Ambegaokar-Eckern-Schön theory for a collective spin: geometric Langevin noise

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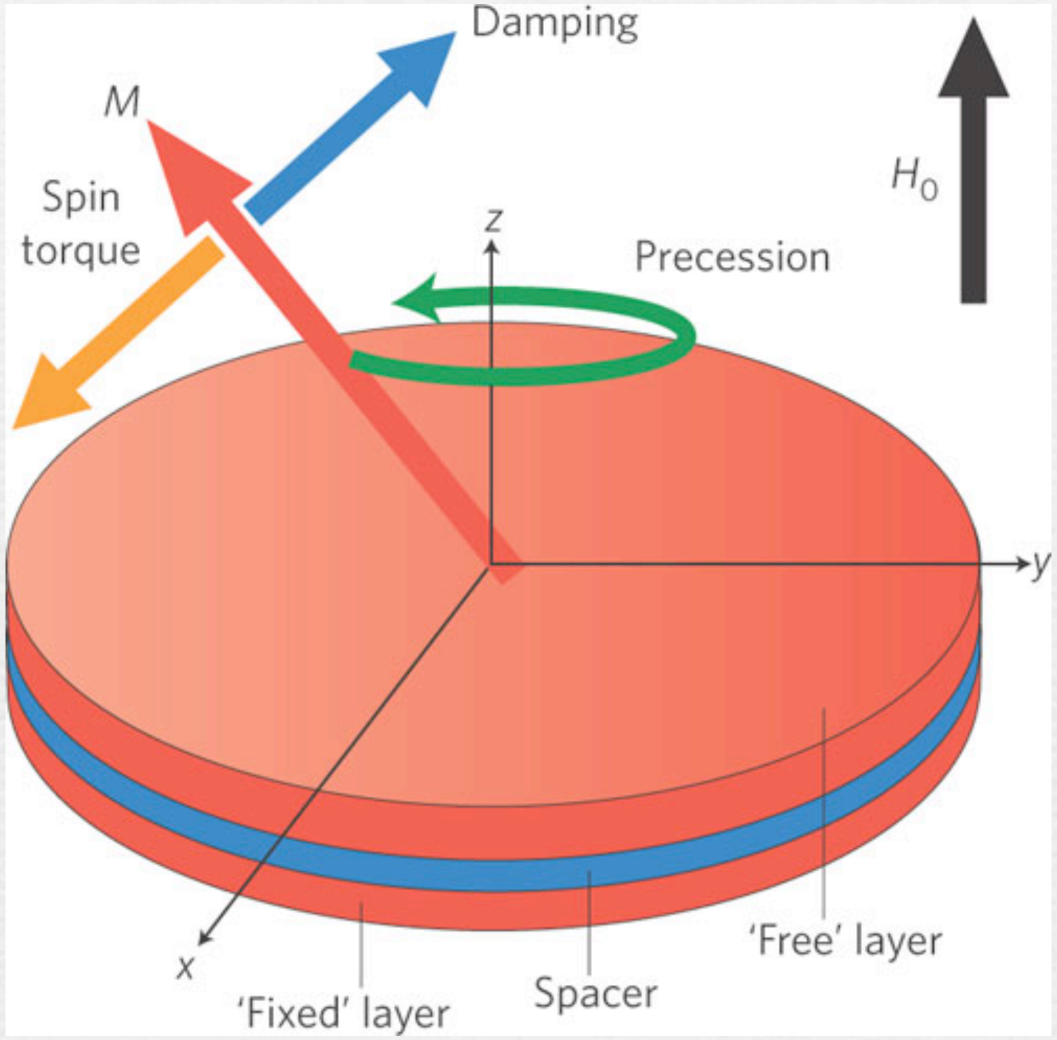
arXiv:1409.0150

Physical systems considered

- Small ferromagnetic particles (super-paramagnets)
- Magnetic tunnel junctions (free layer dynamics)
- Quantum dots close to Stoner transition

Example: magnetic tunnel junctions (MTJ)



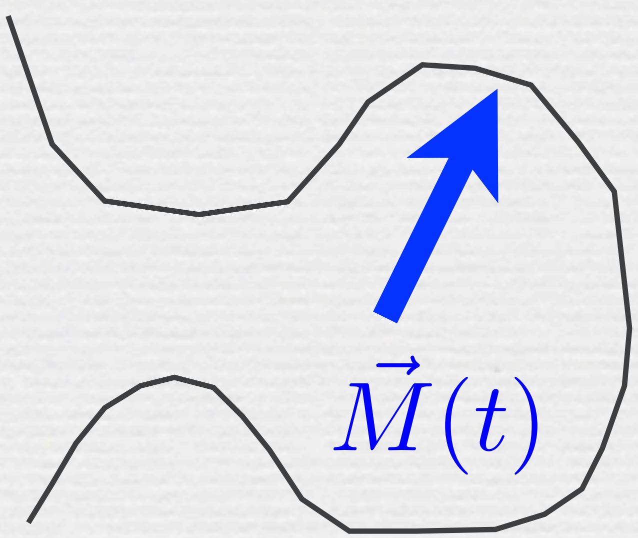
J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).

L. Berger, Phys. Rev. B **54**, 9353 (1996).

Goal

Proper description in terms of slow collective variable: magnetization $\vec{M}(t)$

Landau-Lifshitz-Gilbert (LLG) equation



$$\frac{d\vec{M}}{dt} = \vec{B}_{\text{eff}} \times \vec{M} + \eta \frac{\vec{M}}{|\vec{M}|} \times \frac{d\vec{M}}{dt}$$

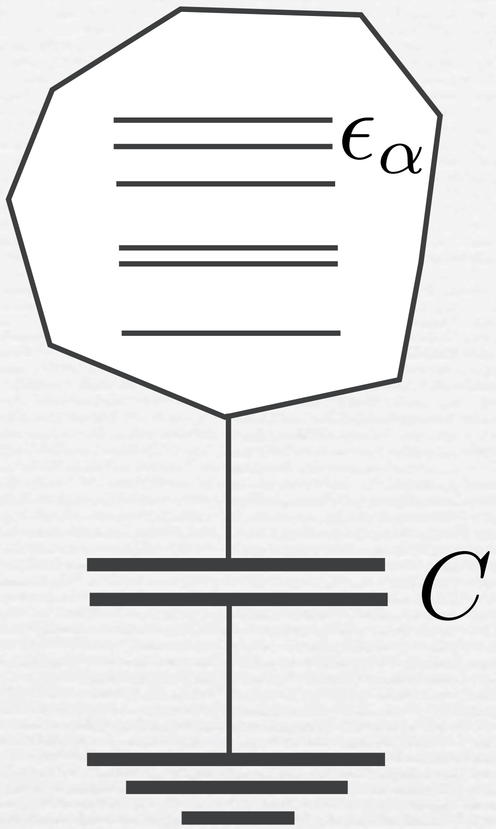
Landau & Lifshitz, Phys. Z. Sowietunion 8, 153 (1935)

T.L. Gilbert (1955)

AES action: Abelian U(1) case

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. **48**, 1745-1748 (1982)

U(1) case



Quantum dot with "zero mode" interaction

$$H = \sum_\alpha \epsilon_\alpha \psi^\dagger_\alpha \psi_\alpha + \frac{(eN)^2}{2C} \qquad N \equiv \sum_\alpha \psi^\dagger_\alpha \psi_\alpha$$

Hubbard-Stratonovich

$$\mathcal{Z} = \int D\bar{\psi} D\psi \, e^{i\mathcal{S}}$$

$$i\mathcal{S}_{\psi,V} = i \int dt \left[\sum_\alpha \bar{\psi}_\alpha \left[i\partial_t - \epsilon_\alpha - eV(t) \right] \psi_\alpha + \frac{CV^2(t)}{2} \right]$$

$$i\mathcal{S}_V = \sum_\alpha \operatorname{tr} \ln \left[i\partial_t - \epsilon_\alpha - eV(t) \right] + i \int dt \, \frac{CV^2(t)}{2}$$

U(1) case

$$i\mathcal{S}_\alpha = \mathrm{tr}\ \ln\left[i\partial_t - \epsilon_\alpha - eV(t)\right]$$

$$i\mathcal{S}_\alpha = \mathrm{tr}\ \ln\left[R^{-1}\left\{i\partial_t - \epsilon_\alpha - eV(t)\right\}R\right] \quad \longleftrightarrow \quad \psi(t) \to R(t)\psi(t)$$

$$R(t) = e^{-i\phi(t)} \in U(1)$$

$$i\mathcal{S}_\alpha = \mathrm{tr}\ \ln\left[i\partial_t - \epsilon_\alpha - eV(t) + iR^{-1}\partial_t R\right]$$

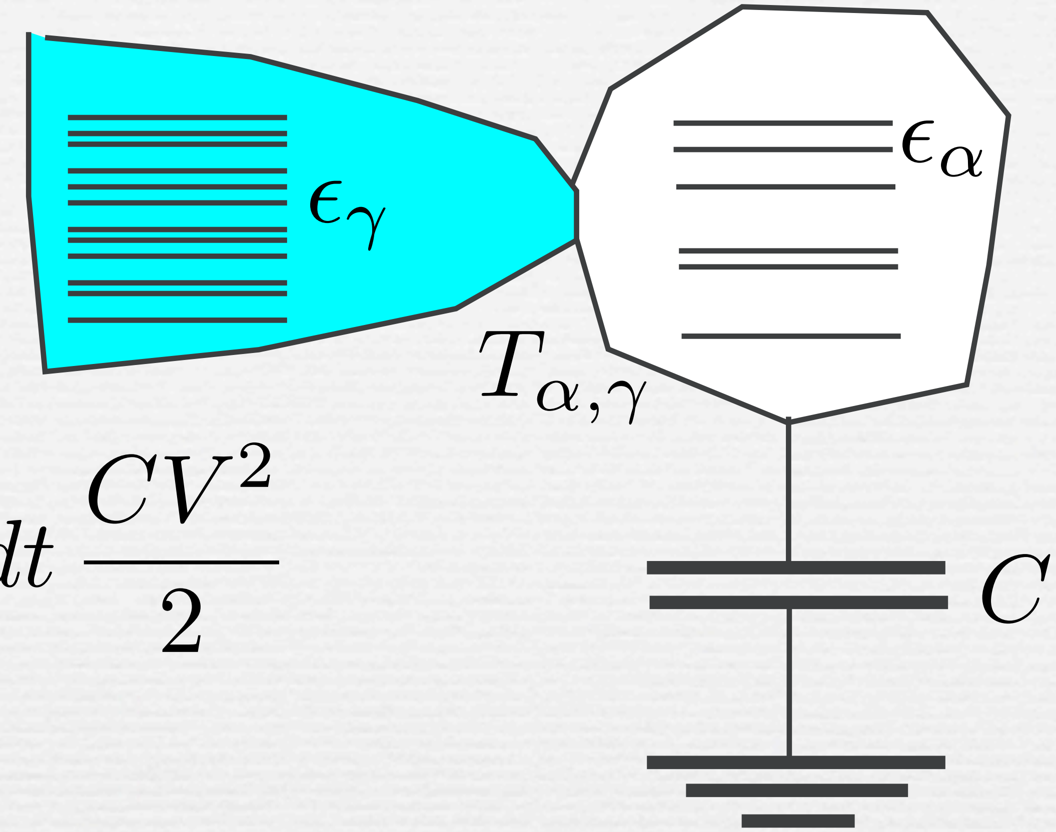
Choose R so that $iR^{-1}\partial_t R = eV \quad \longleftrightarrow \quad \dot{\phi}(t) = eV(t)$

$$i\mathcal{S}_V = \sum_\alpha \mathrm{tr}\ \ln\left[i\partial_t - \epsilon_\alpha\right] + i\int dt\, \frac{C\dot{\phi}^2}{2e^2} = const. + i\int dt\, \frac{C\dot{\phi}^2}{2e^2}$$

Attention: boundary conditions $\quad R(\tau + \beta) = R(\tau)$

A. Kamenev, Y. Gefen, Phys. Rev. B 54, 5428 (1996)

U(1) case



$$i\mathcal{S}_V = \mathrm{tr}\ln\begin{pmatrix} G_{dot}^{-1} & -T \\ -T^\dagger & G_{lead}^{-1} \end{pmatrix} + i\int dt\,\frac{CV^2}{2}$$

$$G_{dot}^{-1} = i\partial_t - \epsilon_\alpha - eV(t)$$

$$G_{lead}^{-1} = i\partial_t - \epsilon_\gamma$$

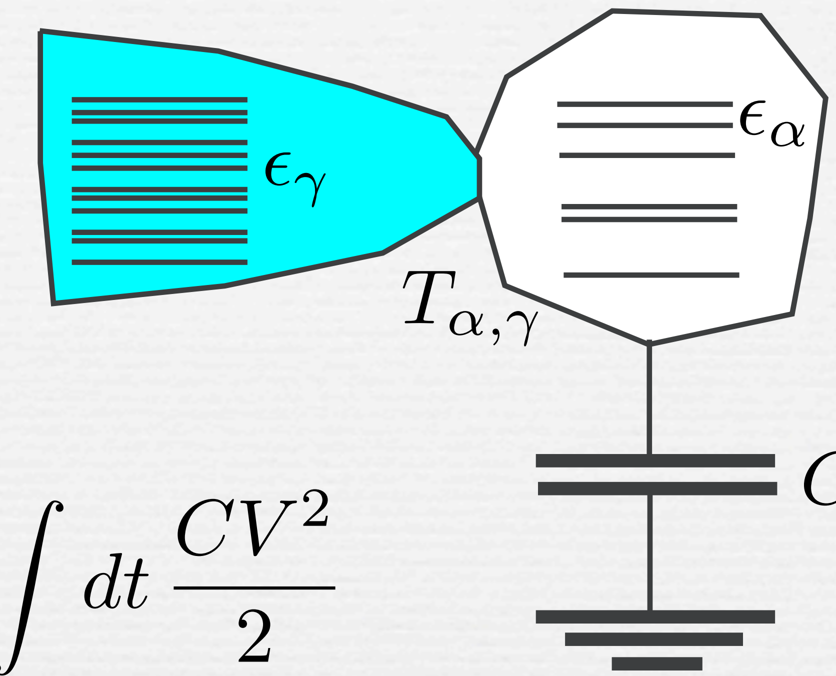
$$i\mathcal{S}_V = \mathrm{tr}\ln\left[i\partial_t - H_{dot}^0 - eV(t) - \Sigma\right] + i\int dt\,\frac{CV^2}{2}$$

$$H_{dot}^0 \equiv \sum_\alpha \epsilon_\alpha\,|\alpha\rangle\langle\alpha|$$

$$\Sigma(t_1, t_2) \equiv T G_{lead}(t_1, t_2) T^\dagger$$

Self-energy due to reservoir

U(1) case



Eliminating V(t)

$$i\mathcal{S}_V = \mathrm{tr}\ \ln\left[R^{-1}\left\{i\partial_t - H^0_{dot} - eV(t) - \Sigma\right\}R\right] + i\int dt\,\frac{CV^2}{2}$$

$$R(t) = e^{-i\phi(t)} \qquad \dot{\phi}(t) = eV(t)$$

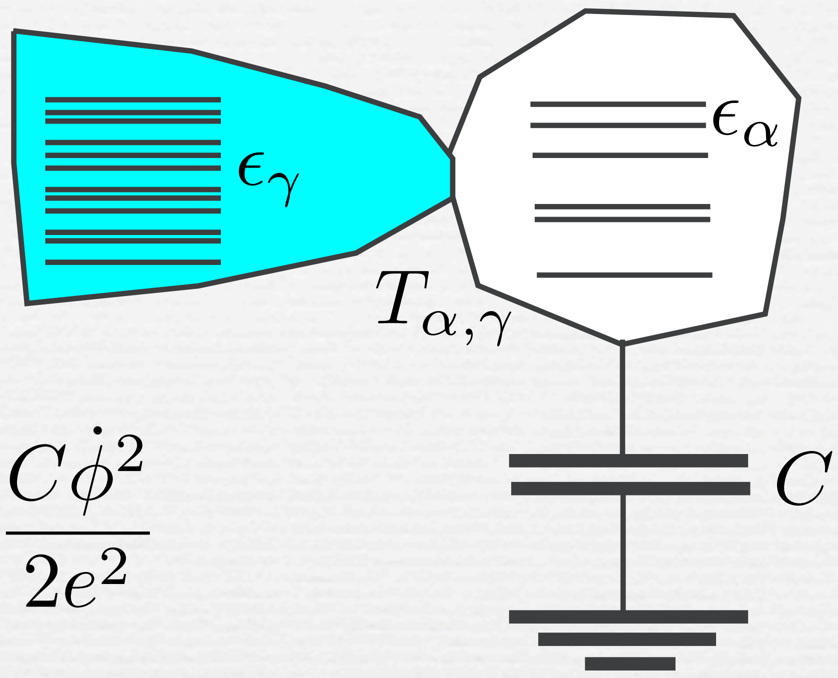
$$i\mathcal{S}_V = \mathrm{tr}\ \ln\left[i\partial_t - H^0_{dot} - R^{-1}(t_1)\Sigma(t_1,t_2)R(t_2)\right] + i\int dt\,\frac{C\dot{\phi}^2}{2e^2}$$

Expansion in tunneling amplitudes

$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\,\alpha(t_1,t_2)R^{-1}(t_1)R(t_2) + i\int dt\,\frac{C\dot{\phi}^2}{2e^2}$$

$$\alpha(t_1,t_2) \equiv \mathrm{tr}\left[G_{dot}(t_2,t_1)TG_{lead}(t_1,t_2)T^\dagger\right]$$

U(1) case



$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\, \alpha(t_1,t_2) R^{-1}(t_1) R(t_2) + i\int dt\, \frac{C\dot{\phi}^2}{2e^2}$$

$$i\mathcal{S}_{AES} = -\int dt_1 dt_2\, \alpha(t_1,t_2) \cos\left[\phi(t_1) - \phi(t_2)\right] + i\int dt\, \frac{C\dot{\phi}^2}{2e^2}$$

Matsubara

$$\alpha(\tau) = \frac{\pi g}{\sin^2(\pi\tau/\beta)}$$

Tunneling conductance

$$g = \pi \rho_{lead} \rho_{dot} |T|^2$$

AES vs. Caldeira-Leggett (CL) action in mesoscopic physics

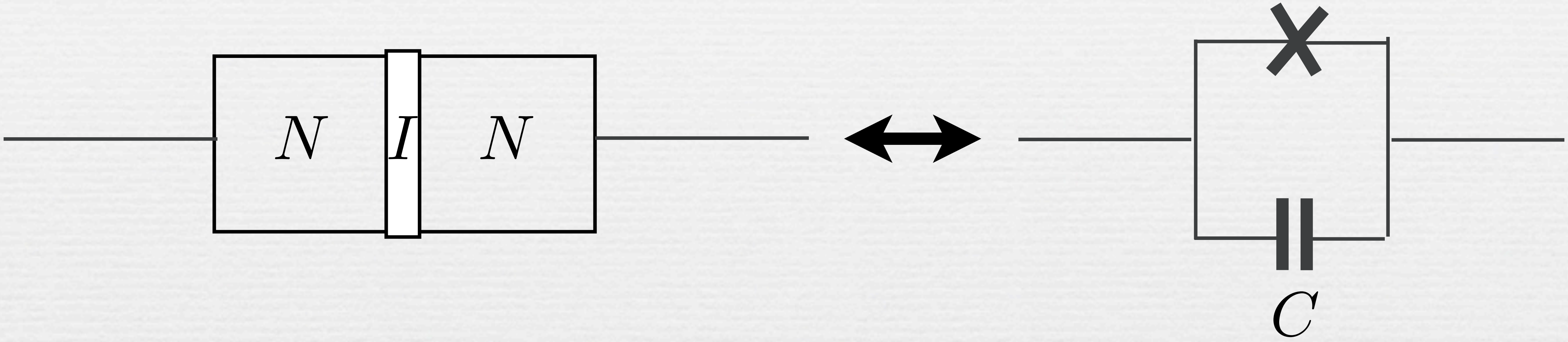
V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. 48, 1745-1748 (1982)

A.O. Caldeira and A.J. Leggett,
Annals of Physics 149, 374 (1983)

AES for tunnel junctions

1) Normal tunnel junction (NIN)

$$R_T = \frac{1}{2g}\frac{2\pi\hbar}{e^2}$$



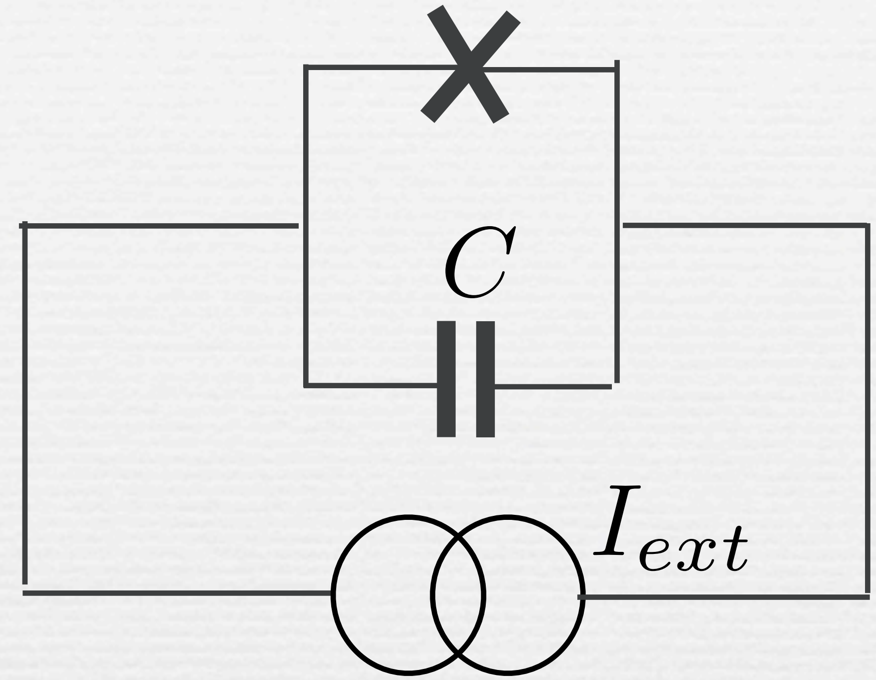
$$i\mathcal{S}_{AES} = i\int dt\,\frac{C\dot{\phi}^2}{2e^2} - \int dt_1 dt_2\,\alpha(t_1,t_2)\,\cos\left[\phi(t_1)-\phi(t_2)\right]$$

$$\dot{\phi}(t) = eV_L(t) - eV_R(t)$$

AES for tunnel junctions

$$R_T = \frac{1}{2g}\frac{2\pi\hbar}{e^2}$$

Normal tunnel junction (NIN)



$$i\mathcal{S}_{AES} = i\int dt\left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e}\right]$$

$$-\int dt_1 dt_2\, \alpha(t_1, t_2)\, \cos\left[\phi(t_1) - \phi(t_2)\right]$$

Langevin eq. of motion

$$C\ddot{\phi} + \frac{\dot{\phi}}{R_T} = I_{ext} + \underbrace{\xi_1 \cos\phi + \xi_2 \sin\phi}_{\delta I}$$

$$\langle \xi_n \xi_m \rangle = \delta_{n,m}\frac{\hbar\omega}{R_T}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}$$

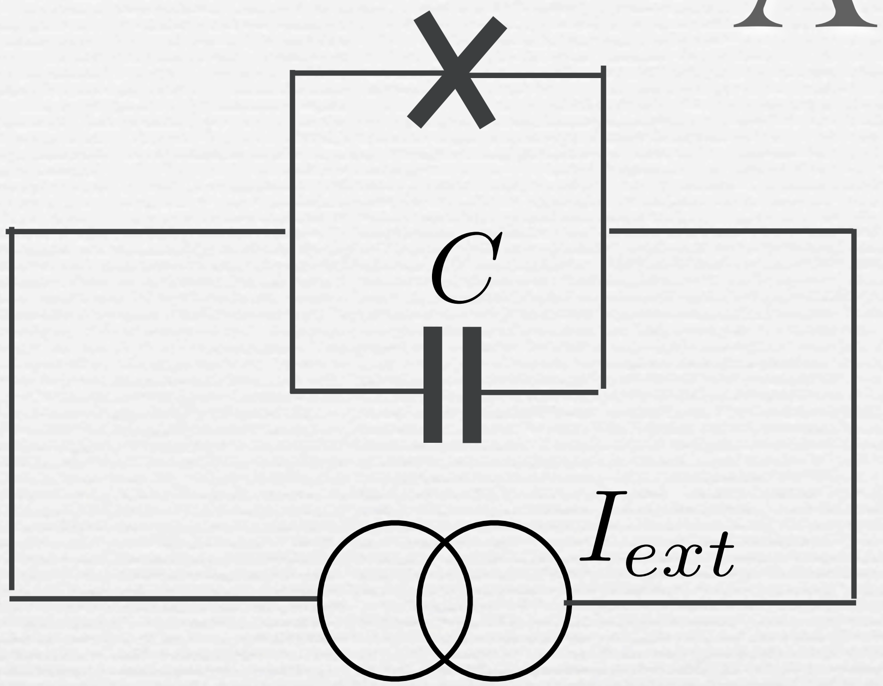
$$\phi = Vt + \delta\phi$$

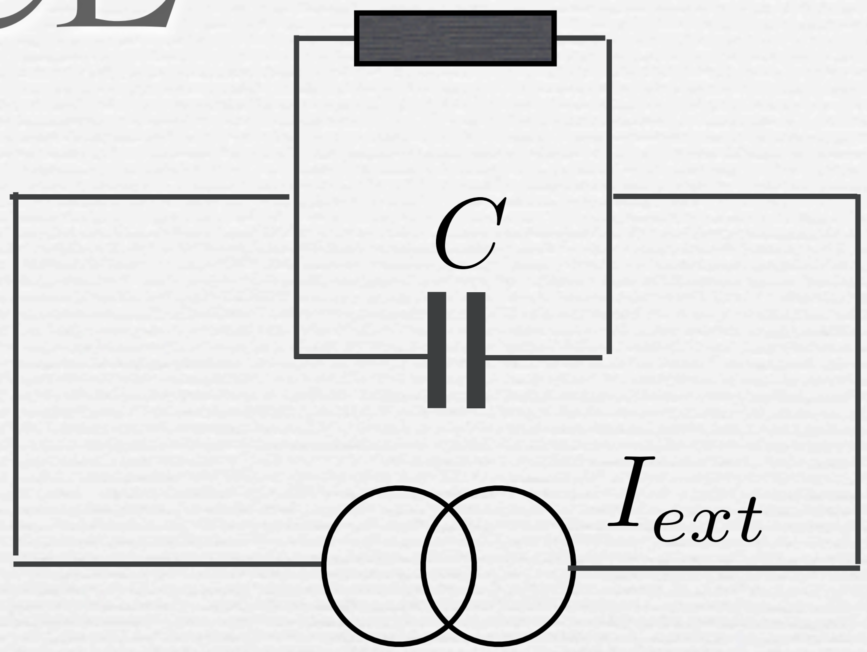
$$V \approx R_T I_{ext}$$

shot noise

$$\langle \delta I \delta I \rangle \sim \frac{eV}{R_T} \sim eI$$

AES vs. CL





$$i\mathcal{S}_{AES} = i \int dt \left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e} \right]$$

$$- \int dt_1 dt_2\, \alpha(t_1, t_2) \cos\left[\phi(t_1) - \phi(t_2)\right]$$

$$C\ddot{\phi} + \frac{\dot{\phi}}{R_T} = I_{ext} + \xi_1 \cos\phi + \xi_2 \sin\phi$$

shot noise

$$i\mathcal{S}_{CL} = i \int dt \left[\frac{C\dot{\phi}^2}{2e^2} + \frac{I_{ext}\phi}{e} \right]$$

$$+ \int dt_1 dt_2\, \alpha(t_1, t_2) \frac{\left[\phi(t_1) - \phi(t_2)\right]^2}{2}$$

$$C\ddot{\phi} + \frac{\dot{\phi}}{R} = I_{ext} + \xi$$

no shot noise

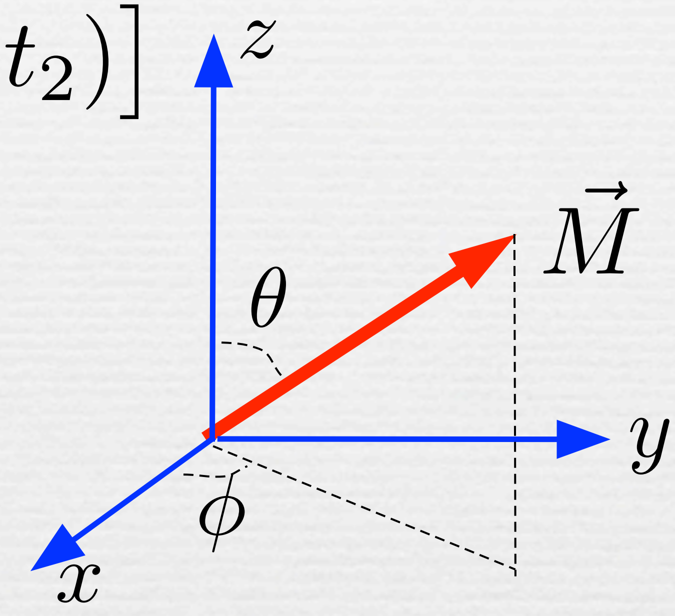
Non-Abelian SU(2) case

Results

1) Dissipation described by SU(2), AES - like action

$$i\mathcal{S}_{AES} = -\int dt_1\, dt_2\, \alpha(t_1, t_2)\, \mathrm{tr}\left[R(t_1) R^{-1}(t_2)\right]$$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right] \exp\left[-\frac{i\theta}{2}\sigma_y\right] \exp\left[\frac{i\psi}{2}\sigma_z\right]$$



2) Naively: non-gauge invariant

ψ - arbitrary

Fixing gauge freedom - technical challenge leading to the main result:

3) LLG - Langevin equations with geometric noise spin diffusion with effective geometric temperature

Quantum dot, exchange interaction

$$H = \sum_{\alpha,\sigma} \epsilon_\alpha \psi^\dagger_{\alpha,\sigma}\psi_{\alpha,\sigma} - J\hat{\mathbf{S}}^2 \qquad \hat{\mathbf{S}} = \sum_\alpha \psi^\dagger_{\alpha,\sigma_1}\mathbf{S}_{\sigma_1\sigma_2}\psi_{\alpha,\sigma_2}$$

$$i\mathcal{S}_{\Psi,M} = i\int dt \left[\sum_\alpha \bar{\psi}_\alpha \left(i\partial_t - \epsilon_\alpha - \mathbf{M}(t)\cdot\hat{\mathbf{S}}\right)\psi_\alpha - \frac{|\mathbf{M}|^{\mathbf{2}}}{4J}\right]$$

$$i\mathcal{S}_M = \sum_\alpha \mathrm{tr}\,\ln\left(i\partial_t - \epsilon_\alpha - \mathbf{M}(t)\cdot\hat{\mathbf{S}}\right) - i\int dt\frac{|\mathbf{M}|^2}{4J}$$

Non-Abelian

M.N. Kiselev, Y. Gefen, Phys. Rev. Lett. 96, 066805 (2006)

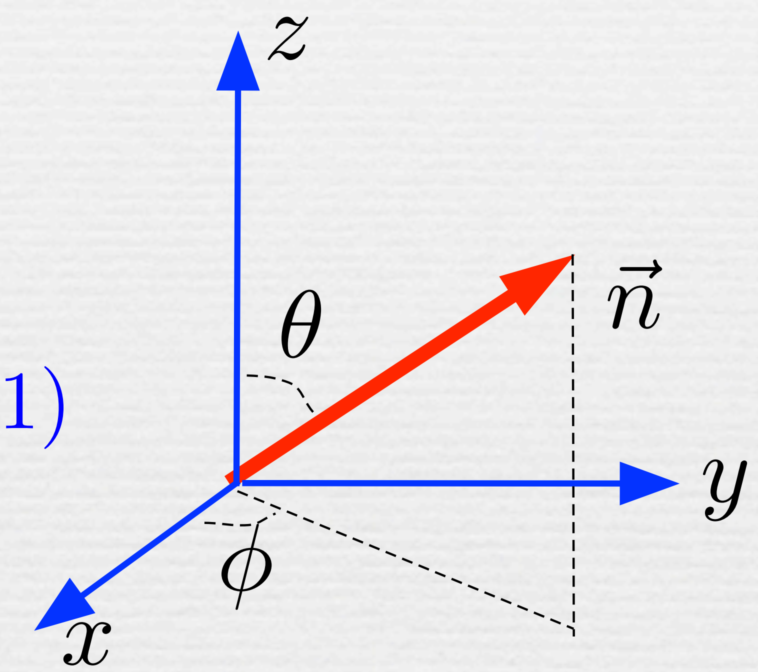
I.S. Burmistrov,Y. Gefen, M.N. Kiselev, Pis'ma v ZhETF 92, 202 (2010)

I.S. Burmistrov, Y. Gefen, and M. N. Kiselev, Phys. Rev. B 85, 155311 (2012)

Geometric adiabatic solution

$$i\mathcal{S}_M = \sum_\alpha \mathrm{tr}\,\ln\left(i\partial_t - \epsilon_\alpha - M(t)\mathbf{n}(t)\cdot\hat{\mathbf{S}}\right) - i\int dt\,\frac{M^2(t)}{4J}$$

Transform to rotating frame



$$\mathbf{n}\cdot\hat{\mathbf{S}} = R\,\hat{S}_z\,R^\dagger$$

$R \in SU(2)/U(1)$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[-\frac{i\psi}{2}\sigma_z\right]$$

$$i\mathcal{S}_M = \sum_\alpha \mathrm{tr}\,\ln\left(i\partial_t - \epsilon_\alpha - M(t)\,S_z + i\,R^{-1}\partial_t R\right) - i\int dt\,\frac{M^2}{4J}$$

Non-Abelian vector potential
i.a., Berry phase

Adibatic expansion

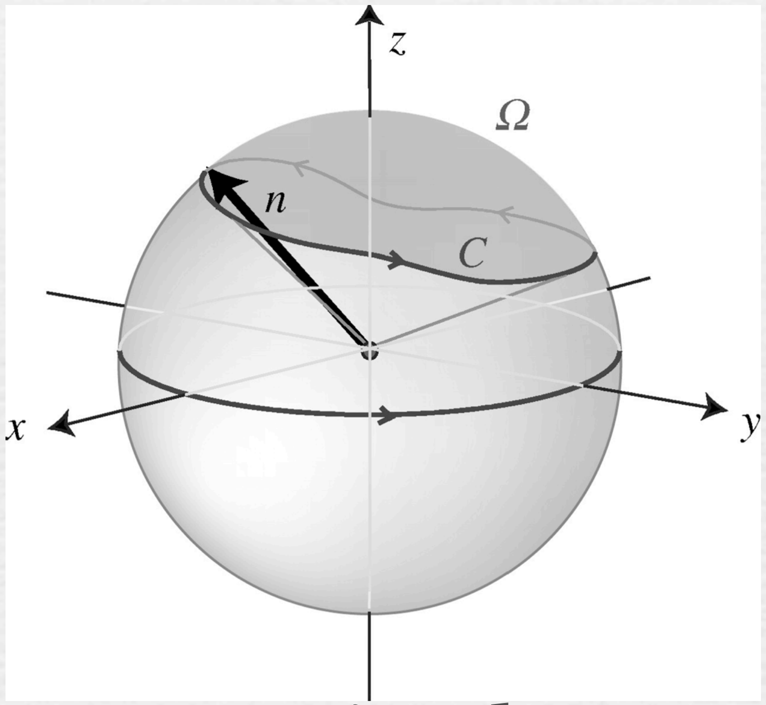
$$\mathcal{S}_M = \mathrm{tr}\ln\left(-\partial_\tau - H^{(0)}_{dot} - M(\tau)\,S_z - R^{-1}\partial_\tau R\right) - \int_0^\beta d\tau\,\frac{M^2}{4J}$$

Stoner ferromagnet or close

$\rho_{dot} J > 1$ $\quad$ $0 < 1 - \rho_{dot} J \ll 1$

$$\mathcal{S}_M \approx -\beta\Omega(M_0) + iS\int_0^\beta d\tau\,(1-\cos\theta)\dot{\phi}$$

$$S \approx \frac{\bar{\rho}_{dot} M_0}{2} \gg 1$$

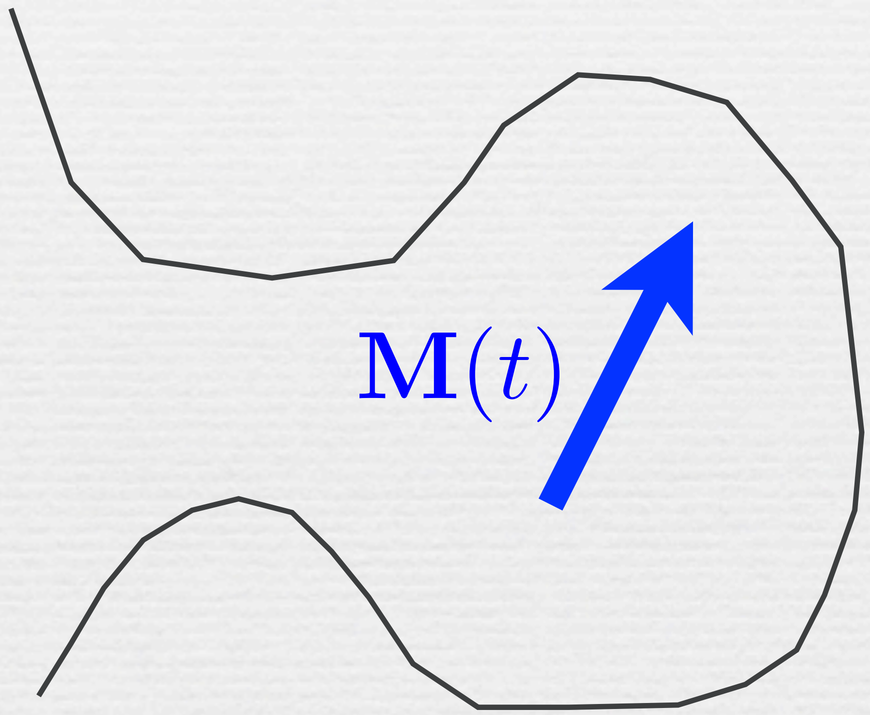


Berry's phase
WZNW action

A. Saha et al. Annals of Phys., 327 (10), 2543 (2012)

Open magnetic quantum dot AES tunnel action

Open dot, "AES" action



$$H = H_{dot} + H_{lead} + H_t$$

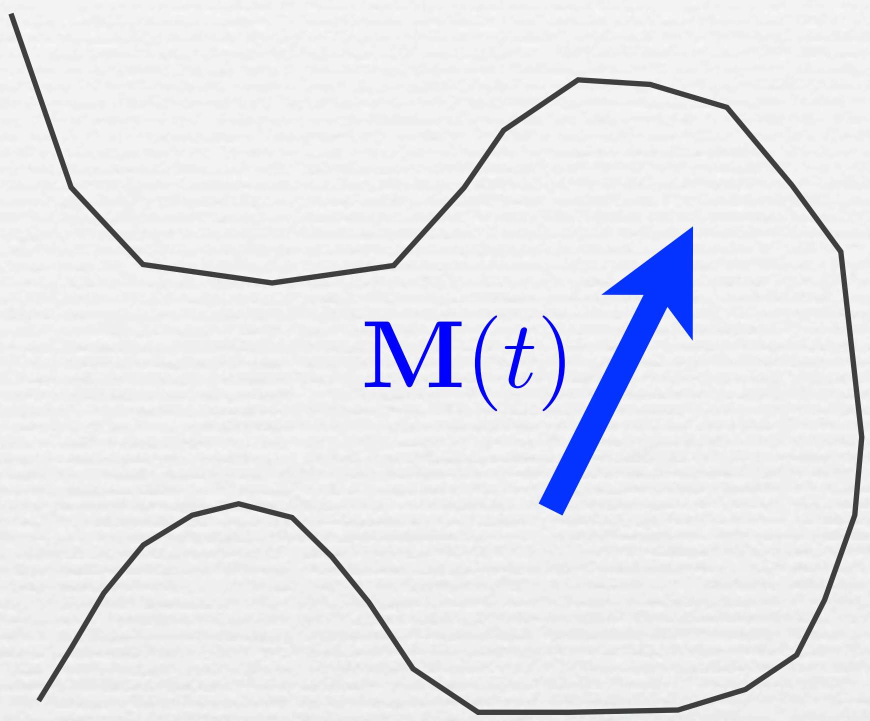
$$H_{dot} = \sum_{\alpha,\sigma} \epsilon_\alpha \psi^\dagger_{\alpha,\sigma}\psi_{\alpha,\sigma} - J\hat{\mathbf{S}}^2$$

$$H_{lead} = \sum_{\gamma,\sigma} \epsilon_{\gamma,\sigma} c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma}$$

$$H_T = \sum_{\alpha,\gamma,\sigma} T_{\alpha,\gamma}\psi^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + h.c.$$

A. L. Chudnovskiy, J. Swiebodzinski, and A. Kamenev, Phys. Rev. Lett. 101, 066601 (2008)

Open dot, effective action

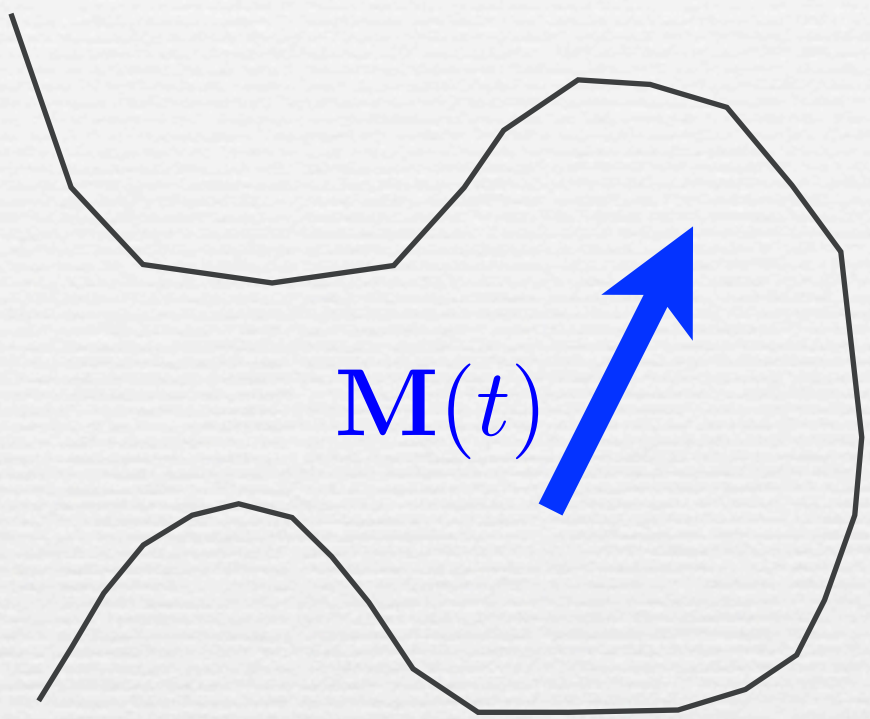


$$\mathcal{S}_M = \mathrm{tr}\,\ln \begin{pmatrix} G^{-1}_{dot} & -T \\ -T^{\dagger} & G^{-1}_{lead} \end{pmatrix} - \oint_K dt\, \frac{M^2}{4J}$$

$$G^{-1}_{dot} = i\partial_t - \epsilon_\alpha - \mathbf{M}(t)\cdot\mathbf{S}$$

$$G^{-1}_{lead} = i\partial_t - \epsilon_\gamma$$

Open dot, effective action



Non-Abelian

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - \mathbf{M}(t)\cdot\mathbf{S} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$H^0_{dot} \equiv \sum_\alpha \epsilon_\alpha\,|\alpha\rangle\langle\alpha|$

$$\Sigma \equiv T G_{lead} T^\dagger$$

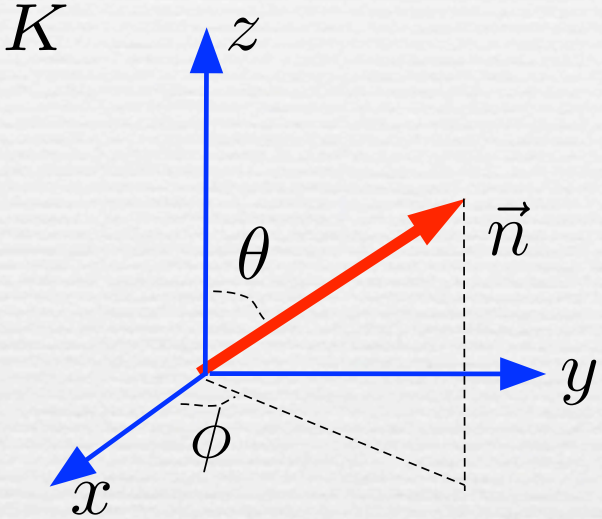
Self-energy due to reservoir

Open dot, rotating frame

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[i\partial_t - H^0_{dot} - M(t)\,\vec{n}(t)\cdot\vec{\mathbf{S}} - \Sigma\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$$\vec{n}\cdot\vec{\mathbf{S}} = R\,S_z\,R^\dagger \qquad R \in SU(2)/U(1)$$

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[\frac{i(\phi-\chi)}{2}\sigma_z\right]$$



$$i\mathcal{S}_\Phi = \mathrm{tr}\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger\Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

Geom.
vector potential

$$Q \equiv R^\dagger(-i\partial_t)R$$

Rotated
tunneling self-
energy

Rotation: convenient representation

$$R = \exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[-\frac{i\psi}{2}\sigma_z\right]$$

$R \in SU(2)/U(1)$

$$\Psi(\tau) \to R(\tau)\Psi(\tau)$$

$$R(\tau+\beta) = R(\tau)$$

$$R = \underbrace{\exp\left[-\frac{i\phi}{2}\sigma_z\right]\exp\left[-\frac{i\theta}{2}\sigma_y\right]\exp\left[\frac{i\phi}{2}\sigma_z\right]}_{\text{periodic}}\exp\left[-\frac{i\chi}{2}\sigma_z\right]$$

$\psi = -\phi + \chi$

$\chi(\tau+\beta) = \chi(\tau) + 4\pi n$

Open dot, vector potential

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[i\partial_t - H^0_{dot} - M\cdot S_z - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$

$$Q \equiv R^\dagger(-i\partial_t)R = Q_\parallel + Q_\perp$$

$$Q_\parallel \equiv \frac{1}{2}\left[\dot\phi(1-\cos\theta) - \dot\chi\right]\sigma_z$$

Berry's phase, gauge dependent

$$Q_\perp \equiv -\frac{1}{2}\left[\dot\theta\,\sigma_y - \dot\phi\sin\theta\,\sigma_x\right]\exp\left[i(\phi-\chi)\,\sigma_z\right]$$

Landau-Zener, neglected

Tunneling expansion, "AES"

$$i\mathcal{S}_M = \mathrm{tr}\,\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right] - i\oint_K dt\,\frac{M^2}{4J}$$ Gauge invariant

$$G_0^{-1} = i\partial_t - H^0_{dot} - M\cdot S_z$$

Expansion

$$i\mathcal{S}_M^{Berry} = -\mathrm{tr}\;[G_0 Q] = iS\oint_K (1-\cos\theta)\dot{\phi}dt$$ Berry phase

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[G_0 R^\dagger \Sigma R\right]$$ Gauge non-invariant

in original U(1) AES

$$R^\dagger(t)R(t') \sim e^{i\left[\varphi(t)-\varphi(t')\right]}$$

V. Ambegaokar, U. Eckern, G. Schön, Phys. Rev. Lett. **48**, 1745-1748 (1982)

Explicit form for non-magnetic lead

$$i\mathcal{S}_M^{AES} = -\int dt_1\, dt_2\, \alpha(t_1 - t_2)\, \mathrm{tr}\left[R(t_1)R^{-1}(t_2)\right]$$

Matsubara

$$\alpha(\tau) = \frac{\pi g}{\sin^2(\pi\tau/\beta)}$$

Tunneling conductance

$$g = \pi \rho_{lead}\rho_{dot}|T|^2$$

$$\mathrm{tr}\left[R(t_1)R^{-1}(t_2)\right] =$$

$$\cos\frac{\theta(t_1)}{2}\cos\frac{\theta(t_2)}{2}\cos\left(\frac{\chi(t_1)-\chi(t_2)}{2}\right)$$

$$+\sin\frac{\theta(t_1)}{2}\sin\frac{\theta(t_2)}{2}\cos\left(\phi(t_1)-\phi(t_2)-\frac{\chi(t_1)-\chi(t_2)}{2}\right)$$

Not gauge invariant

Tunneling expansion, gauge fixing

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[G_0^{-1} - Q - R^\dagger \Sigma R\right]$$

Gauge invariant expansion

$$i\mathcal{S}_M^{AES} = -\mathrm{tr}\left[(G_0^{-1} - Q)^{-1} R^\dagger \Sigma R\right]$$

Would be nice to choose gauge such that $Q = 0$

$$Q_\parallel \equiv \frac{1}{2}\left[\dot{\phi}(1-\cos\theta) - \dot{\chi}\right]\sigma_z = 0$$

$$\dot{\chi} = \dot{\phi}(1-\cos\theta)$$

Would be nice, but ...

AES action on Keldysh contour

U. Eckern, G. Schön, V. Ambegaokar, Phys. Rev. B 30, 6419-6431 (1984)

$$i\mathcal{S}_{AES} = -g\int dt_1 dt_2 \,\mathrm{tr}\left[\begin{pmatrix} R_c^\dagger(t_1) & \frac{R_q^\dagger(t_1)}{2}\end{pmatrix}\begin{pmatrix} 0 & \alpha_A \\ \alpha_R & \alpha_K\end{pmatrix}_{(t_1-t_2)}\begin{pmatrix} R_c(t_2) \\ \frac{R_q(t_2)}{2}\end{pmatrix}\right]$$

$g = \pi\rho_{lead}\rho_{dot}|T|^2$ Tunneling conductance

$\alpha_R(\omega) = \omega + symm.part$ $\alpha_K(\omega) = 2\omega\coth(\omega/2T)$

$$R_c \equiv \frac{R_u + R_d}{2}$$

$$R_q \equiv R_u - R_d$$

Semiclassical equations of motion

Gauge fixing

$Q_{\parallel} = 0$ → $\dot{\chi} = \dot{\phi}(1 - \cos\theta)$

Would be nice, but impossible

Berry phase different on two contours

$$\dot{\chi}_c(t) = \dot{\phi}_c(t)\,(1 - \cos\theta_c(t)) \quad\rightarrow\quad Q_{\parallel,c} = 0$$

$$\chi_q(t) = \phi_q(t)\,(1 - \cos\theta_c(t))$$

$$\rightarrow\quad Q_{\parallel,q} = \frac{1}{2}\,\sigma_z\,\sin\theta_c\,\left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

$$i\mathcal{S}_{WZNW} = iS\int dt\,\sin\theta_c\,\left[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q\right]$$

Keldysh Berry phase action

Equations of motion

$$i\mathcal{S}_{total} \equiv i\mathcal{S}_{WZNW} + i\mathcal{S}_{B} + i\mathcal{S}^{R}_{AES} + i\mathcal{S}^{K}_{AES}$$

$$i\mathcal{S}_{WZNW} = iS \int dt \, \sin\theta_c \left[\dot{\phi}_c \theta_q - \dot{\theta}_c \phi_q\right] \qquad i\mathcal{S}_{B} = -iS\gamma\, B \int dt\, \theta_q \, \sin\theta_c$$

$$i\mathcal{S}^{R}_{AES} = -2ig \int dt_1\, dt_2\, \mathrm{Im}\, \alpha_R(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^c_n(t_2)$$

$$i\mathcal{S}^{K}_{AES} = -\frac{g}{2} \int dt_1\, dt_2\ \alpha_K(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^q_n(t_2)$$

$$A_0 \equiv \cos\left[\tfrac{\theta}{2}\right]\ \cos\left[\tfrac{\chi}{2}\right],\ A_x \equiv \sin\left[\tfrac{\theta}{2}\right] \sin\left[\phi - \tfrac{\chi}{2}\right],$$
$$A_y \equiv -\sin\left[\tfrac{\theta}{2}\right] \cos\left[\phi - \tfrac{\chi}{2}\right],\ A_z \equiv -\cos\left[\tfrac{\theta}{2}\right]\ \sin\left[\tfrac{\chi}{2}\right]$$

Keldysh part - Langevin terms

$$i\mathcal{S}^K_{AES} = -\frac{g}{2}\int dt_1\, dt_2\; \alpha^K_{AES}(t_1 - t_2) \sum_{n=0,x,y,z} A^q_n(t_1) A^q_n(t_2)$$

$$e^{i\mathcal{S}^K_{AES}} = \int \left(\prod_{n=0,x,y,z} D\xi_n\right) \exp\left[\int dt \left\{ i \sum_{n=0,x,y,z} \xi_n A^q_n \right\} + i\mathcal{S}_\xi\right]$$

$$i\mathcal{S}_\xi = -\frac{1}{2}\sum_n \int dt_1 dt_2 \left[\alpha^K_{AES}\right]^{-1}_{(t_1-t_2)} \xi_n(t_1)\xi_n(t_2)$$

$$i\mathcal{S}_{total} \equiv i\mathcal{S}_B + i\mathcal{S}_{WZNW} + i\mathcal{S}^R_{AES} + \int dt \sum_n i\xi_n A^q_n$$

$$\langle \xi_n(t_1)\xi_m(t_2)\rangle = \delta_{nm}\, \alpha^K_{AES}(t_1 - t_2)$$

Landau-Lifshitz-Gilbert-Langevin equation

$$\dot{\theta} + \frac{g}{S}\,\sin\theta\dot{\phi} = \frac{1}{S}\,\eta_\theta \qquad \sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\dot{\theta} = \frac{1}{S}\,\eta_\phi$$

$$\begin{aligned}
\eta_\theta = & \quad \frac{1}{2}\,\cos\frac{\theta}{2}\,\left[\xi_x\,\cos\left(\phi - \frac{\chi}{2}\right) + \xi_y\,\sin\left(\phi - \frac{\chi}{2}\right)\right] \\
& - \frac{1}{2}\,\sin\frac{\theta}{2}\,\left[\xi_z\,\cos\frac{\chi}{2} + \xi_0\,\sin\frac{\chi}{2}\right]\;, \\
\eta_\phi = & - \frac{1}{2}\,\cos\frac{\theta}{2}\left[\xi_x\,\sin\left(\phi - \frac{\chi}{2}\right) - \xi_y\,\cos\left(\phi - \frac{\chi}{2}\right)\right] \\
& - \frac{1}{2}\,\sin\frac{\theta}{2}\left[\xi_z\,\sin\frac{\chi}{2} - \xi_0\,\cos\frac{\chi}{2}\right]
\end{aligned}$$

$$\langle \xi_n \xi_m \rangle_\omega = 2g\,\omega\coth\frac{\omega}{2T}\,\delta_{n,m} \qquad n, m = 0, x, y, z$$

Gauge fixing $\dot{\chi} = \dot{\phi}(1 - \cos\theta)$

Landau-Lifshitz-Gilbert equation

$$\sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\,\dot{\theta} = 0$$

$$\vec{n} \equiv \frac{\vec{S}}{S} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\dot{\theta} + \frac{g}{S}\,\sin\theta\dot{\phi} = 0$$

$$\frac{d\vec{n}}{dt} = \vec{B} \times \vec{n} + \frac{g}{S}\,\vec{n} \times \frac{d\vec{n}}{dt}$$

AES vs. Caldeira-Leggett

Usual LLG-Langevin equation

W. F. Brown, Phys. Rev. 130, 1677 (1963).

$$\frac{d\vec{n}}{dt} = \left(\vec{B} + \delta\vec{B}\right) \times \vec{n} + \alpha\,\vec{n} \times \frac{d\vec{n}}{dt}$$

$$\langle \delta B_n \delta B_m \rangle_\omega \propto \alpha T\, \delta_{n,m}$$

Three independent Langevin variables
not four !!!

What is the microscopic theory?

Caldeira - Leggett situation

$$\delta H = \vec{S}\vec{X} \equiv -J\vec{S}\cdot\frac{1}{2}\sum_{\alpha,\beta} c^{\dagger}_{\alpha,\sigma_1}\vec{\sigma}_{\sigma_1\sigma_2}c_{\beta,\sigma_2}$$

Large spin $\vec{S}$ interacting with a bath
e.g., with electron-hole continuum, $\alpha \neq \beta$

$$\vec{S} \neq \frac{1}{2}\sum_{\alpha,\beta} c^{\dagger}_{\alpha,\sigma_1}\vec{\sigma}_{\sigma_1\sigma_2}c_{\beta,\sigma_2}$$

E.g., N. Bode, L. Arrachea, G. S. Lozano, T. S. Nunner, F. von Oppen, Phys. Rev. B 85, 115440 (2012)

Caldeira-Leggett action

$$\mathcal{S}_{diss} = \int_0^\beta d\tau_1 \int_0^\beta d\tau_2\,\alpha(\tau_1-\tau_2)\,\vec{S}(\tau_1)\cdot\vec{S}(\tau_2)$$

$$\alpha(\tau_1-\tau_2) \sim \langle X(\tau_1)X(\tau_2)\rangle$$

↓

$$\frac{d\vec{n}}{dt} = \left(\vec{B}+\delta\vec{B}\right)\times\vec{n} + S\,\alpha\,\vec{n}\times\frac{d\vec{n}}{dt} \qquad \langle\delta B_n \delta B_m\rangle_\omega \propto \alpha T\,\delta_{n,m}$$

AES vs. Caldeira - Leggett

AES

$$\dot{\theta} + \frac{g}{S}\,\sin\theta\dot{\phi} = \frac{1}{S}\,\eta_\theta \qquad \sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\,\dot{\theta} = \frac{1}{S}\,\eta_\phi$$

$$\eta_\theta = \frac{1}{2}\,\cos\frac{\theta}{2}\,\left[\xi_x\,\cos\left(\phi - \frac{\chi}{2}\right) + \xi_y\,\sin\left(\phi - \frac{\chi}{2}\right)\right] - \frac{1}{2}\,\sin\frac{\theta}{2}\,\left[\xi_z\,\cos\frac{\chi}{2} + \xi_0\,\sin\frac{\chi}{2}\right]$$

$$\eta_\phi = -\frac{1}{2}\,\cos\frac{\theta}{2}\left[\xi_x\,\sin\left(\phi - \frac{\chi}{2}\right) - \xi_y\,\cos\left(\phi - \frac{\chi}{2}\right)\right] - \frac{1}{2}\,\sin\frac{\theta}{2}\left[\xi_z\,\sin\frac{\chi}{2} - \xi_0\,\cos\frac{\chi}{2}\right]$$

$$\langle\eta_n\eta_m\rangle_\omega \propto gT\,\delta_{n,m}\ \text{for}\ T \gg \omega$$

Caldeira-Leggett

$$\dot{\theta} + S\alpha\,\sin\theta\dot{\phi} = \eta_\theta \qquad \sin\theta\left(\dot{\phi} - B\right) - S\alpha\,\dot{\theta} = \eta_\phi$$

$$\eta_\theta = \frac{1}{2}\left(-\xi_x\sin\phi + \xi_y\cos\phi\right)$$

$$\eta_\phi = \frac{\sin\theta}{2}\,\xi_z - \frac{\cos\theta}{2}\,\left(\xi_x\cos\phi + \xi_y\sin\phi\right)$$

$$\langle\eta_n\eta_m\rangle_\omega \propto \alpha T\,\delta_{n,m}\ \text{for}\ T \gg \omega$$

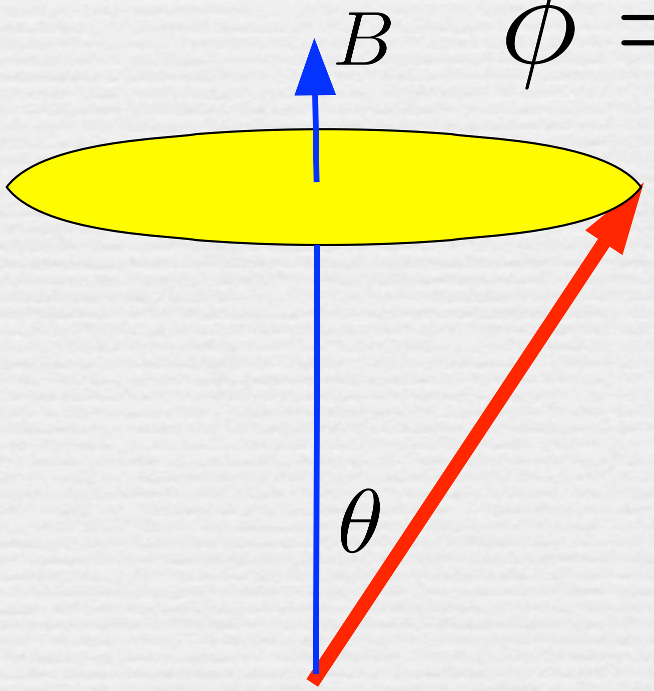
Geometric Langevin terms

Geometric Langevin terms

For example

$$\xi_x \cos\left(\phi - \frac{\chi}{2}\right) \qquad \langle \xi_n \xi_m \rangle_\omega = 2g\,\omega \coth\frac{\omega}{2T}\,\delta_{n,m}$$

$$\phi = Bt$$



gauge fixing

$$\dot{\chi} = \dot{\phi}(1 - \cos\theta) \rightarrow \chi = Bt(1 - \cos\theta)$$

$$\xi_x \cos\left(B\,t\,\cos^2\frac{\theta}{2}\right)$$

noise at $\omega = B\cos^2(\theta/2)$ picked up

Application: geometric spin diffusion

Spin diffusion

$$\dot{\theta} + \frac{g}{S}\,\sin\theta\dot{\phi} = \frac{1}{S}\,\eta_\theta \qquad \sin\theta\left(\dot{\phi} - B\right) - \frac{g}{S}\,\dot{\theta} = \frac{1}{S}\,\eta_\phi$$

$$\langle\eta_{\phi,\theta}(t_1)\eta_{\phi,\theta}(t_2)\rangle_{\omega=0} = \frac{g}{4}\left[\cos^2(\theta_c/2)\,\alpha_K\left(\omega_c\right) + \sin^2(\theta_c/2)\,\alpha_K\left(\omega_s\right)\right]$$

$$\omega_c \equiv B\cos^2(\theta/2) \qquad \omega_s \equiv B\sin^2(\theta/2)$$

Infinitesimal dispersion

$$(\Delta\theta)^2 = \sin^2\theta\,(\Delta\varphi)^2 = D\Delta t$$

Diffusion coefficient

$$D = (g/S^2)T_{eff}$$

$$\begin{aligned} T_{eff} &= \frac{B}{2}\cos^4\left(\frac{\theta}{2}\right)\coth\left[\frac{B}{2T}\cos^2\left(\frac{\theta}{2}\right)\right] \\ &+ \frac{B}{2}\sin^4\left(\frac{\theta}{2}\right)\coth\left[\frac{B}{2T}\sin^2\left(\frac{\theta}{2}\right)\right] \end{aligned}$$

Spin diffusion

Diffusion coefficient $D = (g/S^2)T_{eff}$

$$\begin{aligned} T_{eff} &= \frac{B}{2}\cos^4\left(\frac{\theta}{2}\right)\coth\left[\frac{B}{2T}\cos^2\left(\frac{\theta}{2}\right)\right] \\ &+ \frac{B}{2}\sin^4\left(\frac{\theta}{2}\right)\coth\left[\frac{B}{2T}\sin^2\left(\frac{\theta}{2}\right)\right] \end{aligned}$$

Classical regime: $T \gg B$ ➡ $T_{eff} \approx T$

Quantum regime: $T \ll B$ ➡ $T_{eff} \approx \frac{B}{4}\left(1+\cos^2\theta\right)$

Spin diffusion: AES vs. CL

Classical regime: $T \gg B$ ⇒ $T_{eff} \approx T$

Quantum regime: $T \ll B$

AES

$$T_{eff} \approx \frac{B}{4}\left(1 + \cos^2\theta\right)$$

CL

$$T^{\theta}_{eff} \approx \frac{B}{2}$$

$$T^{\phi}_{eff} \approx \frac{B}{2}\cos^2\theta + T\sin^2\theta$$

Spin diffusion

Diffusion coefficient $D = (g/S^2)T_{eff}$

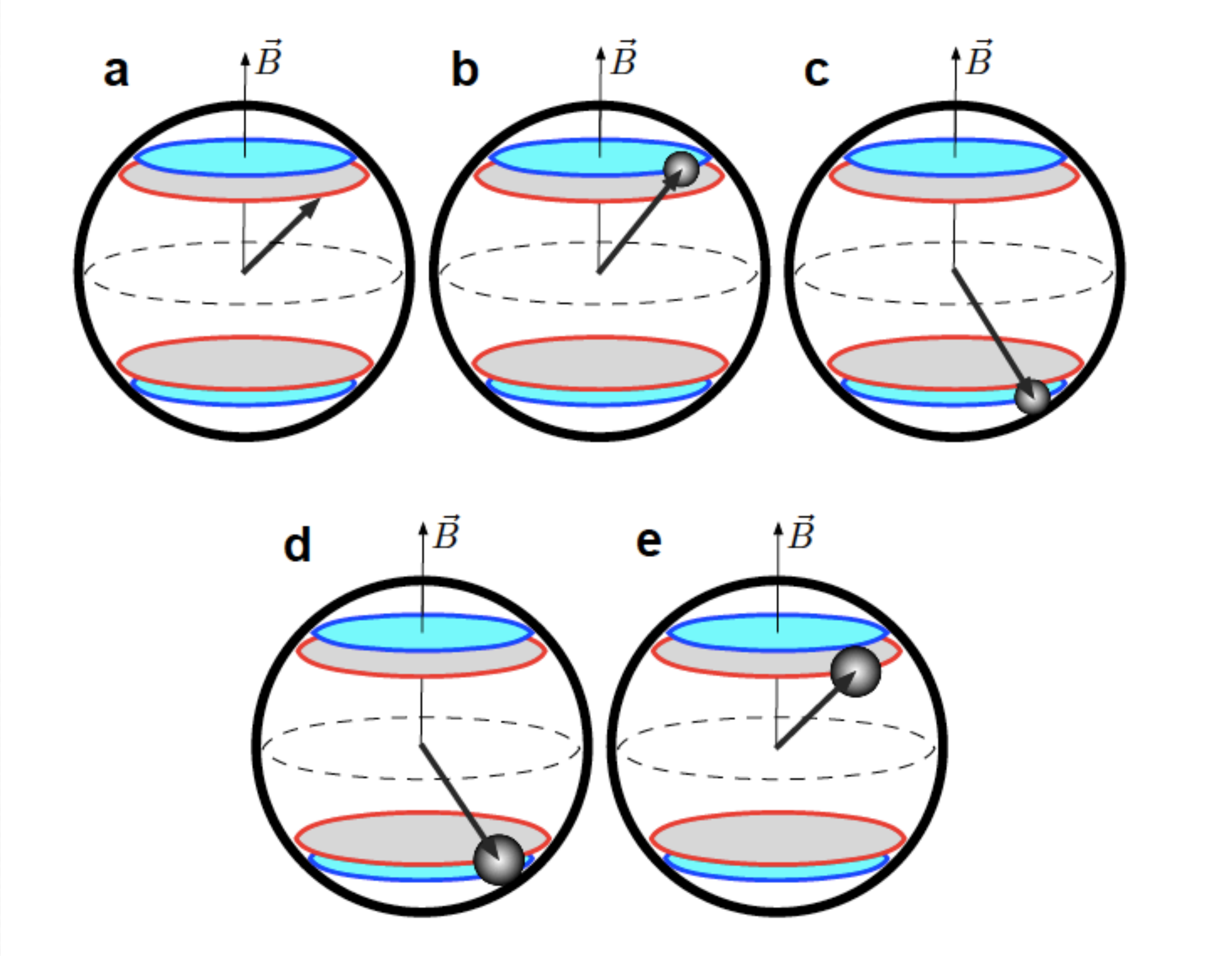
Quantum regime: $T \ll B$ ⟹ $T_{eff} \approx \frac{B}{4}(1 + \cos^2\theta)$

$t_{rel}^{-1} \sim gB/S$

$t_{diff}^{-1} \sim gB/S^2$

Deterministic relaxation much faster than diffusion

Bang-Bang protocol

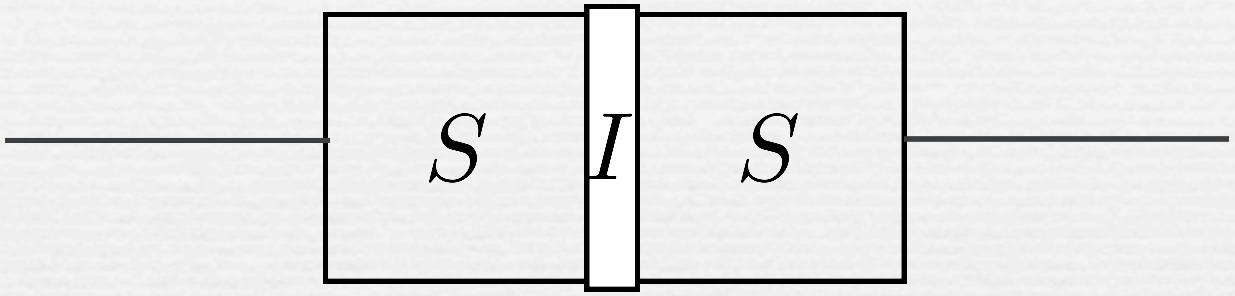


Conclusions

- AES action generalized from U(1) to SU(2)
- Role of the gauge freedom clarified
- Semiclassical LLG-Langevin equations with noise terms influenced by geometrical phase: geometric spin diffusion

AES for tunnel junctions

2) Josephson junction (SIS)



$$i\mathcal{S}_{AES} = i\int dt\,\frac{C\dot{\phi}^2}{2e^2} - \int dt_1 dt_2\,\alpha(t_1,t_2)\,\cos\left[\phi(t_1)-\phi(t_2)\right]$$
$$- \int dt_1 dt_2\,\beta(t_1,t_2)\,\cos\left[\phi(t_1)+\phi(t_2)\right]$$

V. Ambegaokar, U. Eckern, G. Schön
Phys. Rev. Lett. **48**, 1745-1748 (1982)