Charge density depinning in one-dimensional Josephson arrays in the insulating regime

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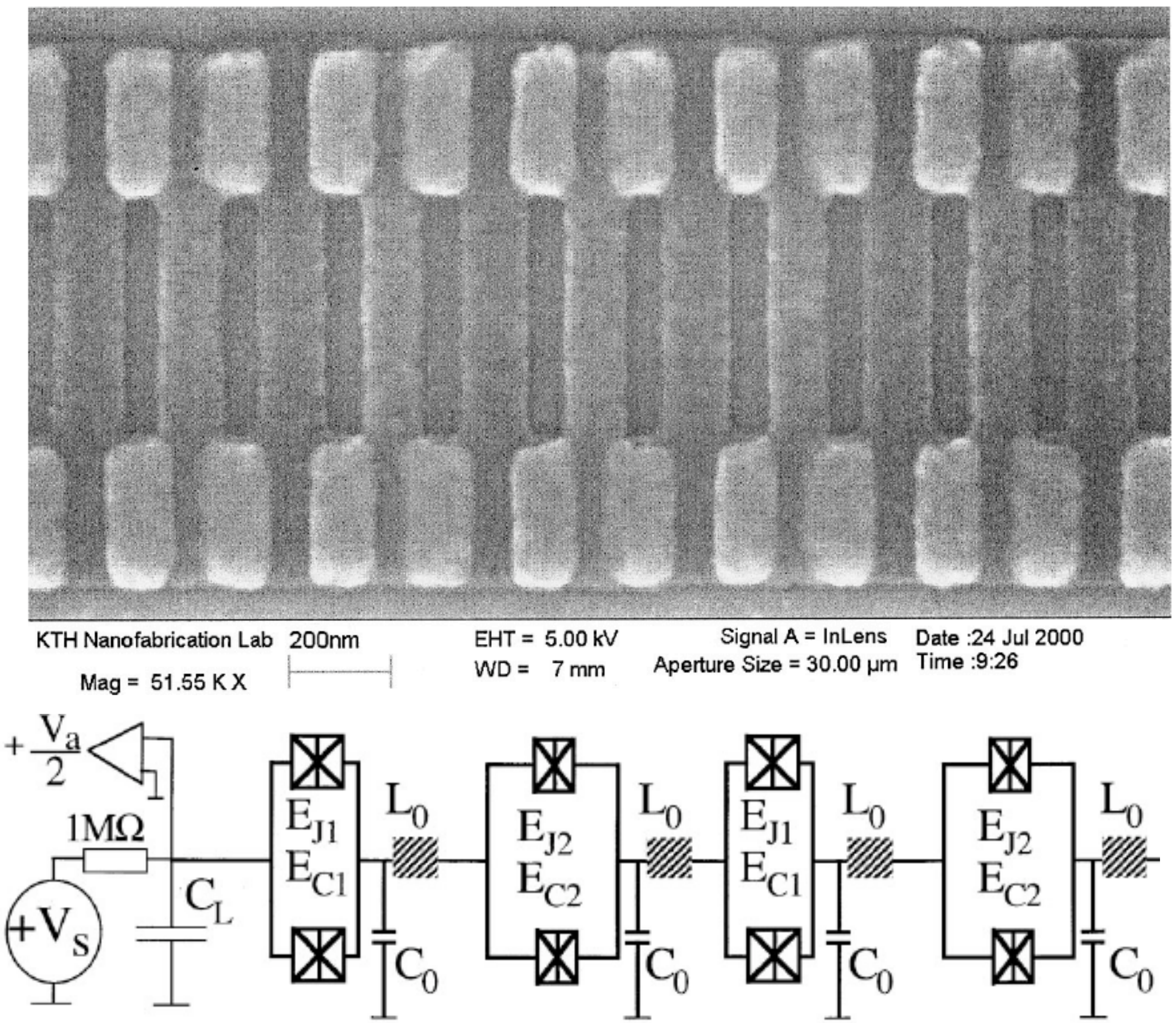
N. Vogt, R. Schäfer, H. Rotzinger,
W. Cui, A. Fiebig, A.S. and A. V. Ustinov,
cond-mat/**1407.3353**

Experimental motivation

Haviland, Delsing, PRB 1996 **Ågren, Andersson, Haviland, JLTP 2000,2001**

R. Schäfer et al., arXiv:1310.4295



30-5000 junctions in array

$$E_J(\Phi) = 2E_{J,0} \cos \frac{\pi\Phi}{\Phi_0}$$

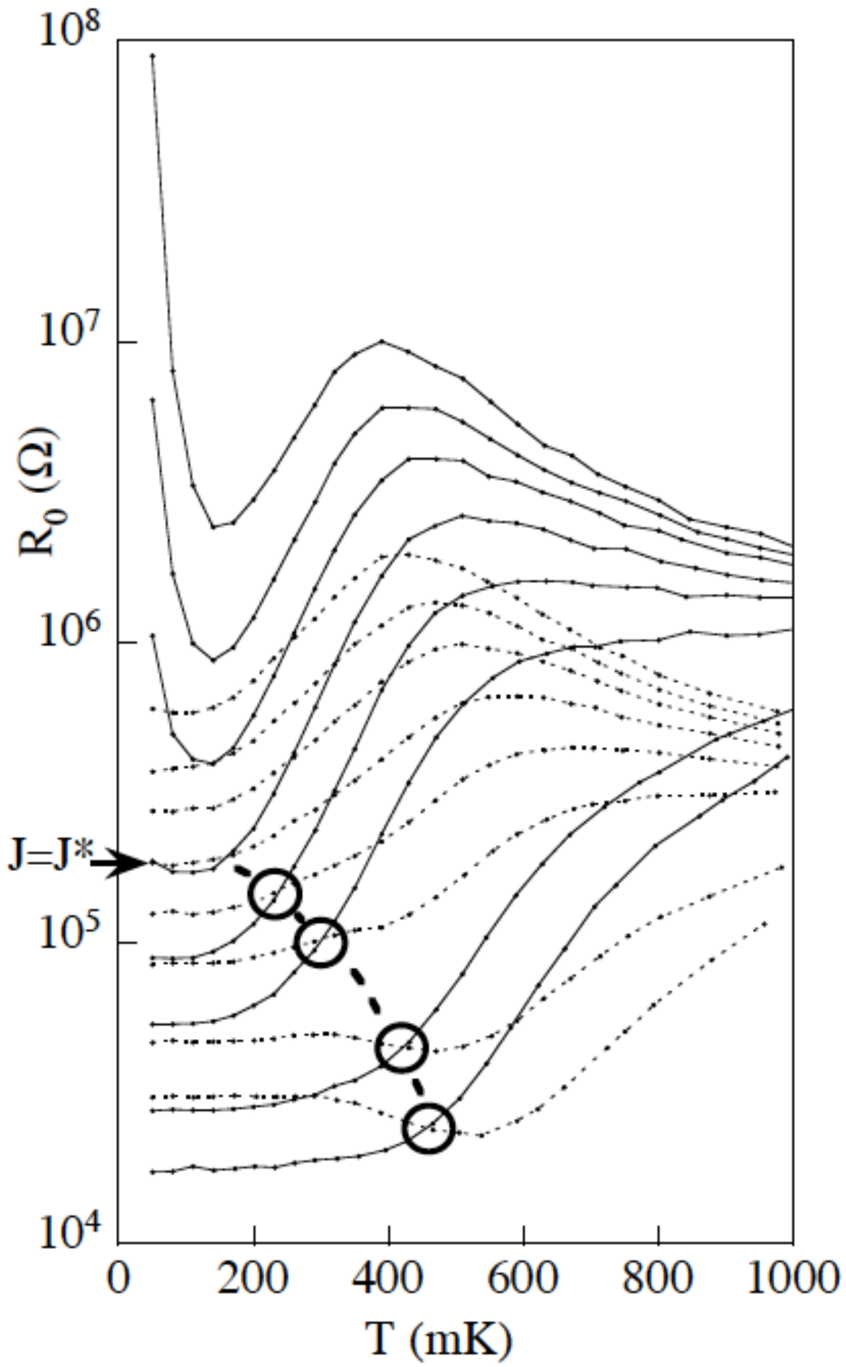
$$E_{J,0} \sim E_C \equiv \frac{(2e)^2}{2C}$$

$$E_{C_0} \equiv \frac{(2e)^2}{2C_0} \gg E_C$$

Poor screening

Experimental motivation

- **(Super)conductor - insulator (Coulomb blockade) transition**
- **Linear response**

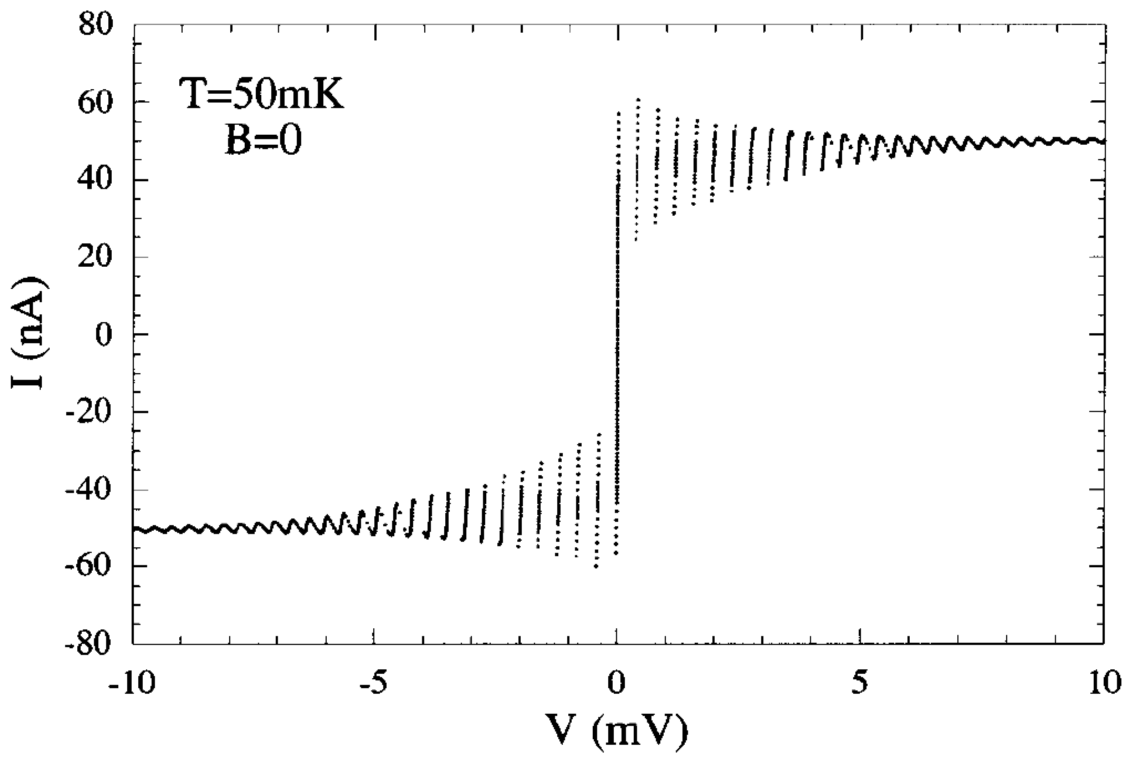


Chow, Delsing , Haviland,, PRL 1997

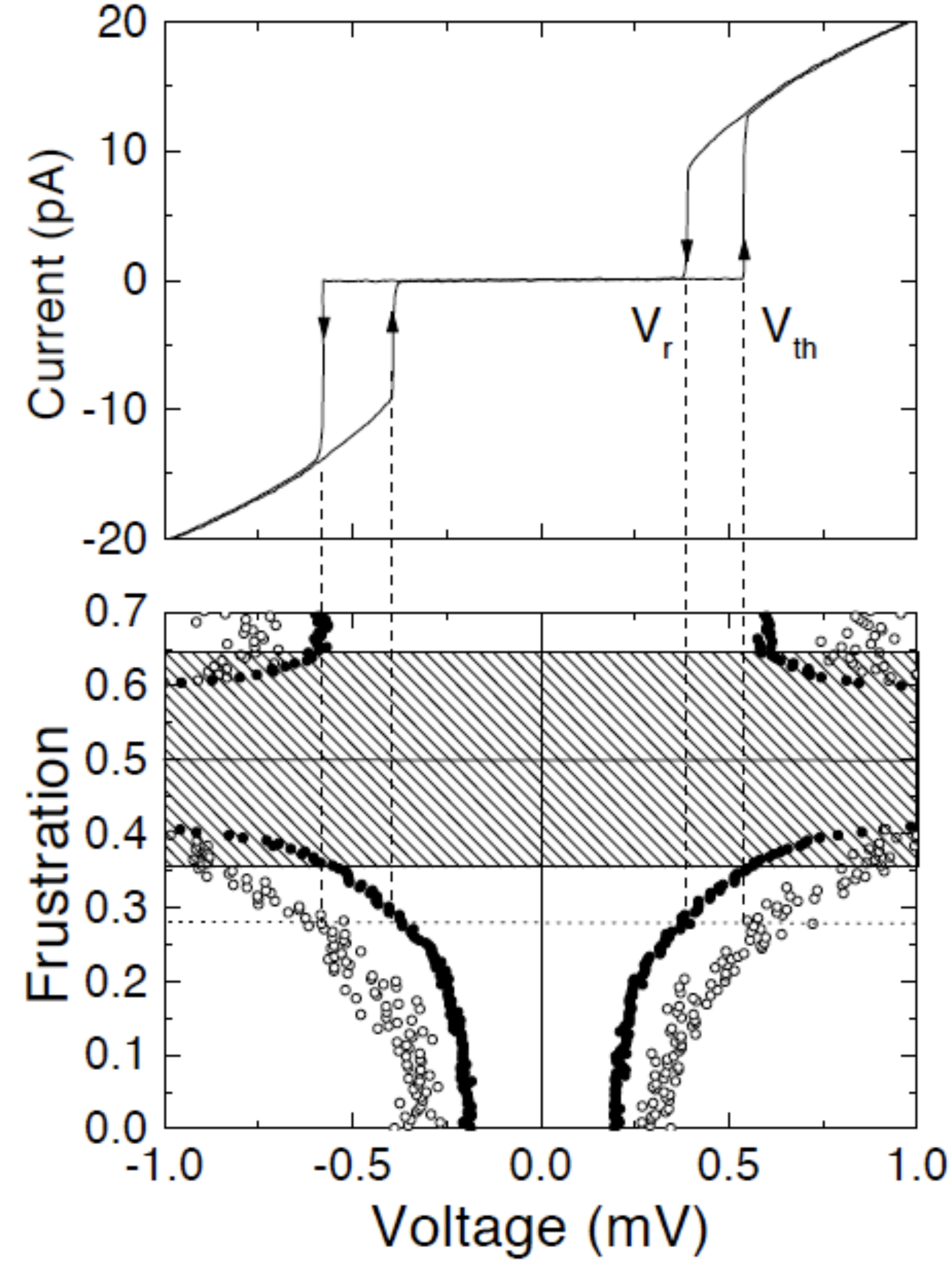
Experimental motivation

- **(Super)conductor - insulator transition: non-linear response**

$$E_J(\Phi) > E_C$$



$$E_J(\Phi) \lesssim E_C$$



- **Flux dependent Coulomb blockade**

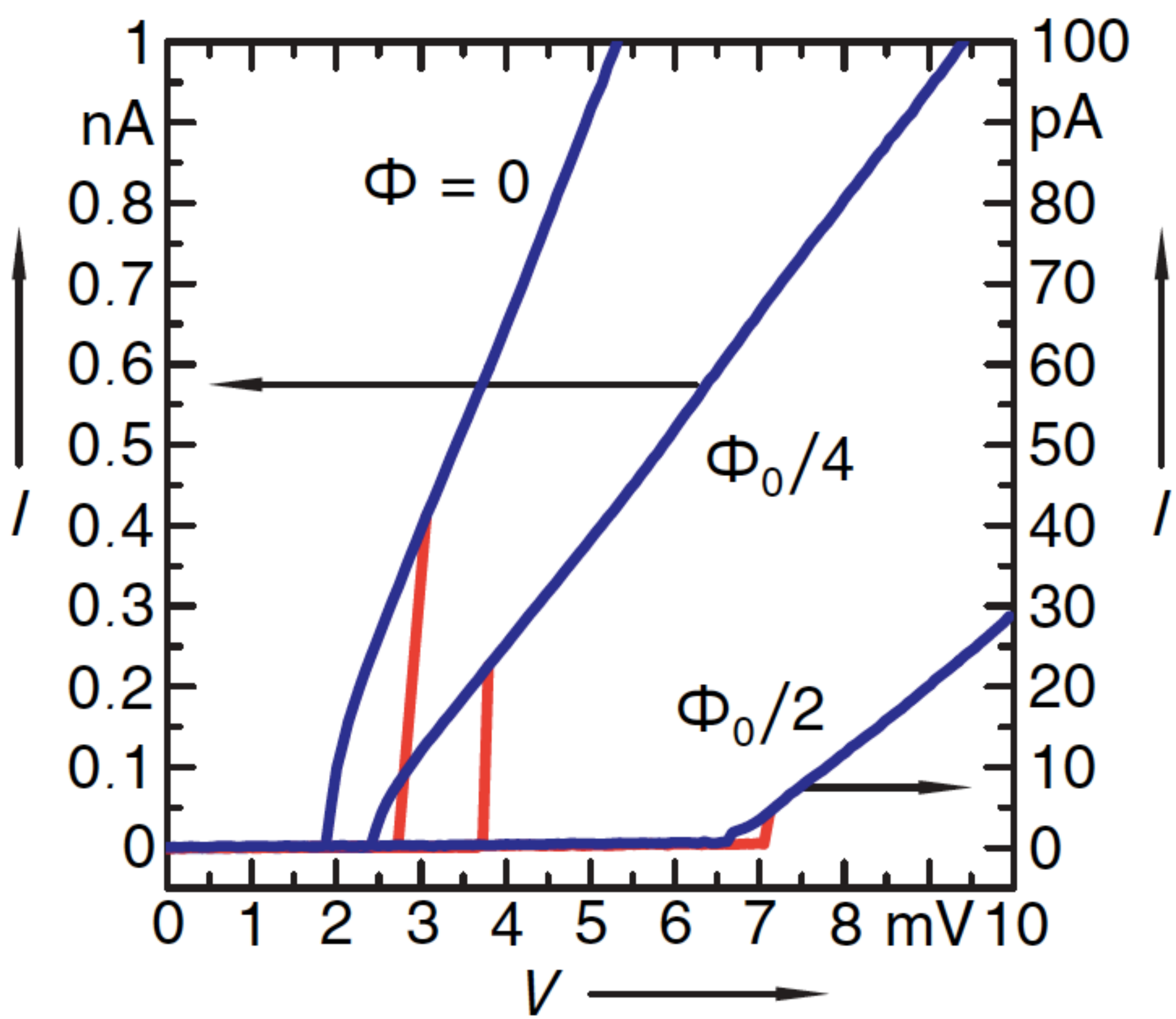
- **Hysteresis:** Unresolved.

Ågren, Andersson, Haviland, JLTP 2000,2001

Experimental motivation

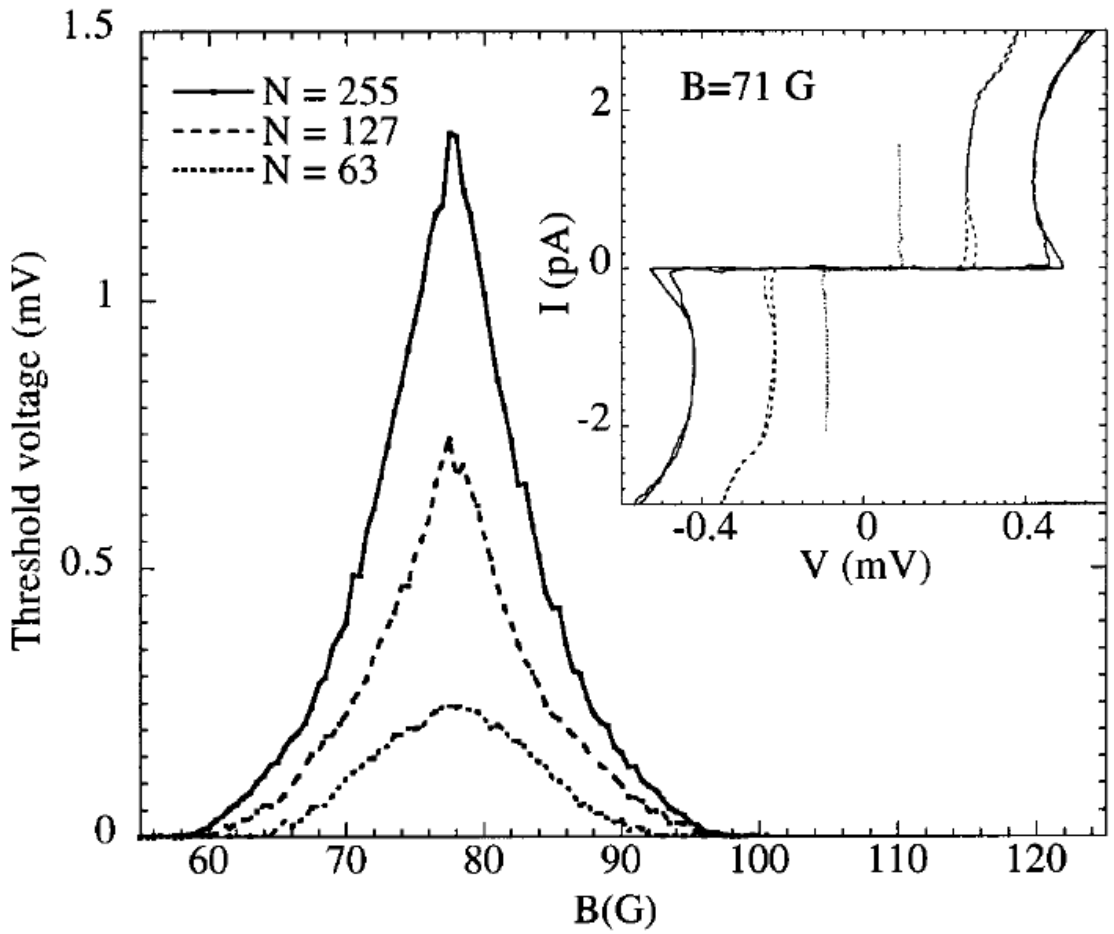
R. Schäfer et al., arXiv:1310.4295

Flux-dependent switching voltage



Experimental motivation

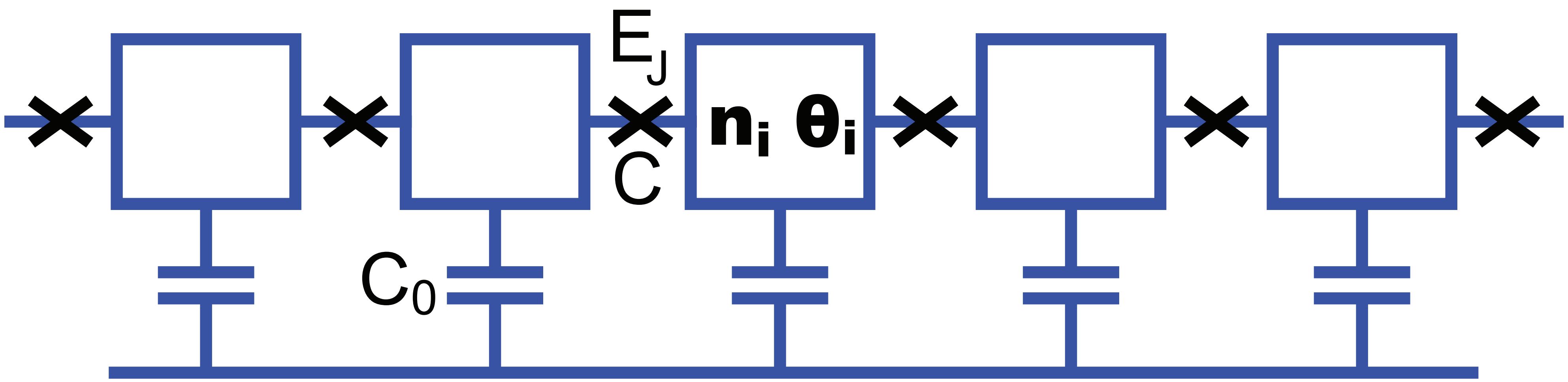
Ågren, Andersson, Haviland, JLTP 2000,2001



Threshold voltage scales linearly with array length
No effect of overall gate voltage

Strong disorder

The model (no disorder)



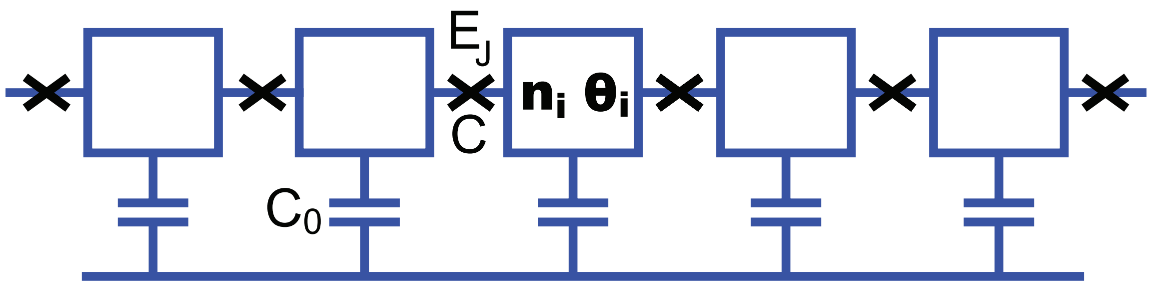
$$H = \frac{1}{2} \sum_{i,j} U(i-j)\, n_i n_j - \sum_i E_\mathrm{J} \cos(\theta_{i+1} - \theta_i)$$

$$[n_j, e^{i\theta_{j'}}] = e^{i\theta_j}\, \delta_{j,j'} \qquad U(i-j) = (2e)^2 (\hat{C}^{-1})_{i,j}$$

$$(\hat{C})_{i,j} = C_0 \delta_{i,j} + C\, [2\delta_{i,j} - \delta_{i,j+1} - \delta_{i,j-1}]$$

Review: R. Fazio and H. van der Zant, Phys. Rep. 355, 235 (2001)

Charging energy



Charging energy $E_C \equiv \dfrac{(2e)^2}{2C}$

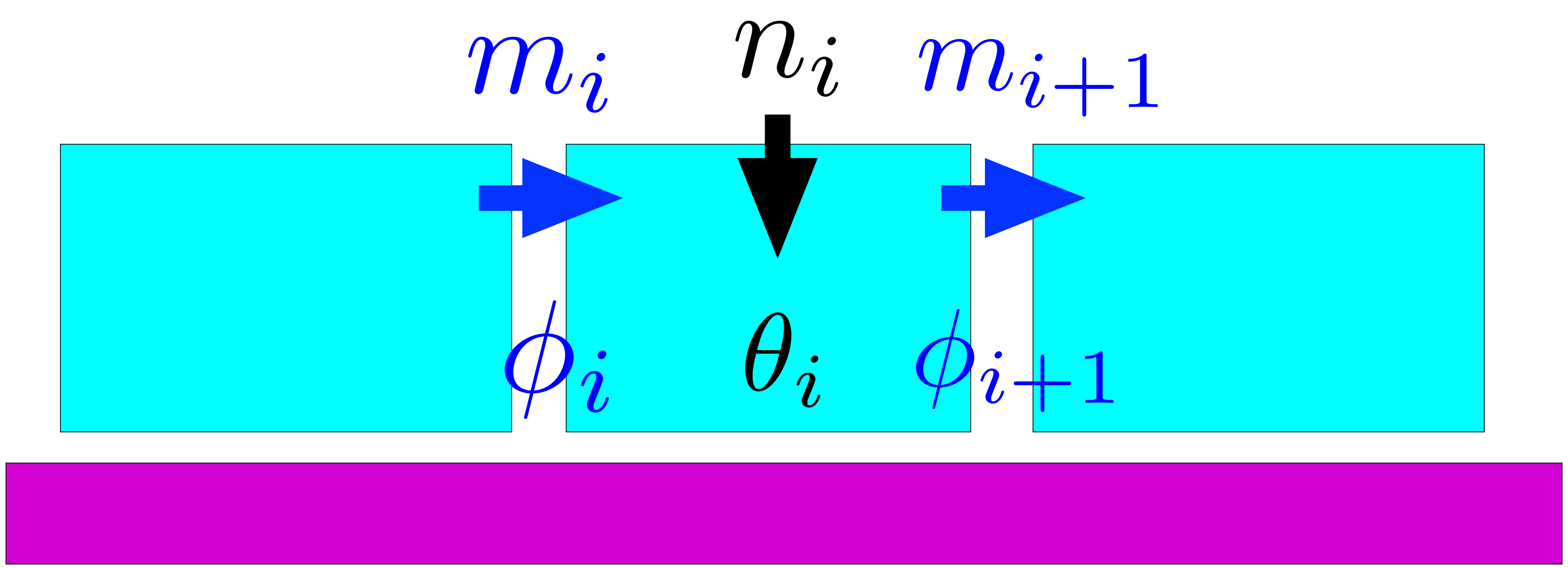
Screening length $\Lambda \equiv \sqrt{\dfrac{C}{C_0}}$

$$E_{C_0} \equiv \frac{(2e)^2}{2C_0} \gg E_C$$

$$H_C = \frac{1}{2} \sum_{i,j} U(i-j)\, n_i n_j$$

$$\Lambda \equiv \sqrt{\frac{C}{C_0}} \gg 1 \quad \Rightarrow \quad U(i-j) \approx \Lambda E_C e^{-\frac{|i-j|}{\Lambda}}$$

BKT quantum phase transition



$$n_i = m_i - m_{i+1} \qquad \phi_i = \theta_i - \theta_{i-1}$$

$$H = \frac{1}{2}\sum_{i,j} V(i-j)\, m_i m_j - \sum_i E_{\rm J} \cos(\phi_i)$$

$$V(k) \sim E_C \Lambda^2 k^2 = E_{C_0} k^2 \qquad \text{for } \Lambda k \ll 1$$

$$V(k) \sim E_C \qquad \text{for } \Lambda k \gg 1$$

BKT quantum phase transition

$$H = \frac{1}{2}\sum_{i,j} V(i-j)\, m_i m_j - \sum_i E_\mathrm{J} \cos(\phi_i)$$

$V(k) \sim E_C \Lambda^2 k^2 = E_{C_0} k^2$ for $\Lambda k \ll 1$

$\cos(\phi_i) \sim 1 - \frac{\phi_i^2}{2}$ for $E_J > E_C$

$$H = \frac{1}{2}\int dx\, [\Lambda^2 E_C (\nabla m)^2 + E_J \phi^2]$$

Long wave length limit

Luttinger parameter $K = \sqrt{\frac{E_J}{E_{C_0}}} = \frac{1}{\Lambda}\sqrt{\frac{E_J}{E_C}} \ll 1$

Relevant perturbation (phase slips) $\sim \cos(2\pi m)$

Insulator

K. A. Matveev, A. I. Larkin, and L. I. Glazman, Phys. Rev. Lett, **89**, 096802 (2002)

R. M. Bradley and S. Doniach, Phys. Rev. B **30**, 1138 (1984)
M.Y. Choi et al., Phys. Rev. B **48**, 15 920 (1993)

BKT quantum phase transition

$$K = \frac{1}{\Lambda}\sqrt{\frac{E_J}{E_C}}$$

M.Y. Choi et al., PRB (1993)

$$K = \sqrt{\frac{E_J}{\Lambda E_C}}$$

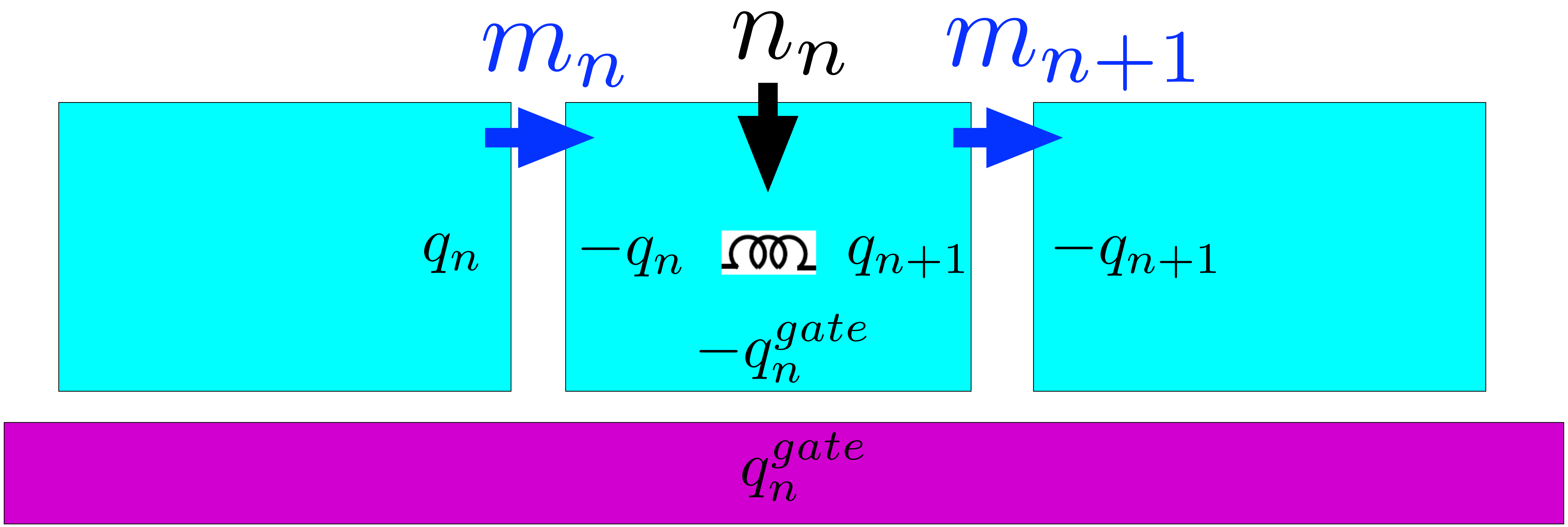
Chow, Delsing, Haviland, PRL (1997)

$$\Lambda \to \infty \Rightarrow \text{insulator for } E_J \approx E_C$$

In experiment: “transition” mostly at $E_J \sim E_C$

Quasi-charge description

Various charge variables



Charge conservation $2en_n = 2e(m_n - m_{n+1}) = q_{n+1} - q_n - q_n^{gate}$

$$Q_n = const. + \sum_{m<n} q_m^{\text{gate}} = q_n + 2em_n$$

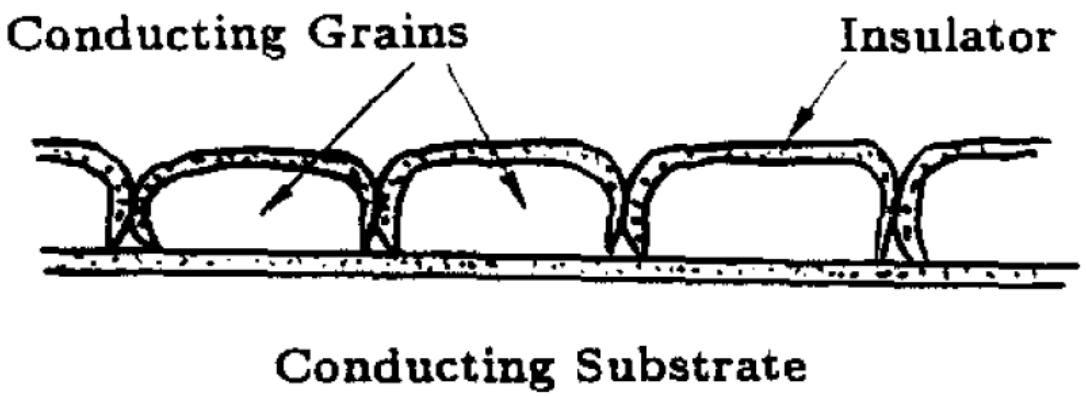
Displacement charge on junction n

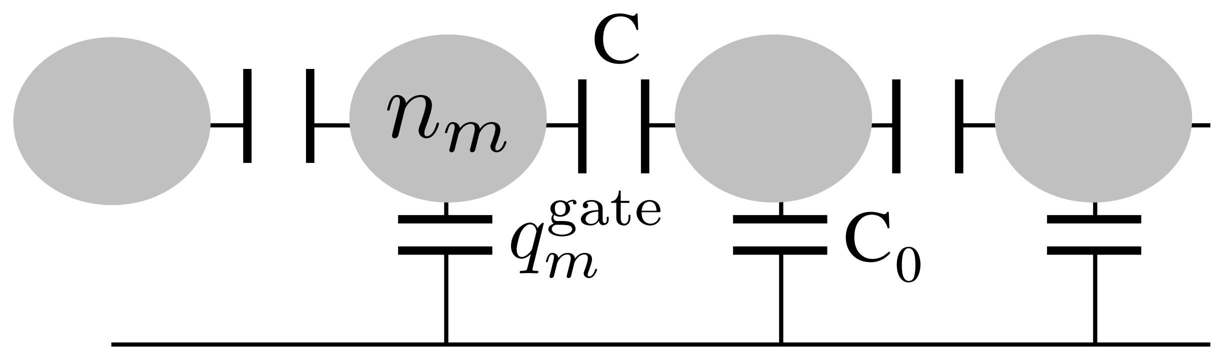
$$q_n = Q_n - 2em_n$$

Charge that has arrived at junction n

$$Q_n(t) = \int^t I_n(t')dt' \qquad const. = 2em_{-\infty}$$

Idea of charge solitons in arrays of tunnel junctions





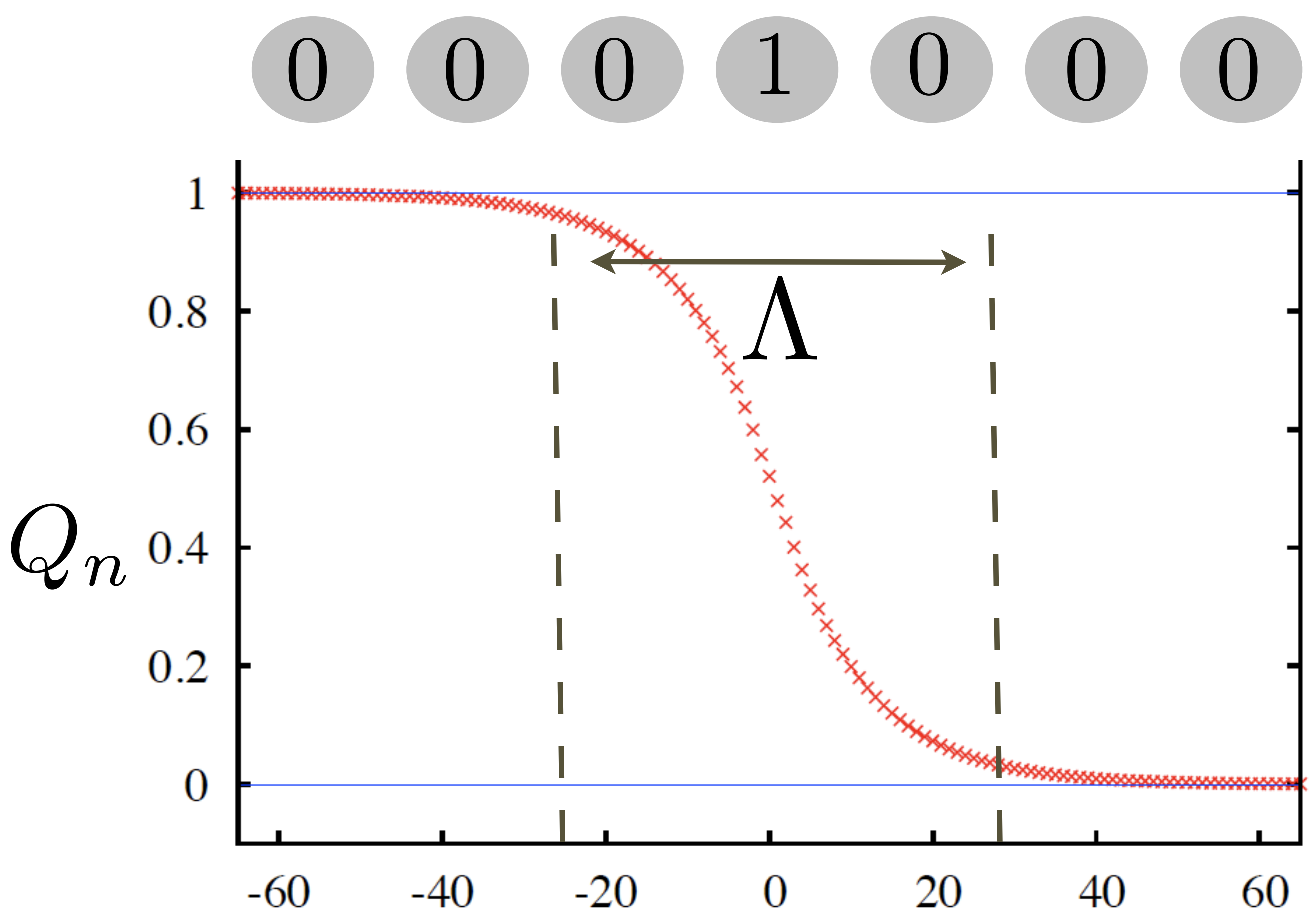
Ben-Jacob, Mullen, Amman (1989)

Likharev et al. (1989)

$$V_n = \frac{Q_n - 2em_n}{C} = \frac{Q_{n+1} + Q_{n-1} - 2Q_n}{C_0}$$

Charge that has arrived at junction n

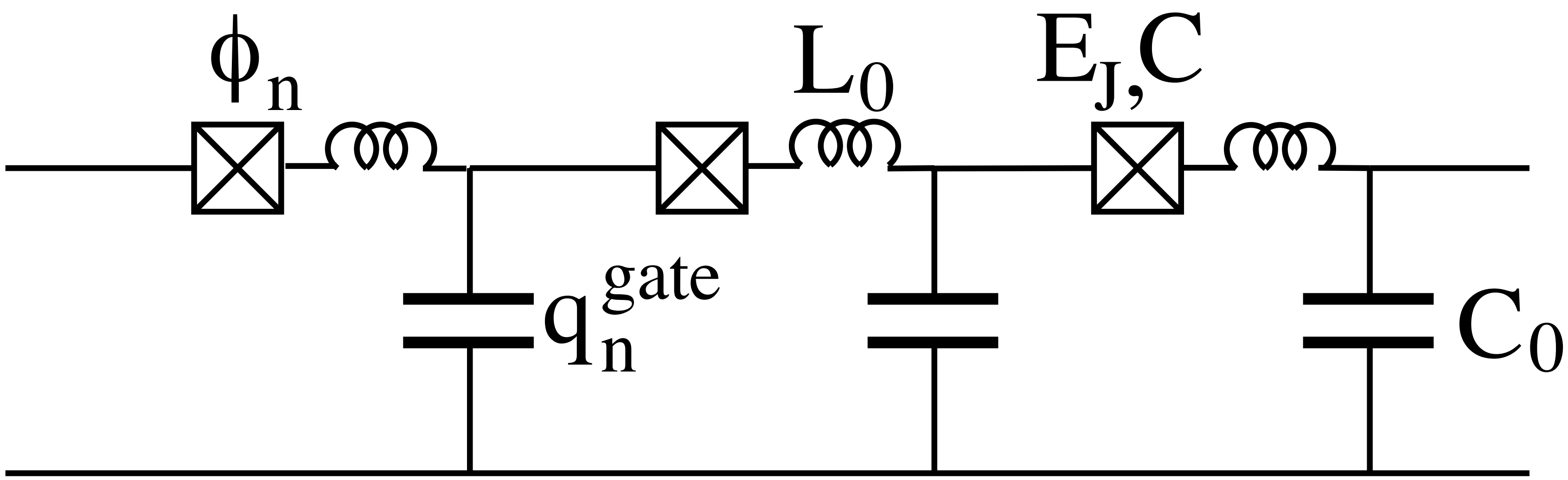
$$Q_n = const. + \sum_{m<n} q_m^{\text{gate}} = \int^t I_n(t')dt'$$



$$\Lambda \equiv \sqrt{\frac{C}{C_0}}$$

Screening length

Quasi-charge description



$$H = \sum_n \left[\frac{(Q_n - 2em_n)^2}{2C} - E_\mathrm{J} \cos\phi_n + \frac{(Q_n - Q_{n-1})^2}{2C_0} + \frac{\Phi_n^2}{2L_0} \right]$$

Continuous charge that has flown into junction n

$$Q_n = const. + \sum_{m<n} q_m^{\mathrm{gate}}$$

$$[\Phi_n, Q_{n'}] = i\hbar\delta_{n,n'}$$

Quantized charge that has tunneled through junction n

$$2em_n$$

$$[m_n, e^{i\phi_{n'}}] = e^{i\phi_n}\delta_{n,n'}$$

Limit of large (kinetic) inductance

Hermon, Ben-Jacob, Schön, PRB 96

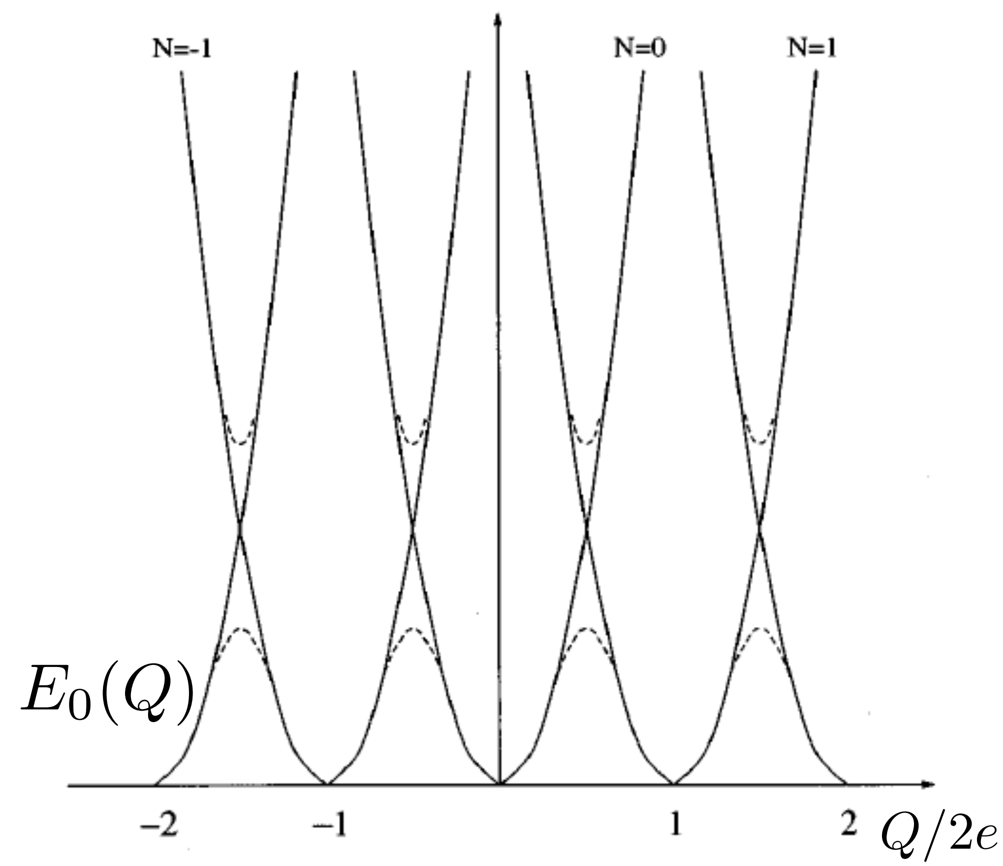
Gurarie, Tsvelik, JLTP 03

$$H = \sum_n \left[\frac{(Q_n - 2em_n)^2}{2C} - E_\mathrm{J} \cos\phi_n + \frac{(Q_n - Q_{n-1})^2}{2C_0} + \frac{\Phi_n^2}{2L_0} \right]$$

$[\Phi_n, Q_{n'}] = i\hbar\delta_{n,n'}$ slow variables

$[m_n, e^{i\phi_{n'}}] = e^{i\phi_n}\delta_{n,n'}$ fast variables

Josephson junction energy bands



Born-Oppenheimer approximation

$$H = \sum_n \left[E_0(Q_n) + \frac{(Q_n - Q_{n-1})^2}{2C_0} + \frac{\Phi_n^2}{2L_0} \right]$$

"sine"-Gordon equation of motion

$$L_0 \ddot{Q}_n + \frac{2Q_n - Q_{n+1} - Q_{n-1}}{C_0} + \frac{\partial E_0}{\partial Q_n} = 0$$

Bloch inductance

Zorin PRL 2006

Single current biased JJ

$$H(Q(t)) = \frac{(2em - Q(t))^2}{2C} - E_{\mathrm{J}} \cos\phi$$

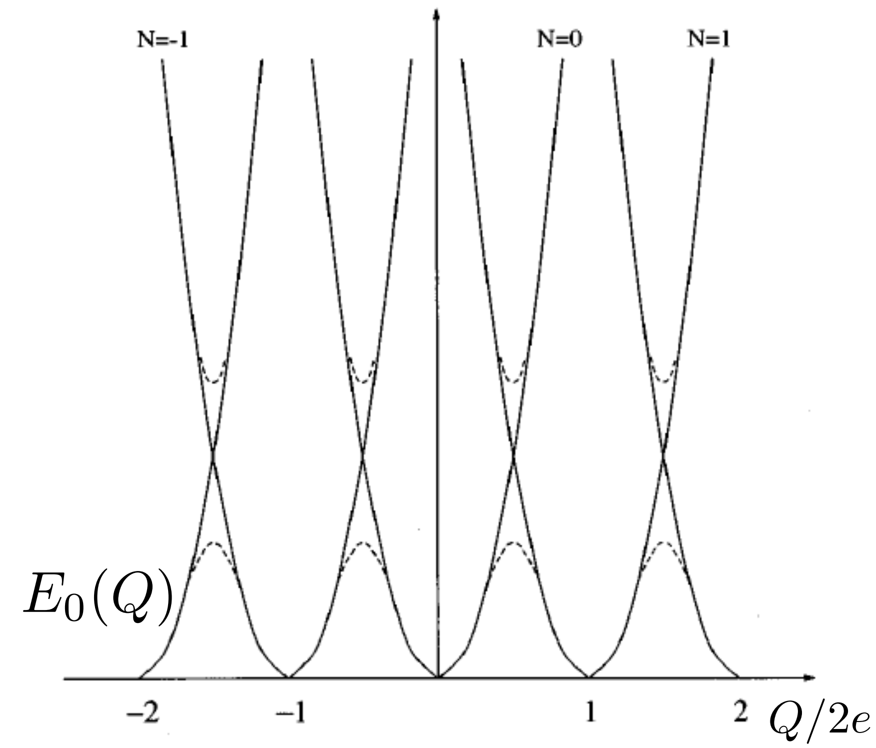
Voltage (adiabatic case)

$$V = \left\langle \frac{Q - 2em}{C} \right\rangle = \left\langle \frac{\partial H}{\partial Q} \right\rangle$$

$$= \frac{\partial E_0}{\partial Q} + L_B(Q)\ddot{Q} + \frac{1}{2}\left[\partial_Q L_B(Q)\right]\dot{Q}^2$$

Euler - Lagrange Eq. for

$$L_{\mathrm{eff}}(Q, \dot{Q}) = \frac{L_B(Q)\dot{Q}^2}{2} - E_0(Q) - VQ$$

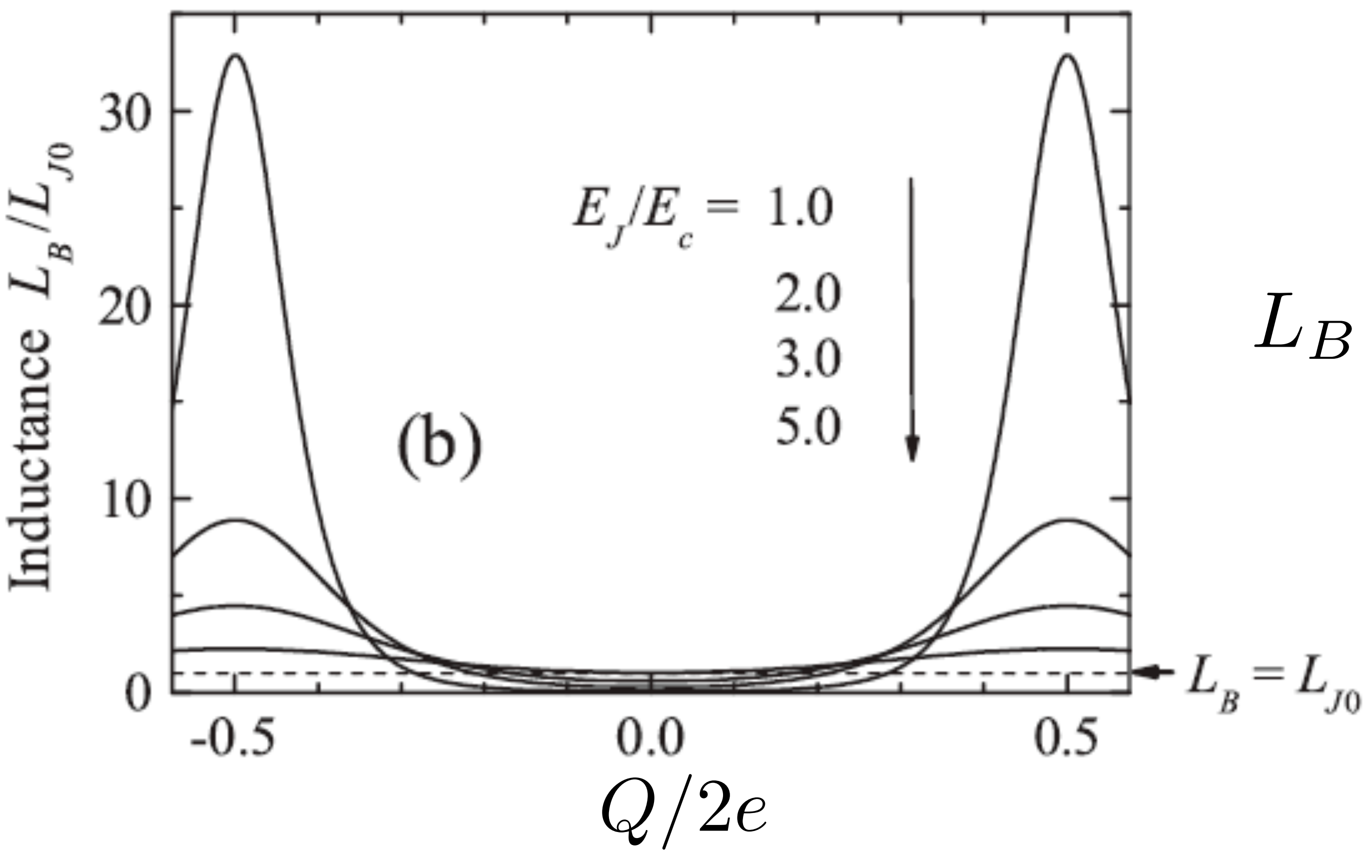


Josephson junction energy bands

J. Homfeld, I. Protopopov, S. Rachel, and A. S., Phys. Rev. B 83, 064517 (2011)

Bloch inductance

Zorin PRL 2006



$$L_B(Q) = \sum_{n>0} 2\hbar^2 \frac{\langle e_n | \partial_Q H | e_0 \rangle^2}{(E_n - E_0)^3}$$

$$L_B(Q) \approx L_{J0} \text{ for } E_J \gg E_C$$

$$L_{J0} = \frac{1}{E_J} \left(\frac{\Phi_0}{2\pi} \right)^2$$

J. Homfeld, I. Protopopov, S. Rachel, and A. S., Phys. Rev. B 83, 064517 (2011)

“sine-Gordon” equation

$$[L_0 + L_B(Q_n)]\,\ddot{Q}_n + \frac{1}{2}\,[\partial_Q L_B(Q_n)]\,\dot{Q}_n^2 + \frac{2Q_n - Q_{n+1} - Q_{n-1}}{C_0} + \frac{\partial E_0}{\partial Q_n} = 0$$

Lagrangian:

$$\mathcal{L} = \sum_n \left[\frac{1}{2}\,[L_0 + L_B(Q_n)]\,\dot{Q}_n^2 - \frac{(Q_n - Q_{n-1})^2}{2C_0} - E_0(Q_n)\right]$$

Question: does it hold for $L_0 \to 0$

Is adiabatic approximation still valid?

J. Homfeld, I. Protopopov, S. Rachel, and A. S., Phys. Rev. B 83, 064517 (2011)

Adiabaticity check

Q-dependent speed of light

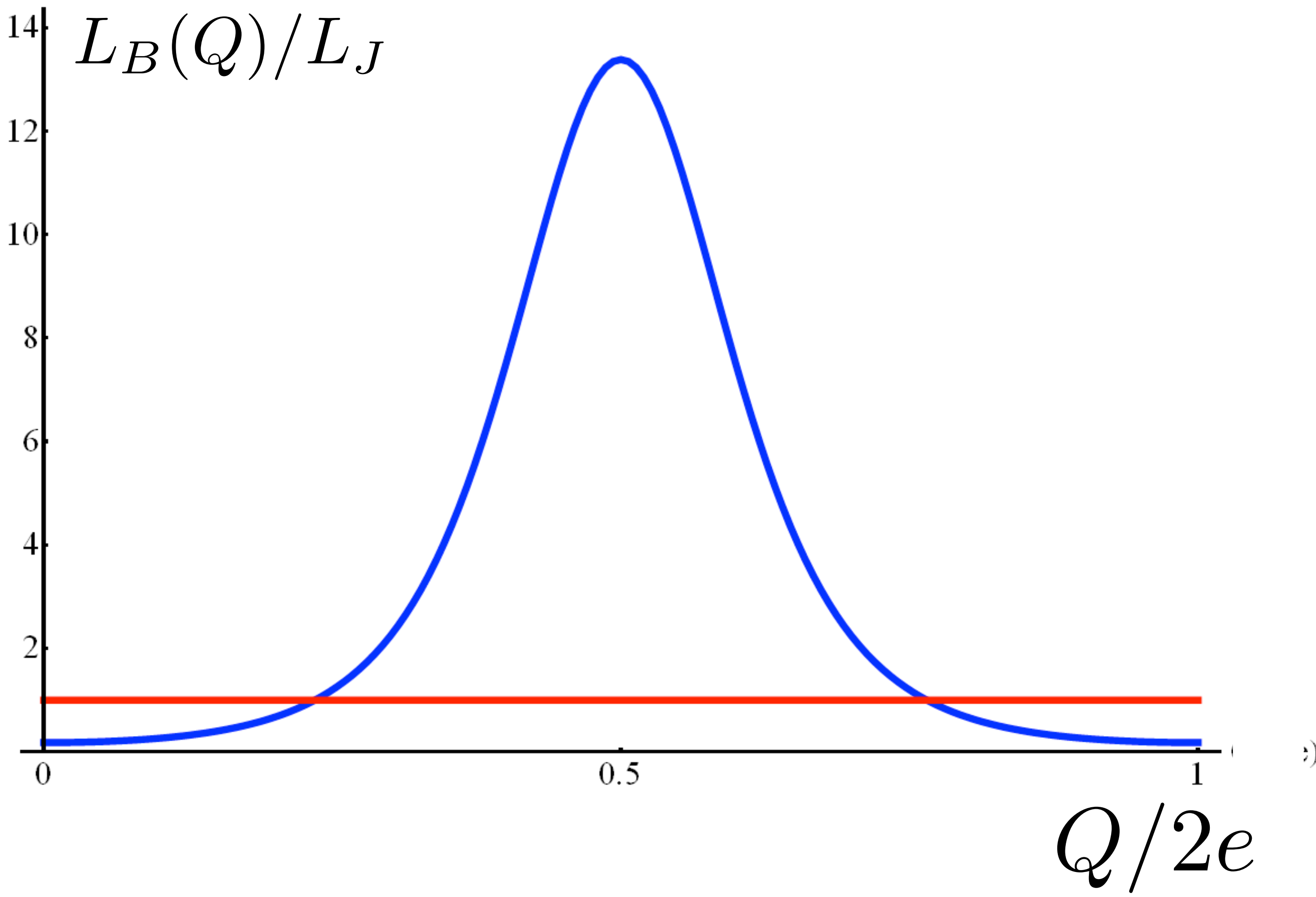
$$c(Q) = \frac{1}{\sqrt{L_B(Q)C_0}}$$

Maximum soliton velocity

$$v < c_{\min} = (L_{\max}C_0)^{-1/2}$$

$$L_{\max} = L_B(Q = e)$$

$$E_C = 2.5E_J$$



for adiabaticity (no Landau-Zener)

$$v^2 < c_{\min}^2 \frac{1}{1 + \frac{E_C}{E_J}}$$

Charge solitons
discrete charges vs. quasi-charge

S. Rachel and A. S., Phys. Rev. B 80, 180508(R) (2009)
J. Homfeld, I. Protopopov, S. Rachel, and A. S., Phys. Rev. B 83, 064517 (2011)

Hierarchy of charging energies

Odintsov, 94,96

$$H_C = \frac{1}{2}\sum_{i,j} U(i-j)\, n_i n_j \qquad U(i-j) \approx \Lambda E_C e^{-\frac{|i-j|}{\Lambda}}$$

	Energy cost of inserting one Cooper pair into the array
$\|\ldots 0\,0\,1_i\,0\,0\ldots\rangle \equiv \|i\rangle$	$E = E_0 = \frac{1}{2}U(0) \approx \frac{\Lambda E_C}{2}$
$\|\ldots 0\,1\,-1_i\,1\,0\ldots\rangle \equiv \|i;1,1\rangle$	$E \approx E_0 + \frac{E_C}{\Lambda}$
$\|\ldots 0\,1\,-1_i\,0\,1\,0\ldots\rangle \equiv \|i;1,2\rangle$ $\|\ldots 0\,1\,0\,-1_i\,1\,0\ldots\rangle \equiv \|i;2,1\rangle$	$E \approx E_0 + \frac{2E_C}{\Lambda}$

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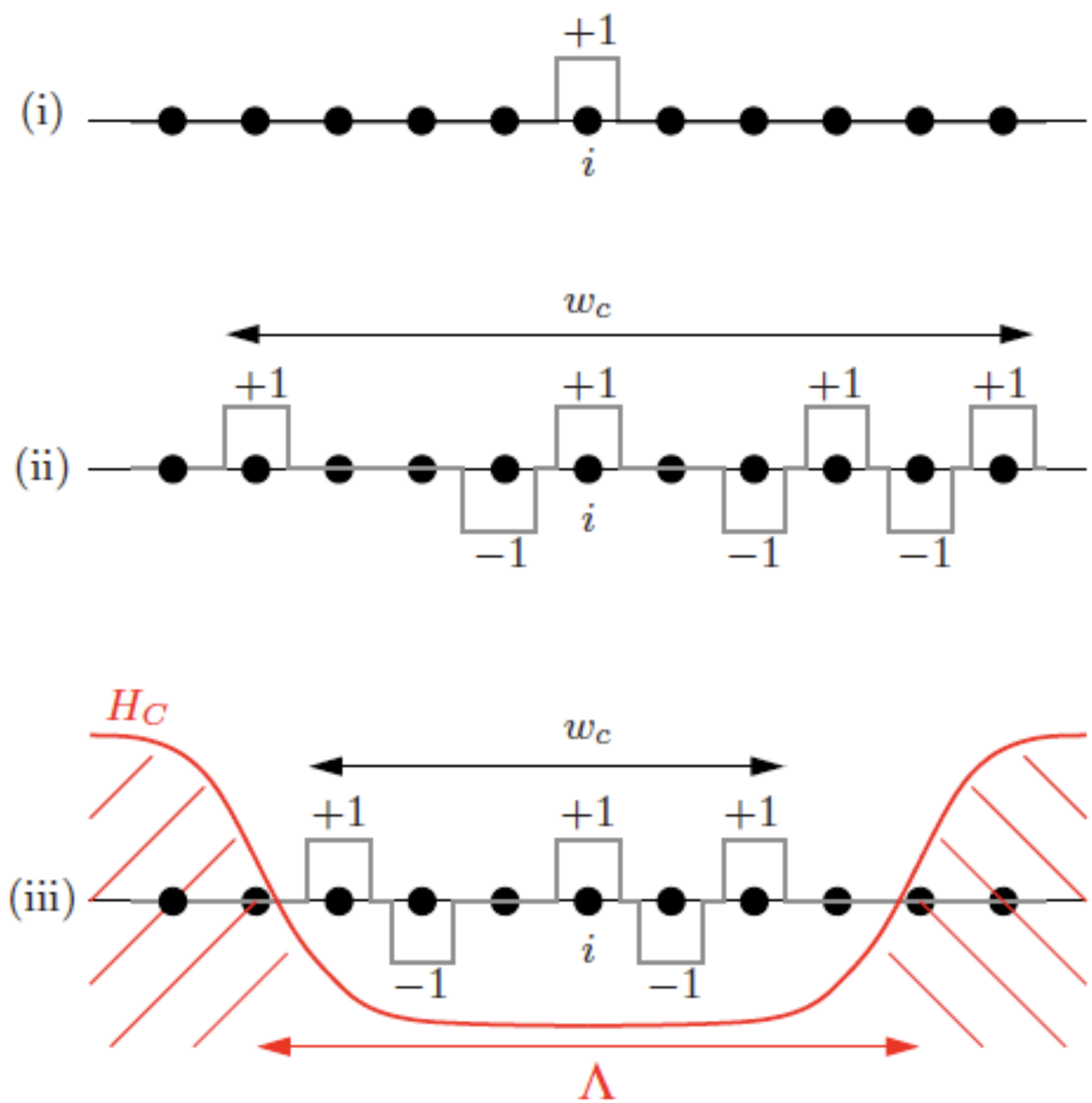
Relevant states



$$\Lambda \equiv \sqrt{C/C_0} \gg 1$$

Interesting regime:

$$\delta E_C \sim \frac{E_C}{\Lambda} < E_{\rm J}$$

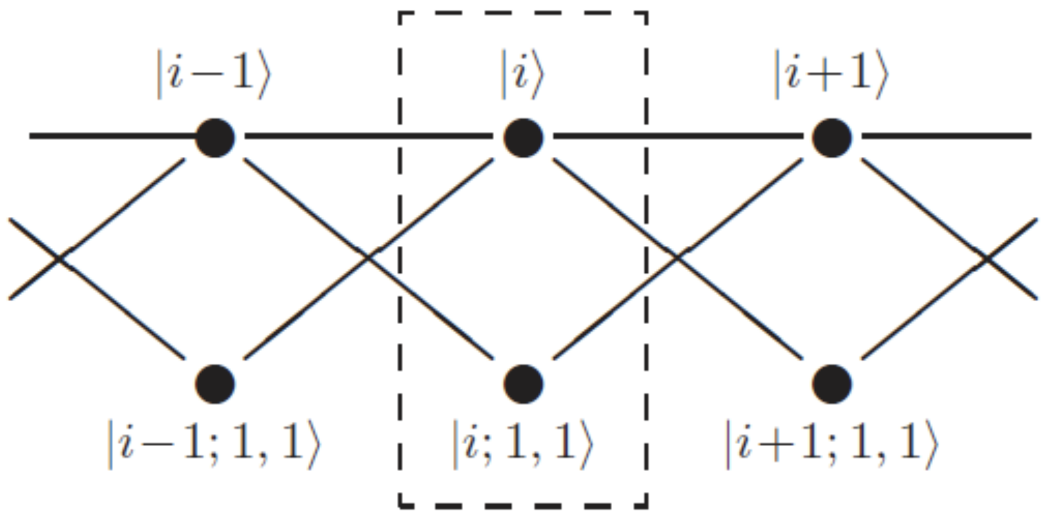
$$\Lambda E_{\rm J} > E_C > E_{\rm J}$$

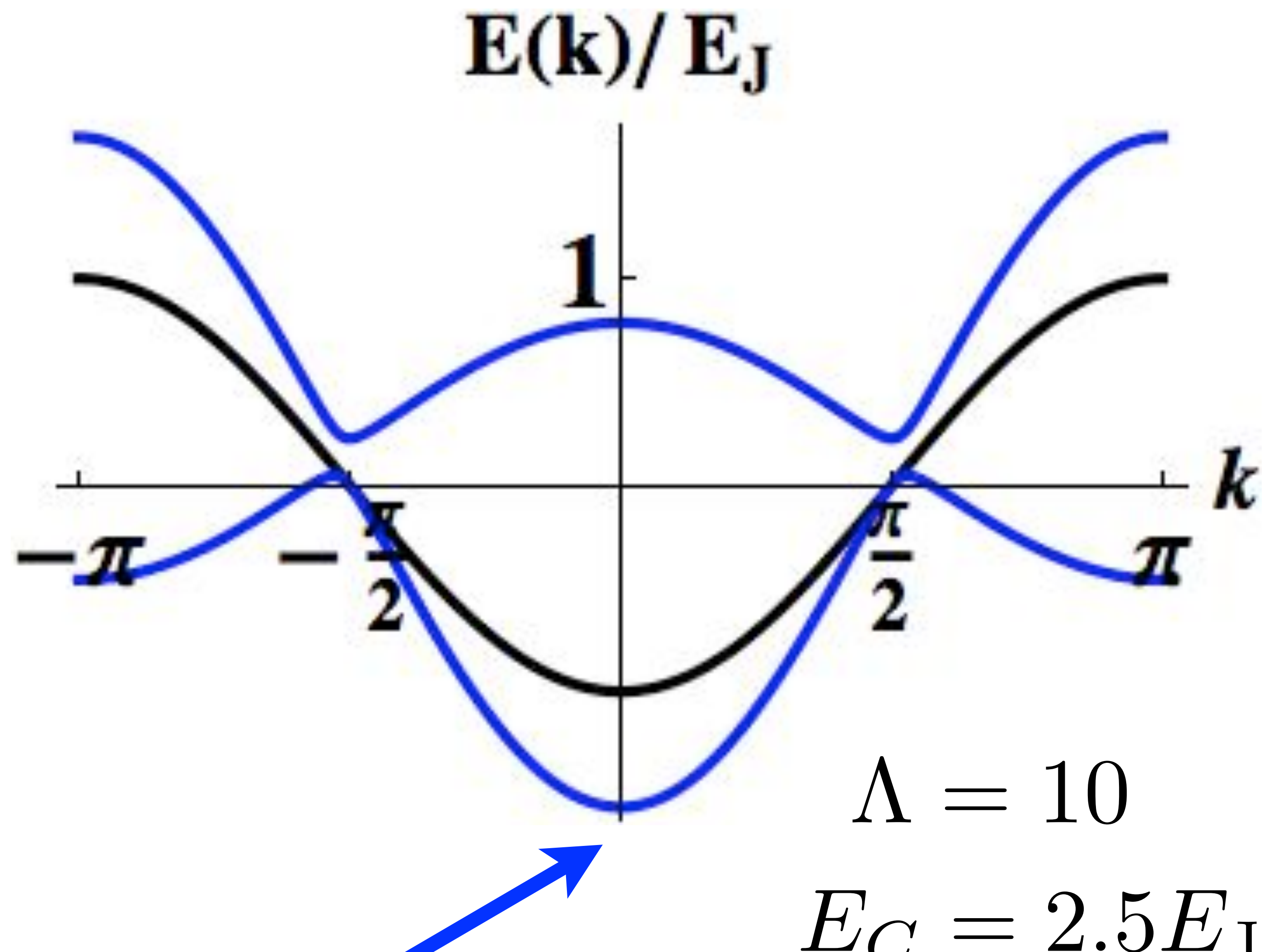
Small solitons (polarons) $\quad \Lambda E_{\rm J} > E_C > E_{\rm J}$

Two-state approximation

$|\ldots 0\,0\,1_i\,0\,0\ldots\rangle \equiv |i\rangle \qquad |\ldots 0\,0\,1\,-1_i\,1\,0\,0\ldots\rangle \equiv |i;1,1\rangle$

Tight binding structure





$$H_k^{(2)} = \begin{pmatrix} -E_{\rm J}\cos k & -E_{\rm J}\cos k \\ -E_{\rm J}\cos k & \frac{E_C}{\Lambda} \end{pmatrix}$$

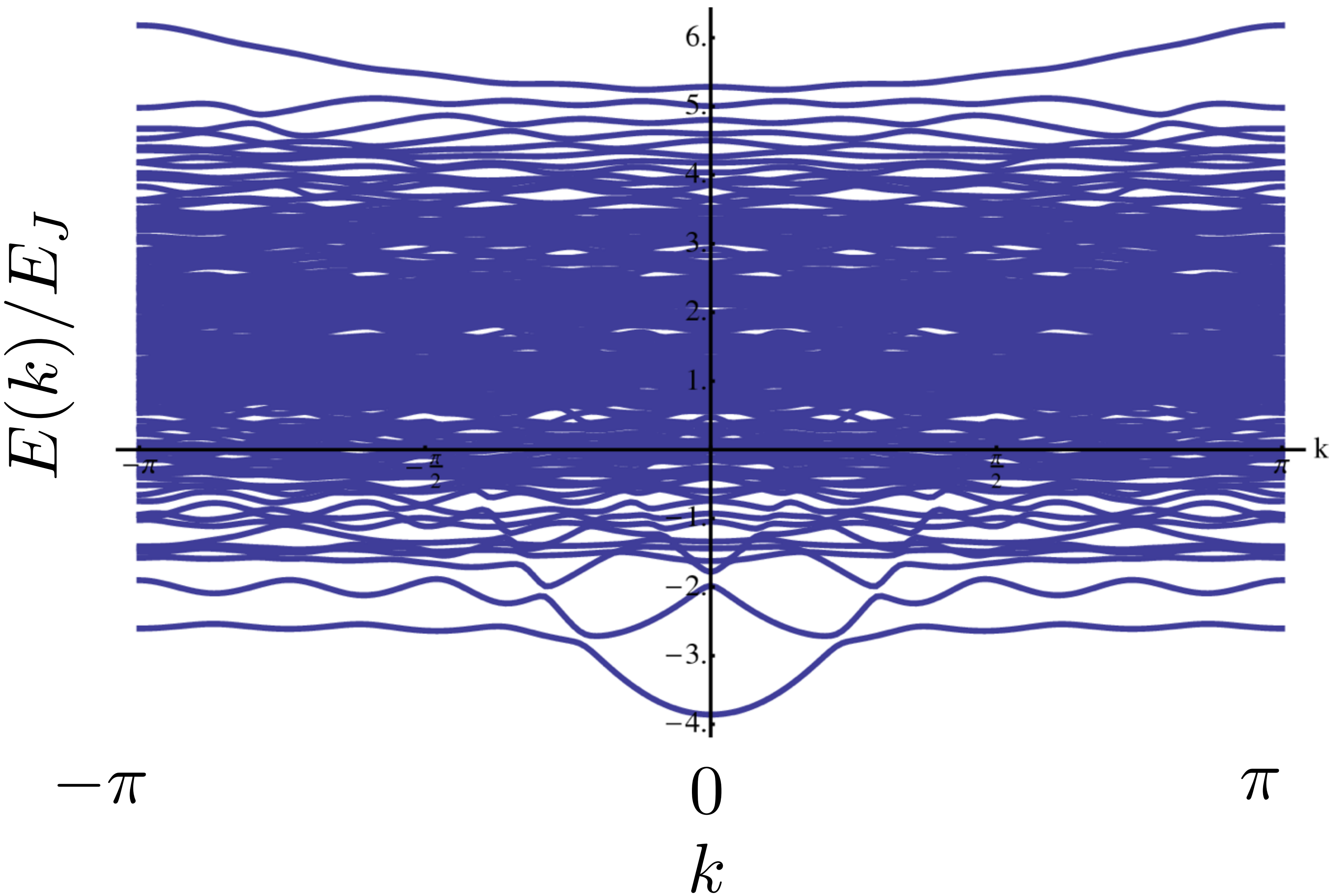
$\Lambda = 10$

$E_C = 2.5 E_{\rm J}$

Mass reduction

Rest energy reduction

Band structure



$\Lambda = 10$

$E_C = 2.5 E_{\mathrm{J}}$

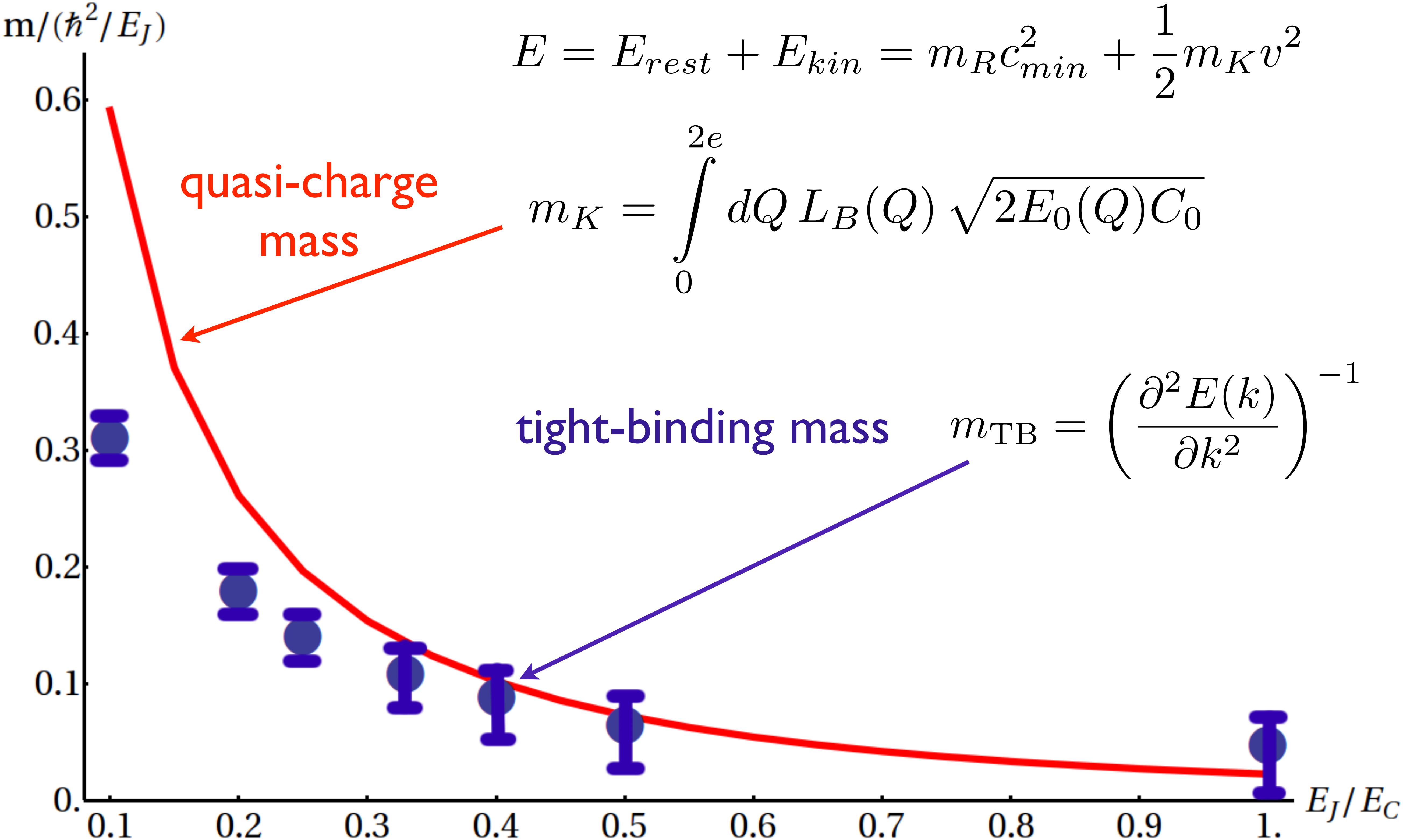
Lowest energy band flattens in the outer part of the Brillouin zone

Reduced dynamical mass $m \sim E_J^{-2}$

$$E = E_{rest} + E_{kin} = m_R c_{min}^2 + \frac{1}{2} m_K v^2$$

$$m_K = \int_0^{2e} dQ\, L_B(Q)\, \sqrt{2E_0(Q)C_0}$$

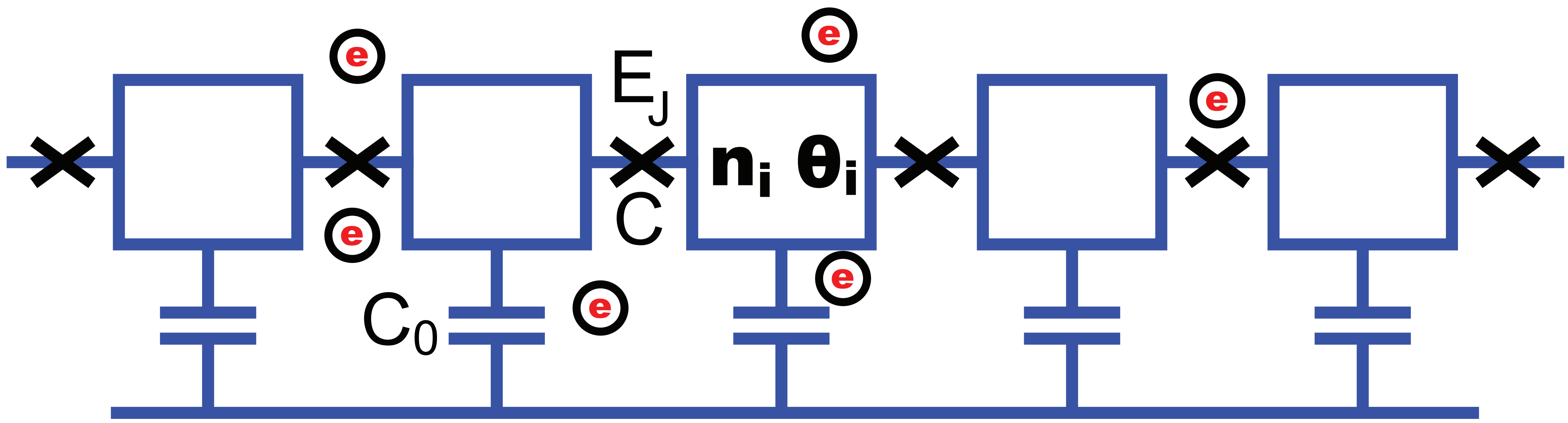
$$m_{\mathrm{TB}} = \left(\frac{\partial^2 E(k)}{\partial k^2}\right)^{-1}$$



Charge disorder: pinning

Gurarie, Tsvelik, JLTP 03

Offset charges



$$H = \frac{1}{2}\sum_{i,j} U_{i,j}\,(n_i + \delta q_i)(n_j + \delta q_j) - \sum_i E_J \cos(\theta_{i+1} - \theta_i)$$

Offset charges

$$H = \frac{1}{2}\sum_{i,j} U_{i,j}\,(n_i + \delta q_i)(n_j + \delta q_j) - \sum_i E_J \cos(\theta_{i+1} - \theta_i)$$

quasi-charge description

$$\mathcal{L} = \sum_n \left[\frac{1}{2} L_B (Q_n + F_n)\dot{Q}_n^2 - \frac{(Q_n - Q_{n-1})^2}{2C_0} - E_0(Q_n + F_n)\right]$$

$$F_n = \sum_{i=-\infty}^{n} \delta q_i$$

Strong disorder:

$$P(\delta q) = const.\ \text{for}\ \delta q \in [-e, e]$$

mod $(2e)$

$$\langle F_n F_m \rangle \sim \delta_{n,m}$$

Charging energy

$$H_c = \sum_i \left[\frac{(Q_i - Q_{i+1})^2}{2C_0} + U\left[Q_i + F_i\right] - E\, Q_i \right] \ ,$$

$$H_c = \int dx \left[\frac{(\partial_x Q(x))^2}{2C_0} + U\left[Q(x) + F(x)\right] - E\, Q(x) \right]$$

$U[Q] \equiv E_0(Q)$ Lowest Bloch band

Larkin length

$$H_c = \int dx \left[\frac{(\partial_x Q(x))^2}{2C_0} + U\left[Q(x) + F(x)\right] \right]$$

$$\langle [Q(x) - Q(0)]^2 \rangle \sim \frac{E_C^2 C_0^2 \, \tilde{R} \, x^3}{3e^2} \sim e^2 \left(\frac{x}{L_L} \right)^3$$

$$R(Q_1, x_1, Q_2, x_2) \equiv \langle U(Q_1, x_1) U(Q_2, x_2) \rangle \approx R(Q_1 - Q_2) \delta(x_1 - x_2)$$

$$\tilde{R}\left[E_J(\Phi)/E_C\right] \equiv \frac{e^2}{E_C^2} \, \frac{\partial^2}{\partial Q^2} R(Q) \Big|_{Q=0}$$

A. I. Larkin, Sov. Phys. JETP 31, 784 (1970)

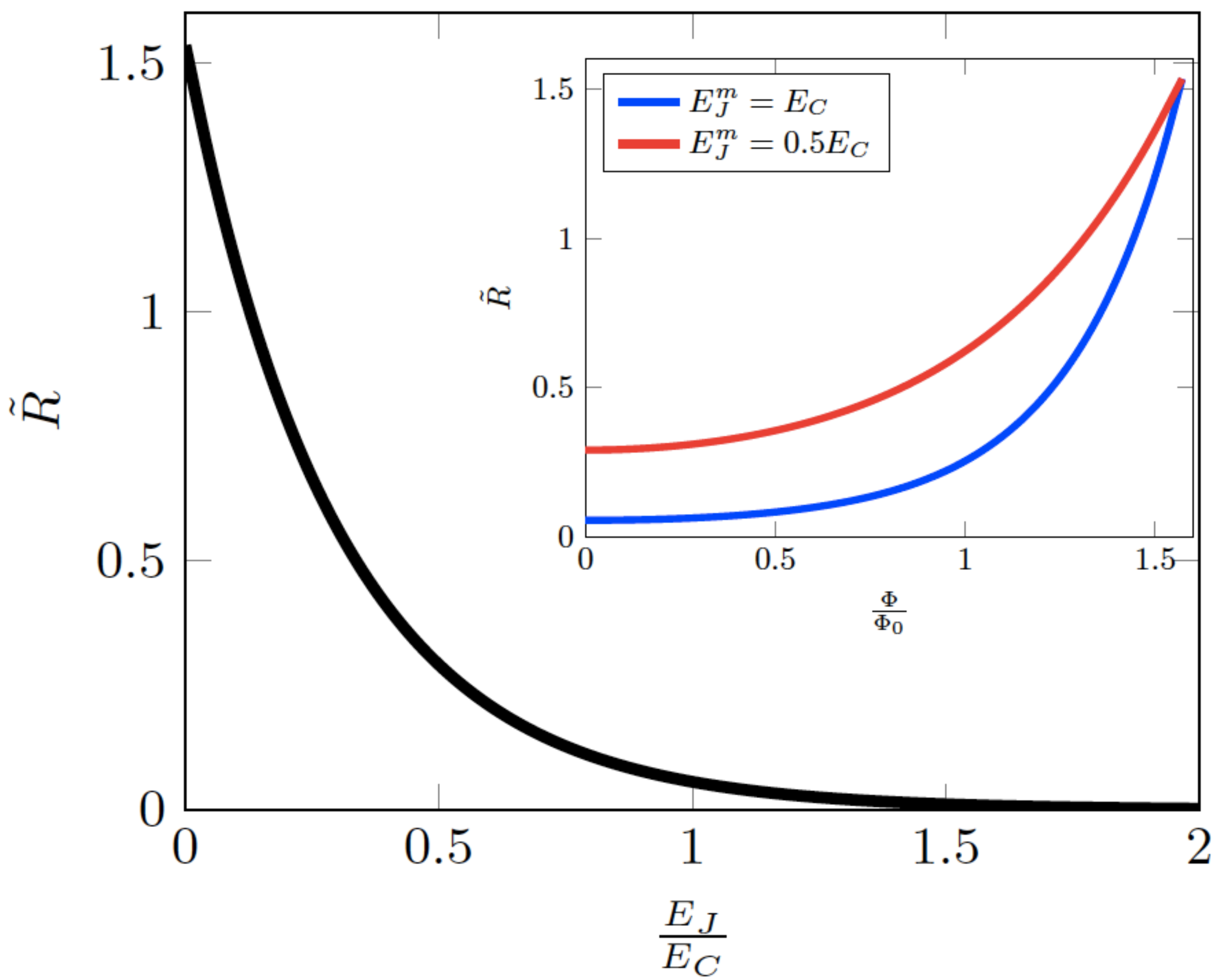
Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975)

H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978)

Larkin length

$$\langle [Q(x) - Q(0)]^2 \rangle \sim e^2 \left(\frac{x}{L_L} \right)^3$$

$$L_L \approx \Lambda^{4/3} \tilde{R}^{-1/3}$$



Depinning field

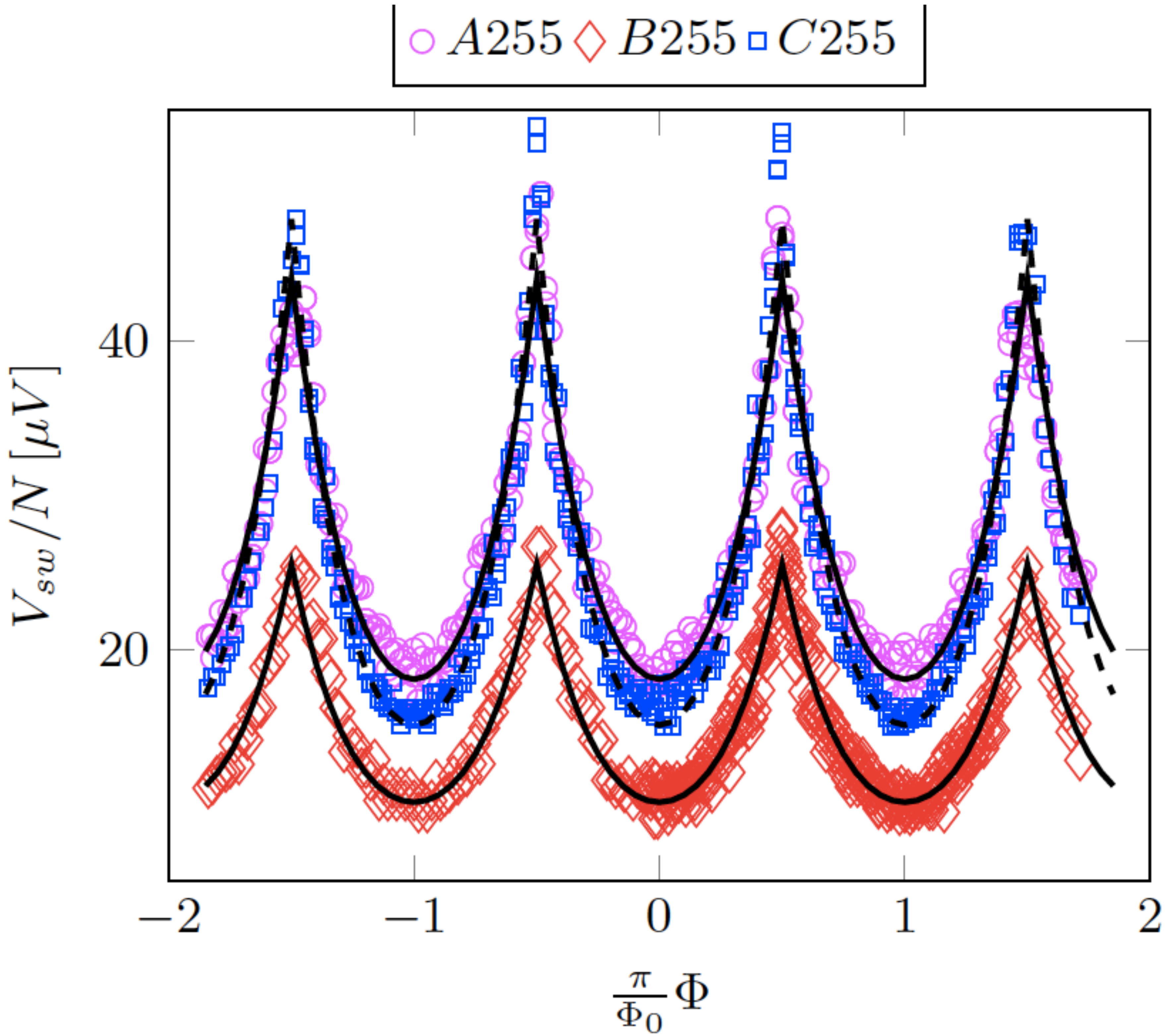
$$E_p \approx \frac{e}{C_0 L_L^2}$$

$$V_{sw} \approx \frac{N E_C}{2e} \Lambda^{-\frac{2}{3}} \tilde{R}^{2/3}$$

for $\Lambda \ll N$

Disorder

$$\frac{V_{sw}}{N} \approx \frac{E_C}{2e}\,\Lambda^{-\frac{2}{3}}\,\tilde{R}^{2/3}\left[E_J(\Phi)/E_C\right]$$



Adiabaticity check

$$Q(x) \approx Q(x') \text{ if } |x - x'| < L_L$$

Piece of order Larkin length is pinned and oscillates as a whole

Pinning frequency

$$\omega_{pin} \sim \sqrt{\frac{E_J E_C}{2\sqrt{L_L}}} \ll \sqrt{2 E_J E_C} \qquad \text{for } E_J \sim E_C$$

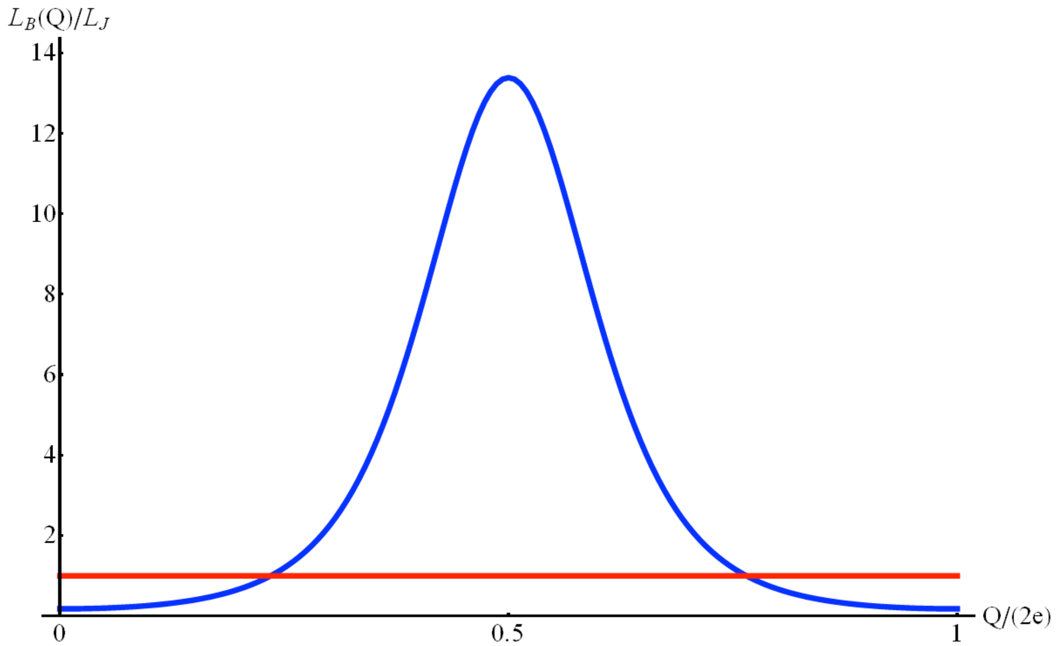
adiabatic if $L_L \gg 1$

$$L_L \approx \Lambda^{4/3} \tilde{R}^{-1/3}$$

$\Lambda \gg 1$ helps

Adiabaticity check

$$\mathcal{L} = \sum_n \left[\frac{1}{2} L_B(Q_n + F_n)\dot{Q}_n^2 - \frac{(Q_n - Q_{n-1})^2}{2C_0} - E_0(Q_n + F_n)\right]$$



$$E_C = 2.5 E_J$$

$$Q(x) \approx Q(x') \text{ if } |x - x'| < L_L$$

inductance sampling

$$L_{\text{eff}} \approx \frac{1}{L_L} \sum_n L_B(Q + F_n)$$

$$L_L \approx \Lambda^{4/3} \tilde{R}^{-1/3}$$

$\Lambda \gg 1$ helps

Conclusions

1) Quasi-charge description stabilized at long screening length

2) Transport onset by depinning