Ioffe Physico-Technical Institute

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Effect of disorder on crumpling transition in graphene

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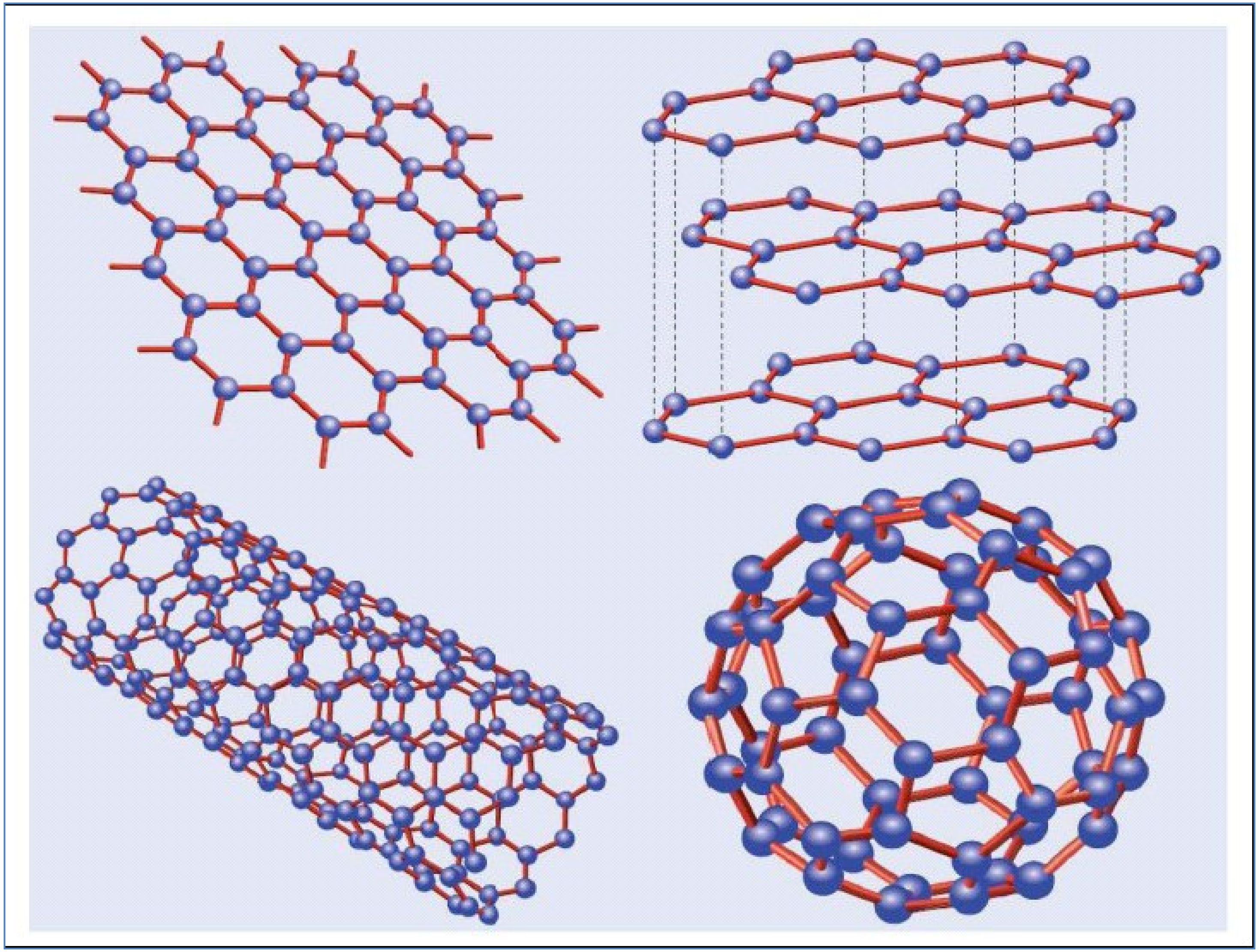
Outline

- *Introduction.* **Graphene as elastic membrane, flexural phonons, ripples.**
- *Formation of flat phase at low temperatures.* **Mean field approximation.**
- *Beyond mean field.* **Softening of membrane due to thermal fluctuations and disorder.**
- *Renormalization of bending rigidity.* **$1/d$ – expansion (d is dimension of space into which membrane is embedded).**
- *Crumpling transition in membrane.* **Scaling of bending rigidity.**
- *Effect of disorder on crumpling transition.* **Increase of critical bending rigidity. Non-monotonous scaling of bending rigidity. Disorder-induced correlation functions**

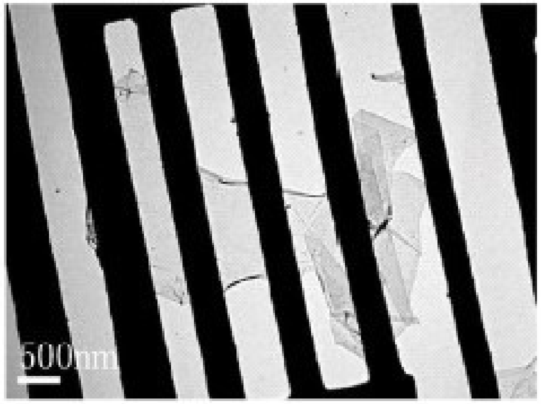
Graphene: monoatomic layer of carbon



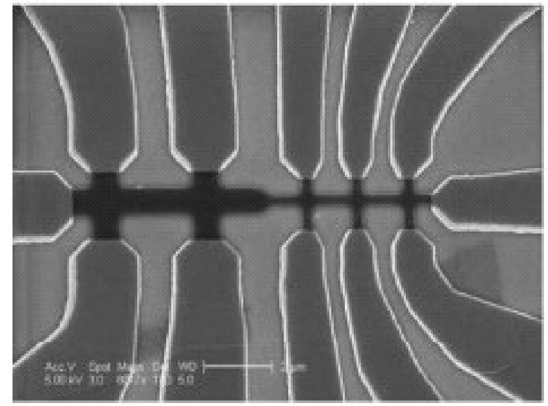
First isolated and explored: Manchester (Geim, Novoselov, et al., 2004)

Nobel Prize 2010 (Andre Geim & Konstantin Novoselov)

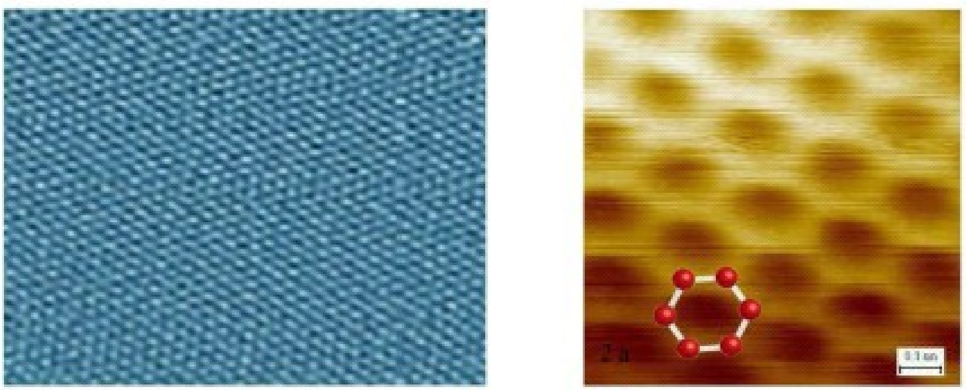
Graphene samples



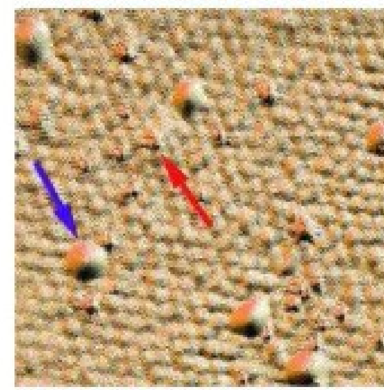
Suspended sample



Hall bar



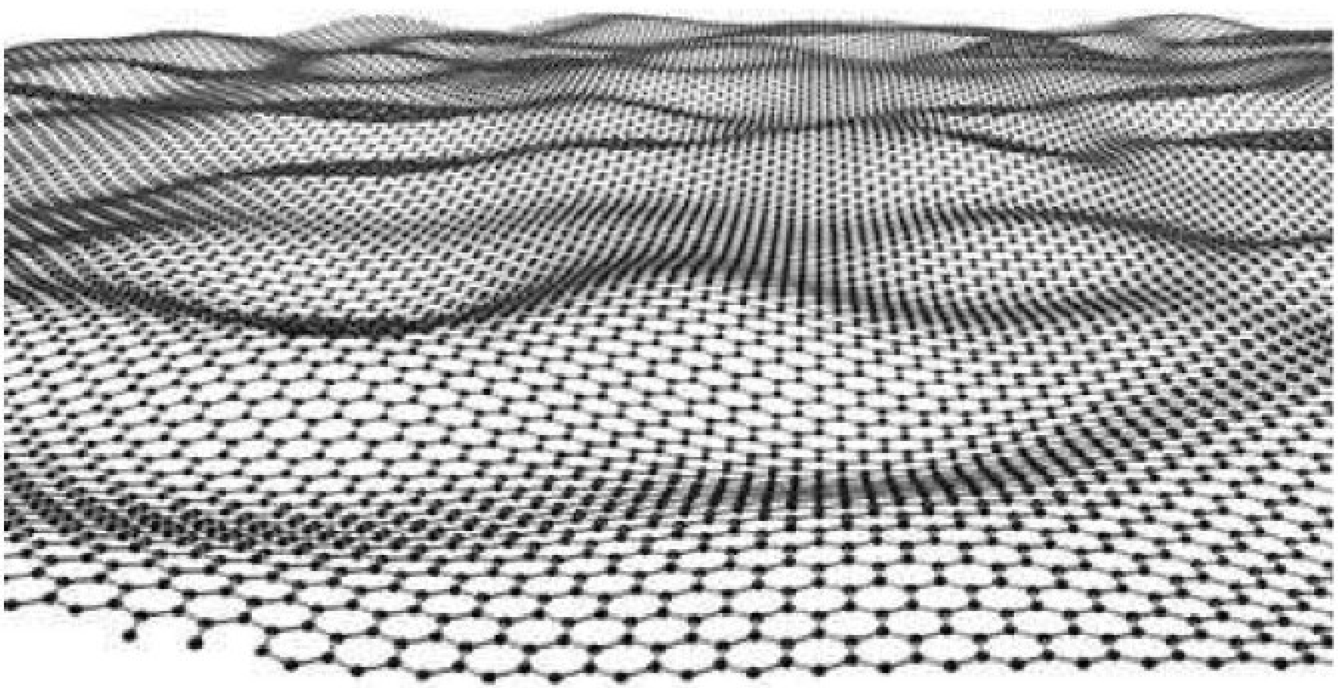
Micro-mechanical cleavage



Epitaxial growth

Suspended graphene: flexural phonons

Ripples = „snapshot" of flexural phonons



Meyer, Geim, Katsnelson, Novoselov, Booth, Roth, Nature'07

Static ripples: frozen disorder ???

Graphene as elastic membrane

Elastic energy

$$E = \frac{1}{2}\int d\mathbf{r}\left[\rho(\dot{\mathbf{u}}^2 + \dot{h}^2) + \varkappa(\Delta h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

$\mathbf{u}(\mathbf{r}), h(\mathbf{r})$ **in-plane and out-of-plane distortions**

$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Strain tensor

$\rho \simeq 7.6 \cdot 10^{-7} \mathrm{kg/m^2}$	**mass density of graphene**
$\lambda \simeq 3 \mathrm{eV/Å^2}$ $\mu \simeq 9 \mathrm{eV/Å^2}$	**elastic constants**
$\varkappa \approx 1 \mathrm{eV}$	**bending rigidity**

Flexural phonons (FP)

$$E_\perp = \frac{1}{2}\int d\mathbf{r}\left[\rho \dot{h}^2 + \varkappa(\Delta h)^2\right]$$

$$h(\mathbf{r}) = \sum_{\mathbf{q}} \sqrt{\frac{\hbar}{2\rho\omega_{\mathbf{q}} S}}(b_{\mathbf{q}} + b^{\dagger}_{-\mathbf{q}})e^{i\mathbf{qr}}$$

out-of-plane flexural mode

$$\omega_q = Dq^2$$

soft dispersion of FP

$$D = \sqrt{\varkappa/\rho}$$

Quasistatic approximation

$$b_{\mathbf{q}} = \sqrt{N_{\mathbf{q}}}e^{-i\varphi_{\mathbf{q}}},$$

$$N_{\mathbf{q}} \approx \sqrt{T/\hbar\omega_{\mathbf{q}}} \gg 1$$

$$h(\mathbf{r}) = \sum_{\mathbf{q}} \sqrt{\frac{2T}{\varkappa q^4 S}} \cos(\mathbf{qr} + \varphi_{\mathbf{q}})$$

$$G(\mathbf{q}) = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = \frac{T}{\varkappa q^4}$$

correlation function of FP

Due to soft dispersion, thermal fluctuations with small *q* are huge

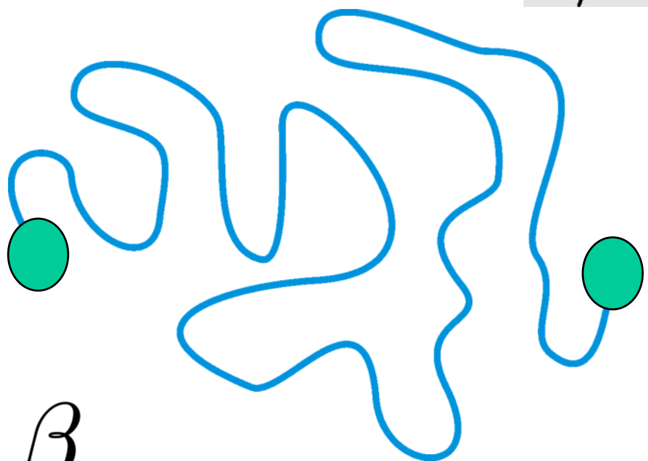
$$\sqrt{\langle h^2(\mathbf{r}) \rangle} \propto \sqrt{\frac{T}{\varkappa} \int \frac{d^2\mathbf{q}}{q^4}} \propto \sqrt{\frac{T}{\varkappa}} L$$

Proportional to the system size !!!

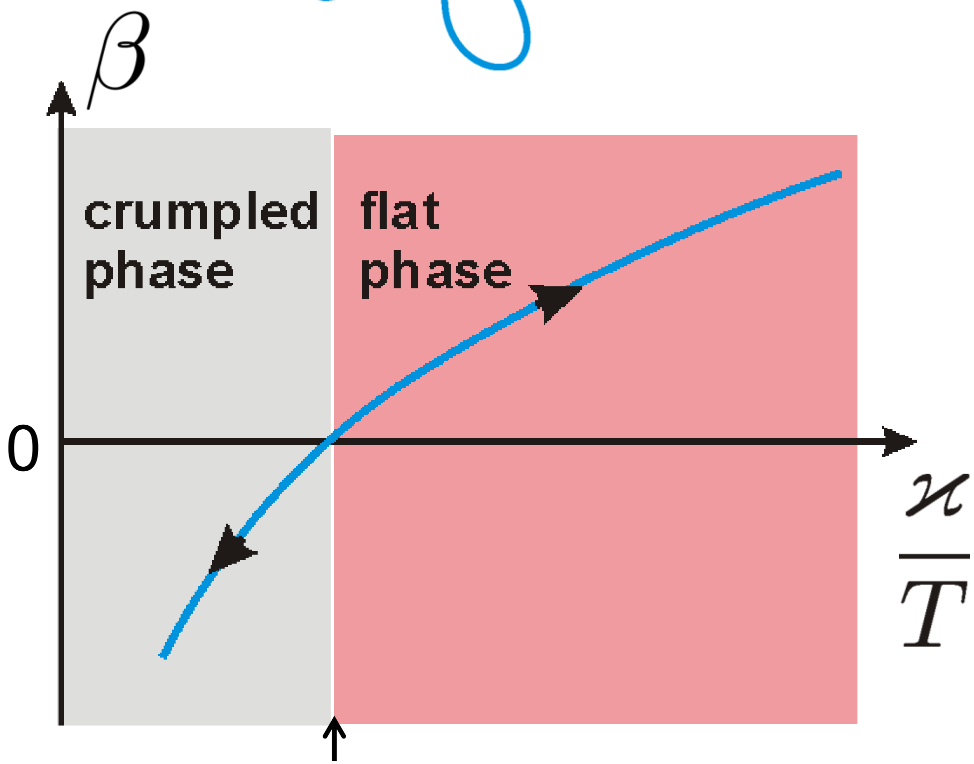
For graphene at room temperature: $\sqrt{T/\varkappa} \approx 0.2$

Crumpling transition of membrane: key parameter $\varkappa/T$

Crumpled phase, $\varkappa/T \to 0$



Flat phase, $\varkappa/T \to \infty$



crumpling phase transition

Scaling of bending rigidity

$$\frac{d(\varkappa/T)}{d\ln L} = \beta\left(\varkappa/T\right)$$

Formation of flat phase at low temperatures

$$F = \int d^D x \left\{ \frac{\varkappa_0}{2} (\partial_\alpha \partial_\alpha \mathbf{R})^2 - \frac{t}{2} (\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R}) + u (\partial_\alpha \mathbf{R} \partial_\beta \mathbf{R})^2 + v (\partial_\alpha \mathbf{R} \partial_\alpha \mathbf{R})^2 \right\}$$

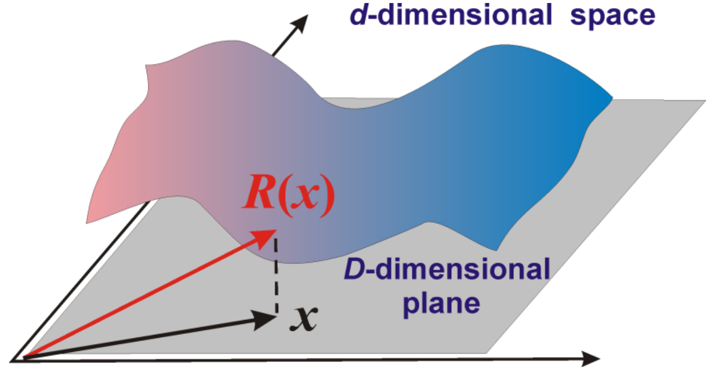
$$\alpha, \beta = 1, ..., D$$

Paczuski, Kardar, Nelson , PRL,1988

$\mathbf{R}(\mathbf{x})$ is d-dimensional vector

$\mathbf{x}$ is D-dimensional vector

For physical membranes d=3, D=2



Mean field → $\mathbf{R} = \xi \mathbf{x}$ → $F = -\xi^2 t + 2\xi^4 (u + Dv)$

$\partial F / \partial \xi = 0$ → $$\xi^2 = \begin{cases} \dfrac{t}{4(u+Dv)}, & \text{for } t > 0 \\ 0, & \text{for } t < 0 \end{cases}$$

flat phase ($t > 0$)

crumpled phase ($t < 0$)

$t \propto T_c - T$ → $\xi^2 \propto T_c - T$

Flat phase $(T<T_c,\ \xi>0)$

$$\mathbf{R} = \xi \mathbf{r} \qquad \mathbf{r} = \mathbf{x} + \underbrace{\mathbf{u} + \mathbf{h}}$$

in-plane and out-of-plane fluctuations

$$\mathbf{u} = (u_1, ..., u_D),\ \ \mathbf{h} = h_1, ..., h_{d-D}$$

$$F = \int d^D x \left\{ \frac{\varkappa}{2} (\Delta \mathbf{r})^2 + \frac{\mu}{4} (\partial_\alpha \mathbf{r} \partial_\beta \mathbf{r} - \delta_{\alpha\beta})^2 + \frac{\lambda}{8} (\partial_\alpha \mathbf{r} \partial_\alpha \mathbf{r} - D)^2 \right\}$$

$$\varkappa = \varkappa_0 \xi^2,\ \ \mu = 4u\xi^4,\ \ \lambda = 8v\xi^4$$

$$\mu, \lambda \propto (T_c - T)^2, \quad \kappa \propto T_c - T$$

Elastic constants turn to zero in the transition point

Strain tensor

$$u_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha \mathbf{r}\partial_\beta \mathbf{r} - \delta_{\alpha\beta}\right) \approx \frac{1}{2}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha + \partial_\alpha \mathbf{h}\partial_\beta \mathbf{h}\right)$$

$$F = \int d^D x \left\{ \frac{\varkappa}{2}(\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

Renormalization of elastic constants

$$d \to \infty, \quad (1/d) - \text{expansion}$$

David, Guitter, Europhys. Lett. (1988), Radzihovsky, Le Doussal, J.Phys. (Paris) (1991)

It is convenient to redefine:

$$\varkappa \to \varkappa d, \;\; \mu \to \mu d, \lambda \to \lambda d$$

Hubbard – Stratonovich transformation → decouples $(\partial r)^4$ terms

$$e^{-F(\mathbf{r})/T} = \int \{d\chi_{\alpha\beta}\}\, e^{-\int d^D\mathbf{x}\left\{\frac{\varkappa d}{2T}(\Delta\mathbf{r})^2 + \frac{id}{2}\chi_{\alpha\beta}(\partial_\alpha\mathbf{r}\partial_\beta\mathbf{r} - \delta_{\alpha\beta}) - \frac{Td}{4\mu}\left(\chi_{\alpha\beta}^2 - \frac{\lambda}{2\mu+\lambda D}\chi_{\alpha\alpha}^2\right)\right\}}$$

$$\mathbf{r} = \xi\mathbf{x} + \delta\mathbf{r}$$

$$\int \{d\delta\mathbf{r}\} e^{-F(\mathbf{r})/T} = e^{-\int d^D\mathbf{x}\left\{\ln\det\hat{M} - \frac{id}{2}\chi_{\alpha\beta}\delta_{\alpha\beta} - \frac{Td}{4\mu}\left(\chi_{\alpha\beta}^2 - \frac{\lambda}{2\mu+\lambda D}\chi_{\alpha\alpha}^2\right)\right\}}$$

$$\hat{M} = -\varkappa\Delta^2 + iT\partial_\alpha\chi_{\alpha\beta}\partial_\beta$$

First, we look for homogeneous solution for χ:

$$\chi_{\alpha\beta} = -i\chi\delta_{\alpha\beta} \quad \rightarrow \quad \ln\det\hat{M} = \int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\ln\left(\varkappa k^4 + T\chi k^2\right)$$

$$F_{eff} \propto \chi(1-\xi^2) + \frac{T\chi^2}{2\mu+\lambda D} - \frac{1}{D}\int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\ln\left(\varkappa k^4 + T\chi k^2\right)$$

$$\partial F_{eff}/\partial\chi = \partial F_{eff}/\partial\xi = 0 \;\Rightarrow\; \chi, \xi$$

Effect of disorder

$$e^{-F(\mathbf{r})/T} = \int \{d\chi_{\alpha\beta}\}\, e^{-\int d^D\mathbf{x} \left\{ \frac{\varkappa d}{2T}[\Delta\mathbf{r} + \boldsymbol{\beta}(\mathbf{x})]^2 + \frac{id}{2}\chi_{\alpha\beta}(\partial_\alpha \mathbf{r}\partial_\beta \mathbf{r} - \delta_{\alpha\beta}) - \frac{Td}{4\mu}\left(\chi^2_{\alpha\beta} - \frac{\lambda}{2\mu + \lambda D}\chi^2_{\alpha\alpha}\right)\right\}}$$

random vector with the statistical weight:

$$P(\boldsymbol{\beta}) = \exp\left[-\frac{d}{2B}\int d^D x \boldsymbol{\beta}^2(\mathbf{x})\right]$$

$$\langle \ln Z\rangle_{\boldsymbol{\beta}} = \lim_{N\to 0}\left\langle \frac{Z^N - 1}{N}\right\rangle_{\boldsymbol{\beta}}$$

$$\hat{M} = \delta_{nm}\left(-\varkappa\Delta^2 + iT\partial_\alpha \chi^n_{\alpha\beta}\partial_\beta\right) + \frac{B\varkappa^2}{T}\Delta^2 \qquad n, m = 1, \ldots, N$$

$$\chi^n_{\alpha\beta} = -i\chi\delta_{\alpha\beta}$$

$$\ln\det\hat{M} = \int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\left[(N-1)\ln\left(\varkappa k^4 + T\chi k^2\right) + \ln\left(\varkappa k^4 + T\chi k^2 - NBk^4\frac{\varkappa^2}{T}\right)\right]$$

$$F_{eff} \propto \chi(1-\xi^2) + \frac{T\chi^2}{2\mu + \lambda D} - \frac{1}{D}\int_0^\Lambda \frac{d^D\mathbf{k}}{(2\pi)^D}\left[\ln\left(\varkappa k^4 + T\chi k^2\right) - \frac{B\varkappa^2 k^2}{T(\varkappa k^2 + \chi T)}\right]$$

disorder-induced contribution

Saddle-point equations

$$\frac{\partial F_{eff}}{\partial \chi} = 0 \Rightarrow \quad 1 - \xi^2 + \chi \frac{2T}{2\mu + \lambda D} = \frac{T}{D} \int \frac{d^D k}{(2\pi)^D} \left[\frac{1}{\varkappa k^2 + \chi T} + \frac{B \varkappa^2 k^2 / T}{(\varkappa k^2 + \chi T)^2} \right]$$

$$\frac{\partial F_{eff}}{\partial \xi} = 0 \Rightarrow \quad \xi \chi = 0$$

both terms logarithmically diverge for D=2

In the flat phase: $\xi \neq 0 \Rightarrow \chi = 0$

$$\frac{d\xi^2}{d\Lambda} = -\frac{1}{4\pi} \left(\frac{T}{\varkappa} + B \right)$$

thermal fluctuations ($\frac{T}{\varkappa}$) **disorder** (B)

$$\Lambda = \ln (L/a)$$

$\xi \to 0,$ **for certain value of** L

Within this approximation flat phase is destroyed both by thermal fluctuations and by disorder

To obtain crumpling transition one should take into account higher order corrections in 1/d

Renormalization of bending rigidity for B=0

David, Guitter, Europhys. Lett. (1988),
Le Doussal, Radzihovsky, PRL (1992)

$$F = \int d^D x \left\{ \frac{\varkappa}{2} (\Delta \mathbf{h})^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

$$G_{ij} = \langle h_i(\mathbf{q}) h_j(-\mathbf{q}) \rangle = \frac{\int h_i(\mathbf{q}) h_j(-\mathbf{q}) e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h}d\mathbf{u}\}}{\int e^{-\frac{F(\mathbf{h},\mathbf{u})}{T}} \{d\mathbf{h}d\mathbf{u}\}} = \delta_{ij} G(q)$$

$$G_0(\mathbf{k}) = \frac{T}{\varkappa k^4}$$

Interaction between in-plane and out-of-plane modes is neglected

However, such interaction dramatically change the small q behavior of $G(q)$ due to strong anharmonicity

Anomalous scaling of bending rigidity

Integrate out the in-plane modes (D=2)

$$F(\mathbf{h}) = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left[\varkappa q^4 \mathbf{h_q}\mathbf{h_{-q}} + \frac{1}{4d_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} \times R(\mathbf{k},\mathbf{k}',\mathbf{q})(\mathbf{h_{-k}}\mathbf{h_{k+q}})(\mathbf{h_{k'}}\mathbf{h_{-q-k'}})\right]$$

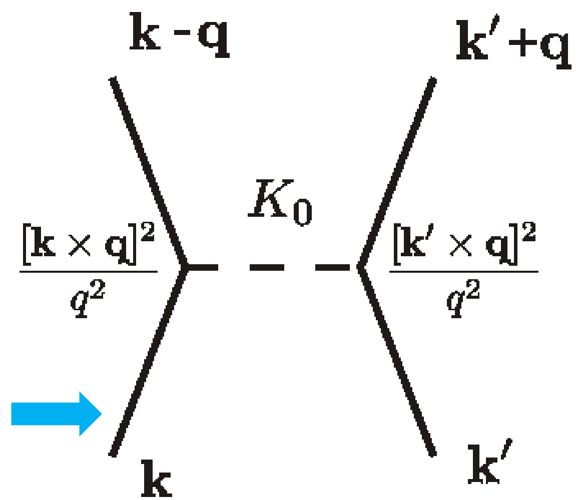
$$R(\mathbf{k},\mathbf{k}',\mathbf{q}) = K_0 \frac{[\mathbf{k}\times\mathbf{q}]^2}{q^2}\frac{[\mathbf{k}'\times\mathbf{q}]^2}{q^2}$$

$$K_0 = \frac{4\mu(\mu+\lambda)}{(2\mu+\lambda)}$$

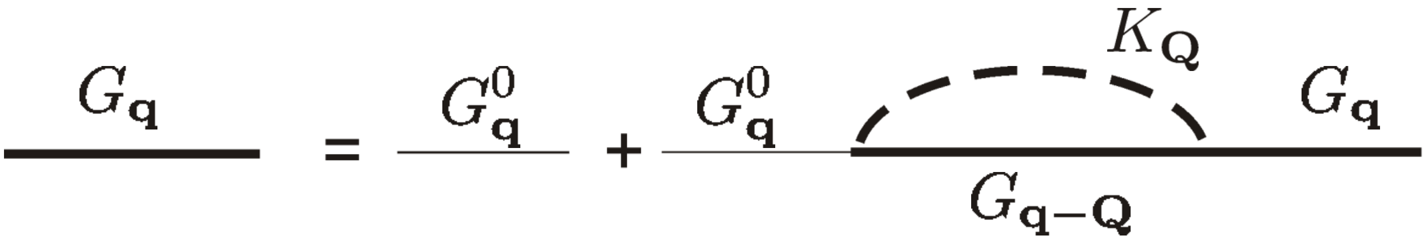
$$d_c = (d-D) \to \infty$$

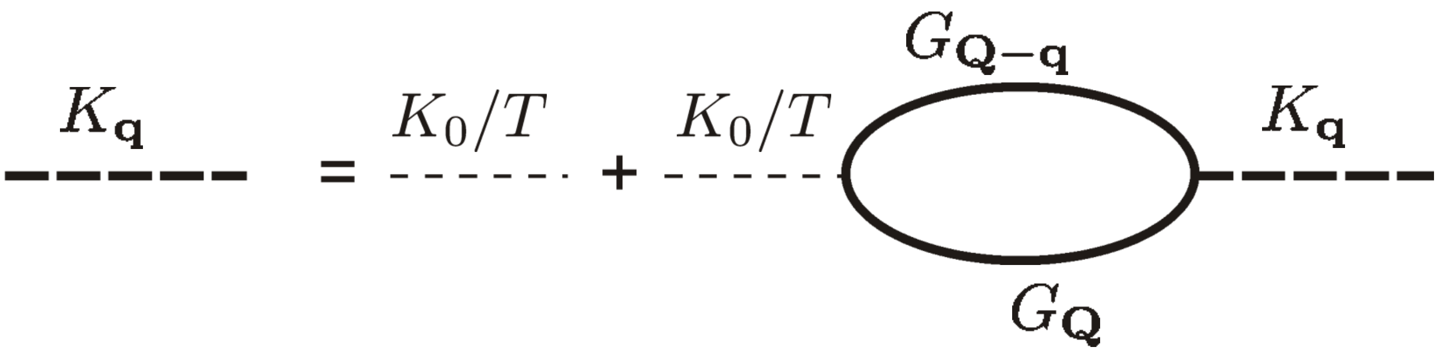
$$G^0_{\mathbf{k}} = \frac{T}{\varkappa k^4}$$

Interaction between out-of-plane modes



Self-Consistent Screening Approximation





$$G_{\mathbf{q}} = \frac{T}{\varkappa q^4 + \Sigma_{\mathbf{q}}}$$

$$\Sigma_{\mathbf{q}} = \frac{2T}{d_c} \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{Q^4} K_{\mathbf{Q}} G_{\mathbf{q}-\mathbf{Q}}$$

Self-energy

$$K_{\mathbf{q}} = \frac{(K_0/T)}{1 + (K_0/T)\Pi_{\mathbf{q}}}$$

$$\Pi_{\mathbf{q}} = \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{q^4} G_{\mathbf{Q}-\mathbf{q}} G_{\mathbf{Q}}$$

Polarization operator

$$d_c = (d - D) \to \infty$$

Weak "anticrumpling" regime: $q_* e^{-d/2} \ll q \ll q_*$

$$q_* = \sqrt{\frac{K_0 T}{\varkappa^2}}$$ **ultraviolet cutoff**

$$\Pi^0_{\mathbf{q}} = \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{q^4} G^0_{\mathbf{Q}-\mathbf{q}} G^0_{\mathbf{Q}} = \int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{q^4} \frac{T}{\varkappa|\mathbf{Q}-\mathbf{q}|^4} \frac{T}{\varkappa Q^4} = \frac{3}{16\pi}\left(\frac{T}{\varkappa}\right)^2 \frac{1}{q^2}$$

$$q \ll q_* \;\Rightarrow\; (K_0/T)\Pi^0_{\mathbf{q}} \gg 1 \;\Rightarrow K_{\mathbf{q}} \approx \frac{1}{\Pi^0_{\mathbf{q}}} = \frac{16\pi}{3}\left(\frac{\varkappa}{T}\right)^2 q^2$$

$$\Sigma_{\mathbf{q}} = \frac{2T}{d_c}\int \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{Q^4} K_{\mathbf{Q}} G^0_{\mathbf{q}-\mathbf{Q}} \approx \frac{32\pi\varkappa}{3d_c}\int_{Q<q_*} \frac{d^2\mathbf{Q}}{(2\pi)^2} \frac{[\mathbf{q}\times\mathbf{Q}]^4}{Q^2|\mathbf{q}-\mathbf{Q}|^4} \approx \varkappa q^4 \frac{2}{d}\ln\left(\frac{q_*}{q}\right)$$

$$\delta\varkappa = \varkappa\frac{2}{d}\ln\left(\frac{q_*}{q}\right) \Rightarrow \frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa$$

Anharmonicity-induced increase of the bending rigidity

Crumpling transition for $d \to \infty$

$$\frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa$$

$$\frac{d\xi^2}{d\Lambda} = -\frac{T}{4\pi\varkappa}$$

$$\tilde{\varkappa} = \varkappa\xi^2$$

rescaled bending rigidity

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \frac{2\tilde{\varkappa}}{d} - \frac{T}{4\pi}$$

$$\tilde{\varkappa}_{cr} = \frac{d\,T}{8\pi}$$

unstable fixed point

agrees with David, Guitter, Europhys. Lett. (1988),

$$\xi_\infty^2 = \xi_0^2\,\frac{\tilde{\varkappa}_0 - \tilde{\varkappa}_{cr}}{\tilde{\varkappa}_{cr}}$$

For $\tilde{\varkappa}_0 > \tilde{\varkappa}_{cr}$, membrane remains in the flat phase in the course of renormalization

Renormalization of disorder

$$F(\mathbf{h}) = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left[\varkappa q^4 \mathbf{h}_\mathbf{q}\mathbf{h}_{-\mathbf{q}} + \frac{1}{4d_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} \times R(\mathbf{k},\mathbf{k}',\mathbf{q})(\mathbf{h}_{-\mathbf{k}}\mathbf{h}_{\mathbf{k}+\mathbf{q}})(\mathbf{h}_{\mathbf{k}'}\mathbf{h}_{-\mathbf{q}-\mathbf{k}'})\right]$$

add disorder, replicate and average over disorder

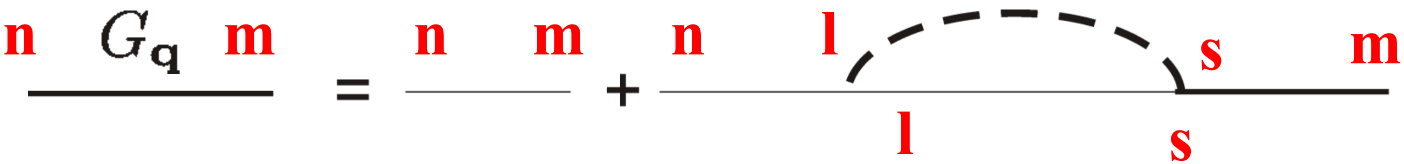
$$F(\mathbf{h}) = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\left[q^4 \sum_{n,m=1}^{N} \mathbf{h}^n_\mathbf{q}\mathbf{h}^m_{-\mathbf{q}}\left(\varkappa\delta_{nm} - \frac{B\varkappa^2}{T}\right) + \frac{1}{4d_c}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} R(\mathbf{k},\mathbf{k}',\mathbf{q})\sum_{n=1}^{N}(\mathbf{h}^n_{-\mathbf{k}}\mathbf{h}^n_{\mathbf{k}+\mathbf{q}})(\mathbf{h}^n_{\mathbf{k}'}\mathbf{h}^n_{-\mathbf{q}-\mathbf{k}'})\right]$$

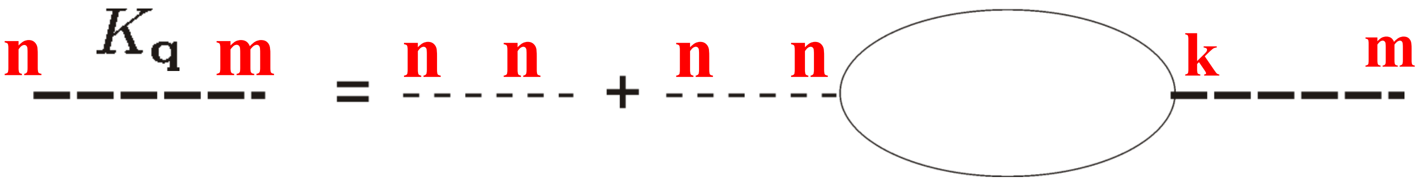
$$\varkappa \longrightarrow \hat{\varkappa} : \quad \varkappa_{nm} = \varkappa\delta_{nm} - \frac{B\varkappa^2}{T}$$

matrix in the replica space

$$\hat{G}^0_{\mathbf{q}} = \frac{T}{\hat{\varkappa} q^4} \to \frac{T}{\varkappa q^4}\left(\delta_{nm} + \alpha\right)$$

$$\alpha = \frac{B\varkappa/T}{1 - NB\varkappa/T}$$





$$\Pi^{nm}_{\mathbf{q}} = \frac{3}{16\pi}\left(\frac{T}{\varkappa}\right)^2 \frac{1}{q^2}\left[(1+2\alpha)\delta_{nm} + \alpha^2\right]$$

$$q \ll q_* \Rightarrow \hat{K}_{\mathbf{q}} = \hat{\Pi}^{-1}_{\mathbf{q}}$$

$$K^{nm}_{\mathbf{q}} = \frac{16\pi}{3}\left(\frac{\varkappa}{T}\right)^2 q^2 \frac{(1+2\alpha+\alpha^2 N)\delta_{nm} - \alpha^2}{(1+2\alpha)(1+2\alpha+\alpha^2 N)}$$

$$\Sigma^{nm}_{\mathbf{q}} = \varkappa q^4 \frac{2}{d} \ln\left(\frac{q_*}{q}\right) \frac{\delta_{nm}[1+3\alpha+\alpha^2(N+1)+\alpha^3 N] - \alpha^3}{(1+2\alpha)(1+2\alpha+\alpha^2 N)}$$

$$\frac{d\left(\varkappa\delta_{nm} - \frac{B\varkappa^2}{T}\right)}{d\Lambda} = \frac{2}{d}\varkappa \frac{\delta_{nm}[1+3\alpha+\alpha^2(N+1)+\alpha^3 N] - \alpha^3}{(1+2\alpha)(1+2\alpha+\alpha^2 N)}$$

RG equations

$$\frac{d\varkappa}{d\Lambda} = \frac{2}{d}\varkappa\frac{1+3B\varkappa/T+B^2\varkappa^2/T^2}{(1+2B\varkappa/T)^2}$$

$$\frac{d}{d\Lambda}\left(\frac{B\varkappa^2}{T}\right) = \frac{2}{d}\varkappa\frac{(B\varkappa/T)^3}{(1+2B\varkappa/T)^2}$$

Rescaled parameters

$$\tilde{\varkappa} = \varkappa\xi^2 \qquad F = \frac{B\varkappa^2\xi^2}{T}$$

$$\Rightarrow \quad f = \frac{F}{\tilde{\varkappa}}$$

$$\frac{df}{d\Lambda} = -\frac{2}{d}\frac{f(1+3f)}{(1+2f)^2}$$

$$\frac{d\tilde{\varkappa}}{d\Lambda} = \frac{2}{d}\tilde{\varkappa}\frac{(1+3f+f^2)}{(1+2f)^2} - \frac{T}{4\pi}$$

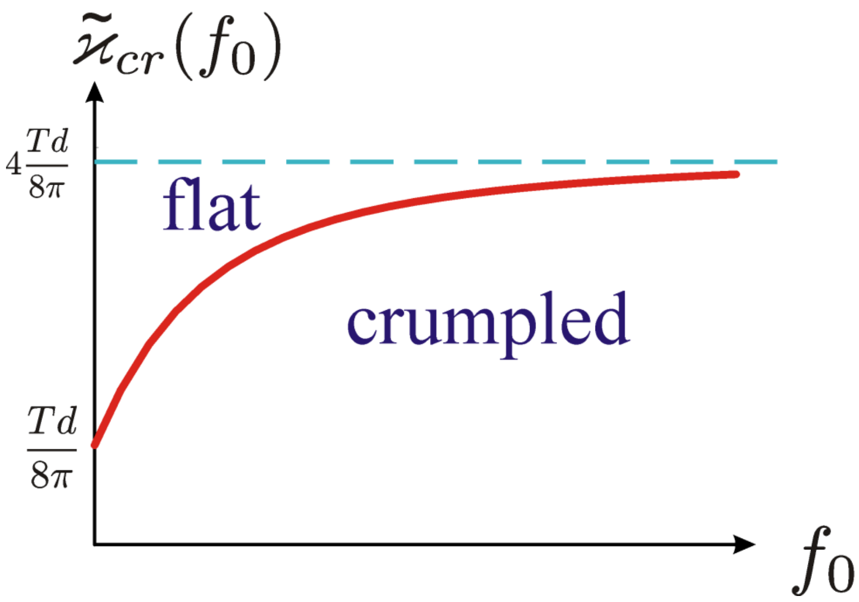
$$\frac{d\xi^2}{d\Lambda} = -\frac{\xi^2(1+f)T}{4\pi\tilde{\varkappa}}$$

$$\Lambda \to \infty \quad \Rightarrow \quad \begin{cases} f \propto \exp\left(-\frac{2}{d}\Lambda\right) \\ \tilde{\varkappa} \propto \exp\left(\frac{2}{d}\Lambda\right) \end{cases}$$

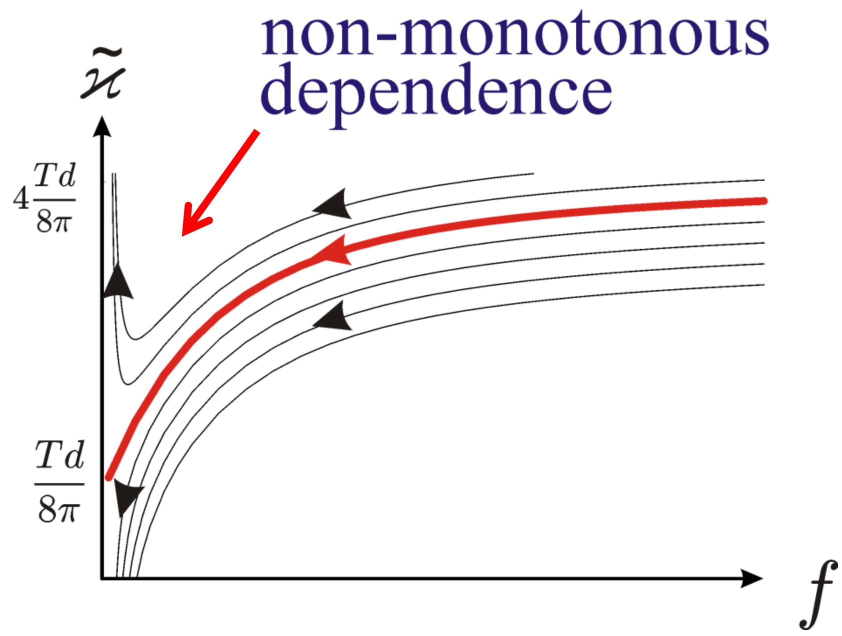
$$F = f\tilde{\varkappa} \quad \to \quad \text{const}$$

Results:

Critical bending rigidity becomes disorder dependent



Non-monotonous scaling of bending rigidity



$$\tilde{\varkappa}_{cr}(f_0) = \frac{Td}{8\pi} \int_0^{f_0} \frac{df}{f} \frac{(1+2f)^2}{1+3f} \exp\left(- \int_f^{f_0} \frac{df'}{f'} \frac{1+3f'+f'^2}{1+3f'} \right)$$

Rescaled disorder strength increases exponentially and then saturates

$$\frac{F_\infty}{F_0} \sim e^{f_0/3}$$

similar result for D=4:
Morse, Lubensky, Grest,
PRA 1992

$$\frac{df}{d\Lambda} = -\frac{2}{d}\frac{f(1+3f)}{(1+2f)^2} \quad \Rightarrow \quad f = f_0 - \frac{3}{2d}\ln\left(Lq_*\right), \ \text{for f} \gg 1$$

⇒ characteristic scale: $L_0 \sim q_*^{-1} e^{2df_0/3}$ ripple size???

Disorder generates new correlation functions

Conventional correlation function

$$\overline{\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle} \propto \frac{1}{q^{4-2/d}} \quad \Rightarrow \quad h_{\text{rms}} \propto L^{1-1/d}$$

flat phase

Disorder-induced correlation function

$$\overline{\langle h_{\mathbf{q}} \rangle \langle h_{-\mathbf{q}} \rangle} \propto \frac{1}{q^{4-4/d}} \quad \Rightarrow \quad \tilde{h}_{\text{rms}} \propto L^{1-2/d}$$

Self consistent screening approximation (SCSA)

(similar to SCBA in the theory of disordered systems)

P. Le Doussal, L. Radzihovsky, PRL (1992)

$$G(\mathbf{q}) = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = \frac{T}{\varkappa q^4 + \Sigma(q)} \quad \Rightarrow \quad G(q) \propto \frac{1}{q^{4-\eta}}$$

Σ(q) is self-energy which should be found self-consistently with the Green function

η is critical exponent

SCSA (D=3): $\eta \approx 0.82$

numerical simulations: $\eta \approx 0.7\text{–}0.8$

Renormalization of bending rigidity

$$\varkappa \rightarrow \varkappa(q) \sim \varkappa \left(\frac{q_c}{q}\right)^{\eta}$$

Physics behind: anharmonic coupling with in-plane modes

$$G_q = \langle h_{\mathbf{q}} h^*_{\mathbf{q}} \rangle = Z \frac{T}{\varkappa q^4} \left(\frac{q}{q_c}\right)^{\eta}, \quad \text{for } q \ll q_c$$

P. Le Doussal, L. Radzihovsky, PRL (1992)

Z≈ 3.5, K. V. Zakharchenko *et al*, PRB (2010)

$$q_c = \frac{\sqrt{T\Delta_c}}{\hbar v}, \quad \Delta_c = \frac{3\mu v^2(\mu+\lambda)\hbar^2}{4\pi\varkappa^2(2\mu+\lambda)} \simeq 18.7 \text{ eV}.$$

In the Dirac point: $q \sim T/\hbar v$

$$q \ll q_c \longleftrightarrow T \ll \Delta_c$$

For all realistic temperatures anharmonic coupling is important !!!

For graphene $\kappa/T \approx 30$ even for T=300 K ➔ flat phase

Bending rigidity increases with increasing the system size (or decreasing the wave vector) :

$$\kappa \sim L^{\eta}, \quad q^{-\eta}$$

η - critical exponent

$$\beta \to \eta, \quad \text{for} \quad \kappa/T \to \infty$$

P. Le Doussal and L. Radzihovsky, PRL (1992)

$$\frac{h}{L} \sim \frac{1}{L^{\eta/2}}$$

in the thermodynamic limit fluctuations are suppressed

$$\omega \sim q^{2-\eta/2}$$

dispersion is modified