

$$k_B T_0 = E_{\max} = 229 \mu\text{eV}$$

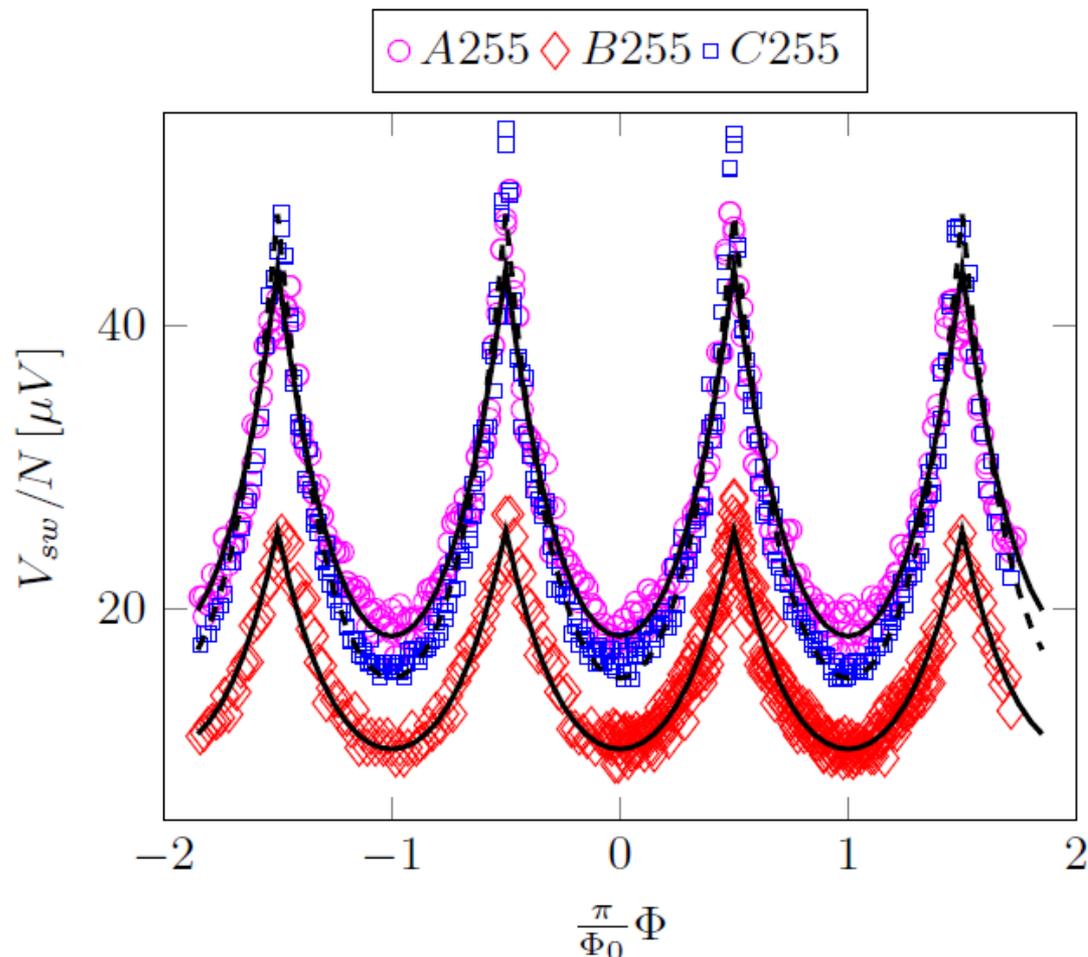
weak disorder estimate

$$\Delta E_C/2 = 220 \mu\text{eV}$$

**! gap of Al  $\Delta \approx 200 \mu\text{eV}$**

thermally generated quasiparticles?

# Charge depinning in arrays of small JJs



The switching voltage normalized to the array length  $N$  as a function of the magnetic flux for 3 arrays of length 255

switching voltage

$$V_{sw} \approx \frac{NE_C}{2e} \Lambda^{-\frac{2}{3}} \tilde{R}^{2/3}$$

Larkin length

$$L_L \approx \Lambda^{4/3} \tilde{R}^{-1/3}$$

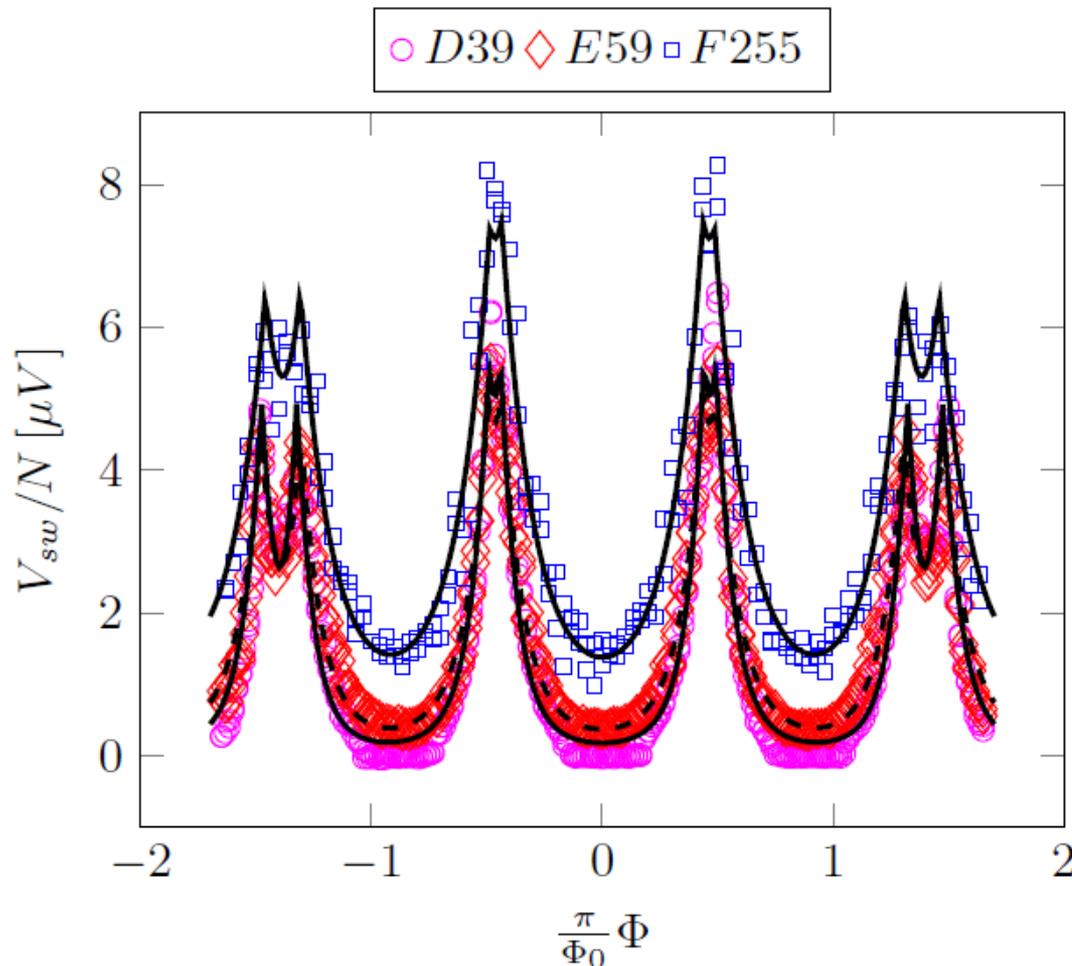
disorder correlation parameter

$$\tilde{R} [E_J(\Phi)/E_C]$$

N. Vogt, R. Schäfer, H. Rotzinger, W. Cui, A. Fiebig, A. Shnirman, and A. V. Ustinov, arXiv (2014)

**=> talk of A. Shnirman today**

# Charge depinning in arrays of small JJs



The switching voltage normalized to array length  $N$  as a function of the magnetic flux for arrays of length 39, 59 and 255

switching voltage

$$V_{sw} \approx \frac{NE_C}{2e} \Lambda^{-\frac{2}{3}} \tilde{R}^{2/3}$$

Larkin length

$$L_L \approx \Lambda^{4/3} \tilde{R}^{-1/3}$$

disorder correlation parameter

$$\tilde{R} [E_J(\Phi)/E_C]$$

N. Vogt, R. Schäfer, H. Rotzinger, W. Cui, A. Fiebig, A. Shnirman, and A. V. Ustinov, arXiv (2014)

**=> talk of A. Shnirman today**

# Summary

- Brief review of experiments with JJ arrays near S/I transition
  - 2D arrays
  - 1D/2D channels – shooting and localizing quantum vortices
  - 1D arrays – searching for ballistic Cooper pairs
- Our recent experiments with 1D arrays
  - tuning  $E_J/E_C$
  - conductance  $\sim E_J^2$ , incoherent Cooper pair tunneling
  - thermal activation of charges
  - depinning of charges => **talk of A. Shnirman today**