Superconductor-Insulator and superconductor-metal transitions

M. V. Feigel'man,   L. D. Landau Institute for Theoretical Physics

1. Introduction. 3 scenario for destruction of superconductivity by disorder
2. Superconductor-metal transitions at T=0
   - Suppression of Tc due to increase of Coulomb repulsion
   - Enchancement of mesoscopic fluctuations near crit point
   - Proximity-coupled array and quantum fluctuations of phases
3. Superconductivity-insulator transitions in homogeneously disordered materials
   - Fractal superconductivity at the mobility edge
   - Pseudo-gaped superconductivity
   - Quantum phase transition between pseudo-gaped superconductor and paired insulator
   - Signatures of the many-body localization

Central Dogma on SIT

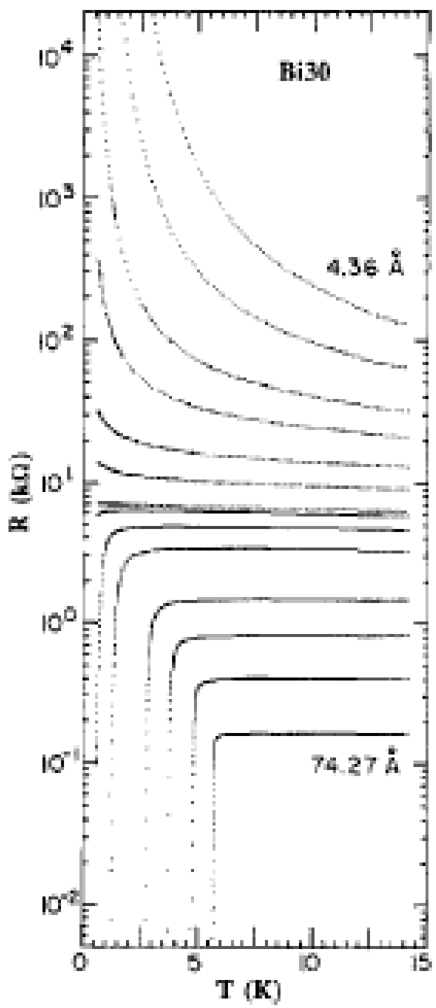
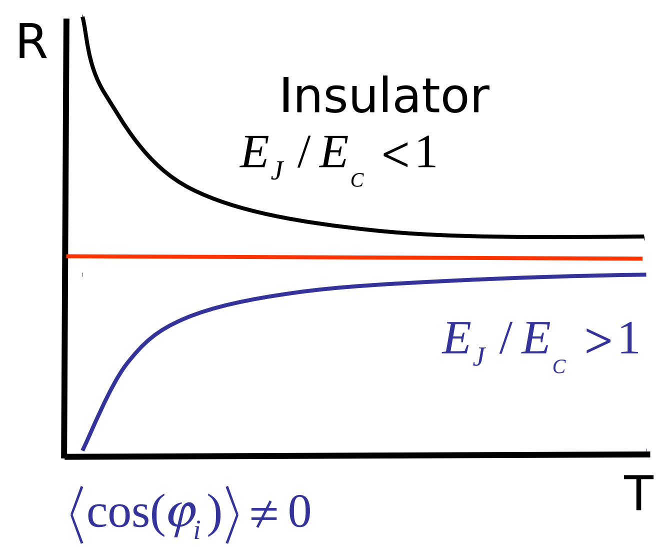


Fig. 5. Temperature dependence of the sheet resistance $R(T)$ for Bi films deposited onto Ge [15]. The films are considered to be homogeneous.

Quantum phase transitions in disordered two-dimensional superconductors
Matthew P. A. Fisher
Phys. Rev. Lett. **65**, 923, 1990

Presence of quantum diffusion in two dimensions: Universal resistance at the superconductor-insulator transition
Matthew P. A. Fisher, G. Grinstein, and S. M. Girvin
Phys. Rev. Lett. **64**, 587, 1990

A.Goldman et al ~1989



Actually, the story is much more complicated

Disordered superconductors (classical results)

- Potential disorder does not affect superconductive transition temperature *(for s-wave)* – *A.A. Abrikosov & L.P.Gor'kov 1958 P.W.Anderson 1959*
- In the "dirty limit" $l << \xi_0$ coherence length decreases as $\xi \sim (l\, \xi_0)^{1/2}$ whereas London length grows as $\lambda \sim l^{-1/2}$

Accuracy limit: semi-classical approx. $k_F l >> 1$

or (the same in another form) $G = \sigma\, (h/e^2)\, \xi^{d-2} >> 1$

What happens if G ~ 1 ?

“Anderson theorem”

$$\Delta(\mathbf{r}) = \int d^d\mathbf{r}'\, K(\mathbf{r},\mathbf{r}')\, \Delta(\mathbf{r}').$$

$$K(\mathbf{r},\mathbf{r}') = \frac{g}{2} \sum_{ij} \eta_{ij}\, \psi_i(\mathbf{r})\psi_j(\mathbf{r})\psi_j(\mathbf{r}')\psi_i(\mathbf{r}'),$$

$$\eta_{ij} = \frac{\tanh(\xi_i/2T) + \tanh(\xi_j/2T)}{\xi_i + \xi_j},$$

Approx. $\Delta(\mathbf{r}) =$ const leads to BCS gap equation

$$1 = g \int N(0)\, (d\xi/\xi)\, \mathrm{th}(\xi/2T)$$

Accuracy limit: semi-classical approx. $k_F l >> 1$

or (the same in another form) $G = \sigma\, (h/e^2)\, \xi^{d-2} >> 1$

What happens if $G \sim 1$?

Superconductivity v/s Localization

Granular systems with Coulomb interaction
K.Efetov *(1980)* M.P.A.Fisher et al *(1990)*
"Bosonic mechanism"
Granular metals or artificial arrays of islands

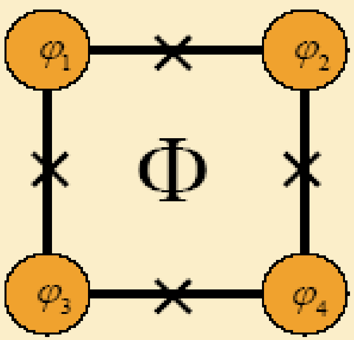
Coulomb-induced suppression of Tc in uniform films
"Fermionic mechanism"
Yu.Ovchinnikov (1973, wrong sign) Mayekawa-Fukuyama (1983) **A.Finkelstein *(1987)*** Yu.Oreg & A. Finkelstein (1999) *Very strongly disordered amorphous metallic alloys* *a*-MoGe, *a*-NbSi, etc

Competition of Cooper pairing and localization (no Coulomb) Imry-Strongin, **Ma-Lee**, Kotliar-Kapitulnik, Bulaevskii-Sadovskii(mid-80's)
Ghosal, Randeria, Trivedi 1998-2001, 2011
Amorphous "poor metals" with low carrier density

Bosonic mechanism

JOSEPHSON ARRAYS

Elementary building block



Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

Control parameter

$x = E_C/E_J$

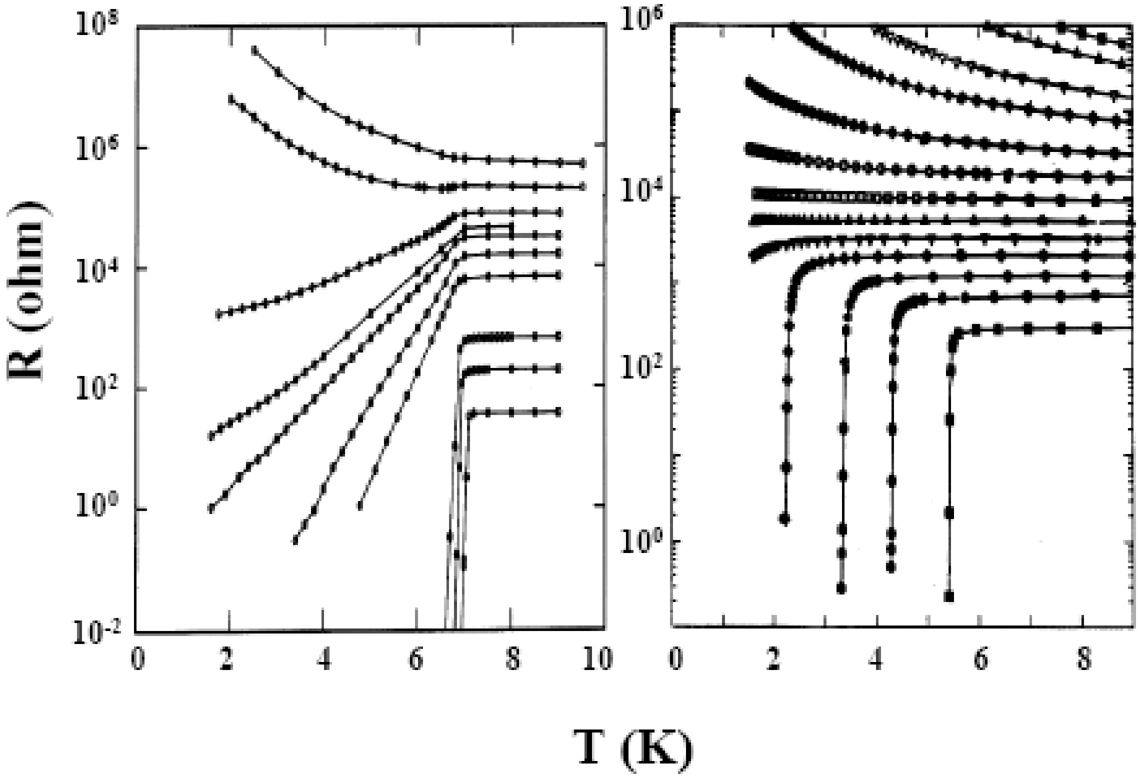
$E_c = e^2/2C$

Artificial arrays: major term in capacitance matrix is n-n capacitance C

- q_i and φ_i are canonically conjugated

Usual SC state with full gap inside each island, irrespectively of the macroscopic state

Granular v/s Amorphous films

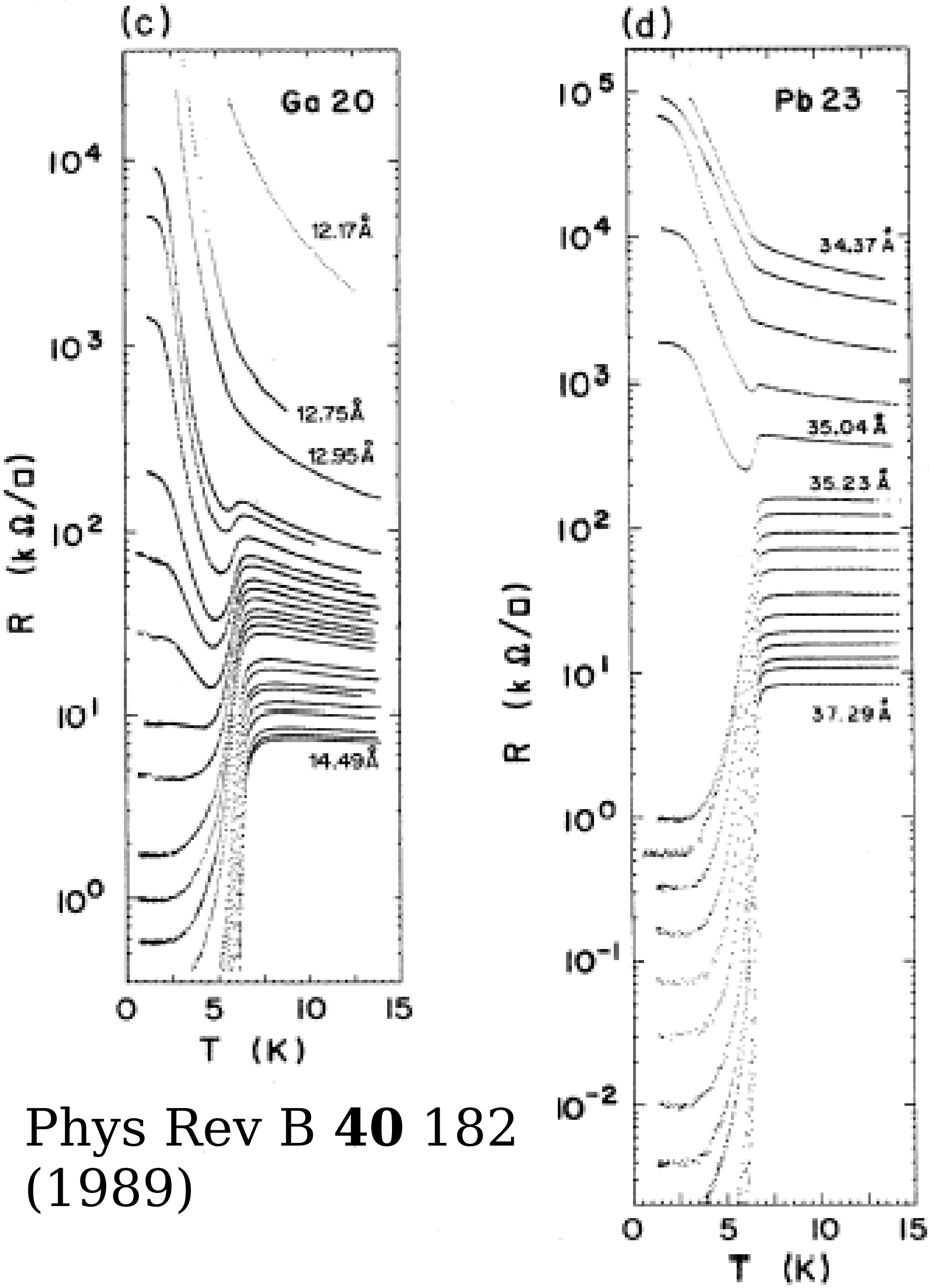


A.Frydman
Physica C
391, 189 (2003)

FIG. 1. Resistance versus temperature for sequential layers of quench-condensed granular Pb (left) and uniform Pb evaporated on a thin Ge layer (right). Different curves correspond to different nominal thickness.

S-I transitions: grains v/s continuous

H. M. Jaeger,* D. B. Haviland, B. G. Orr,† and A. M. Goldman



Phys Rev B **40** 182 (1989)

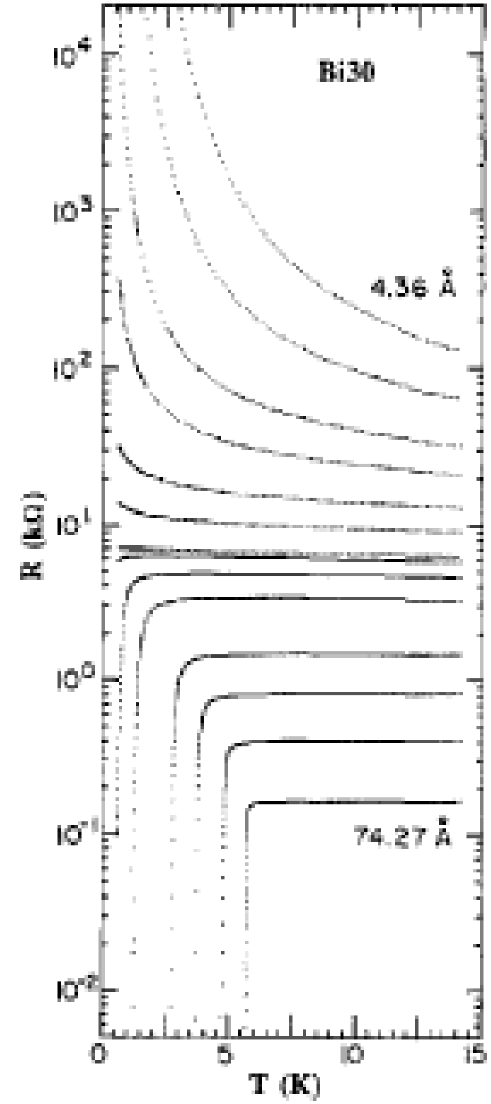


Fig. 5. Temperature dependence of the sheet resistance $R(T)$ for Bi films deposited onto Ge [15]. The films are considered to be homogeneous.

Phys. Rev. Lett. 62, 2180–2183 (1989)

D. B. Haviland, Y. Liu, and A. M. Goldman

Fermionic mechanism: suppression of T_c in amorphous thin films by disorder-enhanced Coulomb interaction

- Theory

S. Maekawa and H. Fukuyama, J. Phys. Soc. Jpn. **51**, 1380 (1982).
H. Takagi and Y. Kuroda, Solid Stat. Comm. **41**, 643 (1982).
A. M. Finkel'stein, Pis'ma ZhETF **45**, 37 (1987), [JETP Lett **45**, 46 (1987)]; A. M. Finkel'stein in *Proc. Int. Symp. on Anderson Localization*, edited by T. Ando and H. Fukuyama (Springer-Verlag,Berlin, 1988), p. 230, Springer Proc. in Physics Vol. 28.

- Experiment

J. M. Graybeal and M. R. Beasly, Phys. Rev. B **29**, 4167 (1984), J. M. Graybeal, M. R. Beasly, and R. L. Green, Physica B+C **126** 731 (1984).

P. Xiong, A. V. Herzog, and R. C. Dynes, Phys. Rev. Lett. **78**, 927 (1997).

- Review: A. M. Finkel'stein, Physica B **197**, 636 (1994).

- Generalization to quasi-1D stripes:
  - Yu. Oreg and A. M. Finkel'stein
  - Phys. Rev. Lett. **83**, 191 (1999)

- Similar approach for 3D poor
- Conductor near Anderson transition:
  - P. W. Anderson, K. A. Muttalib, and
  - T. V. Ramakrishnan,
  - Phys. Rev. B **28**, 117 (1983)

Materials: *a*-MoGe, *a*-NbSi, etc

Fermionic mechanism: qualitative picture

- Disorder increases Coulomb interaction and thus decreases the pairing interaction (sum of Coulomb and phonon attraction). In perturbation theory:

$g = 2\pi \hbar\sigma / e^2$

$$\lambda(\varepsilon) = \lambda_0 - \frac{1}{24\pi g} Log\left(\frac{1}{\varepsilon\tau}\right)$$

Return probability in 2D

Roughly, $$\frac{\delta T_c}{T_c} = -\frac{\delta\lambda}{\lambda^2}$$

It is a "revival" of strong Coulomb repulsion, due to slow diffusion at g ~ 1

Crucial experimental signatures:

1) spectral gap vanishes together with transition temperature T_c

2) non-SC state looks more like a metal than an insulator

Coulomb suppression of T_c

Critical temperature

$$\frac{1}{|\lambda_0|} = \frac{1}{\lambda_g}\tanh(\lambda_g \zeta_c) \Rightarrow T_c\tau = \left(\frac{1 - \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}}{1 + \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}} \right)^{\sqrt{\pi g/2}}$$

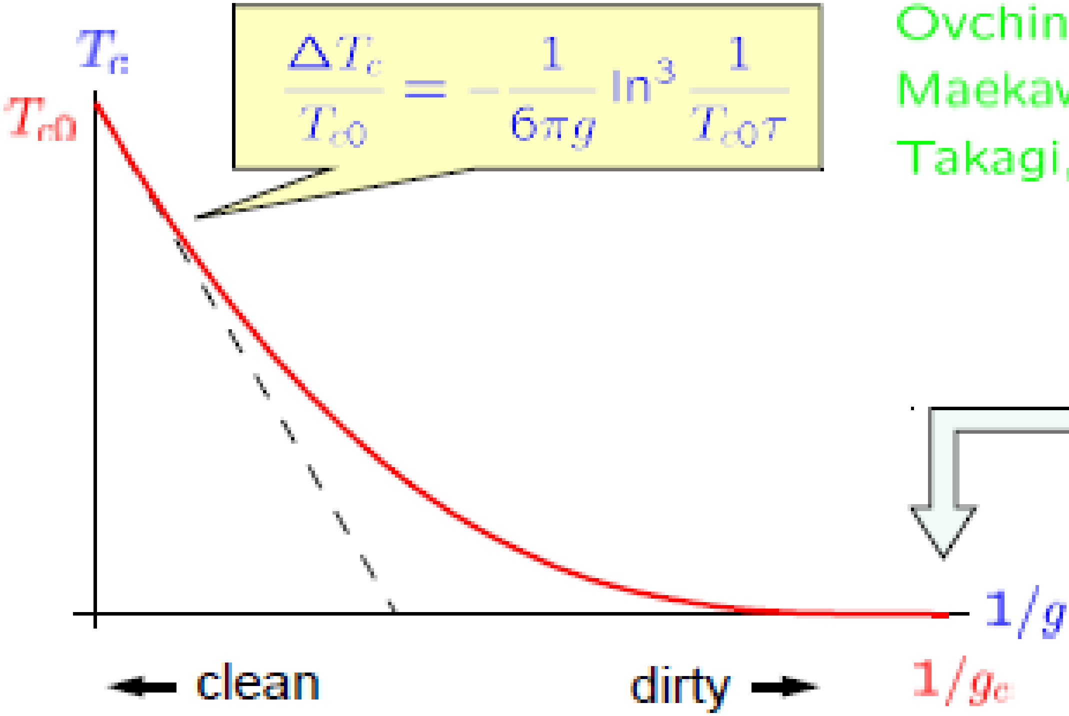
Finkelstein (1987)

$$\frac{\Delta T_c}{T_{c0}} = -\frac{1}{6\pi g} \ln^3 \frac{1}{T_{c0}\tau}$$

Ovchinnikov (1973) **(wrong sign)**

Maekawa, Fukuyama (1982)

Takagi, Kuroda (1982)



$$g_c = \frac{1}{2\pi} \ln^2 \frac{1}{T_{c0}\tau}$$

At $\ln(1/T_{c0}\tau) > 5$

$g_c > 4$ i.e. $R_c < R_Q$

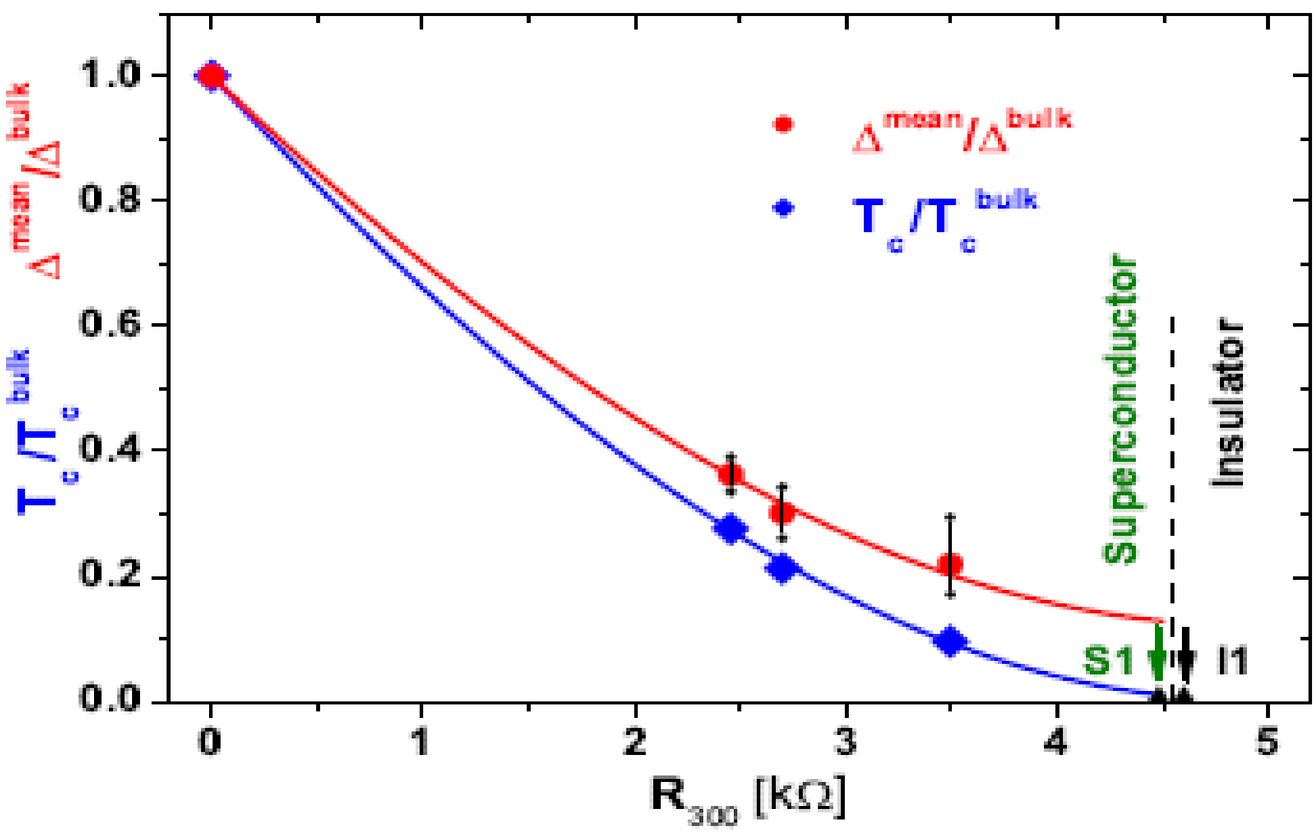
Conclusion:

Superconductor – Metal

Quantum phase transition

Disorder-Induced Inhomogeneities of the Superconducting State Close to the Superconductor-Insulator Transition

B. Sacépé,[1] C. Chapelier,[1] T.I. Baturina,[2] V.M. Vinokur,[3] M.R. Baklanov,[4] and M. Sanquer[1]



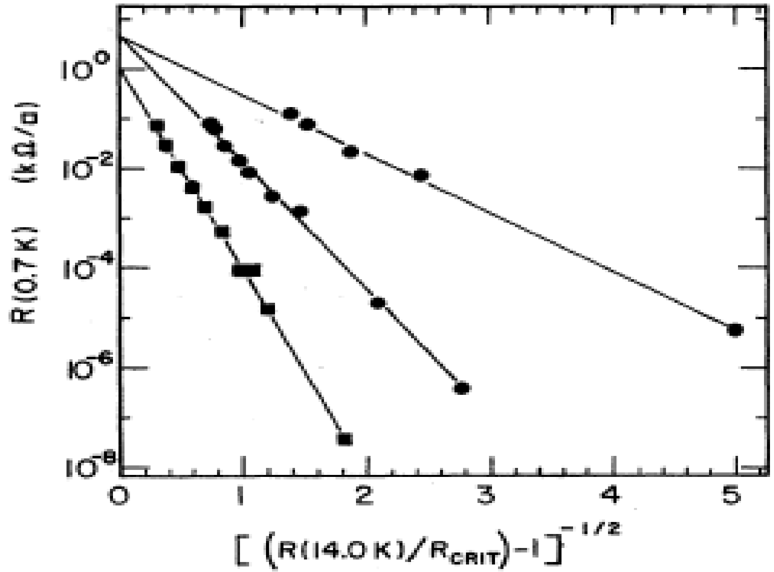
What is the nature of the state on the other side of the T=0 transition ?

Experimental answer: it is a metallic state
with a relatively low resistivity
(sometimes much below its “normal-state” value)

Theoretical answer is unknown.

Onset of superconductivity in ultrathin granular metal films

H. M. Jaeger,* D. B. Haviland, B. G. Orr,† and A. M. Goldman

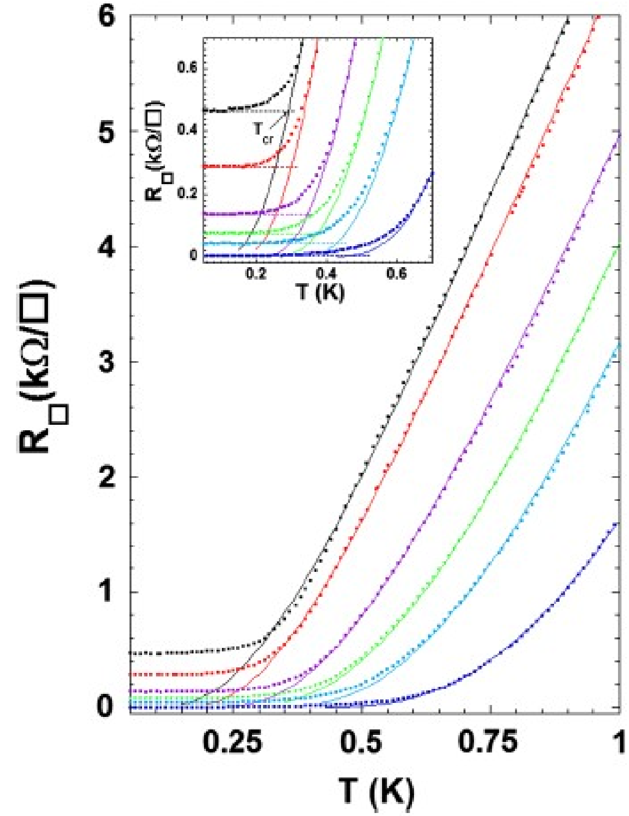


Data for Pb (■) and Ga (●)

$$R(T \to 0) = A \exp[-B(R_N/R_{\text{crit}} - 1)^{1/2}] ,$$

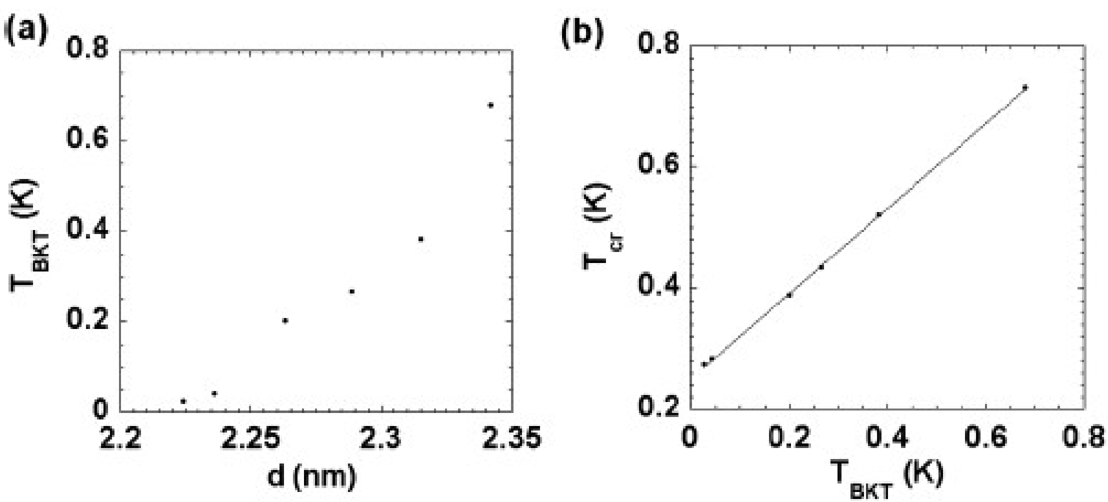
An attempt to use BKT-like analysis

$$R(T) = R_0 \exp[-b/(T - T_{\text{BKT}})^{1/2}],$$



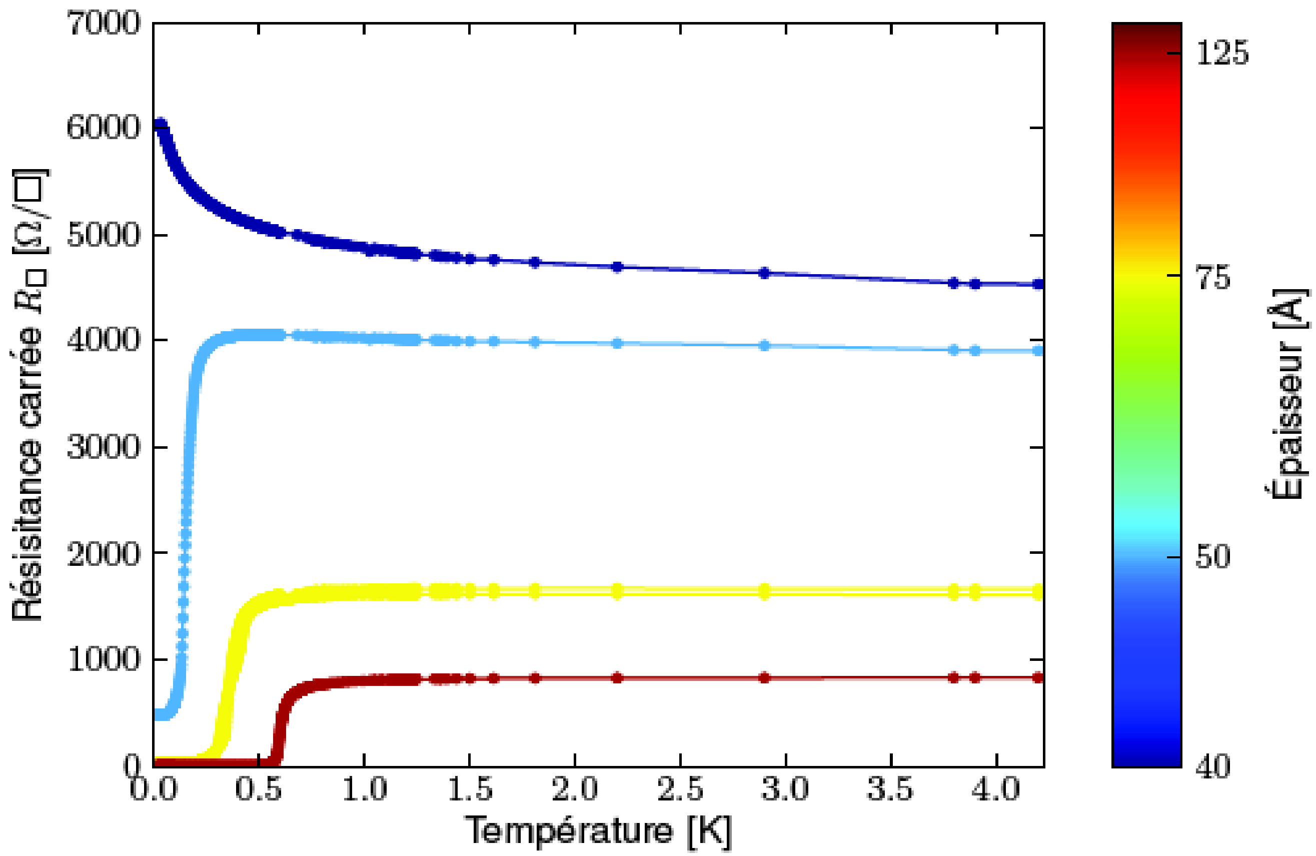
Yen-Hsiang Lin, J. Nelson, and A. M. Goldman
PRL **109,** 017002 (2012)

Ultrathin Bi films on Sb



Nb_xSi_{1-x} thin films

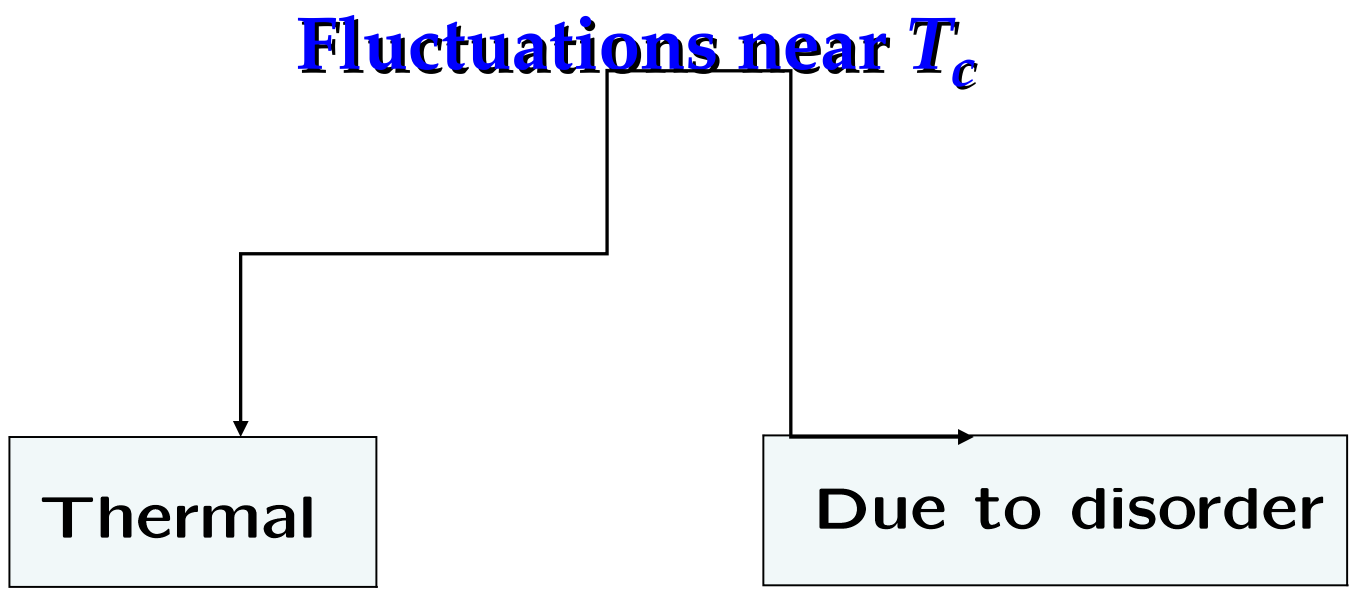
Olivier Crauste THÈSE l'Université Paris – Sud XI (2010)



(b) $x = 17\%$

2) Mesoscopic fluctuations of T_c near the Quantum S-M Phase Transition

M. Feigelman and M. Skvortsov, *Phys. Rev. Lett*. **95** 057002 (2005)



$$\lambda(\varepsilon) = \lambda_0 - \frac{1}{24\pi g} Log\left(\frac{1}{\varepsilon\tau}\right)$$

$$\frac{\delta T_c}{T_c} = -\frac{\delta\lambda}{\lambda^2}$$

Random, due to mesoscopic
Fluctuations of disorder

Ginzburg-Landau expansion: result

$$F[\Delta] = \int \left[\alpha(T/T_c - 1)|\tilde{\Delta}|^2 + \gamma|\nabla\tilde{\Delta}|^2 + \frac{\beta}{2}|\tilde{\Delta}|^4 \right] d\mathbf{r}$$

α, β and γ are the usual GL parameters for dirty superconductors, and

$$\tilde{\Delta} = \Delta w(\zeta_{T_c}) = \frac{\Delta}{\cosh \lambda_g \zeta_{T_c}} \qquad \left(\lambda_g = \frac{1}{\sqrt{2\pi g}}, \;\; \zeta = \ln \frac{1}{E\tau} \right)$$

Gi is the same
as in the absense
of the Coulomb repulsion:

$$\mathrm{Gi} = \frac{\pi}{8\,g}$$

Superconductor with fluctuating T_c

Superconductor with fluctuating T_c (Ioffe, Larkin (1981)): Sov.Phys.JETP **54**, 378 (1981)

$$F = \int \left\{ \left[\alpha(T/T_c - 1) + \delta\alpha(\mathbf{r})\right] |\tilde{\Delta}|^2 + \gamma |\nabla \tilde{\Delta}|^2 + \frac{\beta}{2} |\tilde{\Delta}|^4 \right\} d\mathbf{r}$$

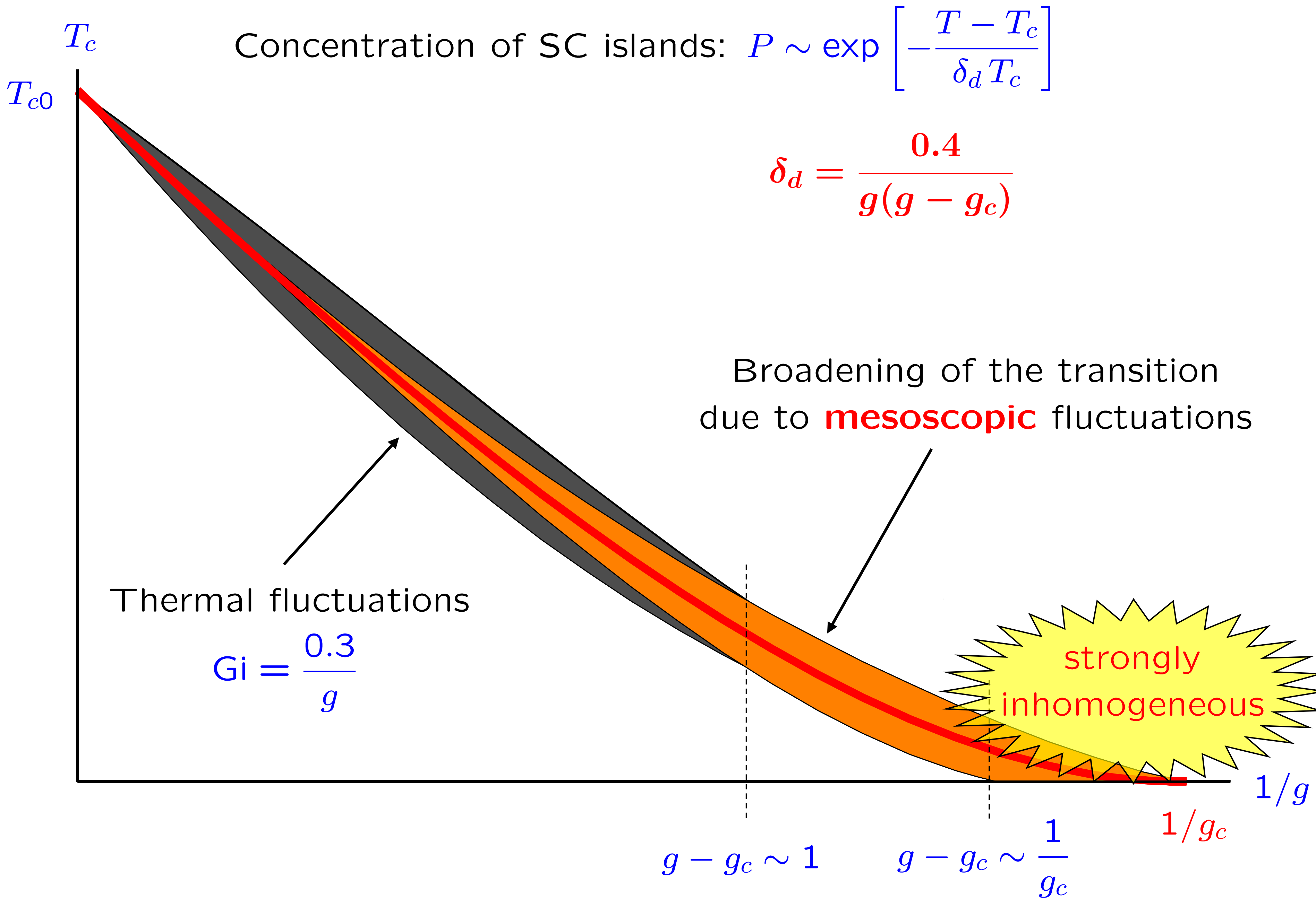
$$\langle \delta\alpha(\mathbf{r}) \delta\alpha(\mathbf{r}') \rangle = \frac{C}{w^4(T)} \delta(\mathbf{r} - \mathbf{r}') = \frac{7\zeta(3)}{8\pi^4 DT} \cosh^2(\lambda_g \zeta_T)\, \delta(\mathbf{r} - \mathbf{r}')$$

Linearized GL equation is similar to a Schrödinger eq. with Gaussian random potential

Localized "tail states" I.Lifshitz, Zittarz & Langer, Halperin & Lax (mid-60's)

(formally equivalent to "instanton solutions" in some effective field theory)

Mesoscopic vs. thermal fluctuations



Intermediate conclusions

Superconducting correlations are extremely inhomogeneous at g near critical conductance g_c

Due to enchancement of mesoscopic fluctuations, a random set of SC islands is formed in the sea of surrounding metal

It does not mean that the system is similar to JJ array since no grains and insulating barriers are present

How important is this "island structure" for the properties of quantum metal state that exists at $g < g_c$?

Consider a model system:

regular array of SC islands siting on dirty metal films

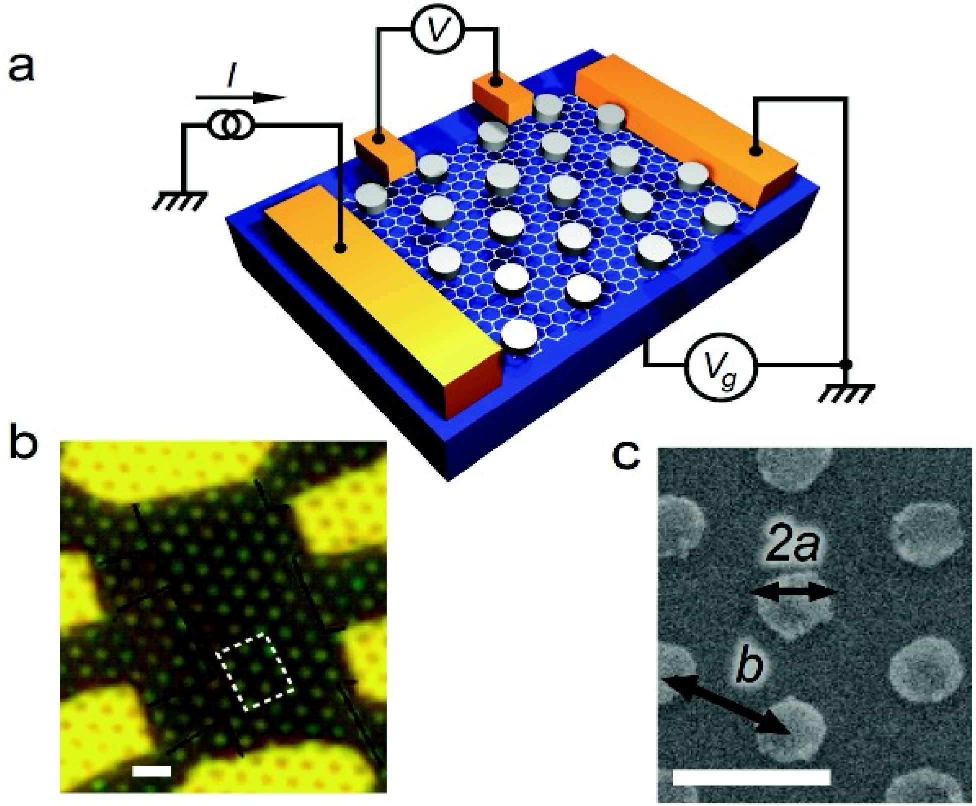
and study its QPT

Experiment: Sn islands on graphene

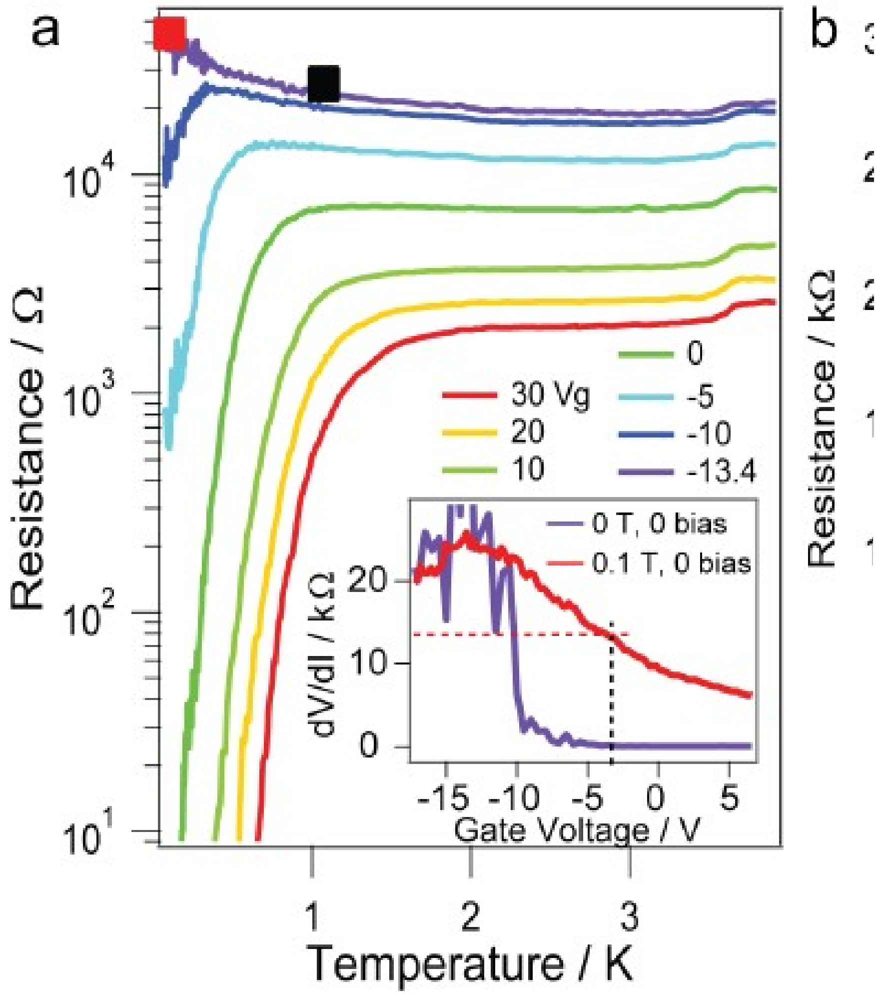
Collapse of superconductivity in a hybrid tin-graphene Josephson junction array

Nature Physics (published 30 March 2014)

Zheng Han[1,2], Adrien Allain[1,2], Hadi Arjmandi-Tash[1,2], Konstantin Tikhonov[3,4], Mikhail Feigel'man[3,5], Benjamin Sacépé[1,2] and Vincent Bouchiat[1,2]



R(T) curves



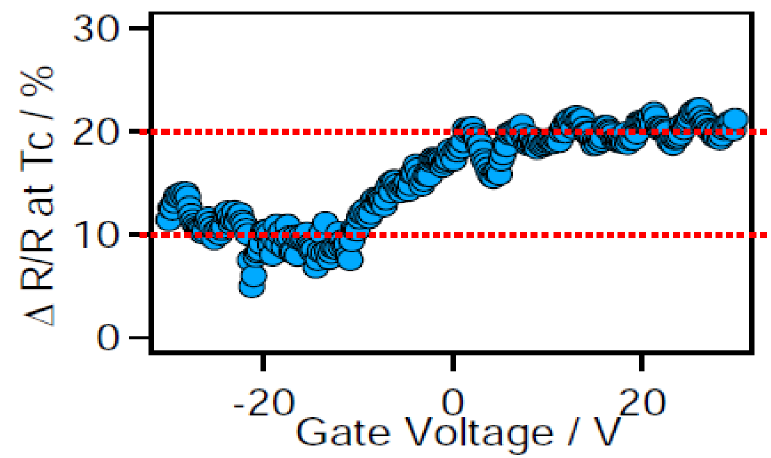
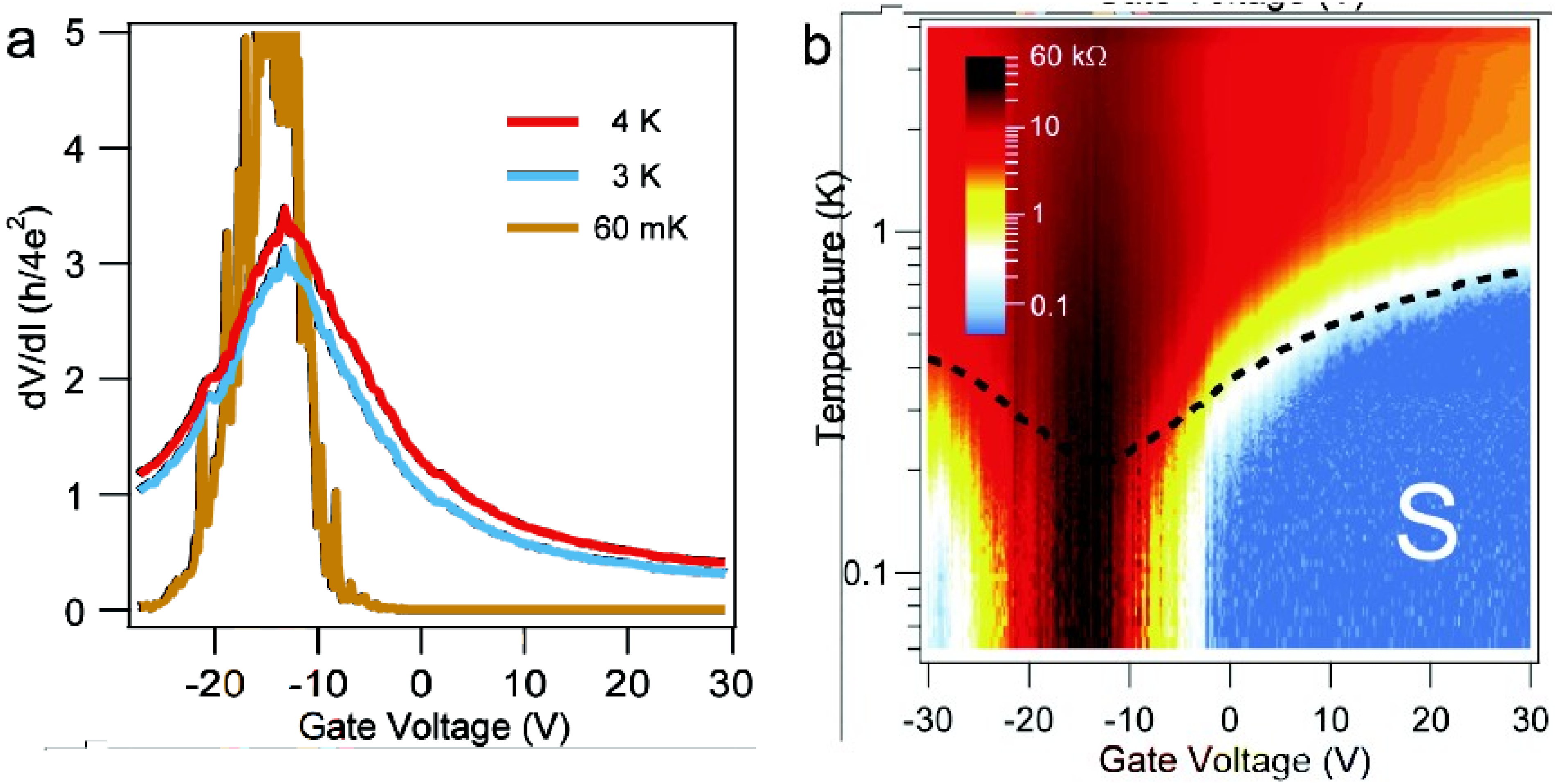


Figure S3: ΔR/R obtained by subtracting field-effect curve at 3.8 K and 3.3 K. Dirac point of the sample is ~ -13 V. ΔR/R at hole side is about half of that at electron side. We understand this as Sn is a electron donor, and once graphene is tuned into hole side, the pinning of Fermi level starts to be significant and a *p*-*n* junction thus formed to reduce the interface transparency.

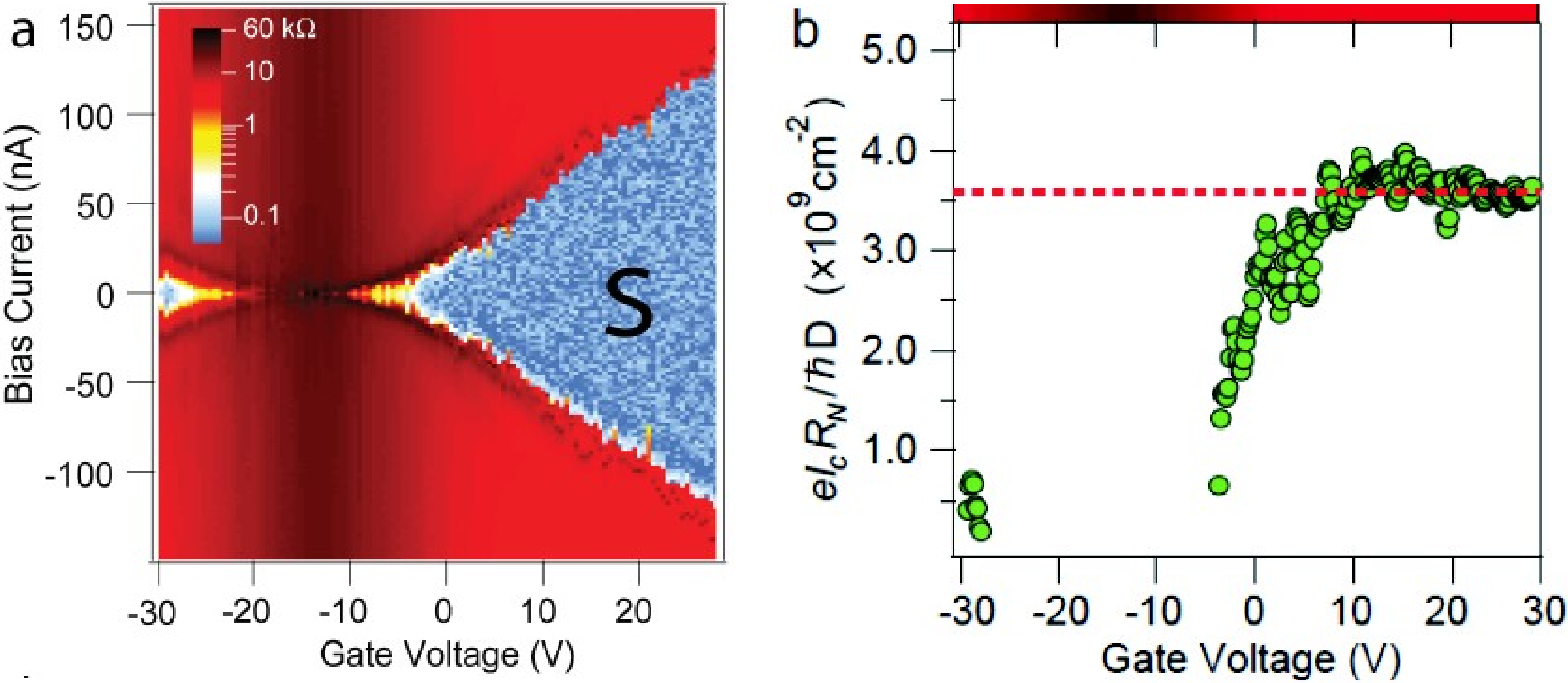


$$E_J(b,T) = 4\pi g T \sum_{\omega_n>0} \left\{\pi/[2\ln(\sqrt{D/2\omega_n}/a)]\right\}^2 P(\sqrt{\omega_n/2E_{Th}}).$$

$$T_c^{BKT} \simeq 1.47 E_J(b, T_c^{BKT}),$$

$$6\pi g \sum_{n=0}^{\infty} \frac{\pi^2}{\ln^2(\hbar D/2a^2\omega_n)} P(\sqrt{\omega_n/2E_{Th}}) = 1$$

$$P(z) = z\int_0^\infty K_0(z\cosh t)K_1(z\cosh t)\,dt.$$



$$E_{J,T=0}(b) = \frac{\pi^3}{4}\frac{g\hbar D}{b^2 ln^2(b/a)} = \frac{\hbar}{2e} I_1 \qquad I_c \approx 6 I_1 = \frac{3\pi^3 g e D}{b^2 ln^2(b/a)}$$

$$eI_cR_N/\hbar D = 3\pi^3/b^2 \ln^2(b/a) = 3.6 \cdot 10^9 cm^{-2}.$$

Experimental value at high gate voltage : 3.8 10^9

The reason for SC suppression: quantum phase fluctuations

Quantum superconductor-metal transition in a proximity array

M. V. Feigel'man[1], A. I. Larkin[1,2] and M. A. Skvortsov[1]

Phys. Rev. Lett. 86, 1869 (2001)

$$\frac{1}{2\hbar}\mathcal{J}(T)\mathcal{C}(T) \geq 1,$$

$$\mathcal{J}(0) = \frac{\pi^4}{2}\frac{gD}{b^2\ln(b/d)}.$$

$$\mathcal{C}(0) = \frac{\mathcal{B}}{\omega_d}e^{2\pi\sqrt{g s_c}},$$

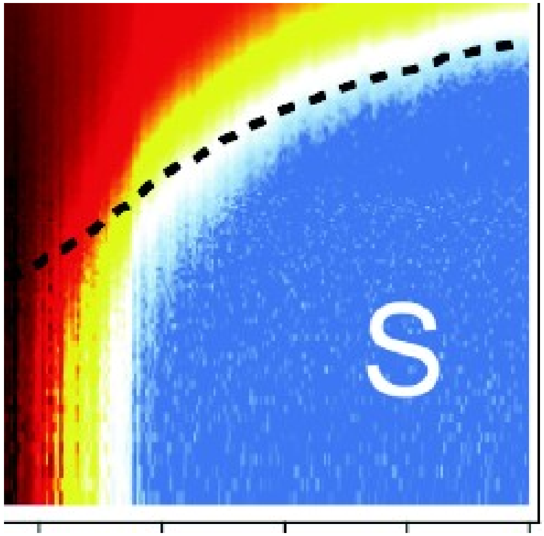
$$\mathcal{J}(T) = \mathcal{J}(0)\frac{\ln(L_T/b)}{\ln(L_T/d)}.$$

$$\ln\frac{T^*}{T_c} \approx \frac{2\ln(b/d)}{b_c^2(g)/b^2 - 1},$$

$$g_c = \mathcal{G}_c\left(\frac{1}{\pi}\ln\frac{b}{\tilde{d}}\right)^2;$$

Neglecting Cooper-channel interaction In graphene

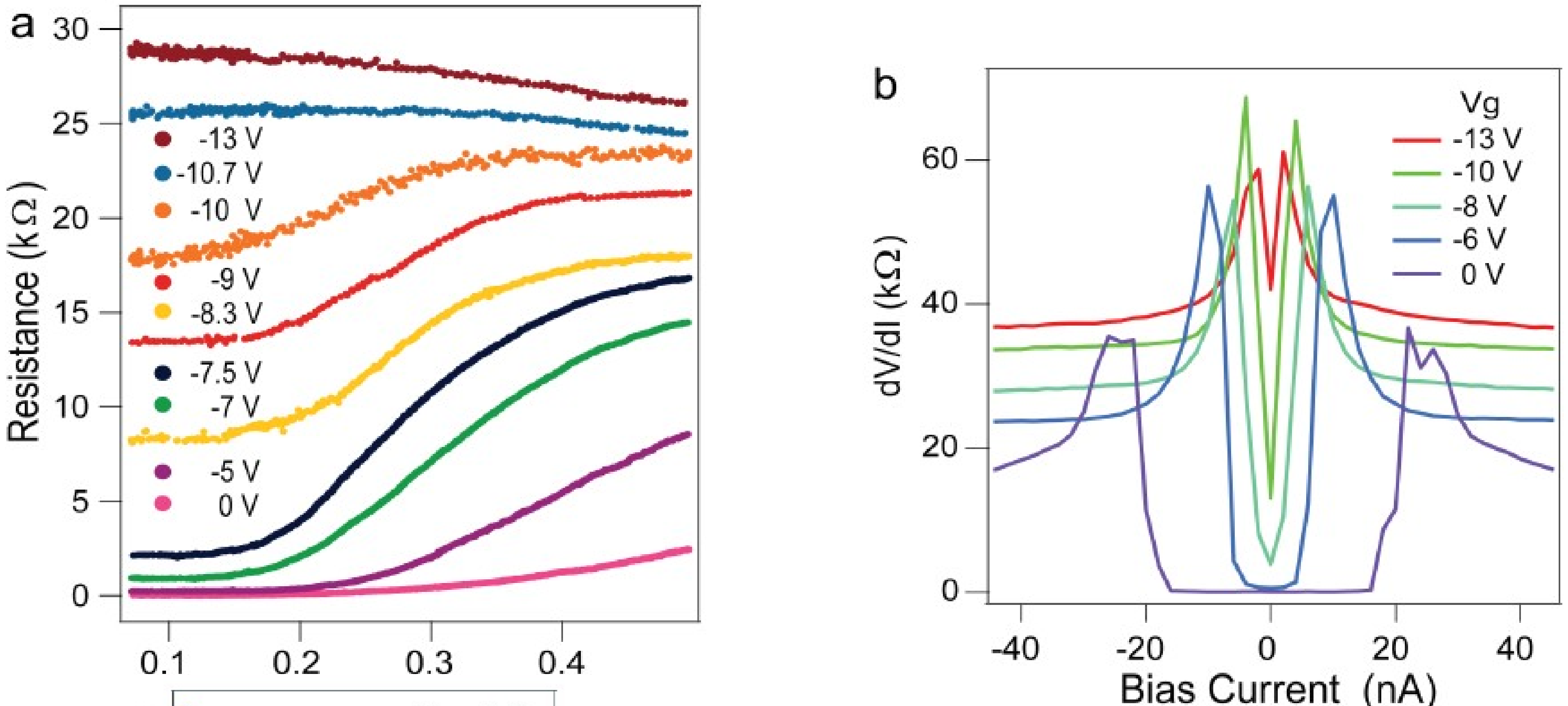
$$T_c \sim E_{Th}\exp\left(-\frac{c}{g-g_c}\right)$$



Otherwise, like in Finkelstein's theory

$$T_c \sim (g - g_c)^{(\pi g/2)^{1/2}}$$

Region close to Dirac point



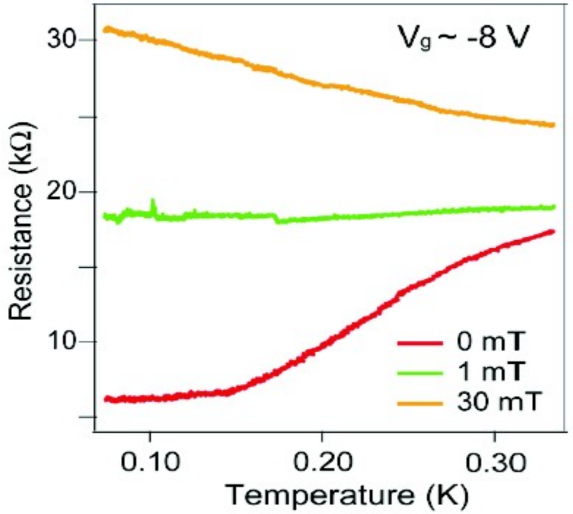
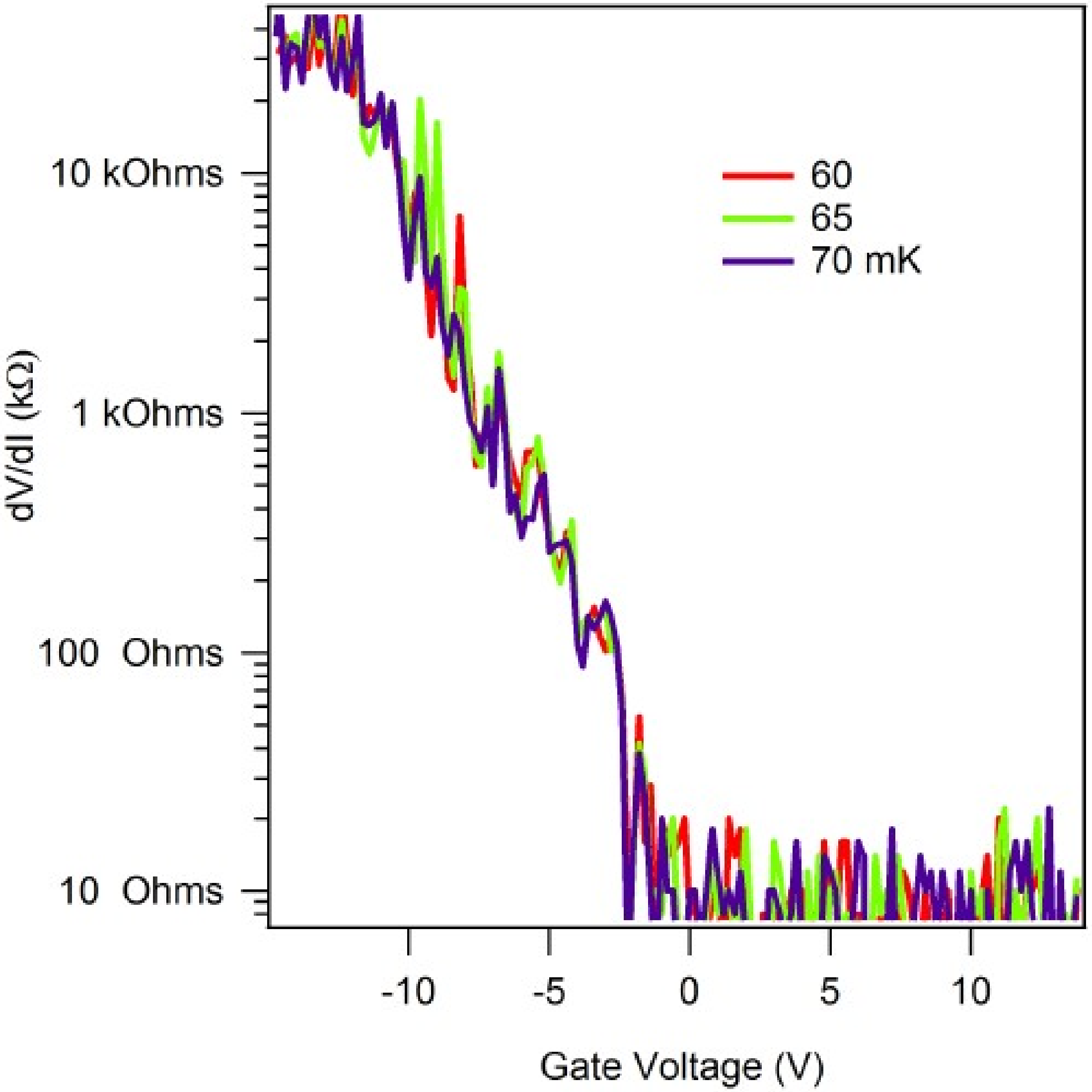


FIG. 3. a) Temperature dependance of resistance at different gate voltages in a temperature range of 70 mK - 500 mK. b) Individual current-bias dV/dI curves at gate voltages from -13 V to 0 V, at 60 mK temperature.

Very large conductivity in the low-T limit

But no superconducting state even at lowest T

Exponentially strong “paraconductivity” at lowest temperatures



At $V_g \leq -8$ V paraconductivity can be treated as fluctuational correction like quantum AL

Near the critical V_g value paraconductivity is exponentially large and the picture of quantum phase slips seems to be relevant

Quantum phase slips in 2D system ??

two options:

1. Actually, the system is of nearly 1D type (percolation-like structure with long 1D chains) leading to finite tunneling action for QPS
2. Genuine quantum version of vortex-driven BKT transition in 2D (still unknown) does exist

Regarding tin-graphene experiment, the 3[d] option is possible: the finite-size effect

Finally, we never know if T is low enough
For this experiment typical energy scale is

$E_{th} = hD/2\pi b^2 \approx 0.1$ Kelvin

Conclusions for Part 1

1. Coulomb repulsion in the Cooper channel is enhanced by disorder and leads to suppression of T_c of homogeneously disordered metal films down to zero at some $g_c >> 1$

2 Near critical conductance superconducting state is very inhomogeneous while the metal itself shows nothing apart weak mesoscopic fluctuations

3. Natural model system for SMT is a model of SC islands on a top of disordered metal film. T=0 quantum phase transition In a such a model can be described by a competition between Proximity-induced Josephson coupling and weak Coulomb blockade

4. “Normal” state of the other side of the T=0 SMT is a characterized by high and nearly T-independent conductance those nature Is unknown.

Part 2: Direct S-I transition and superconductivity in amorphous poor conductors:

fractality, pseudo-gap and new SIT scenario

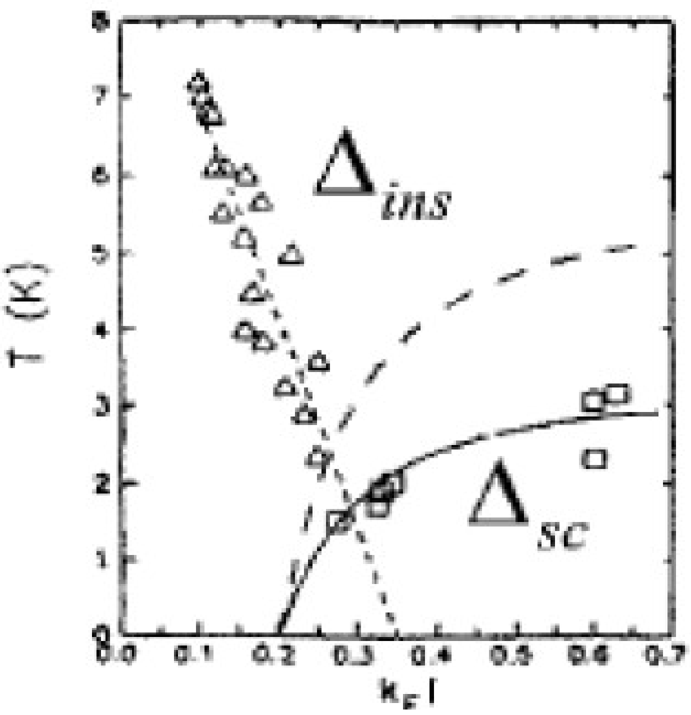
Theoretical approach:
Competition of Cooper pairing and localization
(no Coulomb repulsion)

Imry-Strongin, Ma-Lee, Kotliar-Kapitulnik, Bulaevskii-Sadovskii(mid-80's)

Ghosal, Randeria, Trivedi (1998-2001, 2011) - numerics

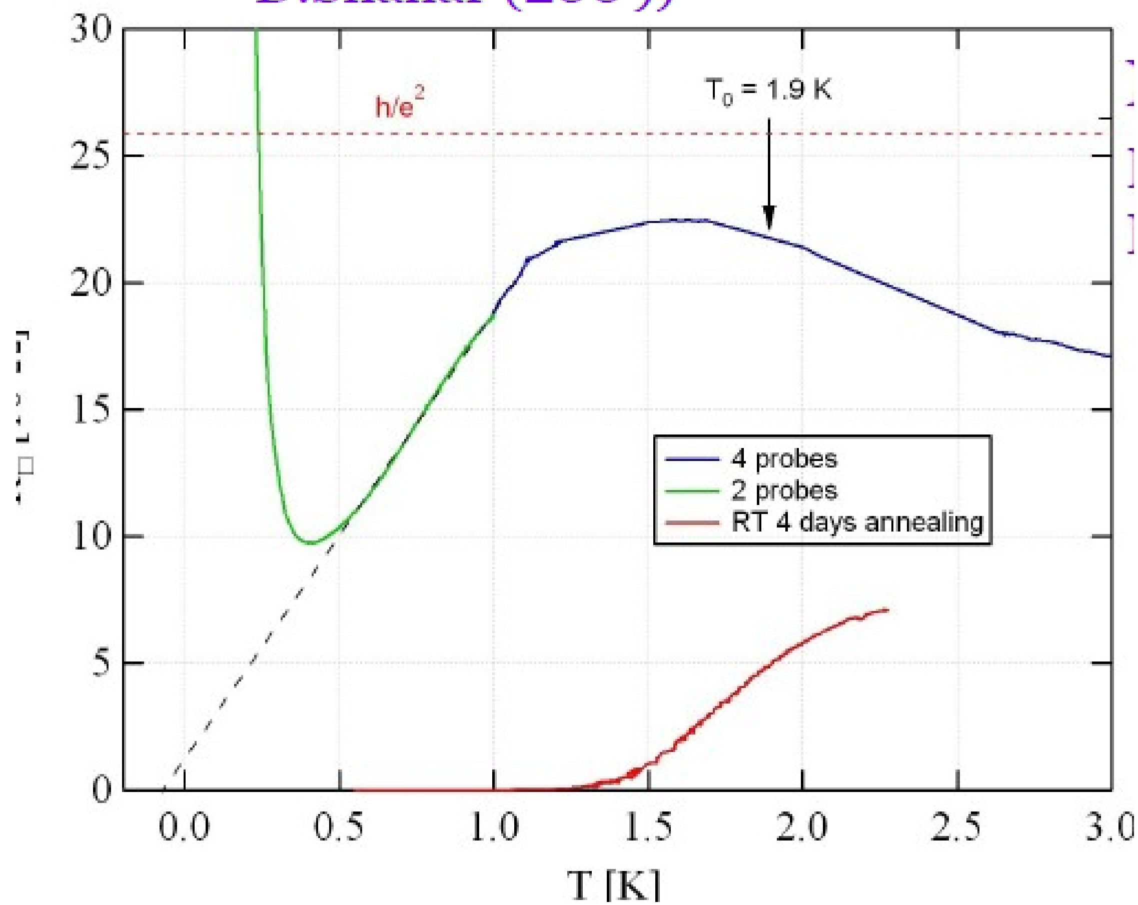
Direct S-I-T - InO_x

D.Shahar & Z.Ovadyahu
amorphous InO 1992

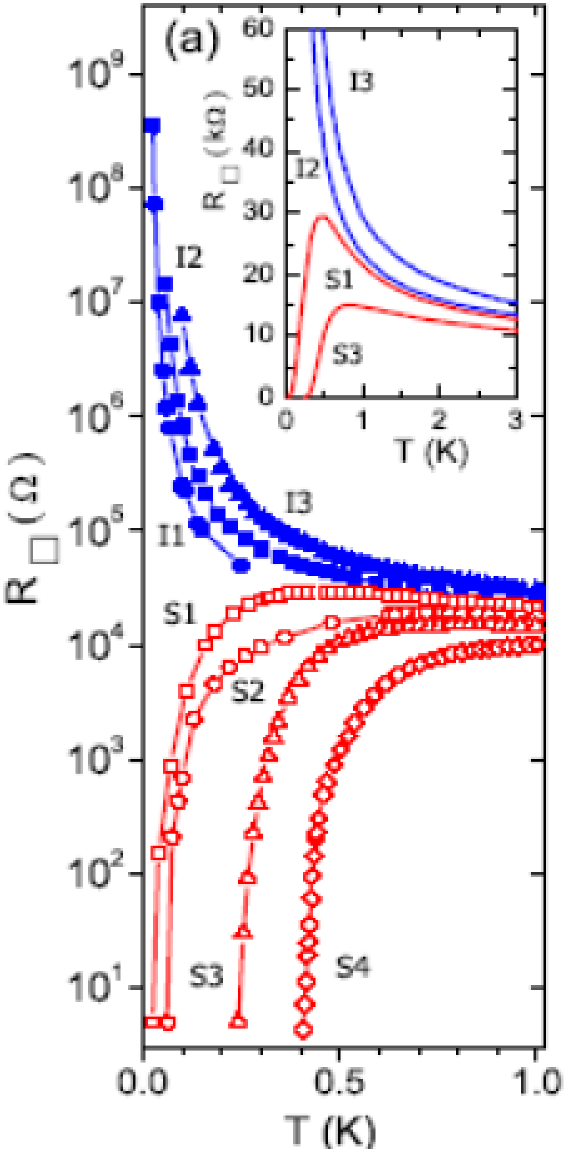


Nearly critical InOx :

B.Sacepe M.Ovadia
D.Shahar (2009)



Direct S-I-T - TiN



Example: Disorder-driven S-I transition in TiN thin films

T.I.Baturina et al Phys.Rev.Lett ***99*** *257003 2007*

Specific Features of Direct SIT:

Insulating behaviour of the R(T) separatrix

On insulating side of SIT, low-temperature resistivity is activated: $R(T) \sim \exp(T_0/T)$

Crossover to VRH at higher temperatures

The quest for the 3d scenario: major challenges from the data

In some materials SC survives up to very high resistivity values. No structural grains are found there.

Preformed electron pairs are detected in the same materials both above T_c and at very low temperatures on insulating side of SIT

- by STM study in SC state
- by the measurement of the activated $R(T) \sim \exp(T_0/T)$ on insulating side

Class of relevant materials

Amorphously disordered (no structural grains)
Low carrier density at helium temperatures
(around 10^{21} cm^{-3} or even less .)

Examples:

amorphous InOx TiN thin films

Possibly similar:

NbN_x B- doped diamond and B-doped Si

$Li_xZrNaCl$ (layered crystalline insulator with carriers due to Li doping)

Bosonic v/s Fermionic scenario ?

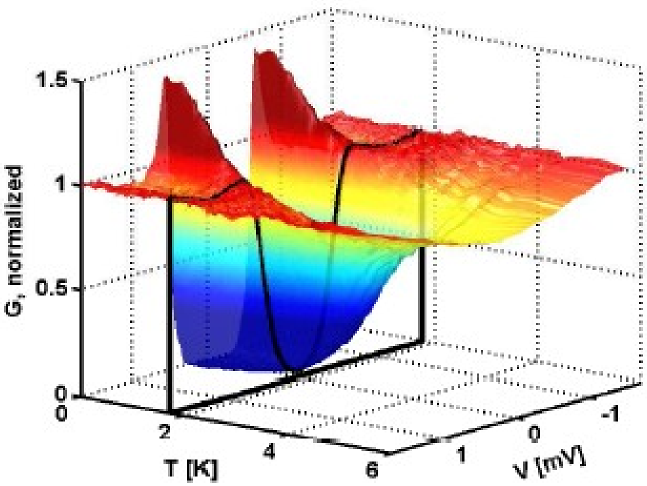
None of them is able to describe InOx data:
Both scenaria are ruled out by STM data in SC state

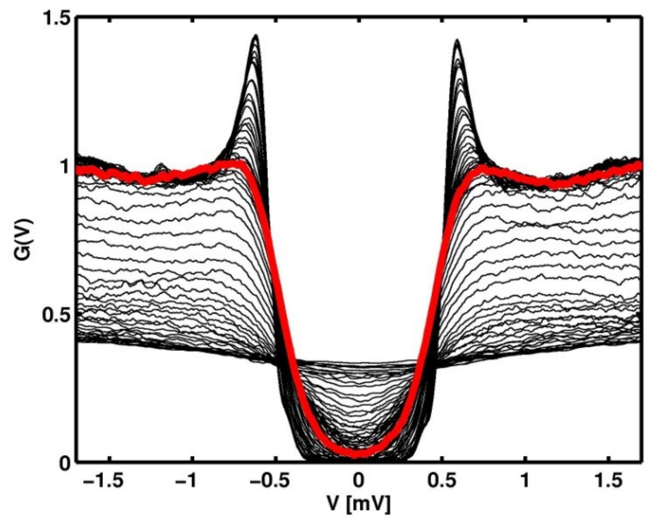
SC side: local tunneling conductance

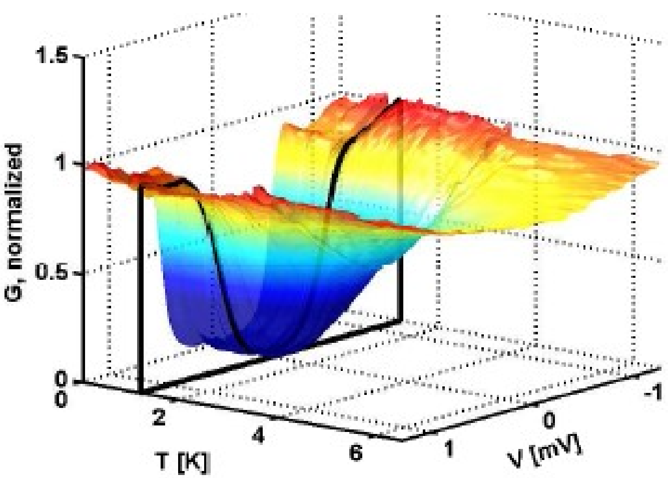
Spectral signature of localized Cooper pairs in disordered superconductors.

Benjamin Sacépé,[1,*] Thomas Dubouchet,[1] Claude Chapelier,[1] Marc Sanquer,[1] Maoz Ovadia,[2] Dan Shahar,[2] Mikhail Feigel'man,[3] and Lev Ioffe[4,3]

Nature Physics **7**, 239 (2011)







The spectral gap appears much before (with T decrease) than superconducting coherence does

Coherence peaks in the DoS appear together with resistance vanishing while T drops

Distribution of coherence peaks heights is very broad near SIT

Theoretical model (3D)

Simplest BCS attraction model, but for critical (or weakly localized) electron eigenstates

$$H = H_0 - g \int d^3r\, \Psi_\uparrow^\dagger \Psi_\downarrow^\dagger \Psi_\downarrow \Psi_\uparrow$$

$$\boldsymbol{\Psi} = \Sigma\, c_j\, \Psi_j\,(r)$$

Basis of exact eigenfunctions of free electrons in random potential

M. Ma and P. Lee (1985) : S-I transition at $\delta_L \approx T_c$

We will see find that SC state is compatible with $\delta_L >> T_c$

$$\delta_L = 1/\nu\, L^3_{loc}$$

Mean-Field Eq. for T_c

$$\Delta(r) = \int K_T(r,r')\Delta(r')d^d r' \tag{9}$$

where kernel $\hat{K}_T$ is equal to

$$K_T(r,r') = \frac{\lambda}{2\nu_0}\sum_{ij}\frac{\tanh\frac{\xi_i}{2T}+\tanh\frac{\xi_j}{2T}}{\xi_i+\xi_j}\psi_i(r)\psi_j(r)\psi_i(r')\psi_j(r') \tag{10}$$

Standard averaging over space $\Delta(r) \to \overline{\Delta}$ leads to "Anderson theorem" result: totally incorrect in the present situation.

The reason: critical eigenstates $\psi_j(r)$ are strongly correlated in real 3D space, they fill some small **submanifold** of the whole space only.

In fact one should define T_c as the divergence temperature of the Cooper ladder

$$\mathcal{C} = \left(1 - \hat{K}\right)^{-1}$$

Thus averaging procedure should be applied to $\mathcal{C}$ instead of K

We expand $\mathcal{C}$ in powers of K and average over disorder realizations. Keeping main sequence of resulting diagramms only, we come to the following equation for determination of T_c:

$$\Phi(\xi) = \frac{\lambda}{2} \int \frac{d\xi' \tanh(\xi'/2T)}{\xi'} M(\xi - \xi')\Phi(\xi') \qquad (11)$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

Fractality of wavefunctions at the mobility edge $E_F=E_c$

For critical eigenstates

$$L_{\rm loc} \to \infty$$

IPR: $M_i = \int |\psi_i(\mathbf{r})|^4 \mathbf{dr}$

one finds

$$\langle M_i \rangle \approx 3\ell^{-(d-d_2)} L^{-d_2}.$$

$$M(\omega) = \left(\frac{E_0}{\omega}\right)^{\gamma}$$

$$E_0 = 1/\nu_0 \ell^3$$

where

$$\gamma = 1 - \frac{D_2}{d}$$

$d_2 \approx 1.3$ in 3D

is a measure of fractality

l is the short-scale cut-off length

Usual "dirty superconductor":

$$M(\omega) = 1 \qquad \gamma = 0$$

3D Anderson model: $\gamma = 0.57$

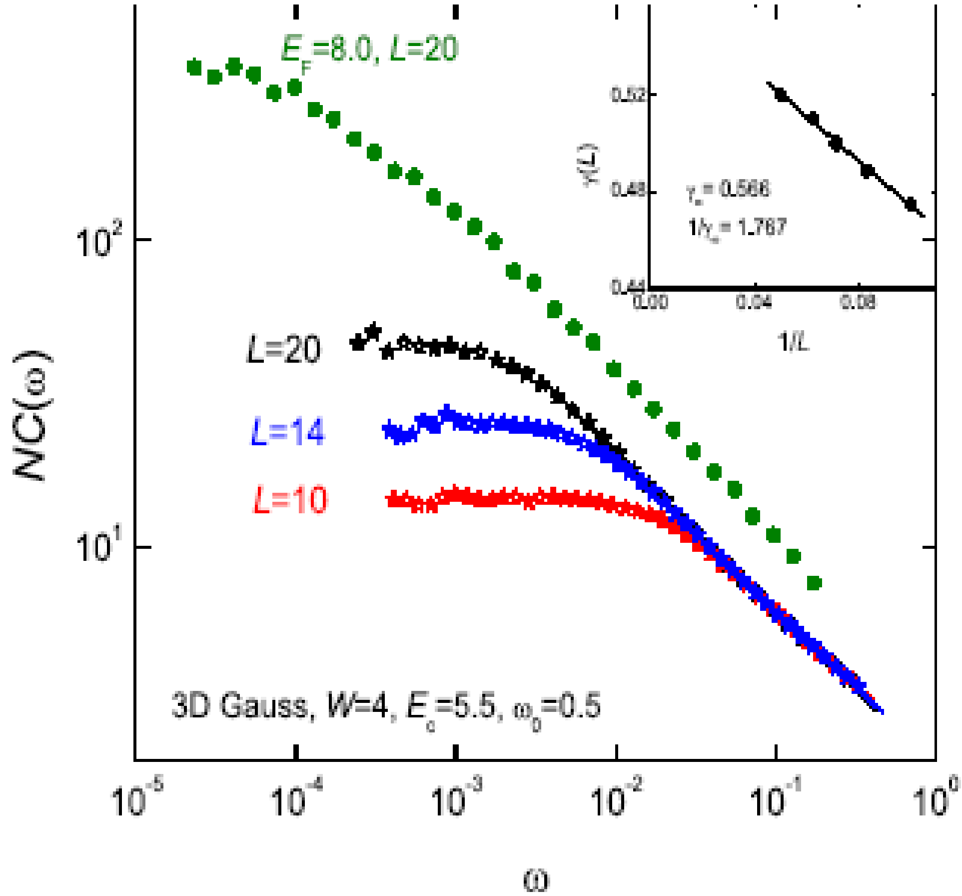


FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Modified BCS-type mean-field approximation for critical temperature T_c

$$\Delta(\xi) = \frac{\lambda}{2} \int d\zeta \eta(\zeta) M(\xi - \zeta) \Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1} \tanh(\xi_i / 2T).$$

$$T_c^0(\lambda, \gamma) = E_0 \lambda^{1/\gamma} C(\gamma)$$

- **For small λ this T_c is higher than BCS value !**

$$\gamma = 1 - \frac{d_2}{d}.$$

The same result was later obtained by Burmistrov , Gorny and Mirlin via RG approach for 2D system

Superconductivity at the Mobility Edge: major features

- Critical temperature T_c is well-defined through the whole system in spite of strong $\Delta(r)$ fluctuations

- Local DoS strongly fluctuates in real space; it results in asymmetric tunnel conductance

  $G(V,r) \neq G(-V,r)$

- Both thermal (Gi) and mesoscopic (Gi_d) fluctuational parameters of the GL functional are of order unity

B. Sacépé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia, D. Shahar, M. Feigel'man, and L. Ioffe, Nature Physics, **7**, 239 (2011)

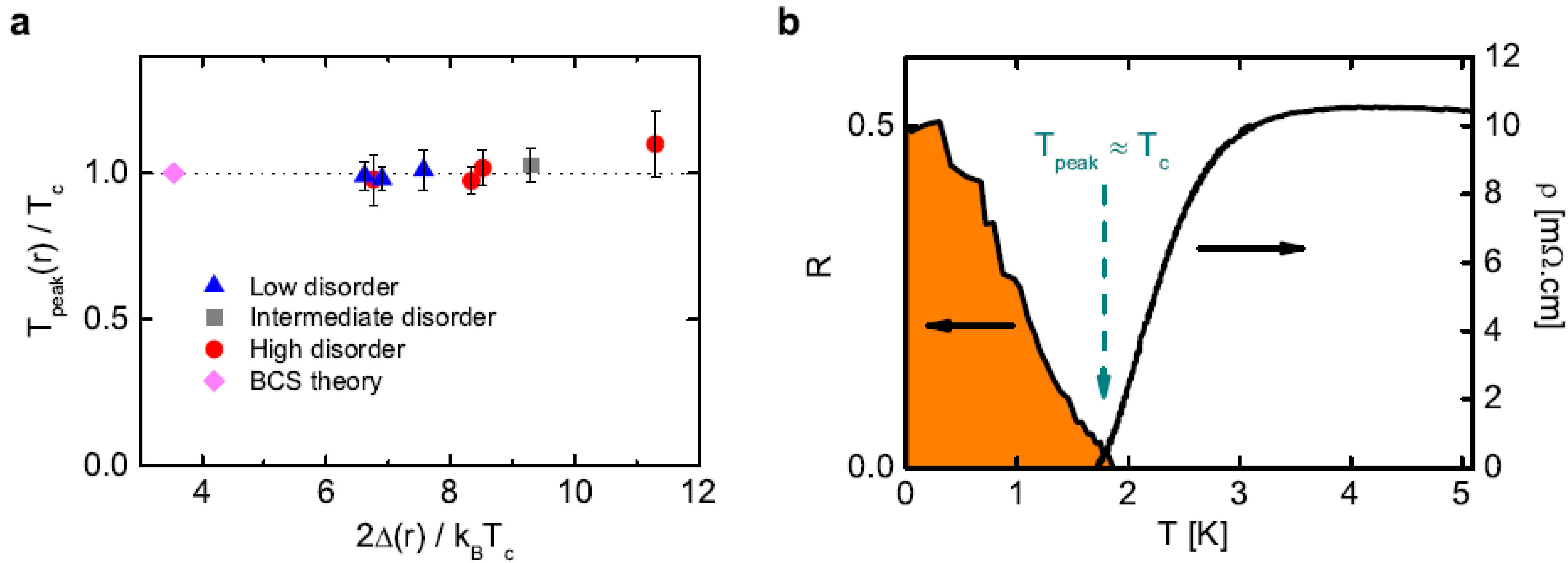


FIG. 4: **Onset of the superconducting phase coherence. a**, Local onset temperature of the coherence peaks, $T_{peak}(r)$, normalized to T_c, versus the ratio $2\Delta(r)/k_B T_c$ where $\Delta(r)$ is the low temperature spectral gap. For comparison, we added the point $T_{peak}(\mathbf{r})/T_c = 1$ corresponding to the theoretical BCS ratio $2\Delta/k_B T_c = 3.52$. **b**, Thermal evolution of the coherence peak height, R (for definition see text), extracted from data of Fig. 1c and of the resistivity ρ of low disorder sample. This plot evidences the coincidence between the appearance of the zero-resistance superconducting state at T_c with macroscopic phase coherence and the onset of the coherence peaks at T_{peak}.

Superconductive state with a pseudogap

Wave-functions with E near E_F are localized,

Localization length L_{loc} is long,

$$n_e L^3_{loc} >> 1$$

Local pairing energy

Parity gap in ultrasmall grains

K. Matveev and A. Larkin 1997

- -------
- ------- E_F
- --↑↓--
- -- ↓--

$\Delta \ll \delta$:

- **No many-body correlations**

$\Delta_P = \frac{1}{2}\lambda\delta$

$\lambda_R = \lambda/(1 - \lambda \log(\epsilon_0/\delta)).$

$\Delta_P = \frac{\delta}{2\ln\frac{\delta}{\Delta}}$

Correlations between pairs of electrons localized in the same "orbital"

Parity gap for Anderson-localized eigenstates

The increase of thermodynamic potential Ω due to addition of *odd* electron to the ground-state is

$$\delta\Omega_{\rm oe} = \xi_{m+1} = \xi_{m+1} - \tilde{\xi}_{m+1} + \tilde{\xi}_{m+1} = \frac{g}{2} M_{m+1} + O(\mathcal{V}^{-1})$$

$$\tilde{\xi}_j = \xi_j - \tfrac{g}{2} M_j.$$

Energy of two single-particle excitations after depairing:

$$2\Delta_P = \xi_{m+1} - \xi_m + gM_m = \frac{g}{2}(M_m + M_{m+1}) + O(\mathcal{V}^{-1})$$

$$\langle M_i \rangle = 3\ell^{-(d-d_2)}\, L_{\rm loc}^{-d_2},$$

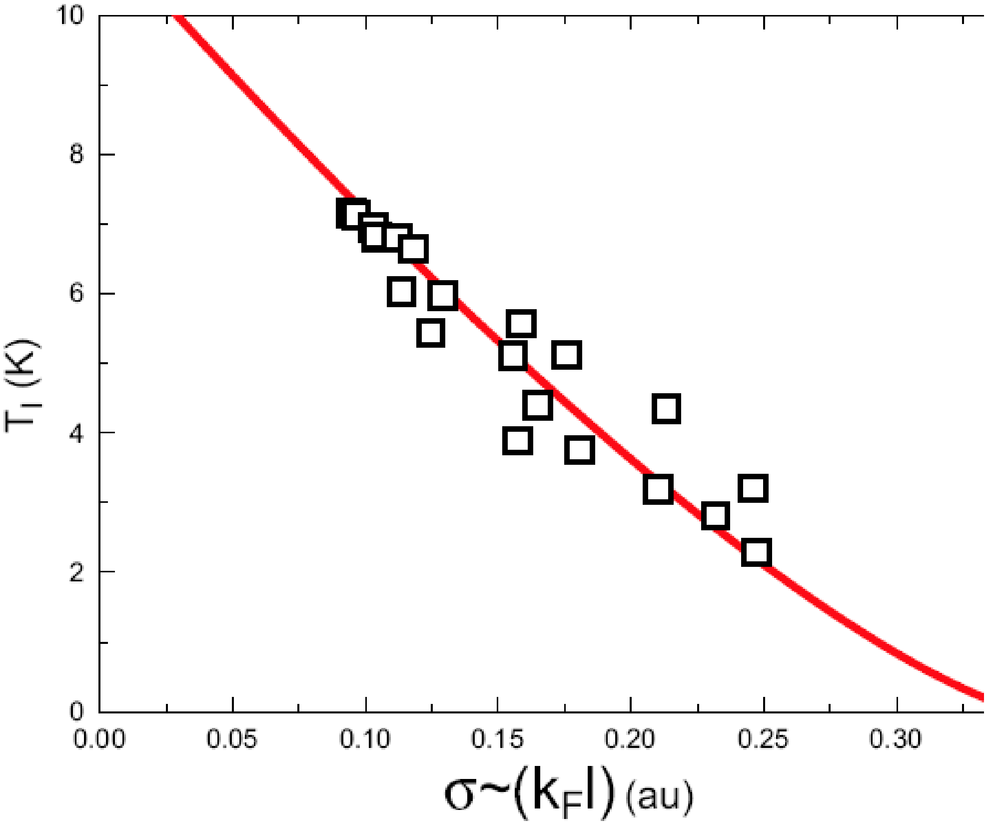
$$\Delta_P = \frac{3}{2} g \ell^{-3} (L_{loc}/\ell)^{-d_2} = \frac{3\lambda}{2} E_0 \left(\frac{E_c - E_F}{E_0} \right)^{\nu d_2}$$

Δ_P - activation gap in transport

$d_2 \approx 1.3$ in 3D

Activation energy T_I from D.Shahar & Z. Ovadyahu (1992) on amorphous InOx and fit to the theory

$$T_I = A(1 - \sigma/\sigma_c)^{\nu d_2}, \quad A \approx 0.5\lambda E_0$$



Example of consistent choice:

λ **= 0.05** E_0 **= 400 K**

No reasonable fit is possible with D=3 instead of d_2

Development of superconducting correlations between localized pairs: equation for T_c

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta \eta(\zeta) M(\xi-\zeta)\Delta(\zeta)$$

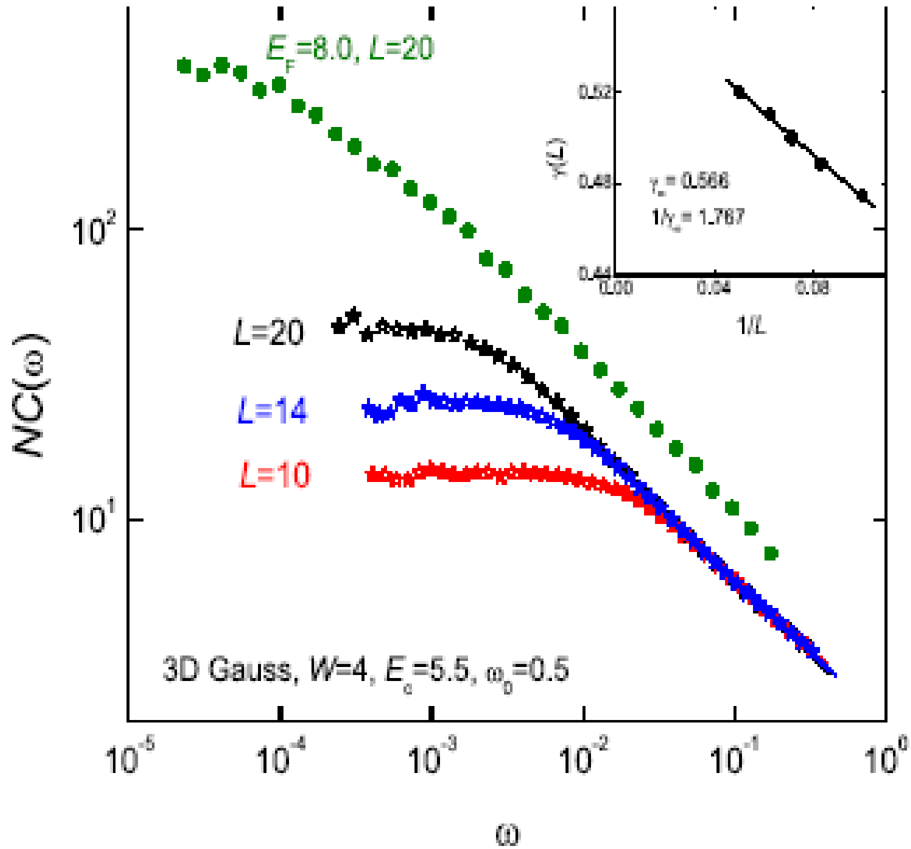
$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

Correlation function

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

should now be determined for localized states

Correlation function M(ω)



No saturation at $\omega < \delta_L$:

$M(\omega) \sim \ln^2 (\delta_L / \omega)$

(Cuevas & Kravtsov PRB,2007)

Superconductivity with

$T_c << \delta_L$ is possible

only with weak coupling !

This region was not noticed previously

Here "local gap" exceeds **SC gap :**

$$\Delta_P = \frac{1}{2D^{\gamma}(\gamma)} \delta_L \left(\frac{\Delta(0)}{\delta_L} \right)^{\gamma}$$

FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Annals of Physics **325,** 1368 (2010)

T_c versus Pseudogap

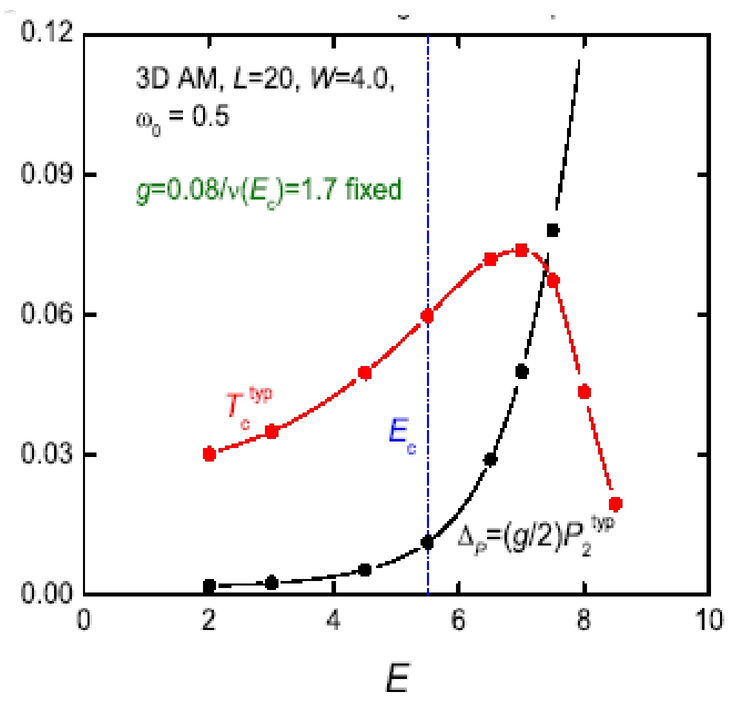


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

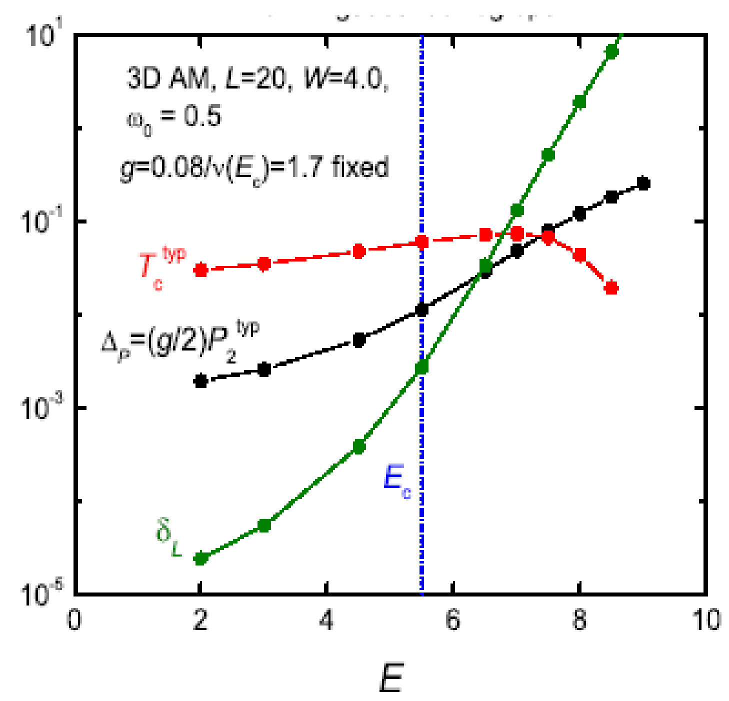


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

Superconductive transition exists even at $\delta_L >> T_{c0}$

Contribution of single-electron states Is suppressed by pseudogap $\Delta_P >> T_c$

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector: all states which are 2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g\sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu}\left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\bar{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

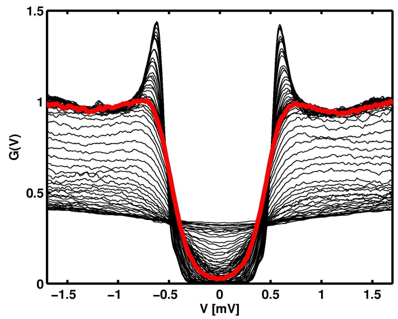
ν: D.o.S, V: total volume

"Pseudospin" approximation

$$Z \sim \nu_0 T_c L^d_{loc}$$

Effective number of interacting "neighbours"

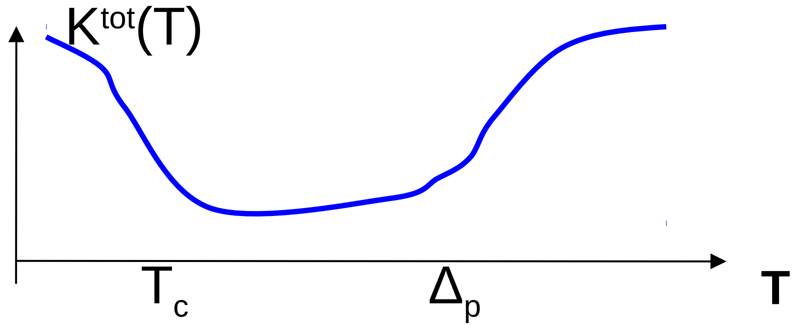
Qualitative features of "Pseudogaped Superconductivity":



STM DoS evolution with T

Double-peak structure in point-contact conductance

Nonconservation of the full spectral weight across T_c

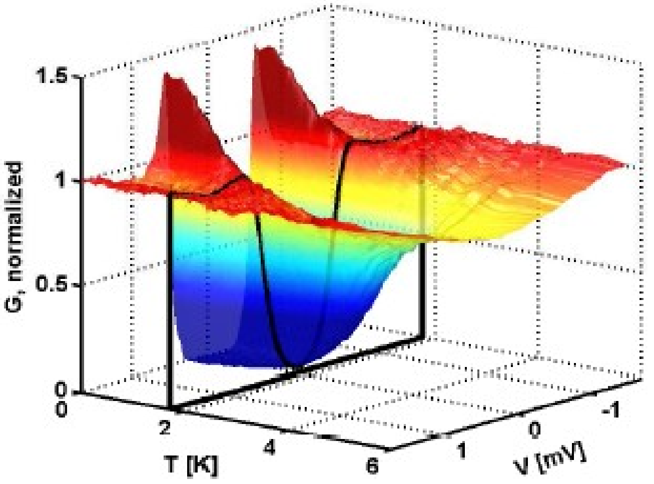


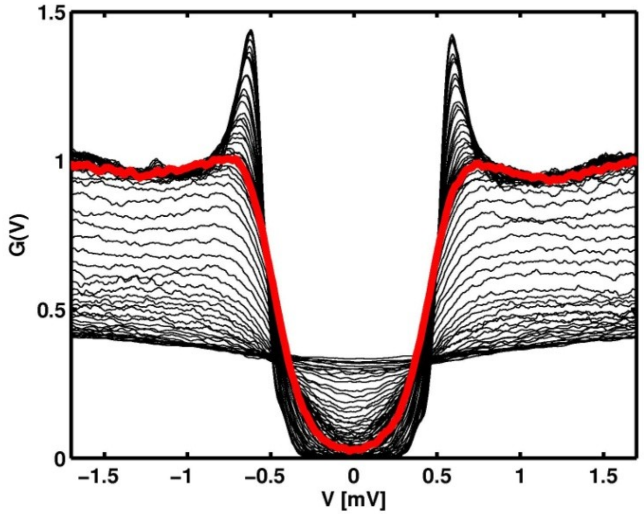
SC side: local tunneling conductance

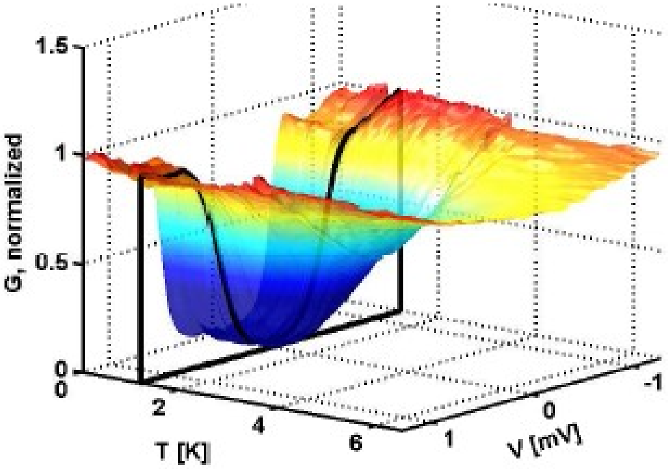
Spectral signature of localized Cooper pairs in disordered superconductors.

Benjamin Sacépé,[1,*] Thomas Dubouchet,[1] Claude Chapelier,[1] Marc Sanquer,[1] Maoz Ovadia,[2] Dan Shahar,[2] Mikhail Feigel'man,[3] and Lev Ioffe[4,3]

Nature Physics **7**, 239 (2011)





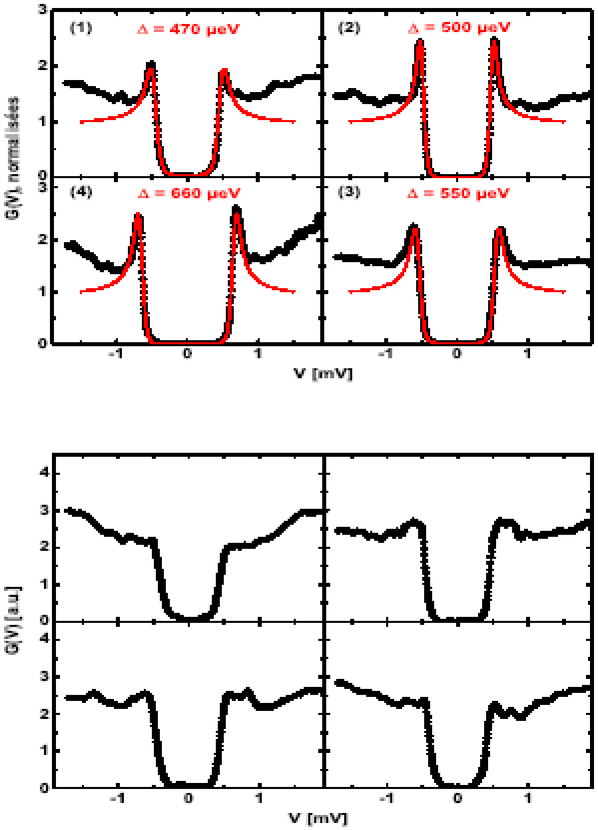


The spectral gap appears much before (with T decrease) than superconducting coherence does

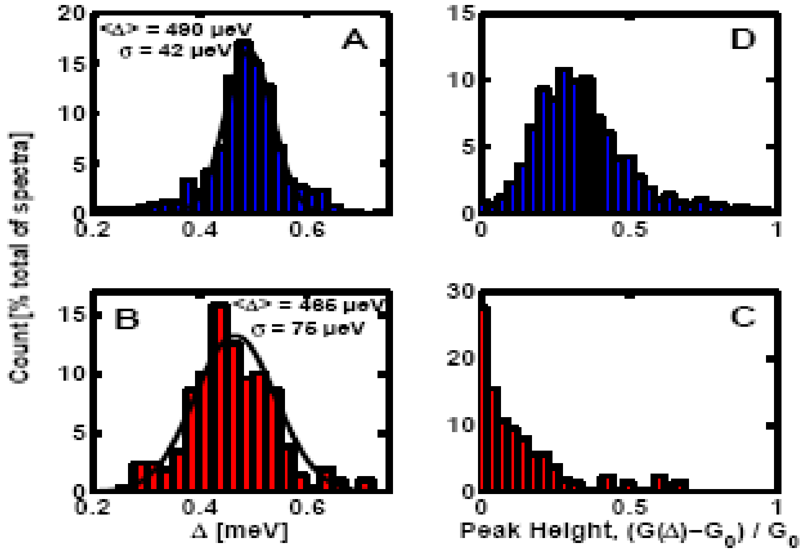
Coherence peaks in the DoS appear together with resistance vanishing while T drops

Distribution of coherence peaks heights is very broad near SIT

Local tunneling conductance-2



Gap widths   Peak heights

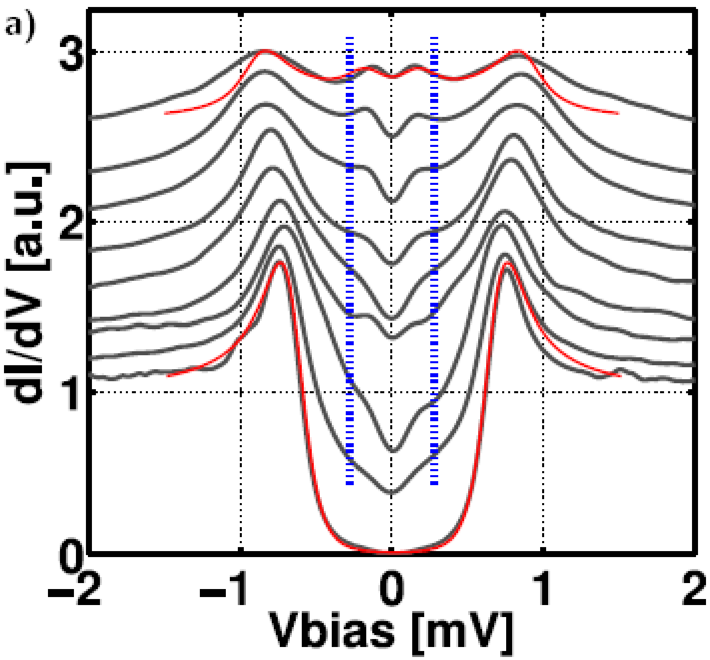


- **Less**
- **disorder**

- **More**
- **disorder**

Andreev point-contact spectroscopy

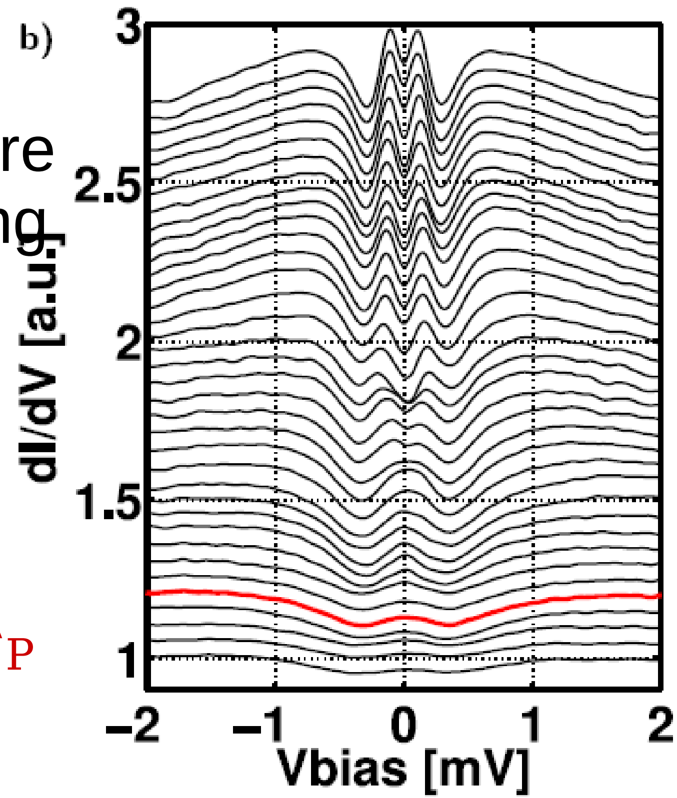
T. Dubouchet,[1,*] C. Chapelier,[1] M. Sanquer,[1] B. Sacépé,[2,3] Maoz Ovadia,[3] and Dan Shahar[3]

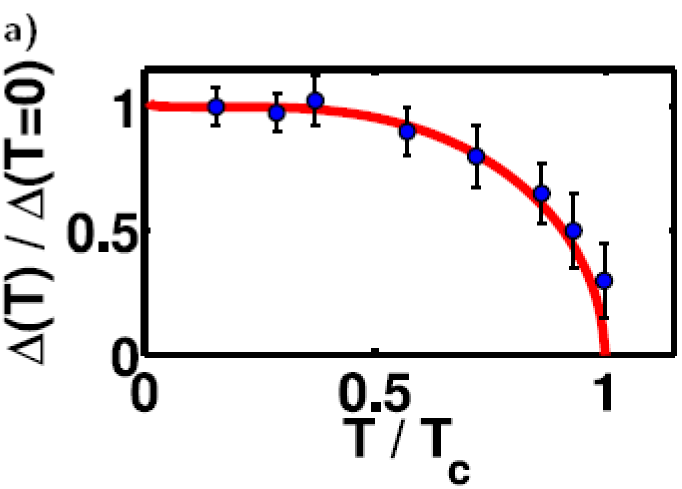


Pair tunnelling does not require to pay depairing energy Δ_P

$2eV_2 = 2\Delta$

$eV_1 = \Delta + \Delta_P$





T.Dubouchet, thesis, Grenoble (11 Oct. 2010)

S-I-T: Third Scenario

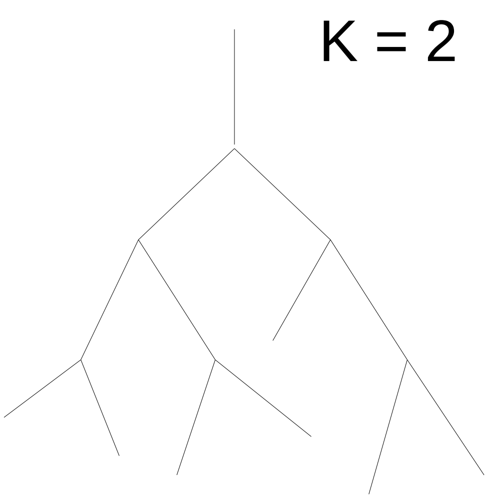
- Bosonic mechanism: preformed Cooper pairs + competition Josephson v/s Coulomb – S I T in arrays
- Fermionic mechanism: suppressed Cooper attraction, no paring – S M T

- Pseudospin mechanism: individually localized pairs

- S I T in amorphous media

SIT occurs at small Z and lead to paired insulator

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$



Cayley tree model is solved

(*M.F.,L.Ioffe & M.Mezard*

Phys. Rev.B **82**, 184534 (2010))

MODEL SOLUTION 1: CAVITY EQUATIONS.

Main idea: cavity equations.
Introduce effective field that simulates the effect of spins at higher levels:

$$H = -\xi_0 \sigma_0^z - h_0 \sigma_0^x \qquad \left\langle \sigma_0^x \right\rangle_0 = \frac{h_0}{\sqrt{h_0^{\ 2} + \xi_0^{\ 2}}} \mathrm{Tanh}\left[\frac{\sqrt{h_0^{\ 2} + \xi_0^{\ 2}}}{T} \right]$$

$$H = -\xi_0 \sigma_0^z - \sum_j (\xi_j \sigma_j^z + \sigma_0^x \sigma_j^x + h_j \sigma_j^x) \quad \text{Choose } h_0 \text{ so that } \left\langle \sigma_0^x \right\rangle_H = \left\langle \sigma_0^x \right\rangle_0$$

Roughly - this approximation is sufficient to get the transition temperature to O(1/K):

$$h_{k+1} = \frac{g}{K} \sum_j \frac{h_{k,j}}{\sqrt{\xi_{k,j}^{\ 2} + h_{k,j}^{\ 2}}} \mathrm{Tanh} \frac{\sqrt{\xi_{k,j}^{\ 2} + h_{k,j}^{\ 2}}}{T}$$

If averaged over uniform distribution of ξ we get usual BCS-like equation:

$$h = g \int_0^\infty \frac{d\xi h}{\sqrt{\xi^2 + h^2}} \mathrm{Tanh}\left[\frac{\sqrt{\xi^2 + h^2}}{T} \right]$$

that tells us that $T_c > 0$ for any $g > 0$.

MODEL SOLUTION 2: EQUATION FOR T_C.

To find T_c we need to find when infinitely small field applied at the boundary leads to large field in the center:

$$h_0 = Zh_N \qquad Z = \sum_{\{i[k]\}} \prod_k \frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i[k]} / T]}{\xi_{k,i[k]}}$$

That is whether $Z=exp(fN)$ with $f>0$ ("magnet" or "superconductor") or $f<0$ (paramagnet)?

Non-trivial physics is due to the fact that Z is not necessarily self-averaging quantity! Consider higher moments:

$$K\left\langle \left[\frac{g}{K} \frac{\mathrm{Tanh}[\xi_{k,i} / T]}{\xi_{k,i}} \right]^n \right\rangle = \sqrt{\frac{3\pi}{4K}} K^{1-n} g^n / T^{n-1}$$

The moments diverge at $T=g/K$ which becomes higher than 'average' $T_c=exp(-1/g)$.

Distribution function for the order parameter

Linear recursion ($T=T_c$)

$$B_i = (g/K)\sum_k (B_k/\xi_k)\tanh(\beta\xi_k) \;,$$

Laplace transform satisfies the equation:

$$\mathcal{P}(s) = \left[\int_0^1 d\xi\; \mathcal{P}\left(s\frac{g}{K}\frac{\tanh\beta\xi}{\xi}\right)\right]^K$$

$$P(B) = \frac{B_0^m}{B^{1+m}}$$

Diverging 1st moment

Solution in the RBS phase: $\mathcal{P}(s) = 1 - As^x$ with $x < 1$

$$1 = K\int \frac{d\xi}{\xi}\left(\frac{g}{K}\frac{\tanh(\beta\xi)}{\xi}\right)^x$$

$$\int_{-1}^{1}\frac{d\xi}{2}\frac{\tanh^x\beta\xi}{\xi^x}\ln\left(\frac{g}{\xi K}\tanh\beta\xi\right) = 0$$

T=0

$$g_c e^{1/(eg_c)} = K$$

$$x = m = 1 - eg_c$$

Vicinity of the Quantum Critical Point

$$T_c(K) = \vartheta(y_c)\left(\frac{K}{K_c} - 1\right)^{1/y_c}$$

$$y_c = eg.$$ **<< 1**

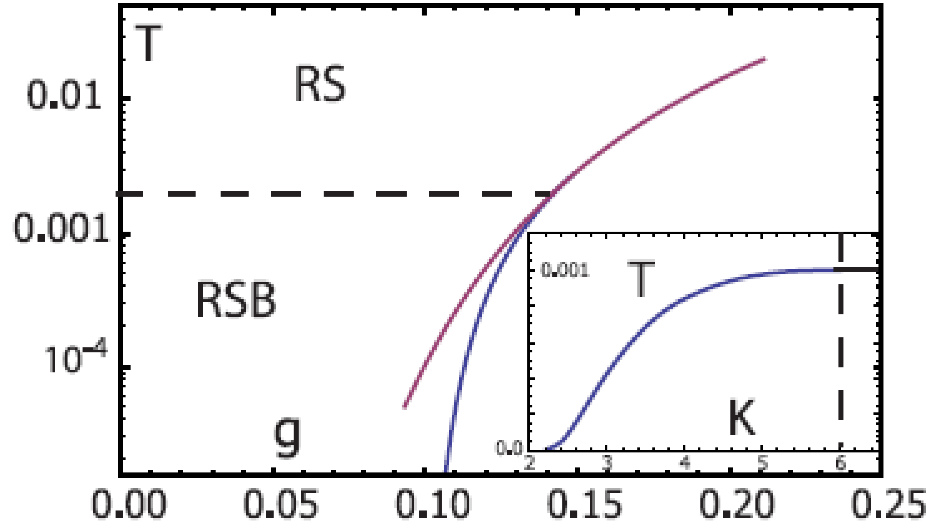


FIG. 2: Main panel: phase diagram in plane (g, T) for $K = 4$. Full lines show the critical temperature as function of g. The low temperature phase is superconducting, the high temperature phase is insulating. The top curve is the naive mean-field prediction which gives the correct result above $T_{RSB} = 0.0207$. The bottom curve is the result of the correct analysis on the Bethe lattice, including the RSB effects in the DP problem, which occur at temperatures $T < T_{RSB}$. The insert shows the phase diagram as function of K for $g = 0.129$. For this value of g the replica symmetric solution gives K-independent transition temperature $T_c = 0.001$; this value roughly correspond to the experimental situation in disordered InO films (see section VI). The prediction of replica symmetric theory is correct for $K > K^{RSB} \simeq 6$. For smaller K the transition temperature starts to drop, the quantum critical point corresponds to $K_c \simeq 2.2$. Notice that in a numerically wide regime the replica symmetry is broken but the effect on transition temperature is small.

Order parameter: scaling near transition

$$B_j = \frac{g}{K}\sum_{k=1}^{K}\frac{B_k}{\sqrt{B_k^2+\xi_k^2}}\tanh\beta\sqrt{B_k^2+\xi_k^2}\ .$$

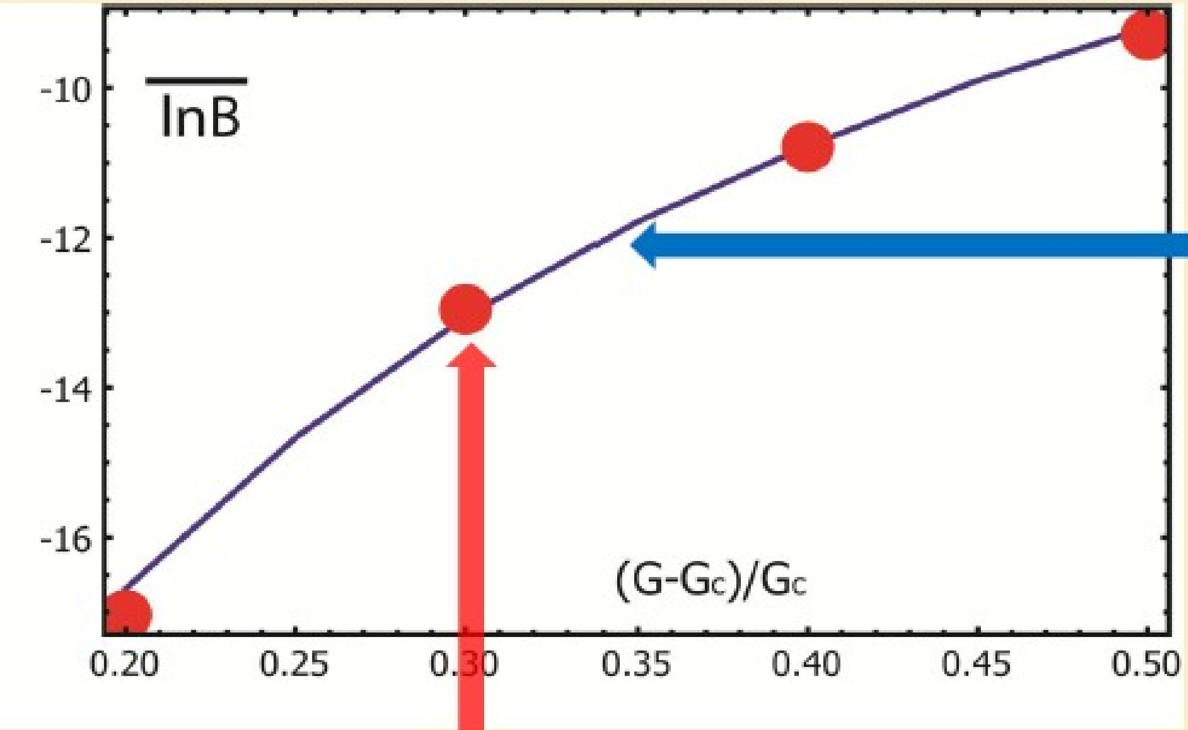
$$P(B) = \frac{B_0^m}{B^{1+m}}$$

$$B_i^m \simeq \sum_{k=1}^{K}\left(\frac{g}{K}\frac{B_k}{\sqrt{B_k^2+\xi_k^2}}\tanh\beta\sqrt{B_k^2+\xi_k^2}\right)^m$$

Typical value near the critical point:

$$B_0 \simeq e^{-1/(eg_c)}\exp\left[-\frac{C}{(g/g_c)^m-1}\right]$$

CHECK FOR IMPORTANCE OF SUBLEADING CORRECTIONS



Typical field obtained from simplified cavity equations

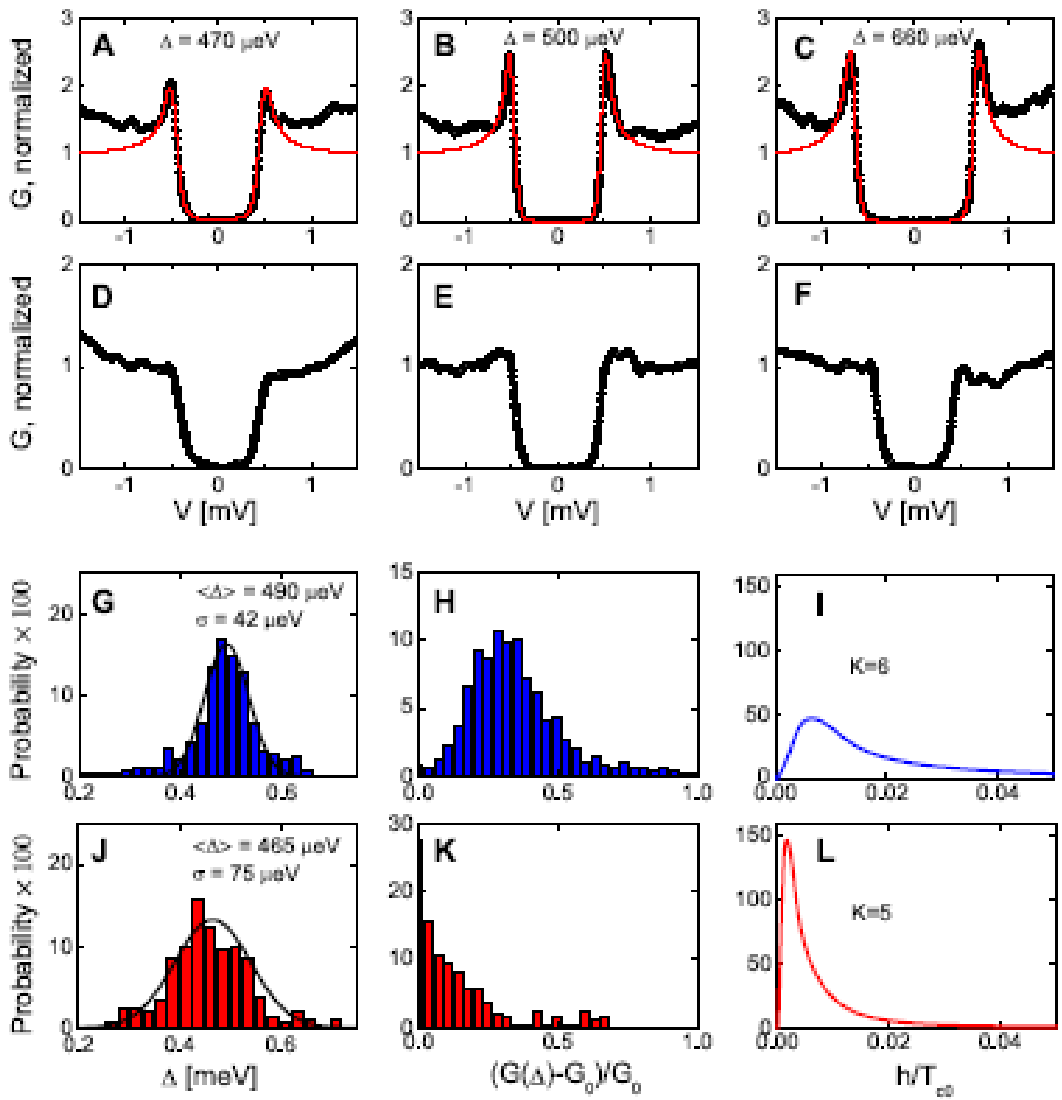
$$h_{k+1} = \frac{g}{K}\sum_j \frac{h_{k,j}}{\sqrt{\xi_{k,j}^2 + h_{k,j}^2}}$$

Typical field obtained from the cavity equations with the exact diagonalization of Hamiltonian at each step

$$H_{arr} = -\xi_0\sigma_0^z - h_0\sigma_0^x$$

$$H_{sn} = -\xi_0\sigma_0^z - \sum_j \left(\xi_j\sigma_j^z + \sigma_0^x\sigma_j^x + h_j\sigma_j^x\right)$$

Choose h_0 so that $\langle\sigma_0^x\rangle_H = \langle\sigma_0^x\rangle_0$



A
Δ = 470 μeV
B
Δ = 500 μeV
C
Δ = 660 μeV
D
E
F
G, normalized
V [mV]
G
<Δ> = 490 μeV
σ = 42 μeV
H
I
K=6
J
<Δ> = 465 μeV
σ = 75 μeV
K
L
K=5
Probability × 100
Δ [meV]
(G(Δ)-G0)/G0
h/Tc0

Insulating phase: continuous v/s discrete spectrum ?

Consider perturbation expansion over M_{ij} in H below:

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$

Within convergence region the many-body spectrum is qualitatively similar to the spectrum of independent spins

↓

No thermal distribution, no energy transport, distant regions "do not talk to each other"

Different definitions for the fully many-body localized state

1. No level repulsion (Poisson statistics of the full system spectrum)
2. Local excitations do not decay completely
3. Global time inversion symmetry is not broken (no dephasing, no irreversibility)
4. No energy transport (zero thermal conductivity)
5. Invariance of the action w.r.t. local time transformations $t \to t + \varphi(t,\mathbf{r})$:
   $d\,\varphi(t,\mathbf{r})/dt = \xi\,(t,\mathbf{r})$ – Luttinger's gravitational potential

Level statistics: Poisson v/s WD

- Discrete many-body spectrum with zero level width: Poisson statistics
- Continuous spectrum (extended states) : Wigner-Dyson ensemble with level repulsion

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Model of interacting fermions (no-conclusive concerning sharp phase transition)

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Level statistics of disordered spin-1/2 systems and materials with localized Cooper pairs

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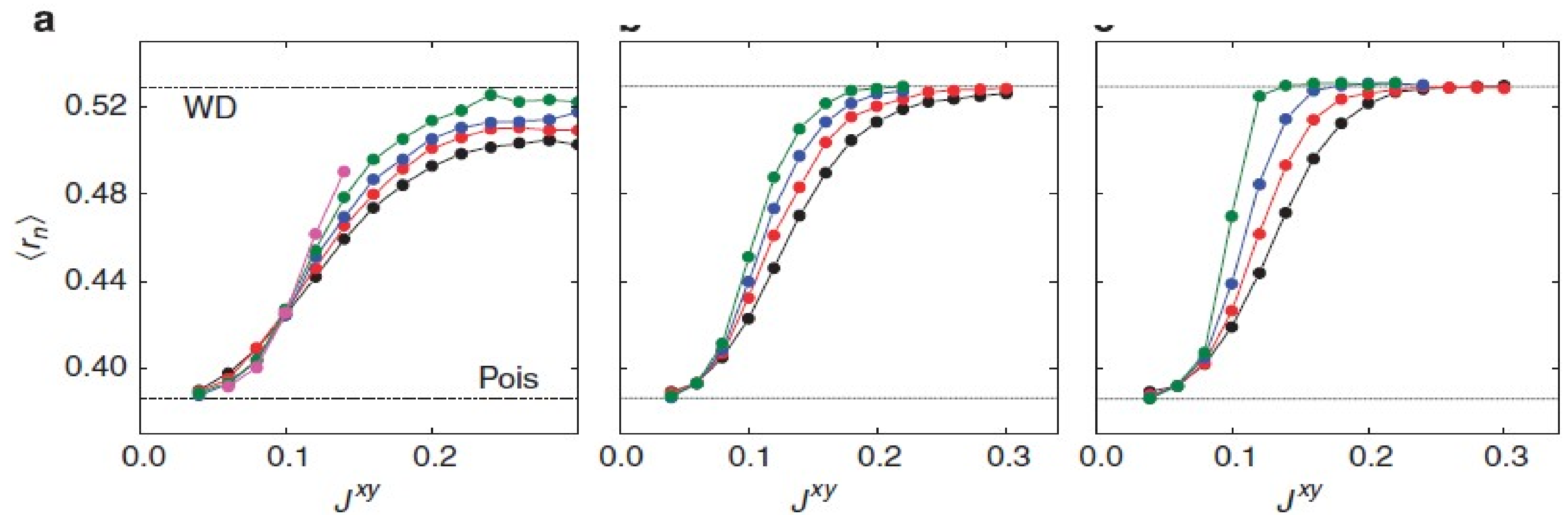
$$0 < r_n = \min(\delta_n, \delta_{n-1}) / \max(\delta_n, \delta_{n-1}) < 1$$



refers to the sector with $S_z^{tot} = 0$ for the model (1) with $J^{zz} = 0$, defined on a $Z = 3$ random graph with bandwidth $W = 1$. Panel **a** shows the statistics of the low-energy excitations in the energy interval (E_{gs}, $E_{gs}+1.5$). Data points are shown for system sizes $N = 14$ (black dots), $N = 16$ (red), $N = 18$ (blue), $N = 2$ (green) and $N = 22$ (violet). The critical value of the coupling $J_c^{xy} = 0.095 \pm 0.003$ is determined via a crossing point analysis. Panel **b** shows similar result for intermediate excitation energies, ($E_{gs}+1.5$, $E_{gs}+2.5$), leading to the critical point $J_c^{xy} = 0.066 \pm 0.002$. Panel **c** corresponds to high energies, close to t centre of the many-body spectrum, with the critical point $J_c^{xy} = 0.061 \pm 0.002$. Each data point represents the average over $N_r = 2{,}000$, 200, 100 and 60 disorder realizations for $N_S = 14$, 16, 18 and 20, respectively. A large (exponential) increase in the number of states implies that larger samples require

Phase diagram (for J^{zz}=0, T=0)

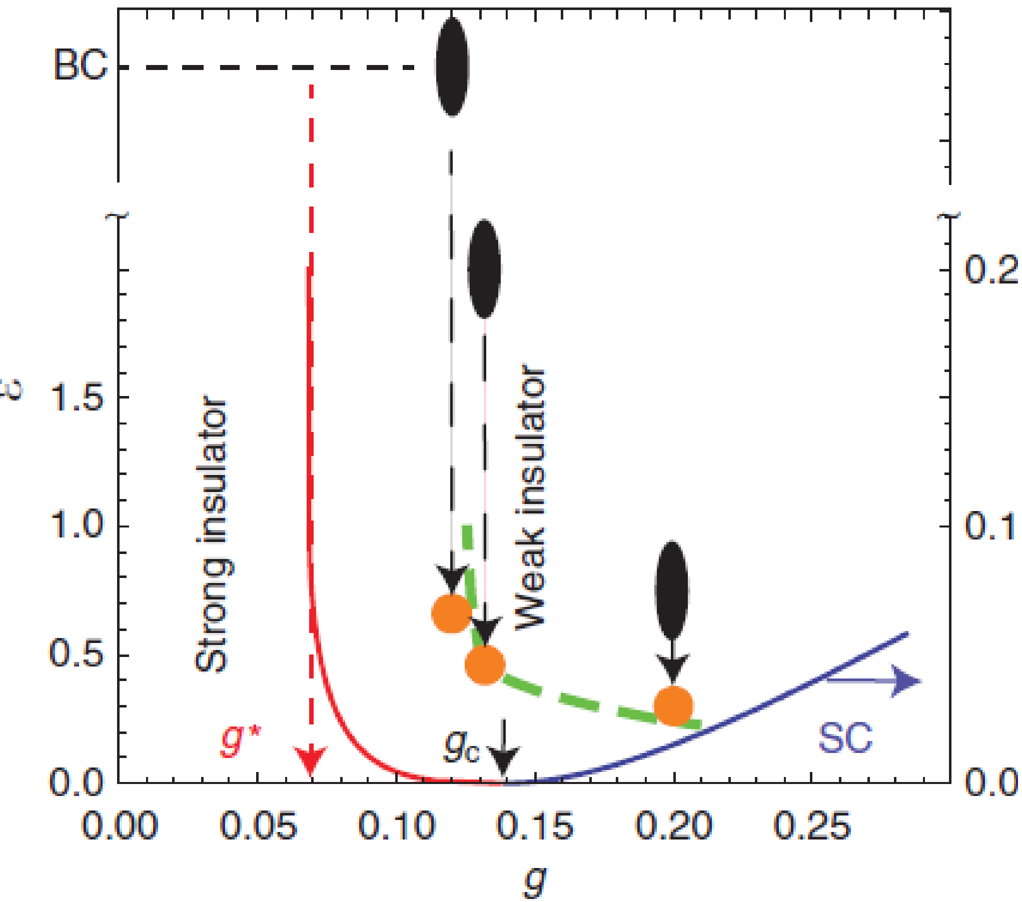
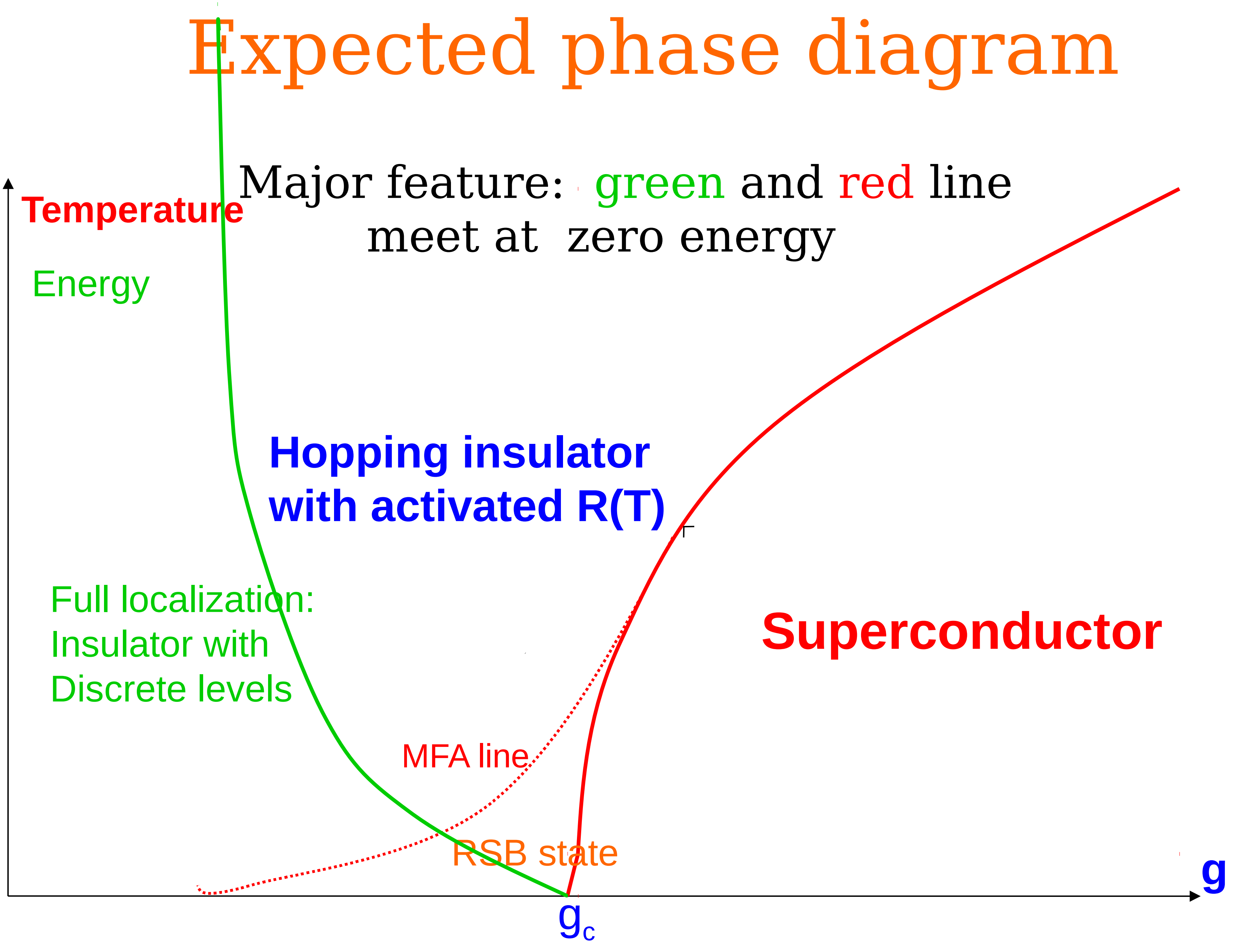


Figure 3 | Phase diagram and finite-size effects. Phase diagram for the model (1) with $J^{ZZ}=0$ as a function of the interaction constant g. The full lines show the predictions of the analytical study of the model (1) for the critical temperature (right vertical axis) and the threshold energy, ε, (left axis) of spin-flip excitations in infinite random graphs with $Z=3$ neighbours. The vertical ovals show the values of the critical coupling constant that correspond to a transition between different types of spectra for different energies E in finite random graphs of small size ($N=16$–20) as determined by direct numerical simulations. The uppermost oval shows the transition at the many-body band centre (corresponding to $E \gg 1$) that sets a lower bound for the critical $g(E)$. The thick dashed line shows the position of the spectral threshold for single-spin excitations with energy ε adjusted by finite-size effects, as explained in the main text and in the Methods section. The small circles show the typical energy of the single-spin excitations, $\varepsilon(E)$, that gives the main contribution to the many body excitations studied in direct numerical simulations. The good agreement between their position and expectations (dashed line) confirms the validity of the cavity method[5,6] that is used to obtain the results in infinite systems. The very small change in the critical value of the coupling constant between excitations at energy $E \approx 2.0$ and the centre of the many-body band implies that all excitations, at high and low energies, become localized when $g < g^*$.

Expected phase diagram

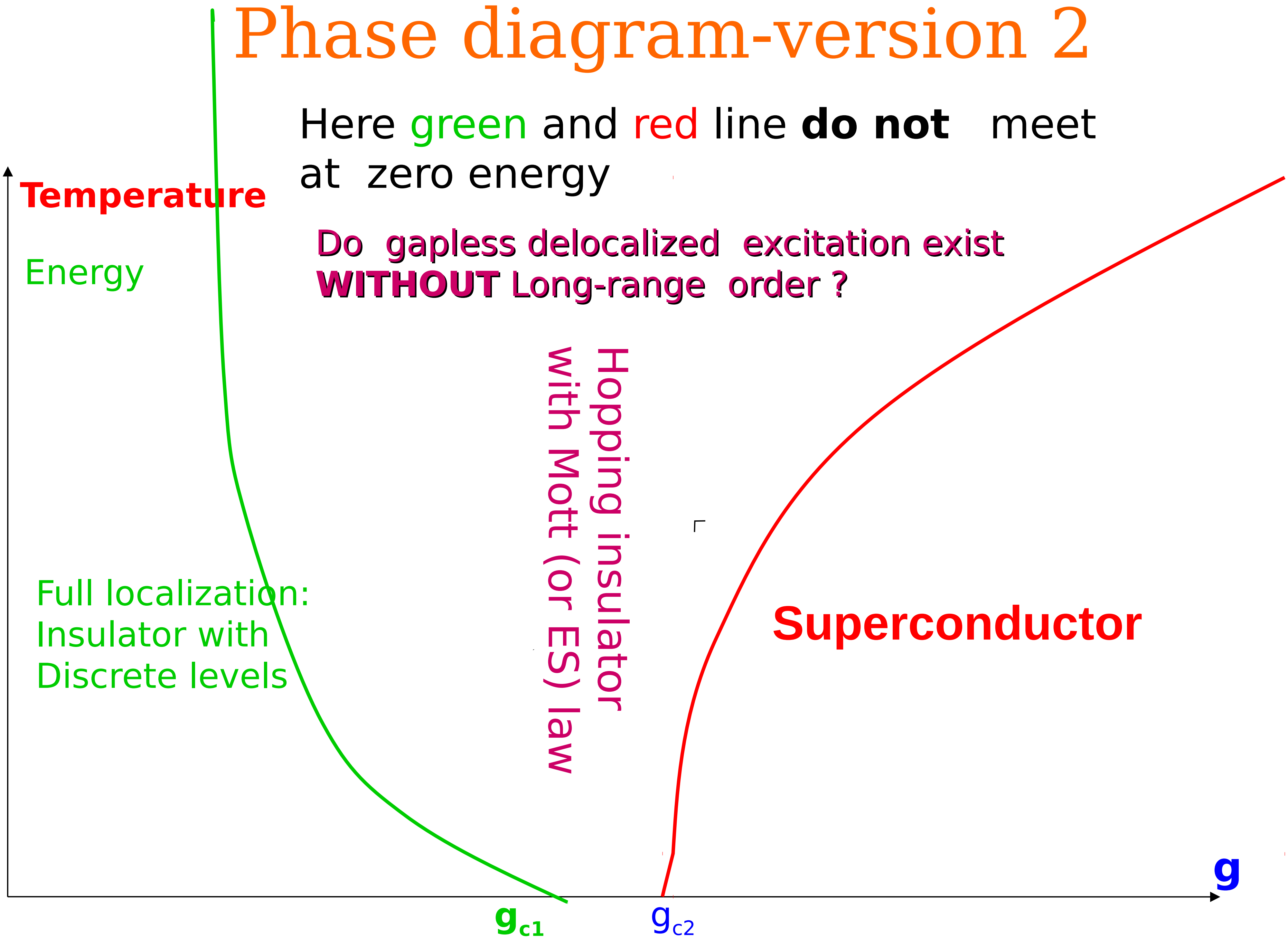
Major feature: green and red line meet at zero energy



Phase diagram-version 2

Here green and red line **do not** meet at zero energy

Do gapless delocalized excitation exist WITHOUT Long-range order ?



Temperature-driven localization transition in presence of $J_z S_i^z S_j^z$ interaction

$$\tilde{H}_{XY} = -2\sum_i \xi_i s_i^z - \sum_{(ij)} J s_i^z s_j^z - \sum_{(ij)} J_{ij}^{XY} (s_i^+ s_j^- + s_i^- s_j^+)$$

$$\nu(E) = CN \exp(a(g)\sqrt{EN})$$

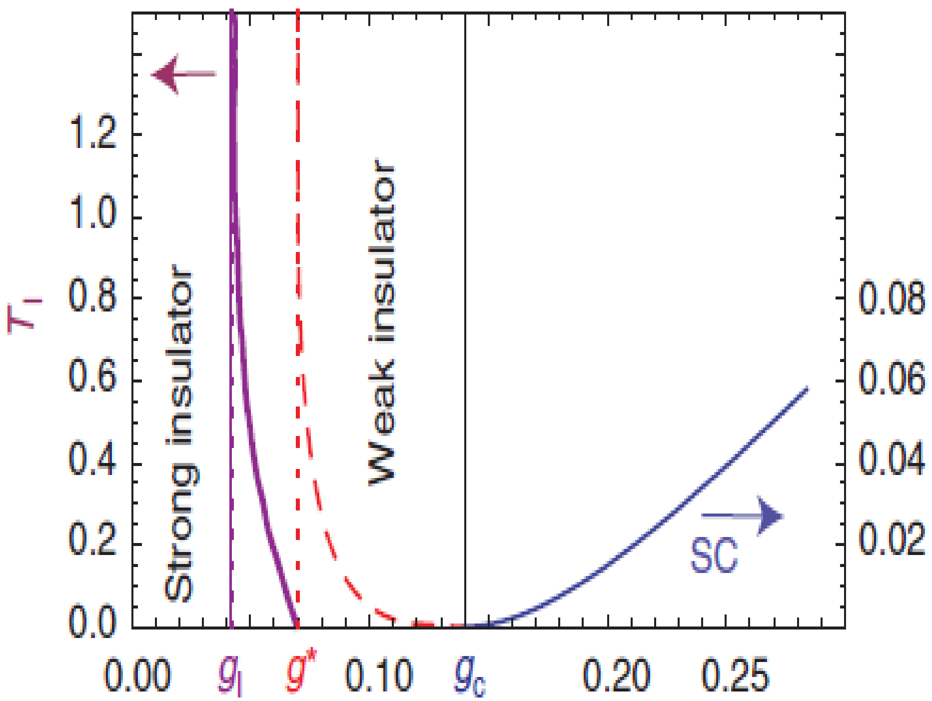
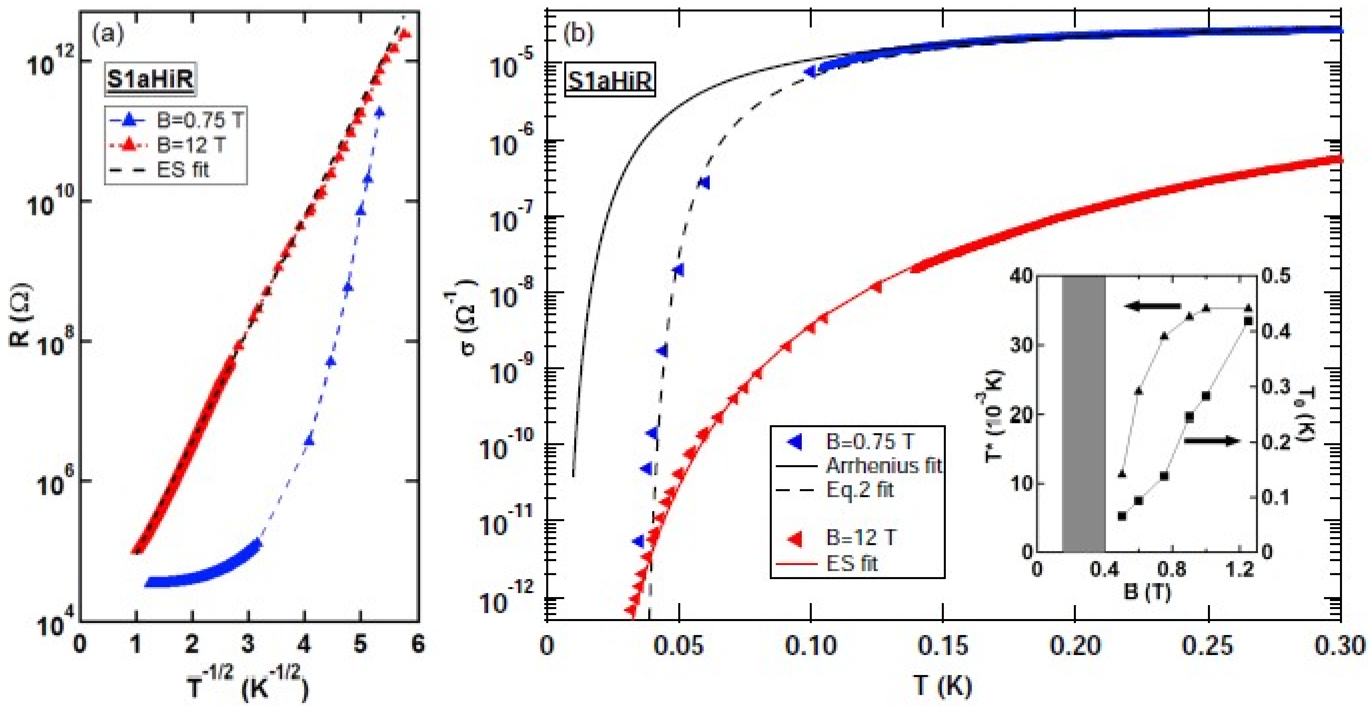


Figure 1 | Phase diagram in the temperature-coupling constant plane. The phase diagram is obtained from the solution of cavity equations for the model (1) with $Z = K+1 = 3$ and confirmed by numerical simulations. Blue line shows the dependence of the critical ordering temperature $T_c(g)$ on the coupling constant at $g > g_c$. The strength of the $s^z s^z$ interaction is $J^{zz} = 0.4$, non-zero J^{zz} results in a g-dependent line (purple) separating weak and strong insulators; in the absence of $s^z s^z$ interaction, this line becomes vertical. In the weak insulator, excitations at sufficiently high energies can decay even at zero temperature (the corresponding energy threshold is shown for $J^{zz} = 0$ by the dashed red line). A non-zero temperature results in non-zero relaxation of all excitations, even the ones of lowest energy. In contrast, in the strong insulator, no excitation with intensive energy can decay. As the interaction constant g is decreased, the temperature separating these phases (purple line) goes to infinity at $g = g_I$. At smaller

Evidence for a Finite Temperature Insulator

M. Ovadia[1,2], D. Kalok[1], I. Tamir[1], S. Mitra[1], B. Sacépé[1,3,4] and D. Shahar[1*]

$$\sigma(T) = \sigma_0 exp[-\frac{T_0}{T - T^*}],$$



Conclusions – part 2

New type of S-I phase transition is described

Pairing on nearly-critical states produces fractal superconductivity with relatively high T_c but small n_s

Pairing of electrons on localized states leads to hard gap and Arrhenius resistivity for 1e transport

Pseudogap behaviour is generic near S-I transition, with "insulating gap" above T_c

On insulating side activation of pair transport is due to ManyBodyLocalization threshold

Superconductivity is extremely inhomogeneous near SIT, for two different reasons:
i) fractality, ii) lack of self-averaging

Conclusions for 1+ 2

1) We don't know how to take into account both Coulomb effects in the Cooper channel and Localization/Fractality effects

It seems that both are relvant for S-I-T in TiN and probably in some other materials.

2) The nature and even the condition for existence of an intermediate “quatum metal” state is unknown

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