Two types of odd-frequency superconductivity:

odd-frequency-diamagnetic

vs.

odd-frequency-paramagnetic superconductivity

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Superconductivity

$G \sim \langle \psi \psi^\dagger \rangle$ – ordinary Green function

$F \sim \langle \psi \psi \rangle$ – anomalous Green function (wave function of Cooper pairs)

$F^+ \sim \langle \psi^\dagger \psi^\dagger \rangle$

$$\hat{G} = \begin{pmatrix} G & F \\ F^+ & G' \end{pmatrix}$$ – Nambu space

Superconductivity = non-diagonal elements in the Nambu space ($\hat{\tau}_1, \hat{\tau}_2$ components)

Simplest case: singlet and s-wave (isotropic)

More complicated: triplet and/or anisotropic (p-wave, d-wave, etc.)

Odd-frequency superconductivity

$F(1;2)$ – anomalous Green function (Matsubara technique)

$F(2;1) = -F(1;2)$ – fermionic antisymmetry (the Pauli principle)

Standard classification of superconducting phases:

even coordinate dependence (s-wave, d-wave, etc.),
odd spin dependence (singlet: $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$)

OR

odd coordinate dependence (p-wave, etc.),
even spin dependence (triplet: $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$)

Berezinskii (1974):
if the imaginary time dependence is odd, then there is a possibility of even coordinate dependence and even spin dependence

$$\tau = \tau_1 - \tau_2 \quad \rightarrow \quad \omega$$

Odd- ω superconductivity! (Example: odd-ω, s-wave, spin-triplet)

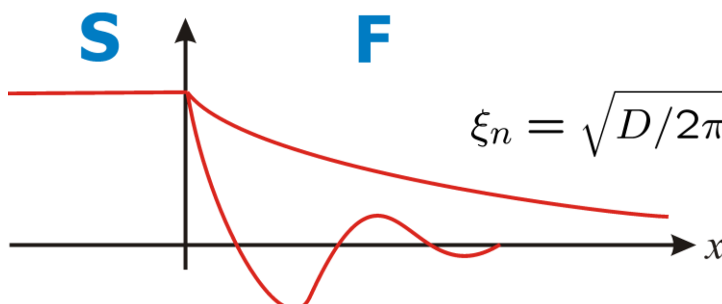
Odd-ω superconductivity

- Berezinskii (1974) [JETP Lett. **20**, 287]:
Possibility of odd-ω pairing in ^{3}He due to retarded paramagnon exchange.
Not realized.

- Abrahams, Balatsky, Scalapino, Schrieffer (1995) [PRB **52**, 1271]:
Studies of bulk properties of hypothetical Berezinskii superconductor.

- Bergeret, Volkov, Efetov (2001) [review – RMP **77**, 1321 (2005)]:
Generation of the Berezinskii component in proximity structures
conventional superconductor / ferromagnet.

Can be spatially separated (noncollinear exchange fields in F):



$$\xi_n = \sqrt{D/2\pi T} \quad \left(\text{for} \quad \begin{matrix} \psi_{t,1} = |\uparrow\uparrow\rangle \\ \psi_{t,-1} = |\downarrow\downarrow\rangle \end{matrix}\right)$$

$$\xi_h = \sqrt{D/h} \quad \left(\text{for } \psi_s = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,,\ \psi_{t,0} = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

- Tanaka, Golubov (2007) [PRL **98**, 037003]:
Generation of the Berezinskii component in proximity structures
triplet superconductor / diffusive normal metal.

Odd-ω superconductivity: dia and para

- Negative superfluid density ("paramagnetic" Meissner effect):

$$\mathbf{j}_S(q) = -\frac{e^2 n_S}{mc}\mathbf{A}(q), \qquad n_S < 0$$

Ok in inhomogeneous proximity systems (except for some special thin-film structures).
But instability in the bulk…

Recent proposals:

- Solenov, Martin, Mozyrsky, Phys. Rev. B **79**, 132502 (2009);
- Kusunose, Fuseya, Miyake, J. Phys. Soc. Jpn. **80**, 054702 (2011); 044711 (2011);
- Matsumoto, Koga, Kusunose, J. Phys. Soc. Jpn. **81**, 033702 (2012);
- Kusunose, Matsumoto, Koga, Phys. Rev. B **85**, 174528 (2012);
- and more:

Model with a retarded interaction, mean-field analysis.
Possibility of a bulk odd-ω–dia phase with $n_s > 0$ and regular diamagnetic Meissner effect in the case when Δ is odd-ω.

General derivation

$$\mathcal{Z} = \int D\psi_\uparrow^* D\psi_\downarrow^* D\psi_\uparrow D\psi_\downarrow \exp(-\mathcal{S}_0 - \mathcal{S}_{\rm int})$$

$$\mathcal{S}_0 = \int_1 \psi_\alpha^*(1)\,(\partial_{\tau_1} + \xi)\,\psi_\alpha(1)$$

$$\mathcal{S}_{\rm int} = \frac{1}{2}\int_1\int_2 V_{\alpha\beta;\gamma\delta}(1-2)\psi_\beta^*(2)\psi_\alpha^*(1)\psi_\gamma(1)\psi_\delta(2)$$

where 1 and 2 are 4-vectors:

$$1 \equiv (\mathbf{r}_1, \tau_1), \quad 2 \equiv (\mathbf{r}_2, \tau_2)$$

Interaction is even with respect to interchanging of two electrons:

$$V_{\alpha\beta;\gamma\delta}(1-2) = V_{\beta\alpha;\delta\gamma}(2-1)$$

Hubbard-Stratonovich transformation:

$$\exp(-\mathcal{S}_{\rm int}) \times \int D\Delta^* D\Delta \exp\left\{\frac{1}{2}\int_1\int_2 \left[V^{-1}(1-2)\right]_{\alpha\beta;\gamma\delta} \Delta_{\alpha\beta}^*(1,2)\Delta_{\gamma\delta}(1,2)\right\}$$

$$\mapsto \int D\Delta^* D\Delta \exp(-\mathcal{S}_{\rm aux} - \mathcal{S}_\Delta)$$

with

$$\mathcal{S}_{\rm aux} = -\frac{1}{2}\int_1\int_2 \left[\Delta_{\alpha\beta}(1,2)\psi_\beta^*(2)\psi_\alpha^*(1) + \Delta_{\alpha\beta}^*(1,2)\psi_\alpha(1)\psi_\beta(2)\right]$$

$$\mathcal{S}_\Delta = -\frac{1}{2}\int_1\int_2 \left[V^{-1}(1-2)\right]_{\alpha\beta;\gamma\delta} \Delta_{\alpha\beta}^*(1,2)\Delta_{\gamma\delta}(1,2)$$

V – attraction, the integral must be convergent (so the sign is fixed)

Mean-field

Mean-field: take a trial path for Δ* and Δ, minimize the free energy

$$\mathcal{F}_{\mathrm{MF}}[\Delta^*, \Delta] = -T \ln \mathcal{Z}_{\mathrm{MF}} = -T \ln \int D\psi^* D\psi \exp\left(-\mathcal{S}_{\mathrm{MF}}[\psi^*, \psi, \Delta^*, \Delta]\right)$$

with respect to the trial path.

Define anomalous averages:

$$F_{\alpha\beta}(1,2) = \langle \psi_\alpha(1)\psi_\beta(2) \rangle_{\mathrm{MF}}$$
$$F^+_{\alpha\beta}(1,2) = \left\langle \psi^*_\alpha(1)\psi^*_\beta(2) \right\rangle_{\mathrm{MF}}$$

From definition:

$$F^*_{\alpha\beta}(\mathbf{q}, \omega) = s_\Delta F^+_{\beta\alpha}(\mathbf{q}, -\omega)$$

where $s_\Delta = \pm 1$ for the even-/odd-ω dependence of Δ

$F^*_{\alpha\beta}(\mathbf{q}, \omega) = +F^+_{\beta\alpha}(\mathbf{q}, -\omega)$ - induced odd-ω—para state

$F^*_{\alpha\beta}(\mathbf{q}, \omega) = -F^+_{\beta\alpha}(\mathbf{q}, -\omega)$ - principal odd-ω—dia state

Induced odd-ω—para state: frequency symmetries of F and Δ are different
(ok since the odd-ω component drops out from the self-consistency equation.

General homogeneous solution

Self-consistency equation: $\Delta_{\alpha\beta}(k) = -\int (dk')V_{\alpha\beta;\gamma\delta}(k-k')F_{\gamma\delta}(k')$

Components:

$$\Delta_{\alpha\beta}(k) = d_0(k)(i\sigma_2)_{\alpha\beta} + \mathbf{d}(k)(i\boldsymbol{\sigma}\sigma_2)_{\alpha\beta}$$

$$G_{\alpha\beta}(k) = G_0(k)\delta_{\alpha\beta} + \mathbf{G}(k)\boldsymbol{\sigma}_{\alpha\beta}$$

$$F_{\alpha\beta}(k) = F_0(k)(i\sigma_2)_{\alpha\beta} + \mathbf{F}(k)(i\boldsymbol{\sigma}\sigma_2)_{\alpha\beta}$$

where

$$G_0(k) = -\frac{(i\omega+\xi_{\mathbf{k}})[\omega^2+\xi_{\mathbf{k}}^2+D_0(k)]}{\left[\omega^2+E_+^2(k)\right]\left[\omega^2+E_-^2(k)\right]}$$

$$\mathbf{G}(k) = \frac{(i\omega+\xi_{\mathbf{k}})\mathbf{D}(k)}{\left[\omega^2+E_+^2(k)\right]\left[\omega^2+E_-^2(k)\right]}$$

$$F_0(k) = \frac{(\omega^2+\xi_{\mathbf{k}}^2)d_0(k)+[d_0^2(k)-\mathbf{d}^2(k)]d_0^*(k)}{\left[\omega^2+E_+^2(k)\right]\left[\omega^2+E_-^2(k)\right]}$$

$$\mathbf{F}(k) = \frac{(\omega^2+\xi_{\mathbf{k}}^2)\mathbf{d}(k)-[d_0^2(k)-\mathbf{d}^2(k)]\mathbf{d}^*(k)}{\left[\omega^2+E_+^2(k)\right]\left[\omega^2+E_-^2(k)\right]}$$

with

$$E_\pm(k) = \sqrt{\xi_{\mathbf{k}}^2+D_0(k)\pm D(k)}$$

$$D_0(k) = d_0^*(k)d_0(k)+\mathbf{d}^*(k)\mathbf{d}(k)$$

$$\mathbf{D}(k) = d_0(k)\mathbf{d}^*(k)+d_0^*(k)\mathbf{d}(k) + i[\mathbf{d}(k)\times\mathbf{d}^*(k)]$$

$$D(k) = \sqrt{\mathbf{D}^2(k)}$$

Simplified examples:

- singlet state, $\mathbf{d}=0$
- triplet state, $d_0 = 0$
- unitary triplet state, $d_0 = 0, \quad [\mathbf{d}\times\mathbf{d}^*]=0$

Superconducting components induced due to spatial inhomogeneity

A single superconducting component.

Then Eilenberger equations:

$$v_F \hat{\boldsymbol{k}} \cdot \nabla g = 2\Delta f_{\mathrm{S}}$$

$$v_F \hat{\boldsymbol{k}} \cdot \nabla f_{\mathrm{B}} = -2\omega_n f_{\mathrm{S}}$$

$$v_F \hat{\boldsymbol{k}} \cdot \nabla f_{\mathrm{S}} = 2(\Delta g - \omega_n f_{\mathrm{B}})$$

$$f_{\mathrm{B}} = \frac{f(\hat{\boldsymbol{k}}) + s_p f(-\hat{\boldsymbol{k}})}{2}$$ - bulk component

$$f_{\mathrm{S}} = \frac{f(\hat{\boldsymbol{k}}) - s_p f(-\hat{\boldsymbol{k}})}{2}$$ - surface component

s_p - (spatial) parity of the superconducting state

f_{B} has the same symmetries as Δ.

f_{S} – opposite frequency symmetry and parity with respect to Δ.

Classification

Frequency	Spin	Parity	Magnetic response	
(a) Even	Singlet	Even	Dia	Bulk
Odd	Singlet	Odd	Para	Induced
(b) Even	Triplet	Odd	Dia	Bulk
Odd	Triplet	Even	Para	Induced
(c) Odd	Singlet	Odd	Dia	Bulk
Even	Singlet	Even	Para	Induced
(d) Odd	Triplet	Even	Dia	Bulk
Even	Triplet	Odd	Para	Induced

mix due to spin-symm. breaking [(a)]

mix due to spin-symm. breaking [(c), (d)]

the two classes do not intermix

Asano, Fominov, Tanaka (2014) [PRB **90**, 094512]

The classification of Cooper pairs in inhomogeneous superconductors. In the absence of spin-dependent potentials, the spin state in the bulk and near an inhomogeneity is the same. At the same time, due to broken translational invariance, the spatial parity can change. This leads to changing of the frequency symmetry, in order to conform with the Pauli principle. ESED states realized in metallic superconductors and high-T_c cuprates in (a) have OSOP states as the subdominant component. ETOD states realized in Sr_2RuO_4 and UPt_3 in (b) have OTEP states as the subdominant component. OSOD states in (c) and OTED states in (d) have never been confirmed in real materials. The subdominant component of OSOD states in (c) is ESEP states. ETOP states appears as a subdominant component of OTED states in (d).

Odd-ω–dia and –para: problem of coexistence

Coexistence of dia and para states:

$$G_{\alpha\beta} = (G_d)_{\alpha\beta} + (G_p)_{\alpha\beta}$$
$$F_{\alpha\beta} = (F_d)_{\alpha\beta} + (F_p)_{\alpha\beta}$$
$$F^+_{\alpha\beta} = (F^+_d)_{\alpha\beta} + (F^+_p)_{\alpha\beta}$$

Unitary pairing:

$$G_{0d} = -\frac{i\omega + \xi}{\omega^2 + \xi^2 + D_{0d}}, \quad G_{0p} = -\frac{i\omega + \xi}{\omega^2 + \xi^2 - D_{0p}}$$
$$\mathbf{F}_d = \frac{\mathbf{d}_d}{\omega^2 + \xi^2 + D_{0d}}, \quad \mathbf{F}_p = \frac{\mathbf{d}_p}{\omega^2 + \xi^2 - D_{0p}}$$

Cross-term in the superfluid density:

$$\frac{\delta n_S}{n} = \pi T \sum_\omega \frac{1}{D_{0d} + D_{0p}} \left[\frac{4\omega^2 + 2D_{0d} + \mathbf{d}_d \mathbf{d}_p^* - \mathbf{d}_d^* \mathbf{d}_p}{\sqrt{\omega^2 + D_{0d}}} - \frac{4\omega^2 - 2D_{0p} + \mathbf{d}_d \mathbf{d}_p^* - \mathbf{d}_d^* \mathbf{d}_p}{\sqrt{\omega^2 - D_{0p}}} \right]$$

Complex!!!

Odd-ω–dia and –para: Josephson junction

Tunneling action (the case of unitary pairing):

$$\mathcal{S}_T = \frac{\pi G}{2} \sum_{\omega} \frac{-\omega^2 + \tilde{\mathbf{d}}_L \tilde{\mathbf{d}}_R (s_L e^{i\varphi} + s_R e^{-i\varphi})/2}{\sqrt{\omega^2 + s_L \tilde{\mathbf{d}}_L^2}\sqrt{\omega^2 + s_R \tilde{\mathbf{d}}_R^2}}$$

$$\mathbf{d}_{L(R)} = \tilde{\mathbf{d}}_{L(R)} e^{i\varphi_{L(R)}}, \quad \varphi = \varphi_R - \varphi_L$$

Phase-dependent part:

$$\frac{s_L e^{i\varphi} + s_R e^{-i\varphi}}{2} = \begin{cases} -\cos\varphi, & \text{odd-}\omega\text{–para/para} \\ \cos\varphi, & \text{odd-}\omega\text{–dia/dia} \\ -i\sin\varphi, & \text{odd-}\omega\text{–para/dia} \end{cases}$$

$s_{L(R)} = \pm 1$ for odd-ω-dia/-para

Odd-ω–para/dia junction – imaginary Josephson current!!!

Can odd-ω–dia state really exist?

The Green functions in the Hamiltonian formulation:

$$F_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2; \tau_1, \tau_2) = \langle T_\tau \hat{\psi}_\alpha(\mathbf{r}_1, \tau_1) \hat{\psi}_\beta(\mathbf{r}_2, \tau_2) \rangle$$

$$F^+_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2; \tau_1, \tau_2) = \langle T_\tau \hat{\psi}^+_\alpha(\mathbf{r}_1, \tau_1) \hat{\psi}^+_\beta(\mathbf{r}_2, \tau_2) \rangle$$

where $\hat{\psi}_\alpha(\mathbf{r}, \tau) = e^{\hat{H}\tau} \hat{\psi}_\alpha(\mathbf{r}) e^{-\hat{H}\tau}, \quad \hat{\psi}^+_\alpha(\mathbf{r}, \tau) = e^{\hat{H}\tau} \hat{\psi}^\dagger_\alpha(\mathbf{r}) e^{-\hat{H}\tau}$

Immediate consequence: $F^*_{\alpha\beta}(\mathbf{q}, \omega) = +F^+_{\beta\alpha}(\mathbf{q}, -\omega)$

if Hamiltonian description exists at any level (in particular, initial many-body $\hat{H}$)

Hence: odd-ω state must be paramagnetic.

Can odd-ω–dia state really exist?

So, what is wrong in the Lagrangian description?
How could it lead to the opposite sign?

Our guess:

No spontaneous gauge symmetry breaking (that could fix the superconducting phase), due to absence of the corresponding physical perturbation
[note that in the conventional case we could write the hermitian perturbation
$\int d\mathbf{r}\left[\delta_0\hat{\psi}^\dagger_\uparrow(\mathbf{r})\hat{\psi}^\dagger_\downarrow(\mathbf{r})+\delta_0^*\hat{\psi}_\downarrow(\mathbf{r})\hat{\psi}_\uparrow(\mathbf{r})\right]$ with $|\delta_0|\to 0$].

Then no mean-field description.
Phase is not fixed, integration over phase, anomalous averages vanish.

Conclusion

Summary:

- Two closed superconducting classes, containing odd-ω–para or odd-ω–dia states.
- Everything is consistent within each class.
- Unphysical results if their coexistence is assumed.

Our conclusion:

- No spontaneous symmetry breaking for the superconducting class containing odd-ω–dia states, so that the anomalous averages vanish.