Role of energy diffusion in ultrasonic attenuation

A. Shtyk [1,2] and M. Feigel'man [1,3]

[1] Landau Institute for Theoretical Physics
[2] Harvard University
[3] MIPT

Acknowledgements: Vladimir Kravtsov

Electron-phonon interaction in disordered conductors

- Pippard ineffectiveness condition (PIC): in disordered metal long-wavelength ultrasonic attenuation is weaker by a $ql \ll 1$ factor

(A. Pippard, 1955)

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- PIC breakdown:
  - Energy diffusion (this talk)
    - in a disordered semiconductor
    - in a superconductor
  - Multiband electron system

    (M.Prunnila et al, 2005; M.Prunnila, 2007)
  - Piezoelectric coupling

    (… ; D.Khveshchenko, M. Reizer, 1997)
  - Static impurities

    (A. Sergeev, V. Mitin, 2000)
  - Weak Coulomb screening

    (A. Sergeev, M. Reizer, V. Mitin, 2005; AS, M. Feigelman, V. Kravtsov, 2013)

Hamiltonian of electron-phonon interaction

(Laboratory frame or reference)

$$H_{e-ph} = \sum_r \overline{\psi}\Big(\underbrace{-(p_F v_F/d)\partial_\alpha u_\alpha}_{} - u_\alpha \partial_\alpha U\Big)\psi$$

Comes from distortions of ionic charge density

$$H_{e-ph-ion} = \sum_{r,r'} -V_0(r-r') n_{ion} \overline{\psi}\psi = \sum_{r,r'} -\underbrace{V(r-r')}_{\text{screening}}\,\underbrace{n_{el}}_{\text{electroneutrality}}\,\overline{\psi}\psi$$

$$V n_{el} = \frac{1}{\nu}\cdot n = \left(\frac{dn}{dE_F}\right)^{-1} n = \left(\frac{d\ln n}{dE_F}\right)^{-1} = \left(\frac{d\ln p_F^d}{dE_F}\right)^{-1} = \frac{p_F v_F}{d}$$

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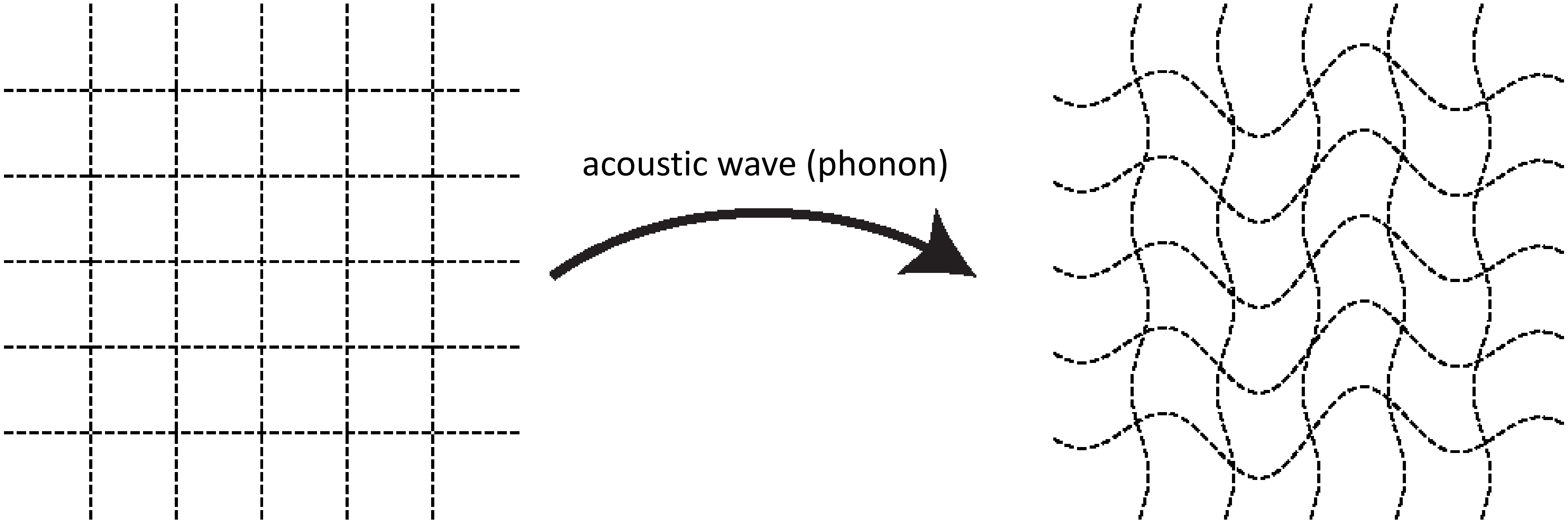
Phonon-induced distortions of disorder potential

$$\sum_i U(r-R_i) \to \sum_i U(r-R_i-u(R_i))$$

Comoving frame of reference



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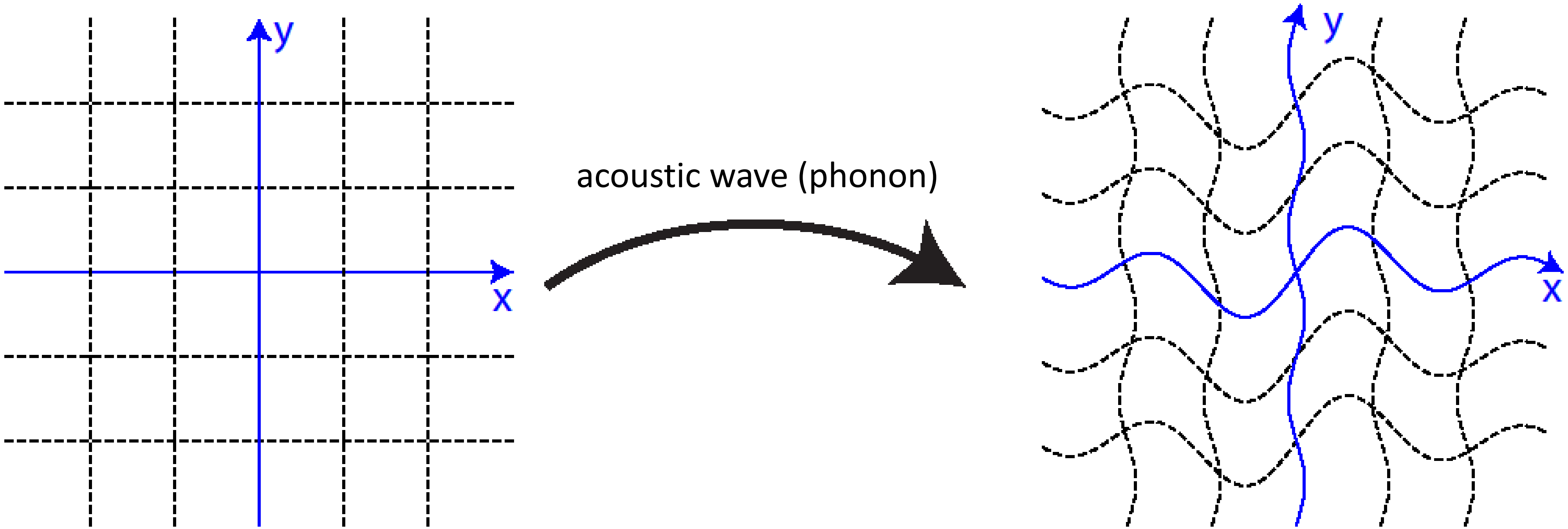
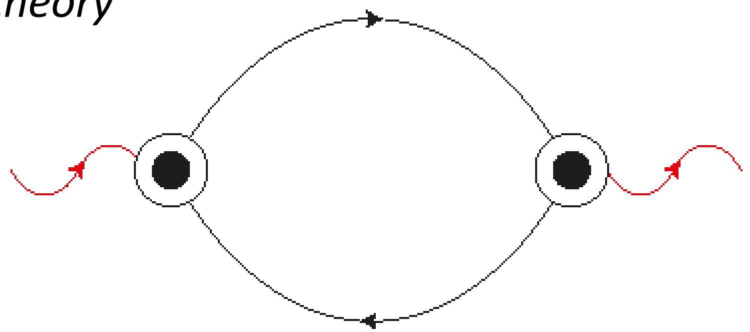


Our new comoving reference frame

$$H^{CFR}_{e-ph} = \sum_r \overline{\psi}\Big[\Big(p_\alpha v_\beta - (p_F v_F/d)\delta_{\alpha\beta}\Big)\partial_\beta u_\alpha\Big]\psi$$ (no disorder in the vertex)

Pippard ineffectiveness condition

standard theory



Weak disorder $ql \gg 1 \quad (\lambda \ll l)$

$$\alpha(\omega) = \frac{1}{\rho_m \omega} \mathrm{Im}\Sigma^R(\omega, q)\Big|_{\omega = sq} \sim \frac{\nu p_F^2}{\rho_m} v_F q$$

Strong disorder $ql \ll 1 \quad (\lambda \gg l)$

$$\alpha(\omega) = \frac{1}{\rho_m \omega} \mathrm{Im}\Sigma^R(\omega, q)\Big|_{\omega = sq} = \frac{8}{15} \frac{\nu p_F^2}{\rho_m} D q^2$$

$$\Gamma_{e-ph} = p_\alpha v_\beta - (p_F v_F / d)\delta_{\alpha\beta} : \quad \left\langle \Gamma_{e-ph} \right\rangle_{\mathbf{p}} \to 0$$ on the FS

The average of the electron-phonon vertex over the Fermi surface is zero. The insertion of the impurity ladder in the diagram kills the vertex by averaging over directions. Diffusion is thus prohibited.

Pippard ineffectiveness condition

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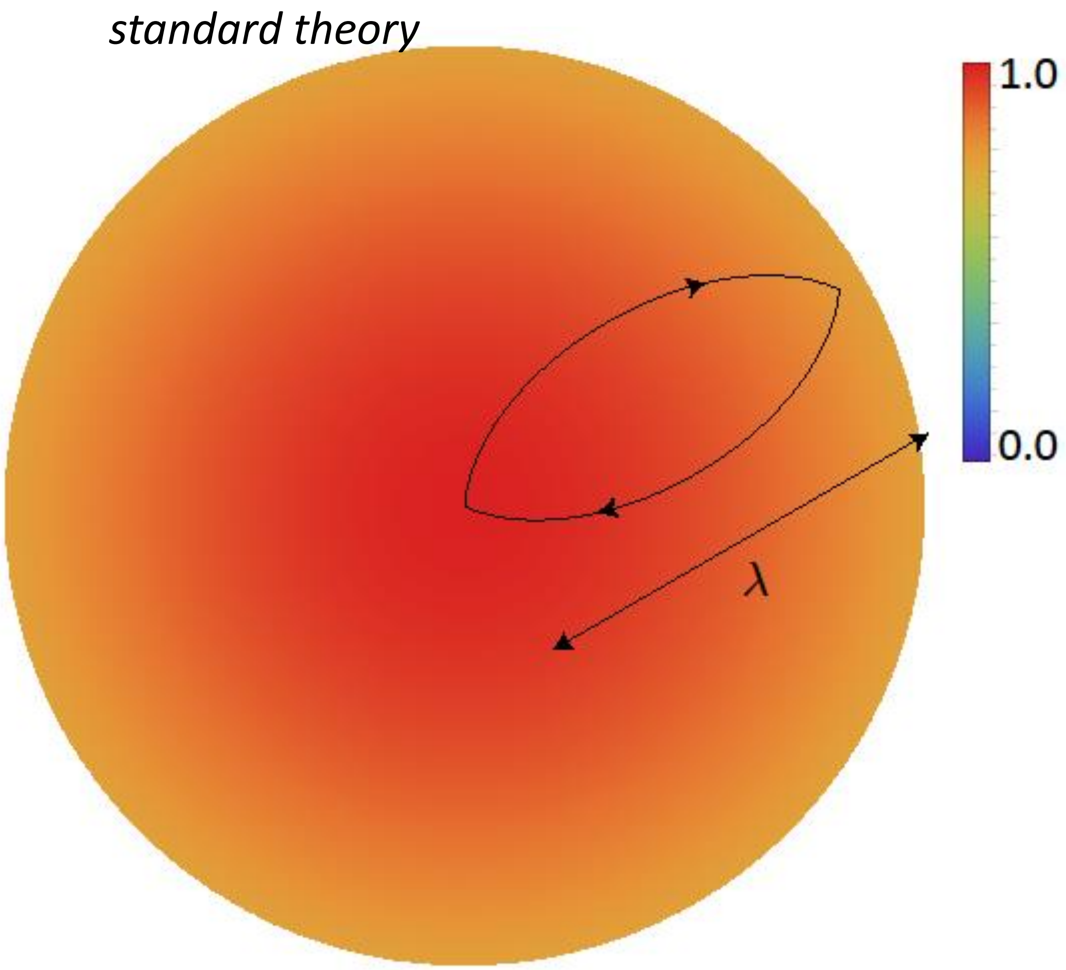
$$G(r) \propto \exp\Big[-r/2l\Big]$$



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$$\int (dr) e^{-r/l} e^{-i\mathbf{q}\mathbf{r}} = 4\pi \begin{cases} \frac{\arctan ql}{q}, & 2d \\ \frac{l}{\sqrt{1+q^2l^2}}, & 3d \end{cases} = \frac{4\pi}{q} \begin{cases} 1, & ql \gg 1 \\ ql, & ql \ll 1 \end{cases}$$

PIC

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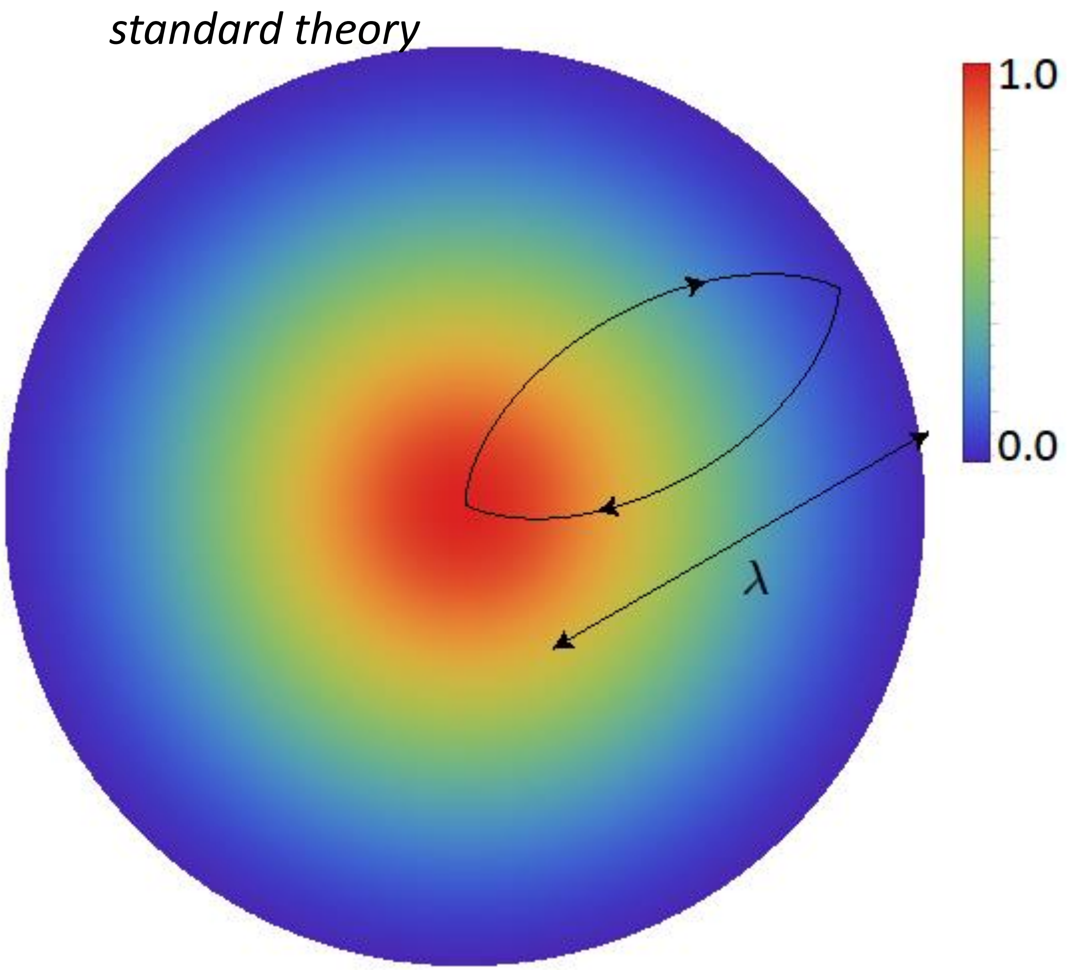

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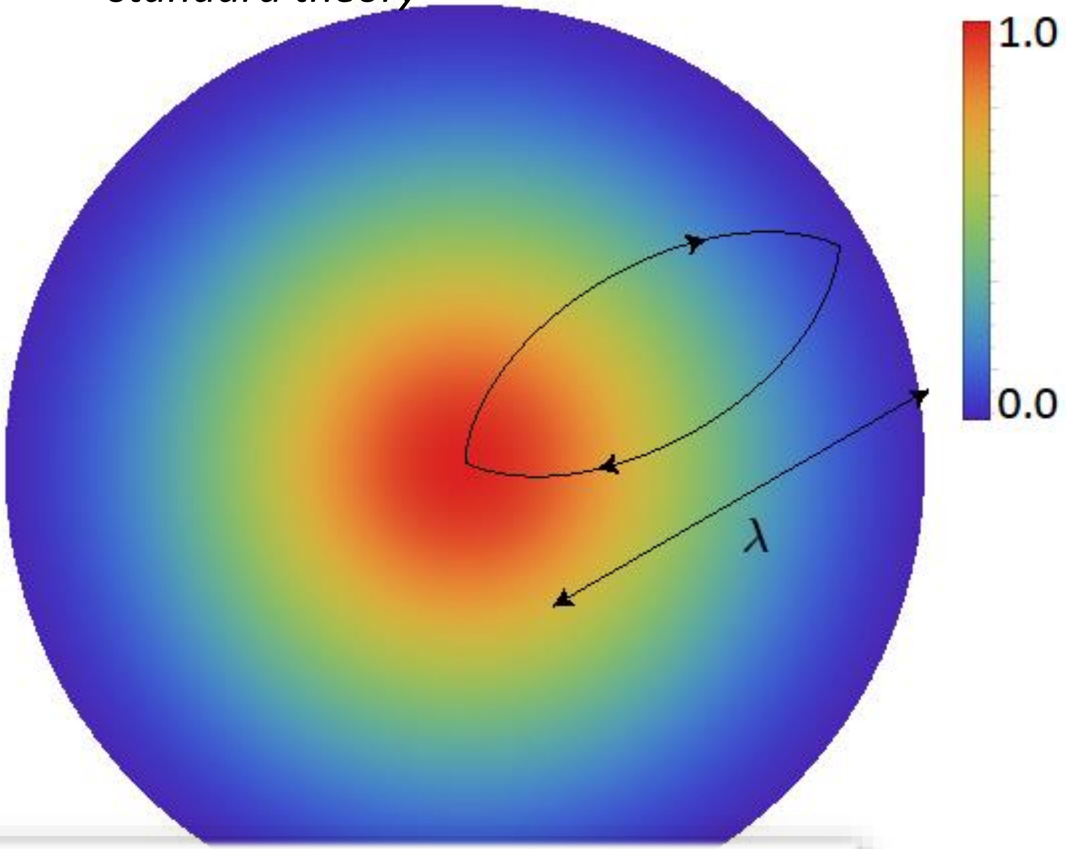

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OK, so we want to activate diffusion channel

r in

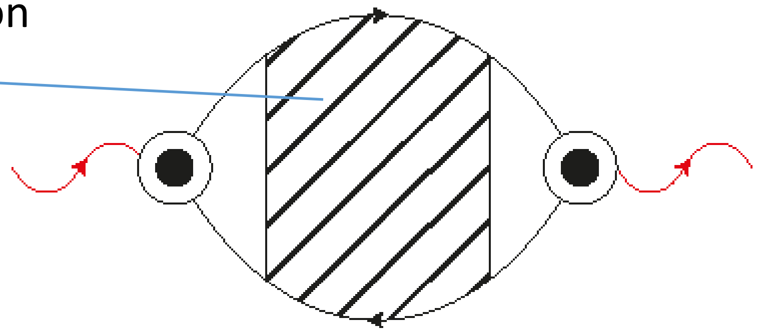
Energy diffusion channel (normal state)

Strong Coulomb interaction (perfect screening). No spin(band) asymmetry.

$$\left\langle \Gamma_{e-ph} \right\rangle_{\mathbf{p}} = \left(pv/d - p_F v_F/d \right) \delta_{\alpha\beta} \sim \varepsilon \sim T$$

$$\frac{\alpha_{E-diff}}{\alpha_{PIC}} \sim \left(\frac{T}{p_F v_F} \right)^2 \times \mathrm{Re} \left[\frac{1}{-i\omega + Dq^2} \right]$$

Enhancement by diffusion



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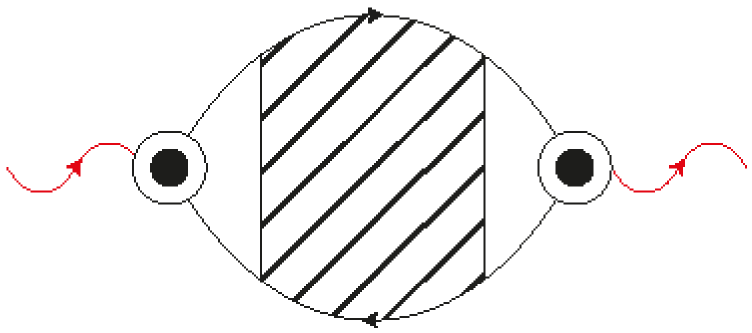
$$\Gamma = \frac{d(pv/d)}{dE} \varepsilon = \left(1 - \frac{p_F v_F}{d\nu} \frac{d\nu}{dE} \right) \varepsilon$$



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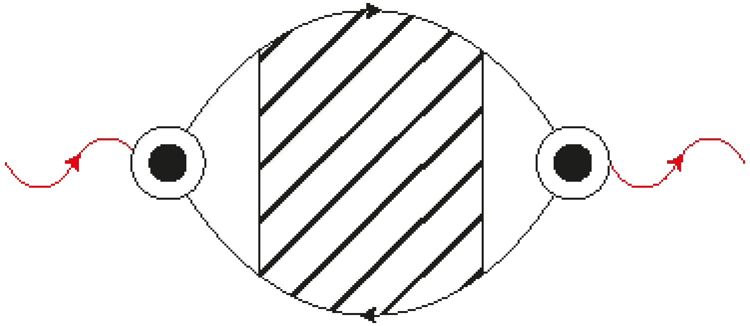
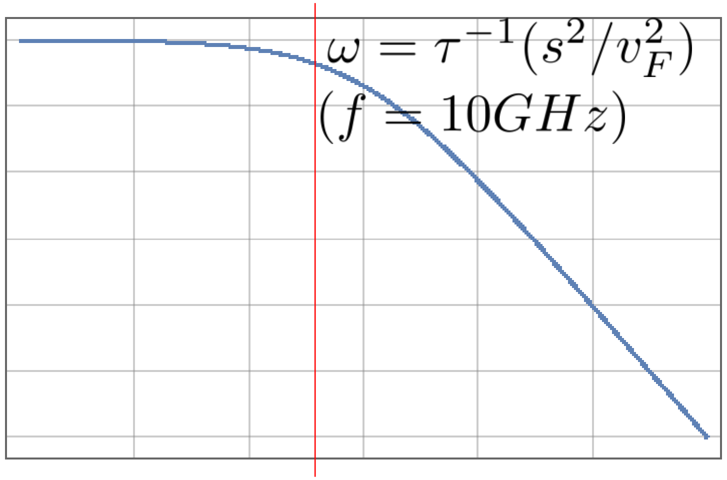
$$\frac{\alpha_{E-diff}}{\alpha_{PIC}} = \frac{5\pi^2}{4} \left(\frac{T}{p_F v_F} \right)^2 \left(1 - \frac{p_F v_F}{d\nu} \frac{d\nu}{dE} \right)^2 \frac{v_F^2}{s^2 + (Dq)^2}$$



For heavily doped Si with $n = 10^{20} cm^{-3}$, $m = 0.36 m_0$, $p_F l = 10$, $s \approx 8 \cdot 10^5 cm/s$ and $T = 0.1 E_F \approx 200K$

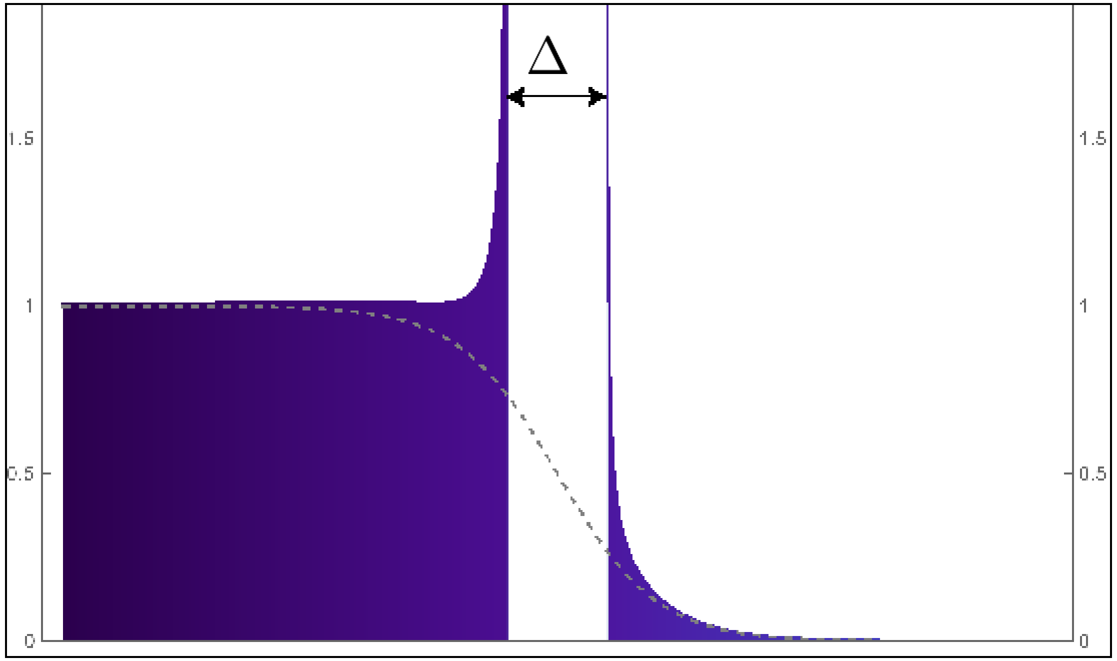
$$\frac{\alpha_{e-diff}}{\alpha_{PIC}} \approx 100, \quad f \leq 10GHz$$



Energy diffusion channel (SC state)

Why to care about SC state? Is there anything different from a mere reduction in normal density as for the local processes?

$$\frac{\alpha_{SC,PIC}}{\alpha_{PIC}} \approx 2\exp\left[-\frac{\Delta}{T}\right]$$ (… ; Tinkham)



There is a gap that forces physics to higher energies!

$$\left\langle \Gamma \right\rangle_{\mathbf{p}} \sim \xi = \sqrt{\varepsilon^2 - \Delta^2} \sim \sqrt{\Delta T} > T$$

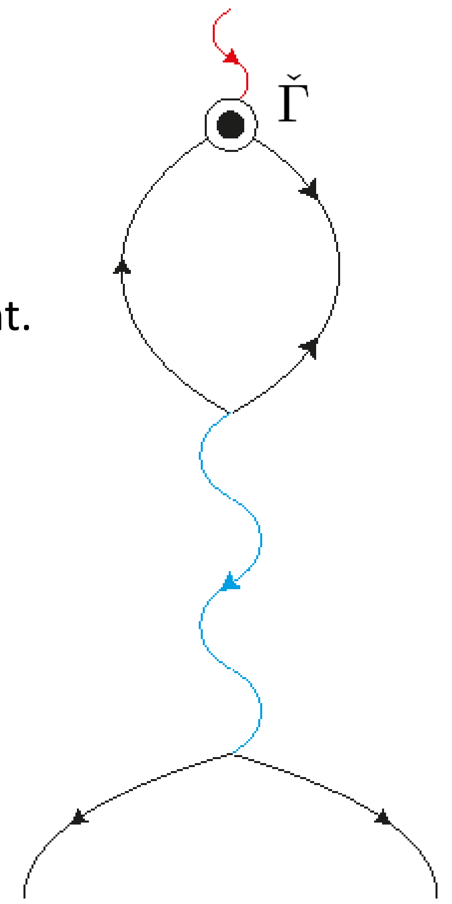
$$\left(\frac{T}{p_F v_F}\right)^2 \to \left(\frac{\sqrt{\Delta T}}{p_F v_F}\right)^2$$

Energy diffusion channel (SC state)

In fact, an additional vertex of the $\check{\Delta}$ structure is generated through dressing by the order parameter fluctuations

$$\check{\Lambda} \propto \check{\Delta} \propto (i\check{\tau}_2)$$

$$\Lambda = \frac{L(\omega,q)}{\lambda}\left(\frac{p_F v_F}{d\nu}\frac{d\nu}{d\varepsilon}+\quad\right)\Delta$$



The vertex arises through the induced changes in the dimensionless $\lambda = \nu g$ coupling constant. It describes induced changes in the order parameter Δ :

$$\Lambda(0,0) \simeq \frac{d\Delta}{d\ln\rho}$$

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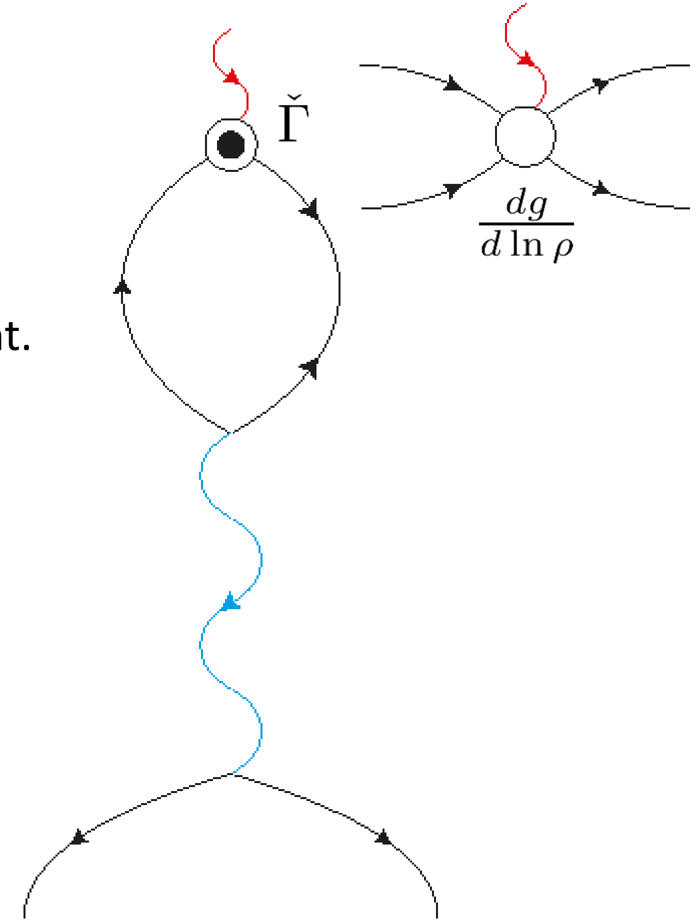

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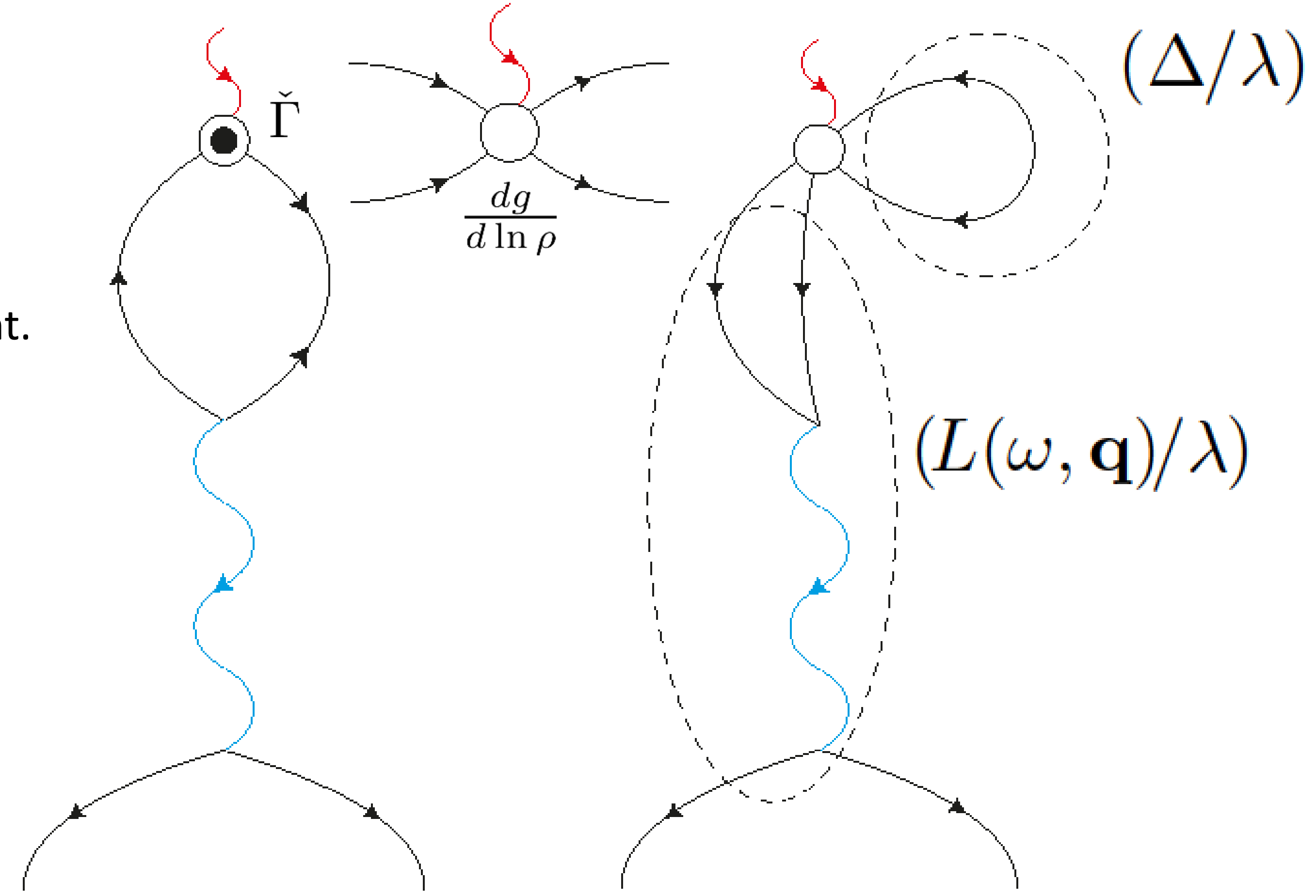
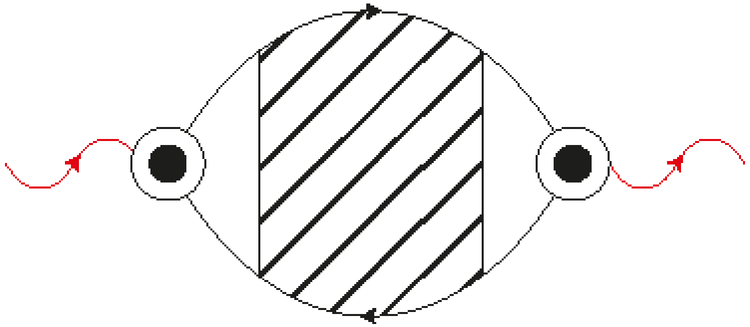


Energy diffusion channel (SC state)

The total vertex in SC state is $\check{\Gamma} + \check{\Lambda}$

$$\frac{\alpha_{SC,E-diff}}{\alpha_{SC,PIC}} \sim \left\langle \left(\frac{\Gamma + \Lambda}{p_F v_F} \right)^2 \times (\text{diffusive enhancement}) \right\rangle_\varepsilon$$



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$$\frac{\alpha_{SC,E-diff}}{\alpha_{SC,PIC}} = \frac{15}{8}\frac{1}{p_F^2 v_F^2}\begin{cases} 3\left(\Lambda^2 \frac{\Delta}{T}\ln\frac{T}{A(\omega)} - 4\Delta\Lambda + 4\Delta T\right) \times \frac{1}{\tau D q^2} & \omega\sqrt{\Delta/T} \ll Dq^2 \\ 2\left(\Lambda^2 - 2\Lambda T + 8T^2\right) \times \frac{v_F^2}{s^2} & Dq^2 \ll \omega\sqrt{\frac{\Delta}{T}} \end{cases}$$

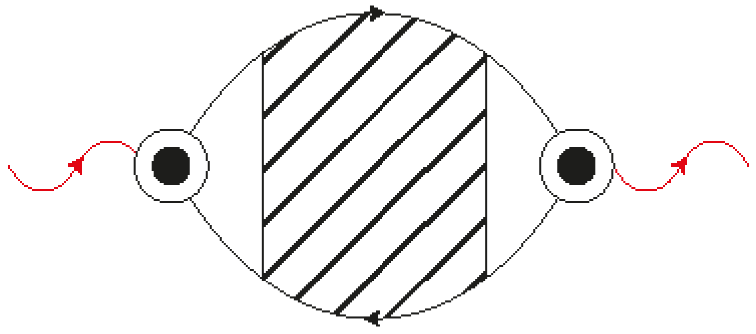
$$A(\omega) = \omega + \Delta\left(\frac{\omega}{Dq^2}\right)^2$$

More complicated behavior as compared to the normal case because

$$E_+ - E_- = \sqrt{\varepsilon_+^2 - \Delta^2} - \sqrt{\varepsilon_-^2 - \Delta^2} = \frac{\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}}\omega$$

As opposed to

$$\varepsilon_+ - \varepsilon_- = \omega$$



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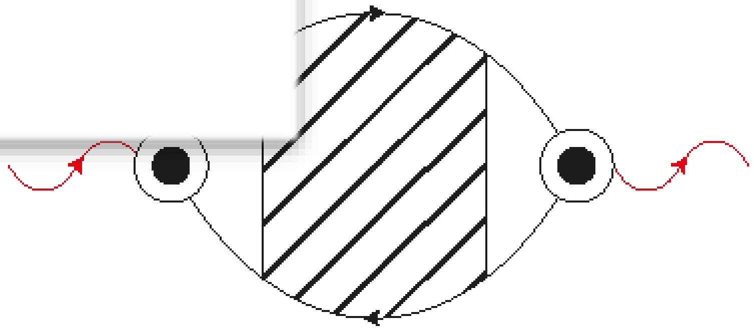
For YPtBi $n = 2 \cdot 10^{18} cm^{-3}$, $m = 0.15 m_0$, electron mean free path $l = 130nm$, $T = 0.2K$, $T_c = 0.77K$, $\omega_D = 200K$ and sound velocity $s = 2 \cdot 10^5 cm/s$

$$\frac{\alpha_{SC,E-diff}}{\alpha_{SC,PIC}} \simeq 0.8, \quad f \leq 1GHz$$

As opposed to

$\varepsilon_+ - \varepsilon_- = \omega$



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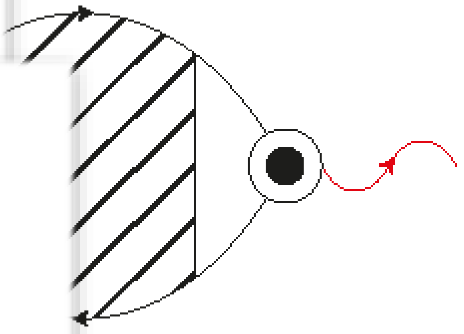
In a superconductor this mechanism might be relevant for electron cooling (thermal phonons) as well!

$$p_F l = 50: \quad f = 1GHz = 50mK$$

$$p_F l = 5: \quad f = 10GHz = 0.5K$$



Conclusions

- Low-frequency phonons experience additional damping due to the coupling between lattice density modulations and non-equilibrium distribution of thermally excited quasiparticles.
- The conditions most convenient for observation of this effect are
  - not too low ratio T/E_F and ultrasound frequencies $\omega \ll T/\hbar$
  - in superconductors with relatively small $k_F\xi$ product, i.e. not far from the crossover from BCS to BEC state.