Department of Physics and Astronomy

Nonequilibrium noise and current fluctuations at the superconducting phase transition

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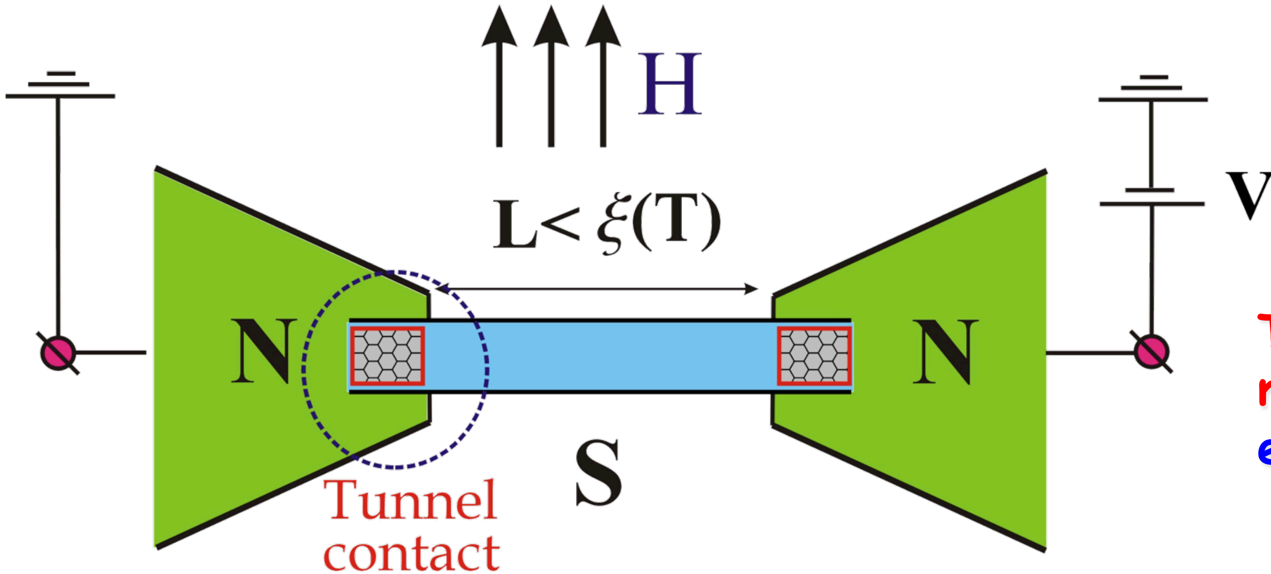
(Landau Institute, December 22, 2014)

In collaboration with Dmitry Bagrets (Cologne)

Reference: PRB 90, 180505(R) (2014)

MICHIGAN STATE UNIVERSITY

N/I/S/I/N Junction



$T > T_c$!

The junction can be made using conventional e-beam lithography !

$$G = \frac{G_L G_R}{G_L + G_R} >> G_Q$$

Conductance

$$1/\tau_H = \frac{D}{L^2}\left(\frac{\Phi}{\Phi_0}\right)^2$$

Dephasing

$$\varepsilon_{\mathrm{Th}} = \left(G/G_Q\right)\delta \le T_c^0$$

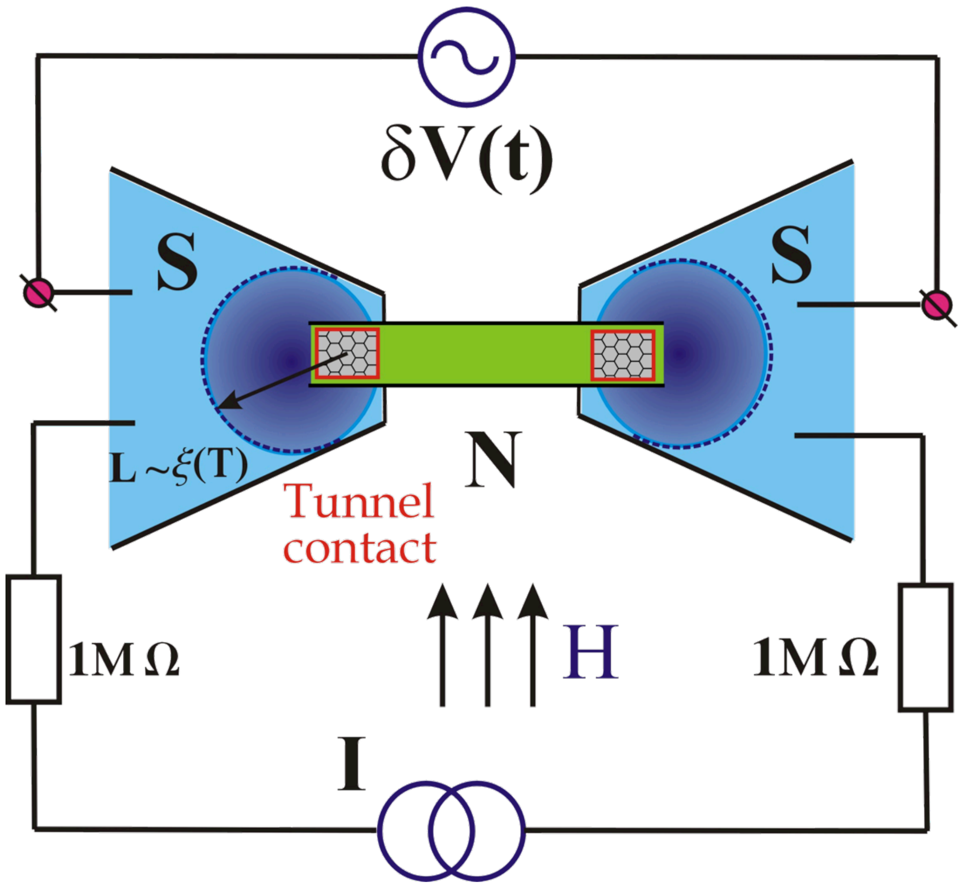
Thouless energy

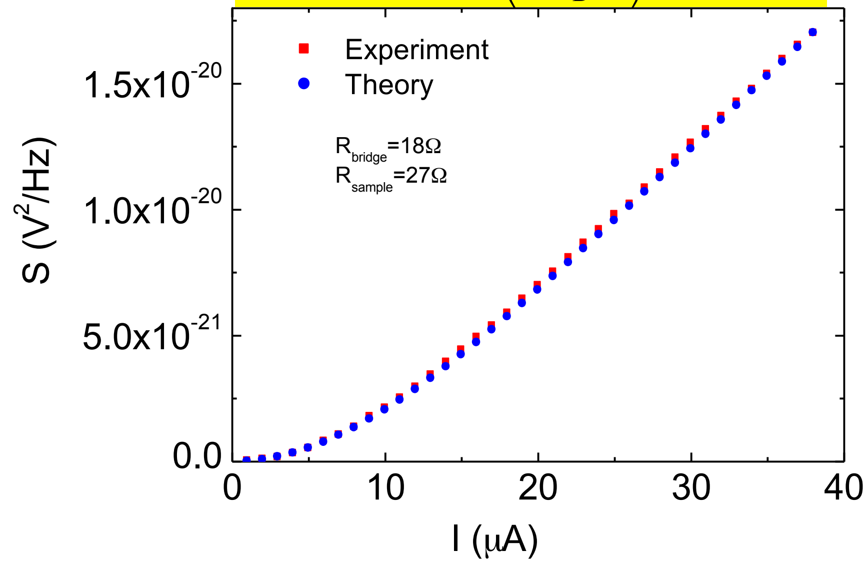
$$\ln\left(\frac{T_c}{T_c^0}\right) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\gamma}{4\pi}\right), \quad \gamma = \left(\varepsilon_{Th} + \tau_H^{-1}\right)/T_c$$

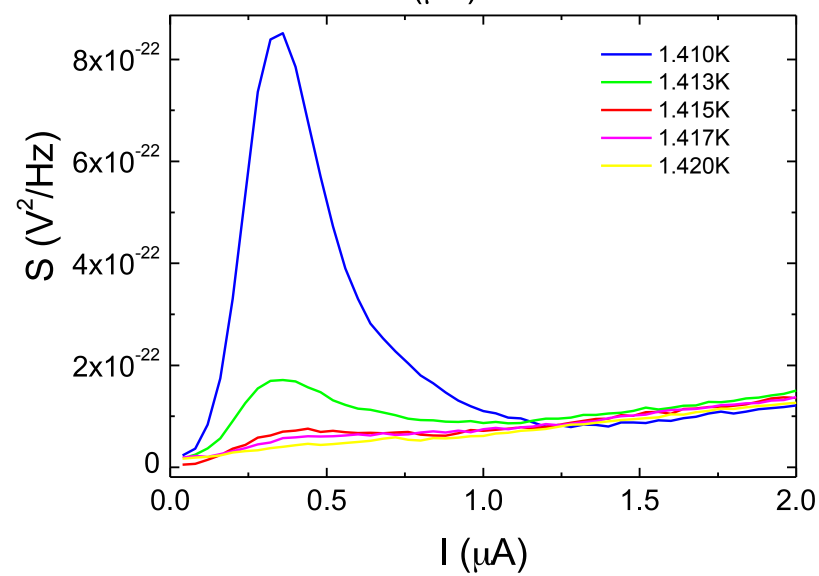
Transition temperature

Alternative Realization

$$S_I = \frac{2}{3} eI \coth\left(\frac{eIR}{2k_B T}\right) + \frac{4}{3}\frac{2k_B T}{R}$$

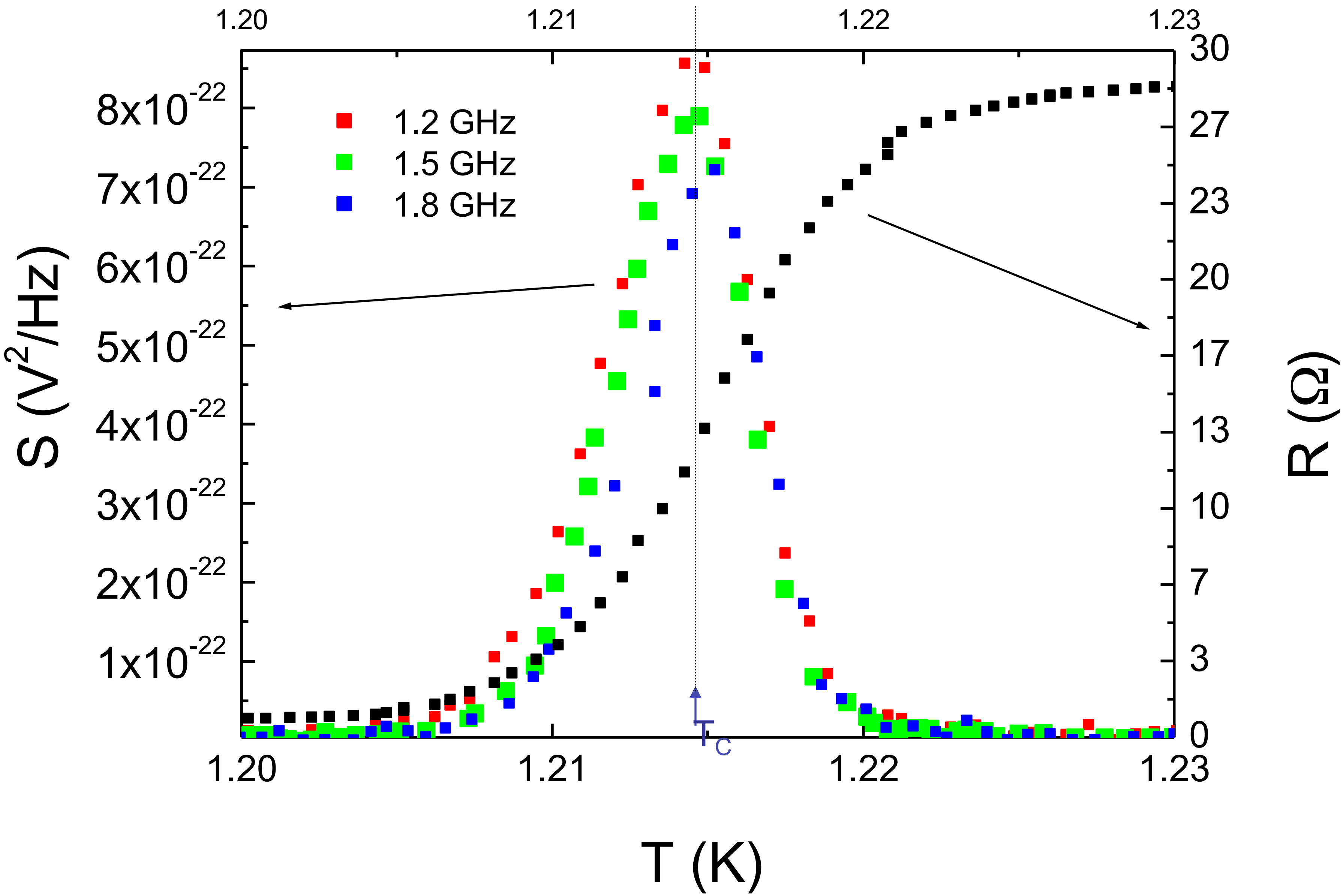




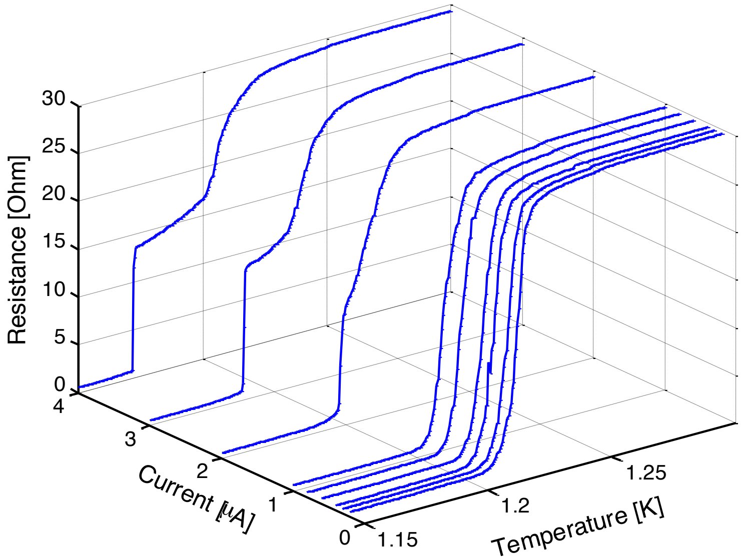


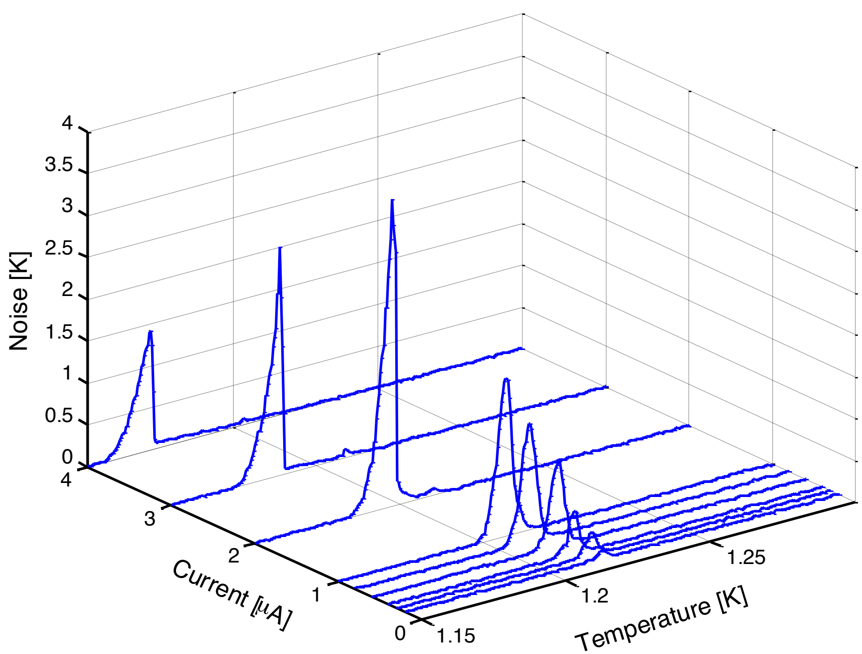
S/N/S contact with T above T_c in the current-bias mode.

M. Reznikov (private communication)



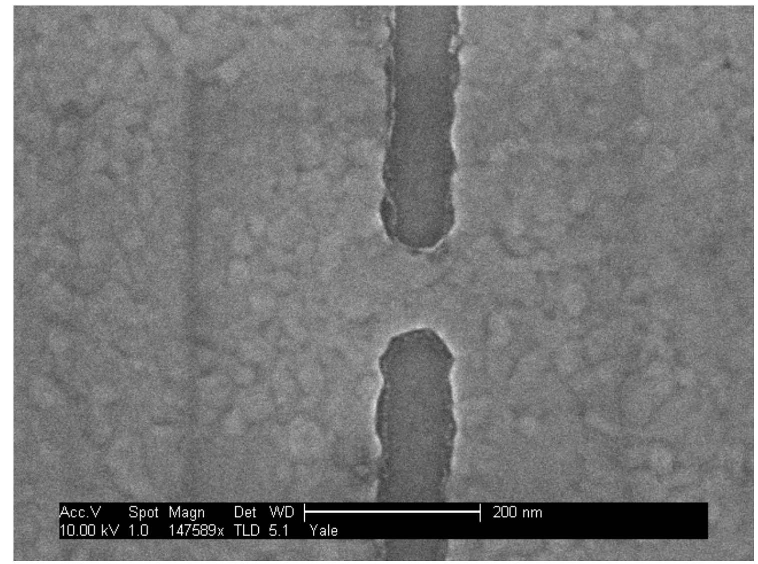
1.20
1.21
1.22
1.23
30
27
23
20
17
13
10
7
3
0
8x10^-22
7x10^-22
6x10^-22
5x10^-22
4x10^-22
3x10^-22
2x10^-22
1x10^-22
1.2 GHz
1.5 GHz
1.8 GHz
S (V^2/Hz)
R (Ω)
T_c
T (K)



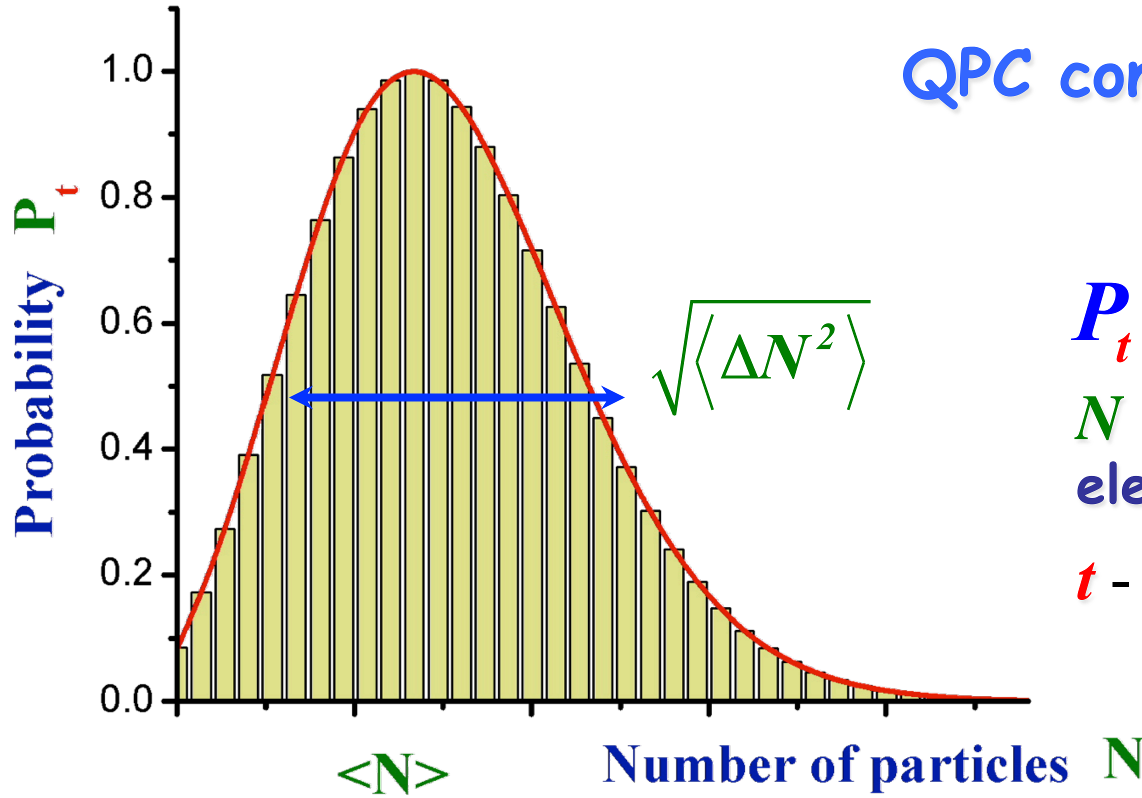


100x100x8 nm, 7 Ohm/□

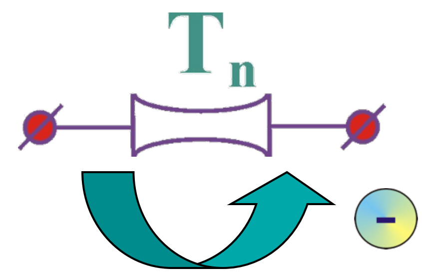
$$\xi(0) \approx 100nm$$



Full Current Statistics



QPC contact



$P_t(N)$ = full info

N - number of electrons transferred

t - time of the measurement

$$P_t(N) = \int Z(\chi) \exp(i\chi N) \frac{d\chi}{2\pi}$$

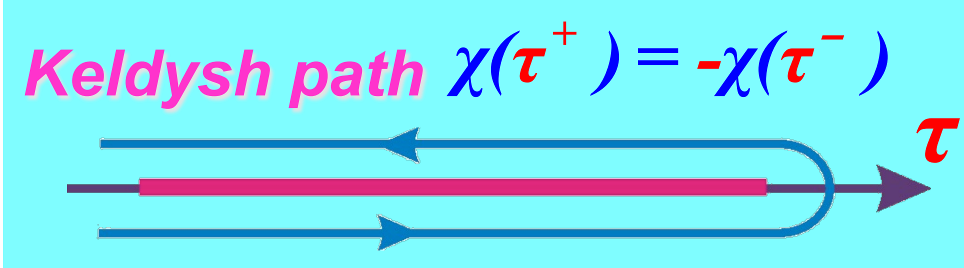
Tails of the distribution P(N) contain the information about rare events

Keldysh Method for $Z(\chi)$

- Effective Hamiltonian

$$\hat{H}_{\chi} = \hat{H} + \frac{1}{2}\chi \hat{I}$$

Quantum phase



- Cumulant Generating Function ("N.E. Free Energy")

$$Z(\chi) = \exp\{-S(\chi)\} = \left\langle e^{i\hat{H}_{-\chi}t} e^{-i\hat{H}_{\chi}t} \right\rangle$$

- Higher order currents cumulants

Give the information on
n-particle correlations !

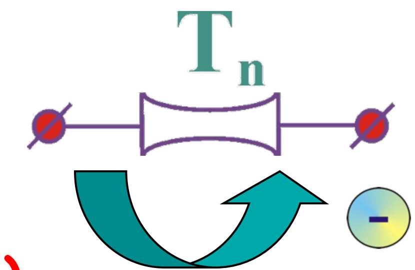
$$\langle\langle I^{n} \rangle\rangle \propto -i^{n} \frac{\partial^{n}}{\partial \chi^{n}} S(\chi)$$

Levitov-Lesovik Formula

- **Method: generating function**

counting field

$$\exp[-S(\chi)] = \sum_N P_t(N) \exp(i N \chi)$$



- **Counting electrons** (Levitov, Lee, Lesovik '95)

N_{at}

$$S(\chi) = -\frac{eVt_0}{2\pi} \sum_n \ln\{1 + T_n[\exp(i\chi) - 1]\}$$

Anti-bunching of e due to Pauli principle !

- **Experimental status (C_3)**

B. Reulet, J. Senzier, D. Prober ' 03

Yu. Bomze, G.Gershon, D.Shovkun, L.S.Levitov, M. Reznikov' 05

Gambling with Electrons

$$Z(\chi) = e^{-S(\chi)} = \prod_n [(1 - T_n) + (T_n e^{i\chi})]^{N_{at}}$$

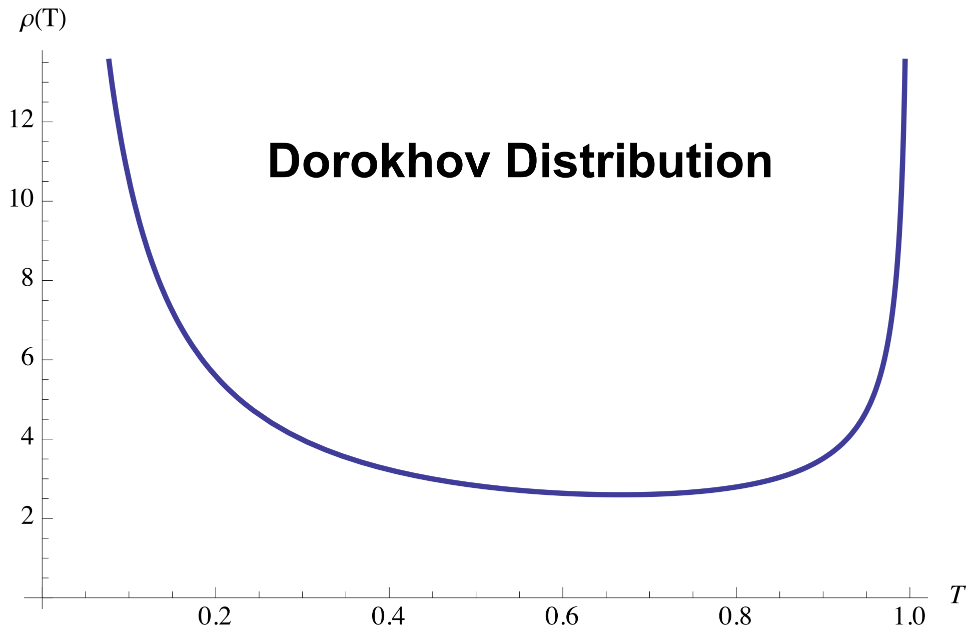
$$Z_n(\chi) = [(1 - T_n) + (T_n e^{i\chi})]^{N_{at}} = \sum_{N=0}^{N_{at}} \begin{pmatrix} N_{at} \\ N \end{pmatrix} T_n^N (1 - T_n)^{N_{at} - N} e^{i\chi N}$$

$P_t(N)$

Probability distribution *P(N)* is binominal like in the theory of gambling

What About Disorder and Interacting Electrons?

Density of Transmission Eigenvalues



$$\rho(T) \propto \frac{1}{T\sqrt{1-T}}$$

$$T = \frac{1}{\cosh^2 \xi}$$

Disorder Averaging $$\frac{\left\langle \sum_{n=1}^{N} f(T_n) \right\rangle}{\left\langle \sum_{n=1}^{N} T_n \right\rangle} = \int_0^{\infty} d\xi \, f(\cosh^{-2}\xi)$$

In practice we want disorder averaging to be done at the first step!

Nonlinear Sigma Model – $Q_{tt'}(r)$

$$iS[Q] = -\frac{\pi\nu}{4}\mathrm{Tr}[D(\partial_r Q)^2 - 4\partial_t Q]$$

$$\partial_r Q = \nabla_r Q - i[A_\chi, Q]$$

$$Q^2 = 1$$

$$Z[\chi] = \int D[Q]\exp(iS[Q, A_\chi])$$

FCS in a Diffusive Wire

Gauge transformation

$$Q \to e^{ix A_\chi} Q e^{-ix A_\chi}$$

Saddle point approximation – Usadel equation

$$D \frac{\partial}{\partial x}\left(Q \frac{\partial Q}{\partial x}\right) = 0$$

FCS Action

$$S(\chi) = N_0 \ln^2[p_\chi + \sqrt{p_\chi^2 - 1}] \quad p_\chi = 2e^{i\chi} - 1$$

Fluctuations→Ginzburg-Landau Functional

Assumes equilibrium (good for linear response calculations)

$$F/T_c = \xi_0^{-D} T_c \int d\tau d^D r \left\{ \tau_{GL} \psi^* \partial_\tau \psi + \frac{1}{2} \xi_0^2 |\nabla \psi|^2 + a|\psi|^2 + b|\psi|^4 \right\}$$

ξ_0 - zero T coherence length

$$a = \Delta T / T_c << 1, \quad b = \left(\frac{\delta_\xi}{T_c} \right) \frac{7\zeta(3)}{8\pi^2}$$

$\delta_\xi = \dfrac{1}{\nu_D \xi_0^D}$ - level spacing in volume ξ_0^D $\quad \tau_{GL} = \dfrac{\pi}{8T_c}$ - GL time

- Condensate correlation function

$$\langle \psi^*_{q,\omega} \psi_{q,\omega} \rangle \propto \frac{1}{i\omega\tau_{GL} + \xi_0^2 q^2 + a}$$

- Ginzburg criterium

If $\dfrac{\Delta T}{T_c} >> \left(\dfrac{\delta_\xi}{T_c} \right)^{1/2}$, then fluctuations are Gaussian

Quantum Keldysh Action…

- 0-D Keldysh s -model

$$S = -\frac{1}{16}\sum_{k=L,R} g_k \mathrm{Tr}\{Q_\chi^{[k]}, Q\} + i\pi\delta^{-1}\mathrm{Tr}\{(i\partial_t + \Delta)Q\} + S_\Delta + S_H$$

- Cooper pairs

$$S_\Delta = -\frac{2i\delta^{-1}}{\lambda}\mathrm{Tr}\Delta^+\hat{\sigma}_x\Delta$$

Keldysh path

- Electrons

$$Q(t_1,t_2) \propto \langle T_C\psi(t_1)\psi^+(t_2)\rangle$$

$$Q^2 = 1$$

Normalization

- In-plane m.field

$$S_H = -\frac{\pi\delta^{-1}}{8\tau_H}\mathrm{Tr}\left(\tau_z Q\right)^2$$

- Twisted boundary conditions

$$Q_L = \mathrm{e}^{i\chi\bar{\tau}_3/2}\,\hat{G}_L^{(0)}\mathrm{e}^{-i\chi\bar{\tau}_3/2}$$

Strategy

- Find saddle point of the action – 0D Usadel equation
- Account for fluctuation around the saddle point
- Parameterize Q-matrix manifold with Cooperon modes
- Integrate out fluctuations at Gaussian level
- Regularize resulting Det[…]

$$Q_0 = \frac{1}{\sqrt{N_\chi}}(\alpha_1 Q_1^{[\chi]} + \alpha_2 Q_2^{[\chi]})$$

$$N_\chi = \frac{1}{(g_1+g_2)^2}(g_1^2 + g_2^2 + g_1 g_2 \{Q_1^{[\chi]}, Q_2^{[\chi]}\})$$

…Nonequilibrium TDGL Action

Integrate out electron fluctuations

$$Q = e^{iC} Q_0\, e^{-iC}, \quad \{C, Q_0\} = 0$$

Cooperon modes

Metallic saddle point

… and obtain the Gaussian action for Δ

$$\delta S[\Delta] \propto \mathrm{Tr}\, \vec{\Delta}^{t}_{-\omega} \begin{bmatrix} i\left(\chi^2 - i\chi\frac{eV}{T_c}\right)\frac{\varepsilon_{Th}}{T_c} f(\gamma) & \pi_A^{-1} - \left(\chi^2 - i\chi\frac{eV}{T_c}\right) g(\gamma) \\ \pi_R^{-1} - \left(\chi^2 - i\chi\frac{eV}{T_c}\right) g(\gamma) & -2i \end{bmatrix} \vec{\Delta}_{\omega}$$

$$\pi^{-1}_{R(A)} = \frac{8\Delta T}{\pi T_c} \pm \frac{i\omega}{T_c} + \frac{14\zeta(3)}{\pi^3}\left(\frac{eV}{T_c}\right)^2$$

$$\frac{\Delta T}{T_c} >> \left(\frac{\delta}{T_c}\right)^{1/2}$$

Ginzburg criterium

Cumulant Generating Function

Junction is "weakly" driven out of equilibrium, $\Delta T \ll eV \ll T_c$

$$\Delta S_\chi = t_0 T_c \varepsilon \sqrt{1-\left(\chi^2 - i\chi \frac{eV}{T_c}\right)\left(\frac{\varepsilon_{Th}}{T_c}\frac{f(\gamma)}{\varepsilon^2} + \frac{g(\gamma)}{\varepsilon}\right)}$$

$$\varepsilon = \frac{8}{\pi}\frac{\Delta T}{T_c} + \frac{14\zeta(3)}{\pi^3}\left(\frac{eV}{T_c}\right)^2 \ll 1$$

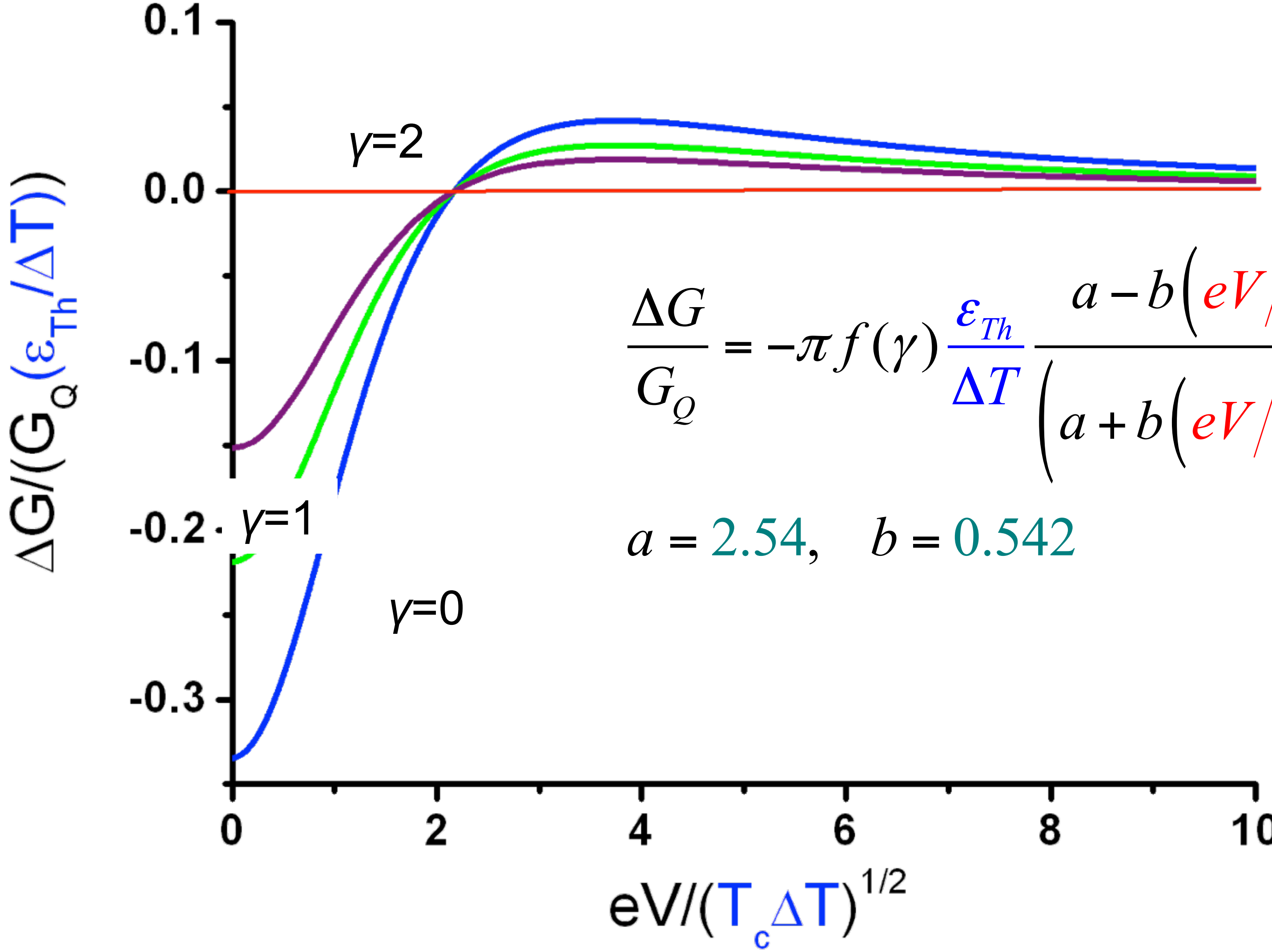
FCS of the long avalanches of charges !

We get bunching of electrons due to time-dependent fluctuations of the order parameter

... the fluctuations are most strong provided

$$\varepsilon_{Th} \gg \Delta T$$

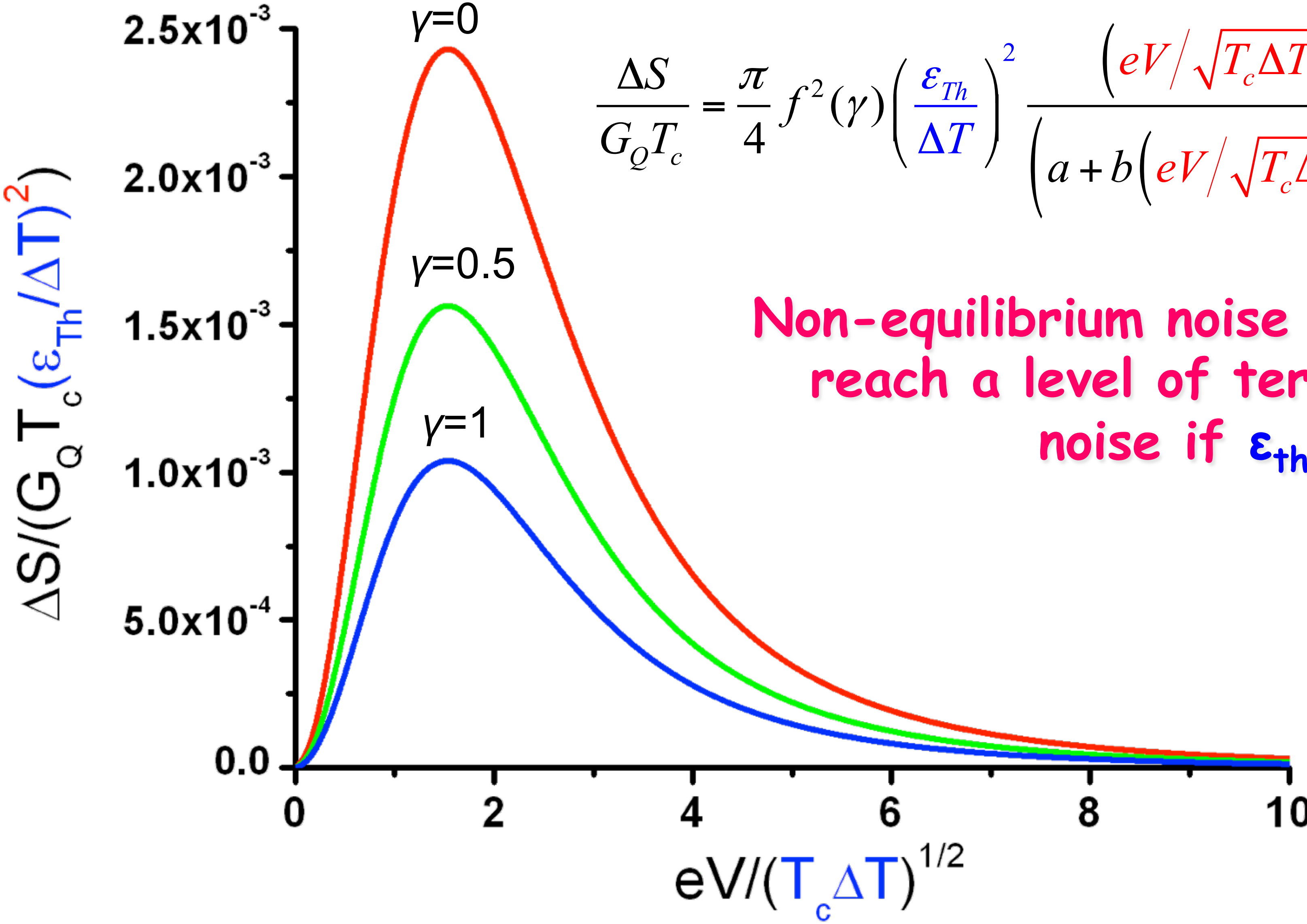
Differential Conductance



$$\frac{\Delta G}{G_Q} = -\pi f(\gamma)\,\frac{\varepsilon_{Th}}{\Delta T}\,\frac{a - b\left(eV/\sqrt{T_c\Delta T}\right)^2}{\left(a + b\left(eV/\sqrt{T_c\Delta T}\right)^2\right)^2}$$

$a = 2.54, \quad b = 0.542$

Nonequilibrium Noise



$$\frac{\Delta S}{G_Q T_c} = \frac{\pi}{4} f^2(\gamma)\left(\frac{\varepsilon_{Th}}{\Delta T}\right)^2 \frac{\left(eV/\sqrt{T_c \Delta T}\right)^2}{\left(a + b\left(eV/\sqrt{T_c \Delta T}\right)^2\right)^3}$$

Non-equilibrium noise can reach a level of termal noise if $\varepsilon_{th} > T_c$

Higher Cumulants of Currents

Current fluctuations are parametrically enhanced close to superconducting transition !

$$\left\langle I(\omega_1)\ldots I(\omega_n)\right\rangle\Big|_{\omega_k\to 0} \approx e^{n-2}G_Q T_c \left(\frac{\varepsilon_{Th}}{\Delta T}\right)^n \left(\frac{T_c}{\Delta T}\right)^{n/2-1}$$

Low frequency dispersion is set by

$$\omega_0 \approx \max\left\{\Delta T, \frac{eV^2}{T_c}\right\}$$

$\Delta T \sim 10$ mK gives
$\omega_0/2\pi \sim 200$ GHz

Estimates for the second moment

$$\left(\frac{\Delta S_I}{GT_{c0}}\right)_{max} = \frac{1}{25g}\left(\frac{\varepsilon_{Th}}{\Delta T}\right)^2$$

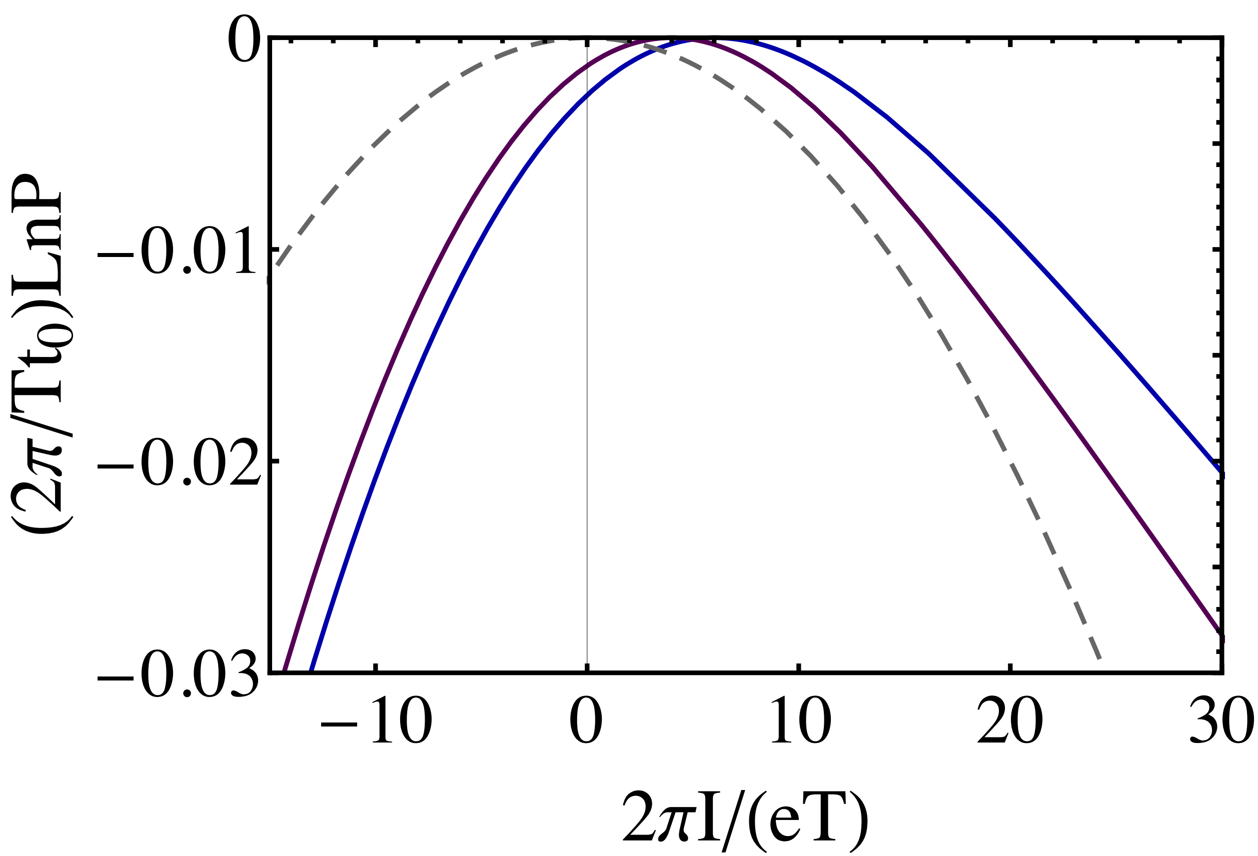
Wire dimensions: L=0.5μm, w=100nm, d=10nm

Wire parameters: $D=10^2 cm^2 s^{-1}$, $v_F=2\times10^8$cm/s, ρ=2μΩ cm

Energy scales: $\varepsilon_{Th}=D/L^2$=0.3K, δ=0.75mK, $\Gamma=0.25T_{c0}$

$$\left(\frac{\Delta S_I}{GT_{c0}}\right)_{max} \simeq 2$$

Probability Distribution Function



Long exponential tail in the distribution of current fluctuations

$$P(I) \propto \exp(-\lambda I t_0 / e)$$