# Anderson Localization:
# Multifractality, Symmetries, Topologies, and Interactions

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# Plan

- **Anderson localization: basic properties, field theory**
- **Wave function multifractality**
- **Symmetries of disordered systems**
- **Manifestations of topology in localization theory**
- **Influence of electron-electron interaction**

# Anderson localization

**Philip W. Anderson**

**1958 "Absence of diffusion in certain random lattices"**

**sufficiently strong disorder $\longrightarrow$ quantum localization**

**$\longrightarrow$ eigenstates exponentially localized, no diffusion**

**$\longrightarrow$ Anderson insulator**

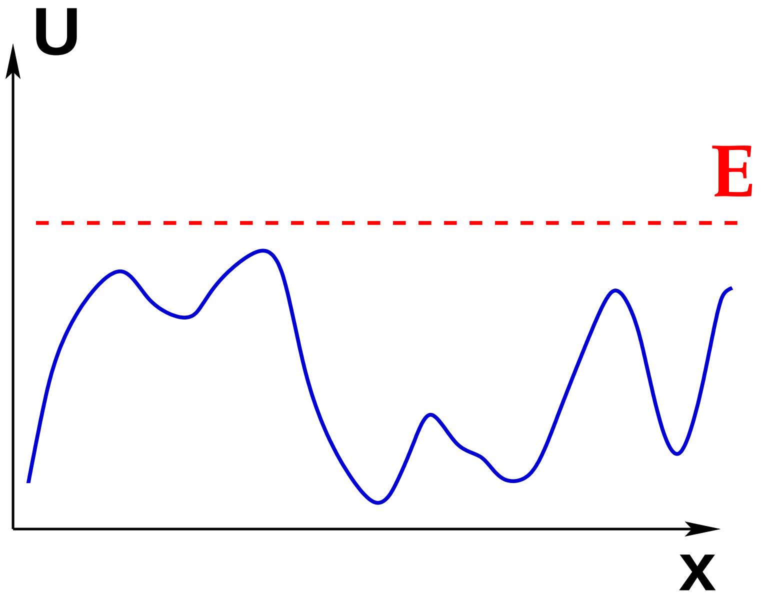
**Nobel Prize 1977**

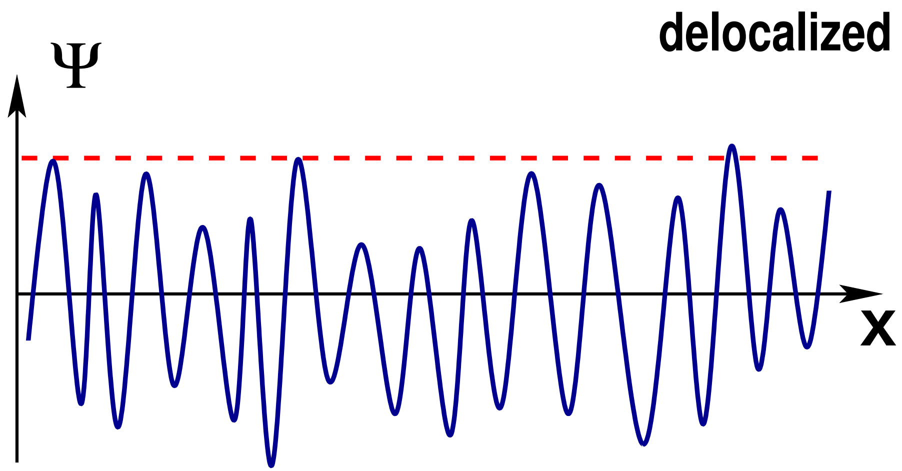
# Anderson Localization: Extended and localized wave functions

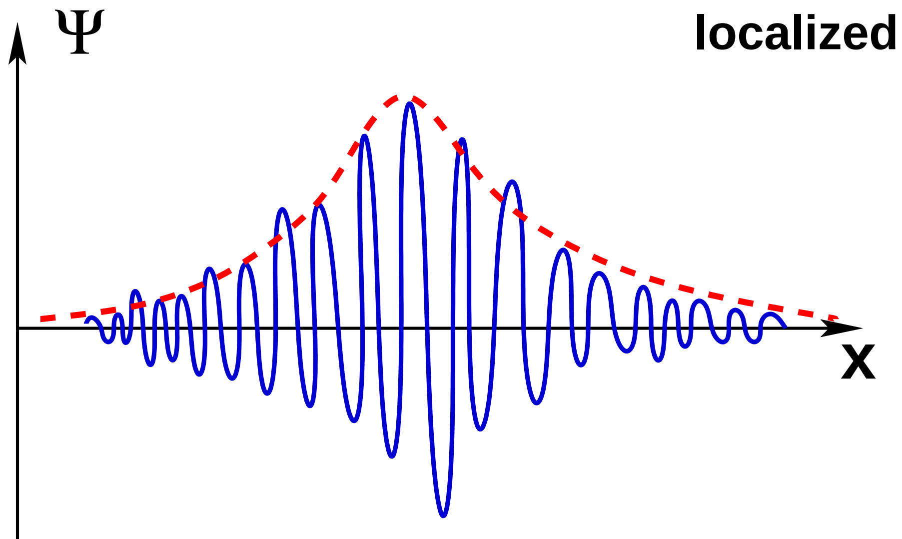
**Schrödinger equation**

**in a random potential**

$$\left[-\hbar^2 \frac{\Delta}{2m} + U(\mathbf{r})\right]\psi = E\psi$$

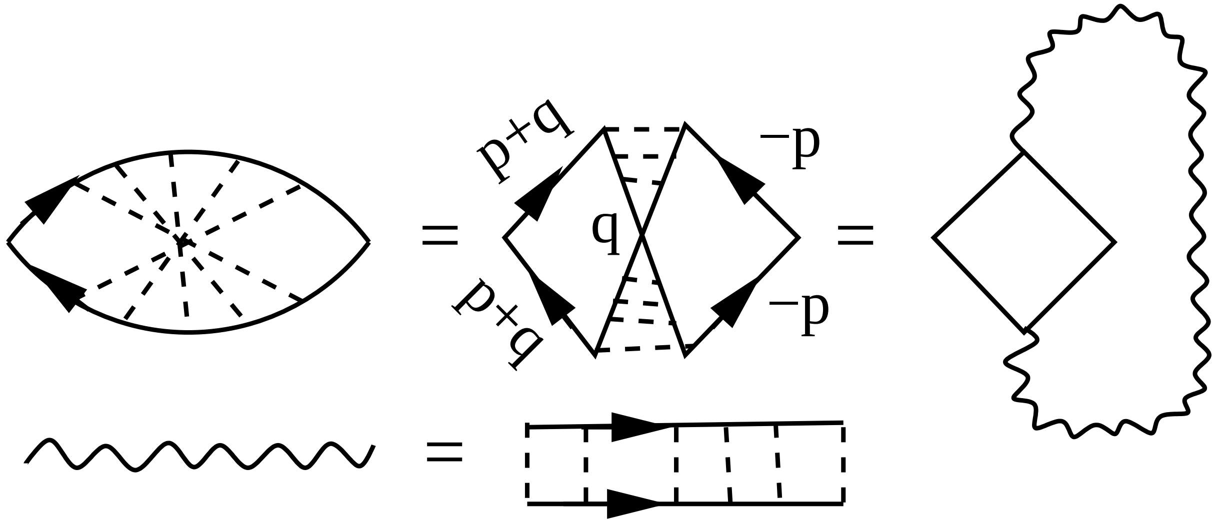


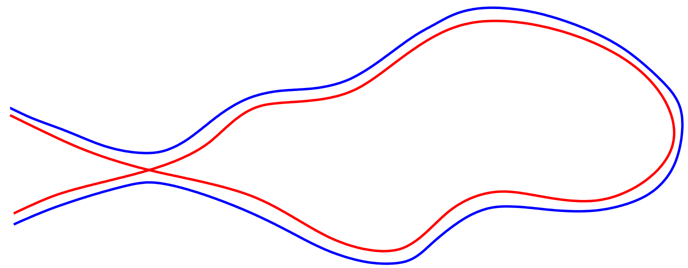




$$|\psi|^2 \sim \exp\{-|r - r_0|/\xi\}$$

# Precursor of strong Anderson localization: Weak localization



**Cooperon loop (interference of time-reversed paths)**

$$\Delta\sigma_{\mathrm{WL}} \simeq \sigma_0 \frac{1}{\pi\nu}\int \frac{(dq)}{Dq^2 - i\omega}$$

$$\Delta\sigma_{\mathrm{WL}} = -\frac{e^2}{2\pi h}\left(\frac{\sim 1}{l} - \frac{1}{L_\omega}\right), \quad \text{3D}$$

$$L_\omega \simeq (D/\omega)^{1/2}$$

$$\Delta\sigma_{\mathrm{WL}} = -\frac{e^2}{\pi h}\ln\frac{L_\omega}{l}\ , \quad \text{2D}$$

**Generally: IR cutoff**

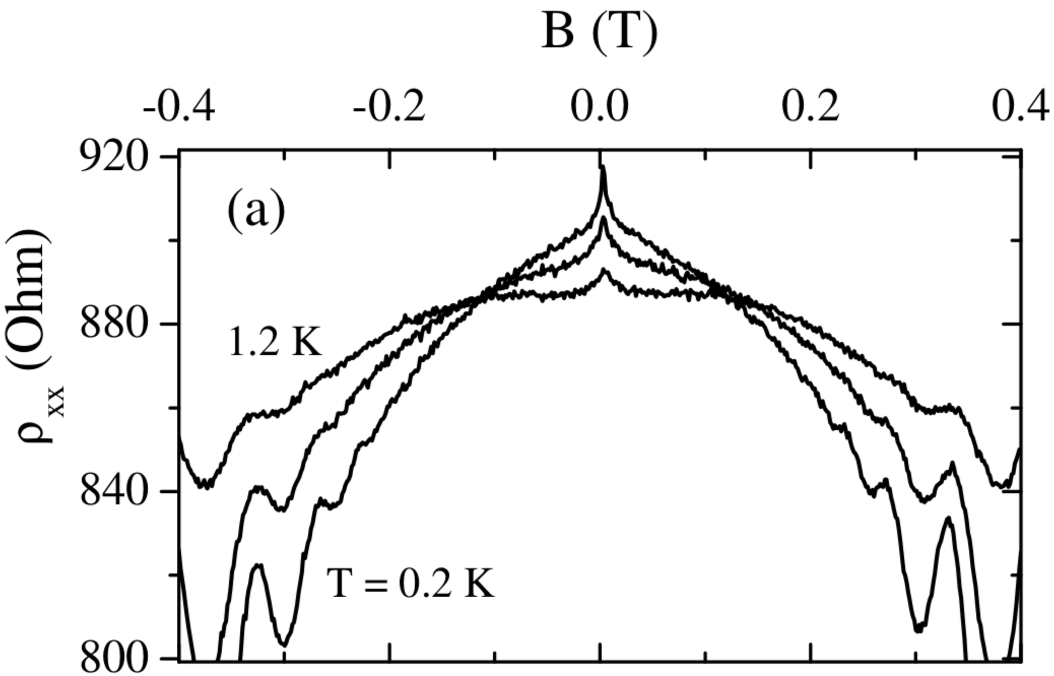
$$L_\omega \longrightarrow \min\{L_\omega, L_\phi, L, L_H\}$$

$$\Delta\sigma_{\mathrm{WL}} = -\frac{e^2}{h}L_\omega\ , \quad \text{quasi-1D}$$

$$e^2/h \ \simeq\ (25\ k\Omega)^{-1}$$

**"conductance quantum"**

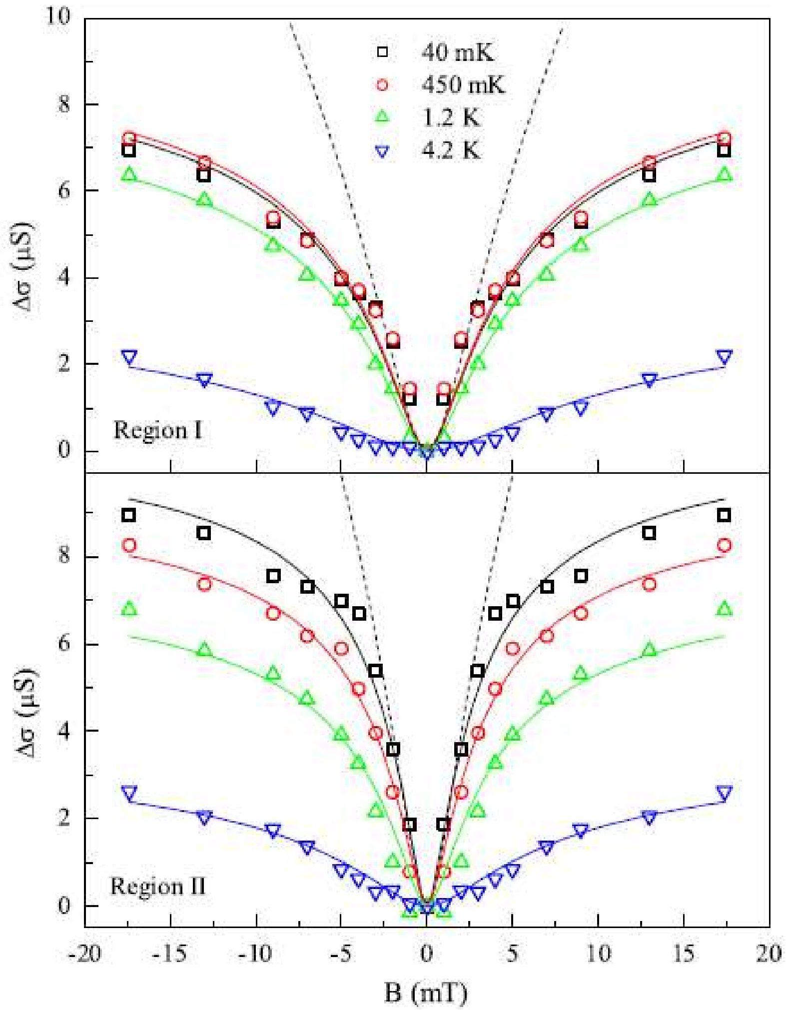
# Weak localization in experiment: Magnetoresistance




Li et al. (Savchenko group), PRL'03

2D electron gas
in GaAs heterostructure

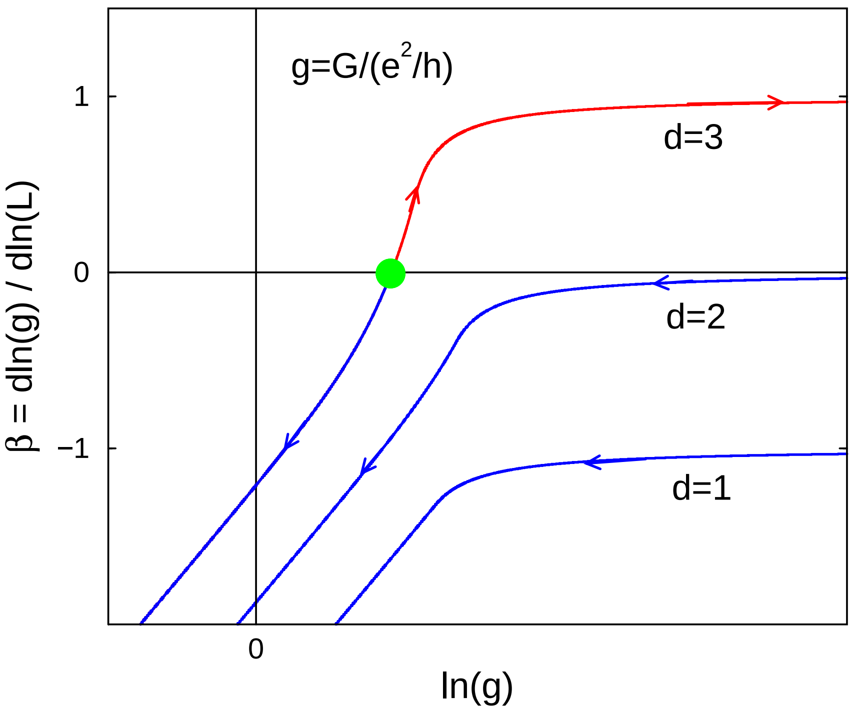
low field: weak localization




Gorbachev et al. (Savchenko group), PRL'07

weak localization in bilayer graphene

# Anderson Insulators & Metals




**Connection with scaling theory of critical phenomena: Thouless '74; Wegner '76**

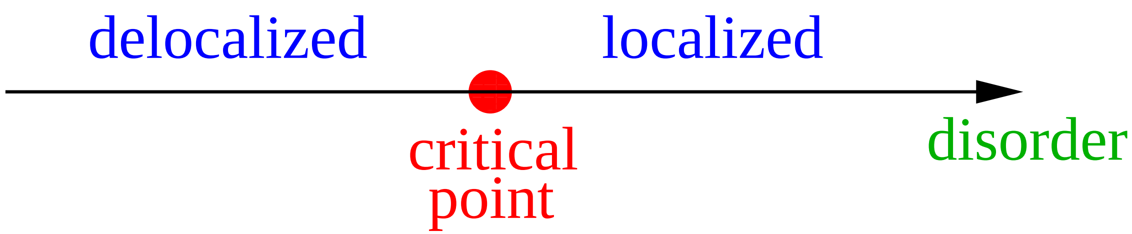
**Scaling theory of localization:**
**Abrahams, Anderson, Licciardello, Ramakrishnan '79**

**scaling variable:**
**dimensionless conductance** $g = G/(e^2/h)$

**RG for field theory ($\sigma$-model)**
**Wegner '79**

**quasi-1D, 2D :**
**all states are localized**

$d > 2$: **Anderson metal-insulator transition**




**review: Evers, ADM, Rev. Mod. Phys. 80, 1355 (2008)**

# Field theory: non-linear $\sigma$-model

**action:**

$$S[Q] = \frac{\pi\nu}{4}\int d^d\mathrm{r}\ \mathrm{Tr}\,[-D(\nabla Q)^2 - 2i\omega\Lambda Q], \qquad Q^2(\mathrm{r}) = 1$$

Wegner'79

**$\sigma$-model manifold:**

e.g., "unitary" symmetry class (broken time-reversal symmetry):

- fermionic replicas: $\mathrm{U}(2n)/\mathrm{U}(n)\times\mathrm{U}(n)$ , $n\to 0$ "sphere"
- bosonic replicas: $\mathrm{U}(n,n)/\mathrm{U}(n)\times\mathrm{U}(n)$ , $n\to 0$ "hyperboloid"

- supersymmetry (Efetov'83): $\mathrm{U}(1,1|2)/\mathrm{U}(1|1)\times\mathrm{U}(1|1)$

{"sphere" $\times$ "hyperboloid"} "dressed" by anticommuting variables

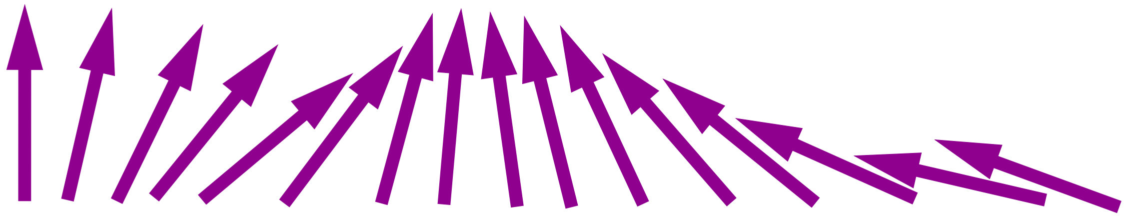
- with electron-electron interaction: Finkelstein'83

# $\sigma$ model: Perturbative treatment

For comparison, consider ferromagnet model in external magnetic field:

$$H[\mathrm{S}] = \int d^d r \left[ \frac{\kappa}{2} (\nabla \mathrm{S}(r))^2 - \mathrm{B S}(r) \right] , \qquad \mathrm{S}^2(r) = 1$$

$n$-component vector $\sigma$-model



Target manifold:

sphere $\mathrm{S}^{n-1} = \mathrm{O}(n)/\mathrm{O}(n-1)$

Independent degrees of freedom: transverse part $\mathrm{S}_\perp$ ; $\quad S_1 = (1 - \mathrm{S}_\perp^2)^{1/2}$

$$H[\mathrm{S}_\perp] = \frac{1}{2} \int d^d r \left[ \kappa [\nabla \mathrm{S}_\perp(r)]^2 + B \mathrm{S}_\perp^2(r) + O(\mathrm{S}_\perp^4(r)) \right]$$

Ferromagnetic phase: broken symmetry,

Goldstone modes – spin waves

$$\langle \mathrm{S}_\perp \mathrm{S}_\perp \rangle_q \propto \frac{1}{\kappa q^2 + B}$$

Similarly

$$S[Q] = \frac{\pi\nu}{4} \int d^d r \, \mathrm{Str}[D(\nabla Q_\perp)^2 - i\omega Q_\perp^2 + O(Q_\perp^3)]$$

theory of “interacting” diffusion modes;

Goldstone mode: diffusion propagator

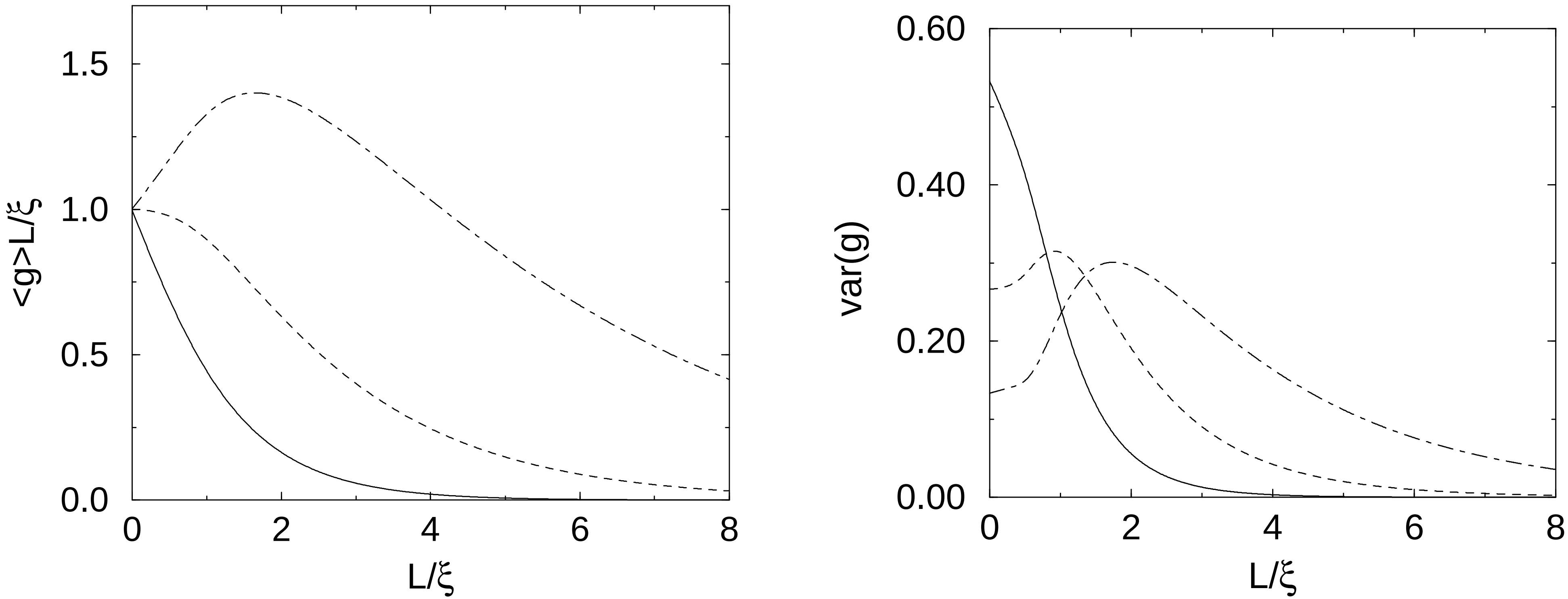
$$\langle Q_\perp Q_\perp \rangle_{q,\omega} \sim \frac{1}{\pi\nu(Dq^2 - i\omega)}$$

# Quasi-1D geometry: Exact solution of the $\sigma$-model

quasi-1D geometry (many-channel wire) $\longrightarrow$ 1D $\sigma$-model

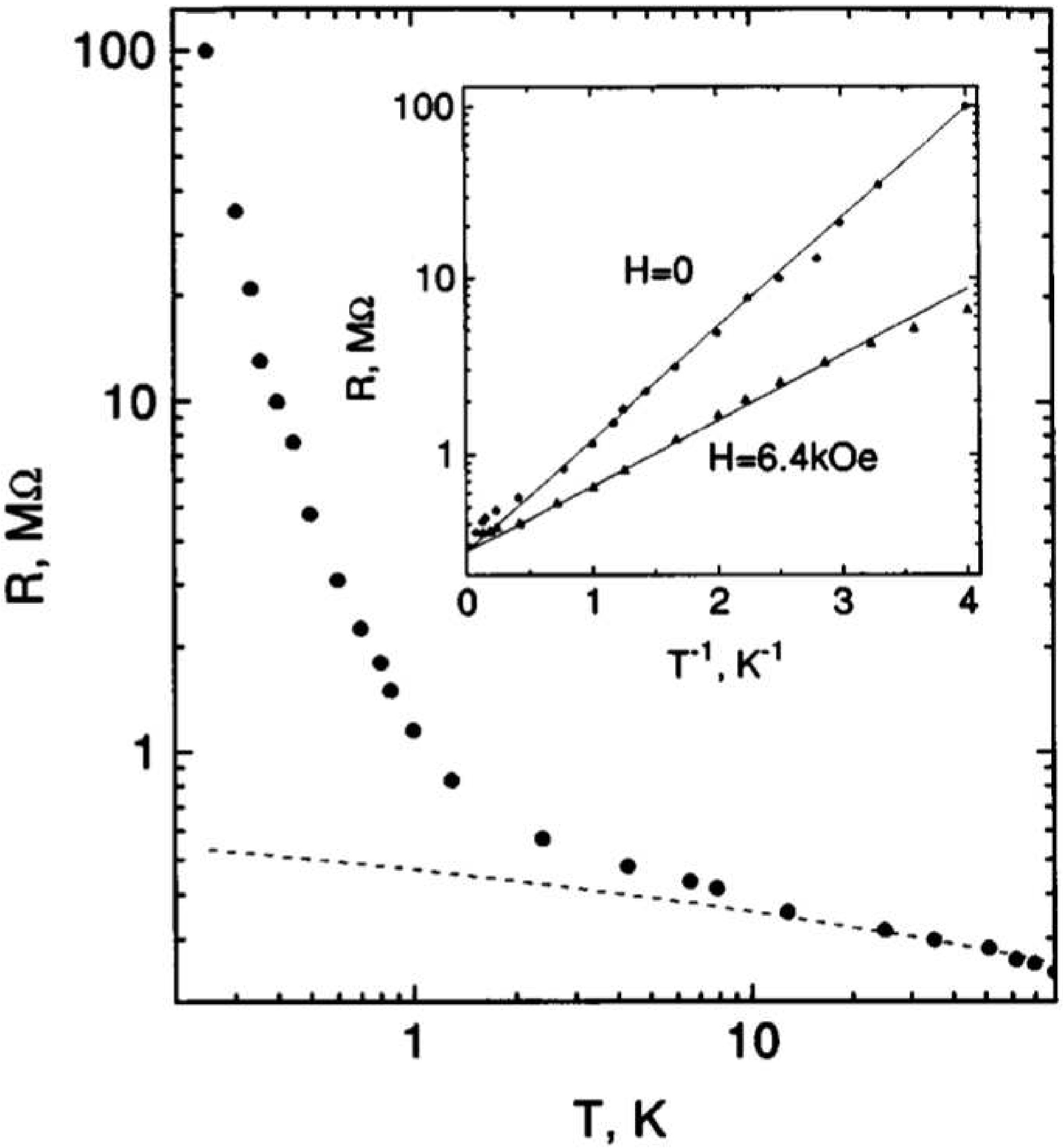
$\longrightarrow$ diffusion on $\sigma$-model curved space $\quad \partial_t W = \Delta_Q W \;, \quad t = x/\xi$

- Localization length Efetov, Larkin '83
- Exact solution for the statistics of eigenfunctions Fyodorov, ADM '92-94
- Exact $\langle g \rangle (L/\xi)$ and $\mathrm{var}(g)(L/\xi)$ Zirnbauer, ADM, Müller-Groeling '92-94




orthogonal (full), unitary (dashed), symplectic (dot-dashed)

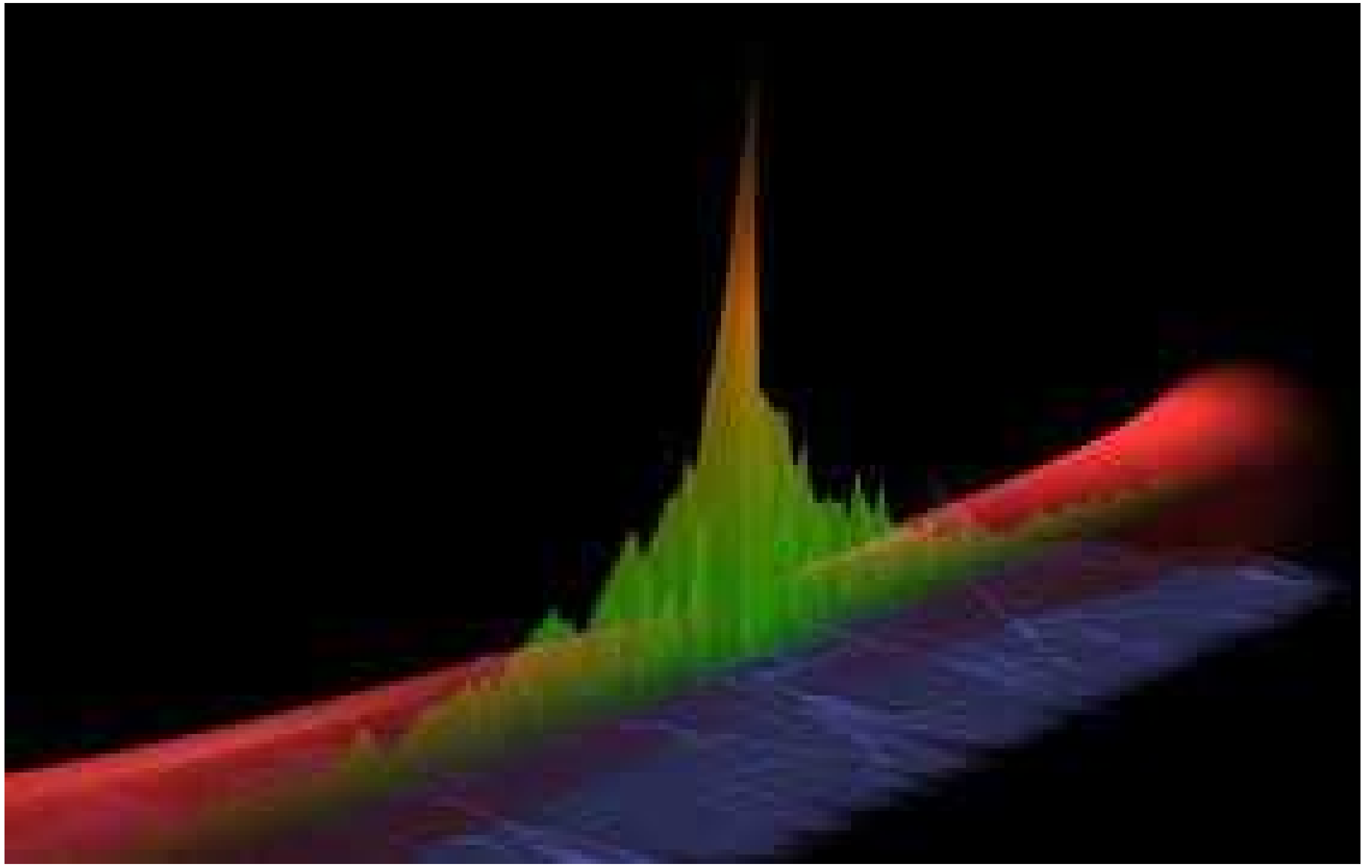
# From weak to strong localization of electrons in wires

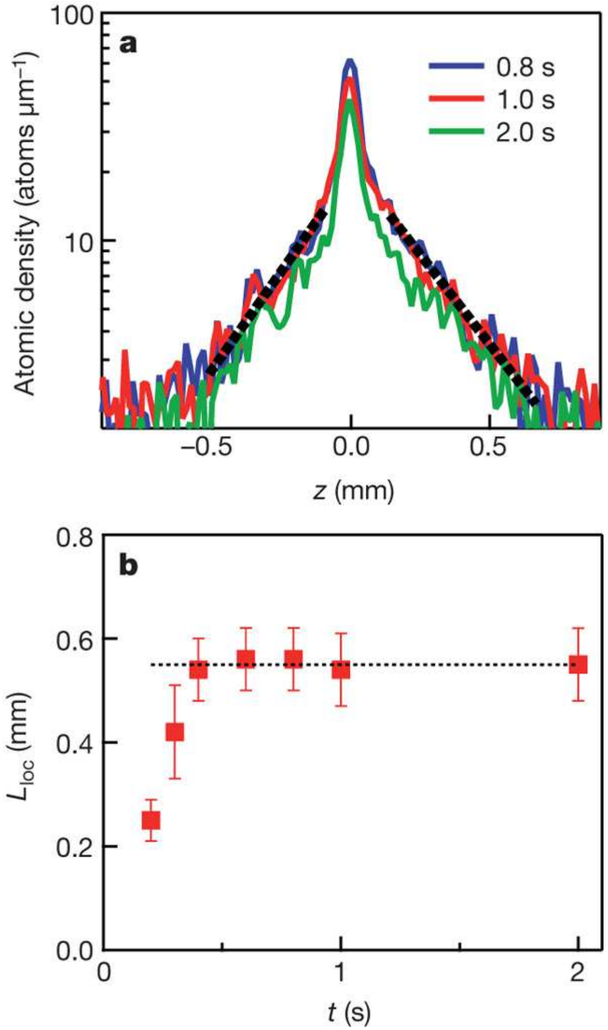



GaAs wires

Gershenson et al, PRL 97

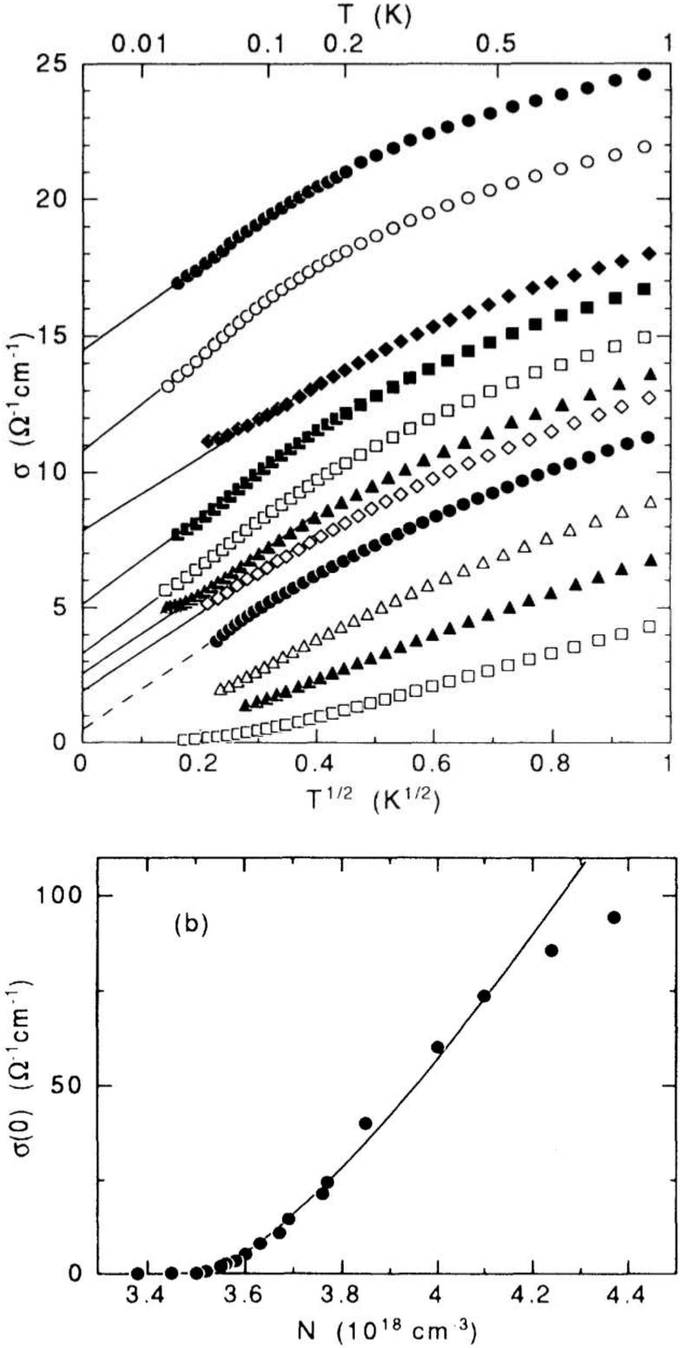
# Anderson localization of atomic Bose-Einstein condensate in 1D

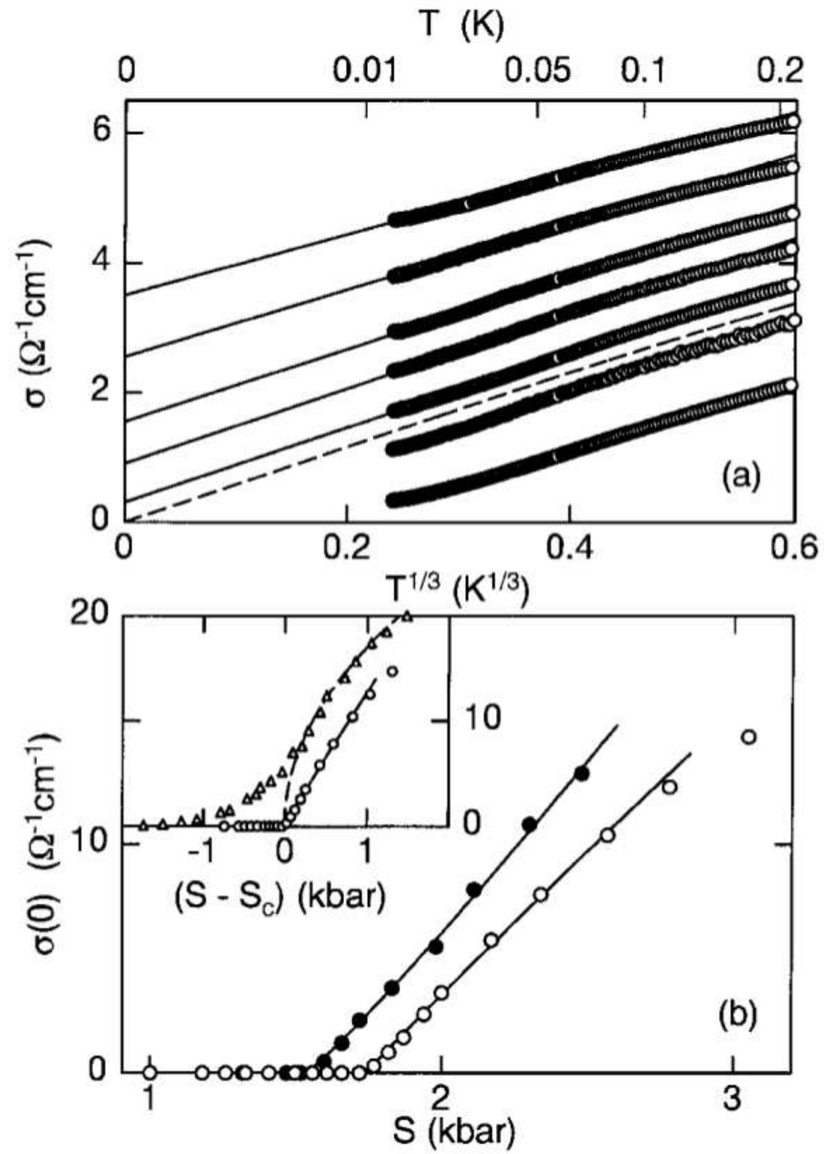





Billy et al (Aspect group), Nature 2008

# 3D Anderson localization transition in Si:P



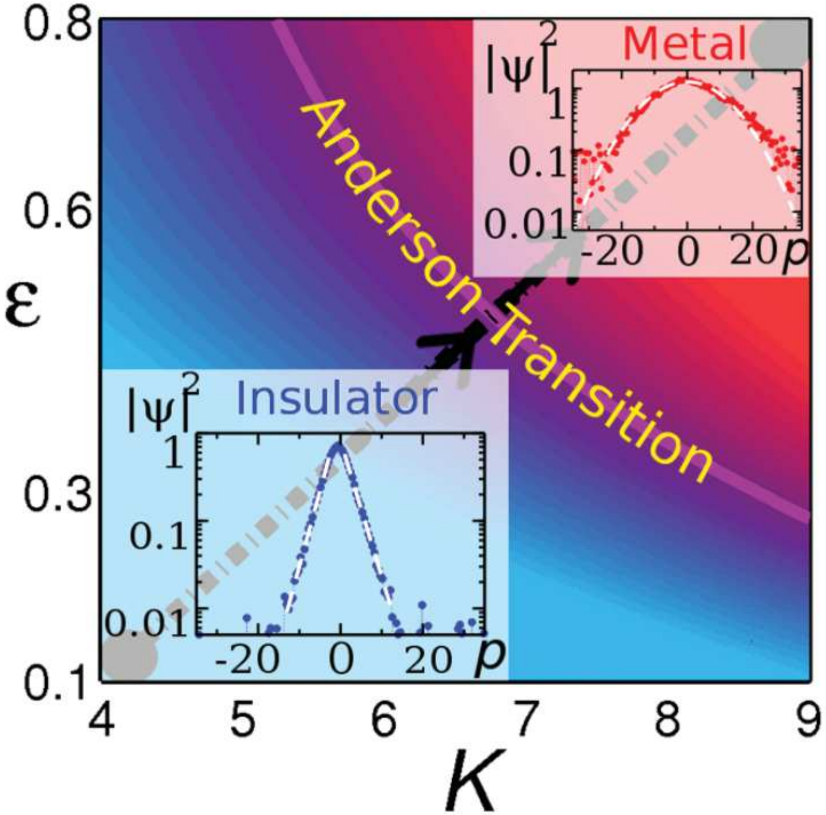

Stupp et al, PRL'93; Wafenschmidt et al, PRL'97
(von Löhneysen group)

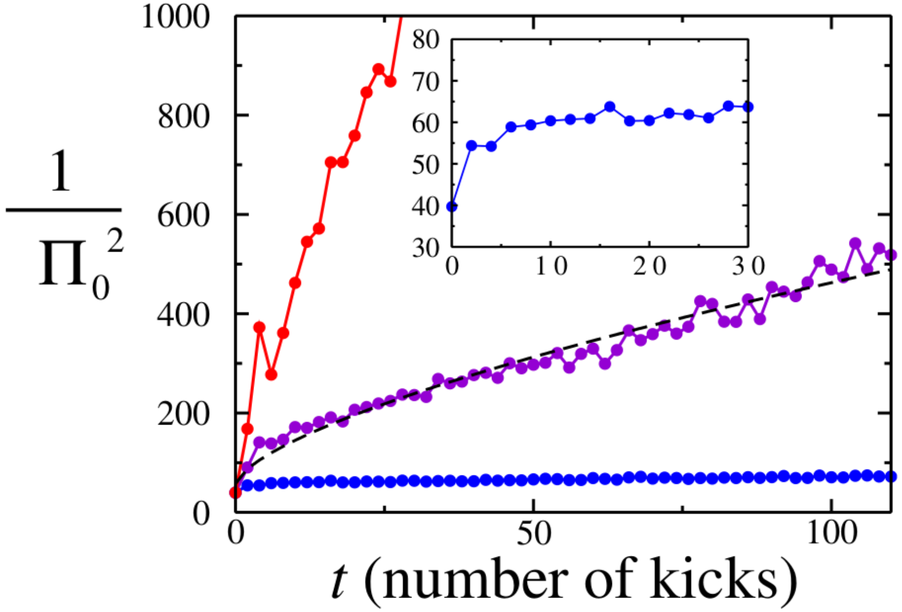
# 3D Anderson localization in atomic “kicked rotor”

**kicked rotor** $$H = \frac{p^2}{2} + K\cos x[1+\epsilon\cos\omega_2 t\cos\omega_3 t]\sum_n \delta(t - 2\pi n/\omega_1)$$

Anderson localization in momentum space. Three frequencies mimic 3D !

Experimental realization: cesium atoms exposed to a pulsed laser beam.




Chabé et al, PRL’08

# Multifractality at the Anderson transition

$P_q = \int d^d r |\psi(\mathrm{r})|^{2\mathrm{q}}$ inverse participation ratio

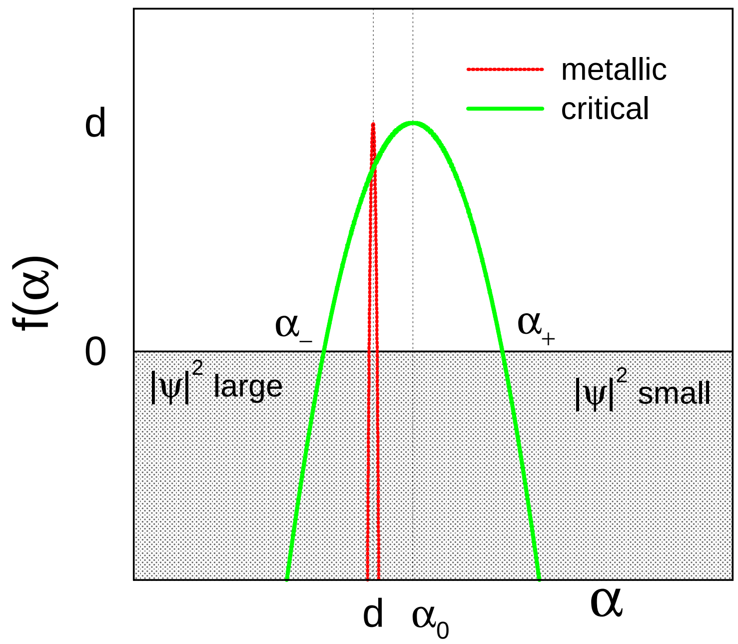
$$\langle P_q \rangle \sim \begin{cases} L^0 & \text{insulator} \\ L^{-\tau_q} & \text{critical} \\ L^{-d(q-1)} & \text{metal} \end{cases}$$

$\tau_q = \quad d(q-1) \quad + \quad \Delta_q \quad \equiv \quad D_q(q-1)$ multifractality

normal anomalous

$\tau_q \longrightarrow$ Legendre transformation

$\longrightarrow$ singularity spectrum $f(\alpha)$

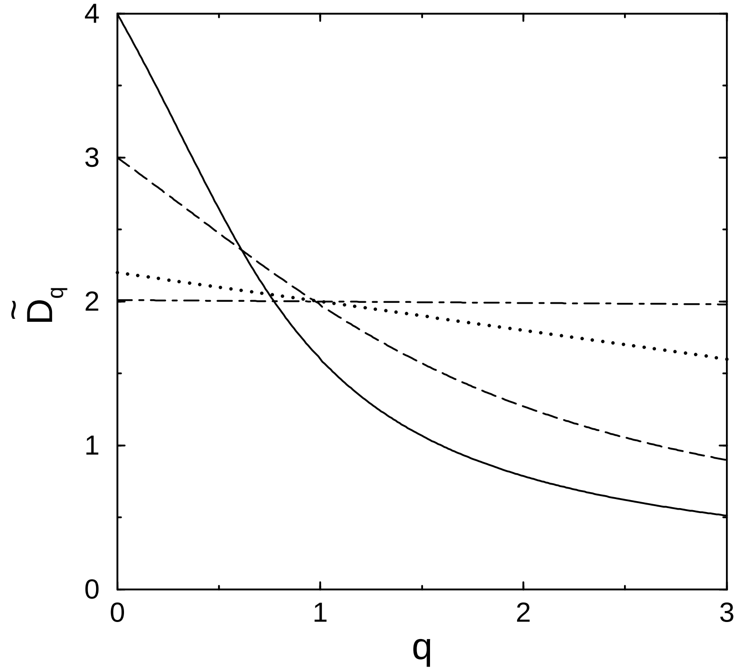


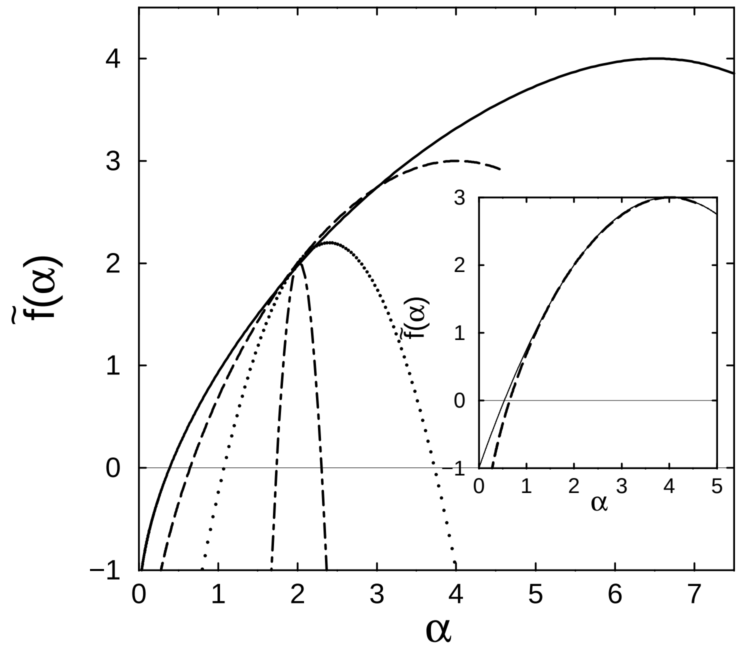

wave function statistics:

$\mathcal{P}(\ln|\psi^2|) \sim L^{-d+f(\ln|\psi^2|/\ln L)}$

$L^{f(\alpha)}$ – measure of the set of points where $|\psi|^2 \sim L^{-\alpha}$

# Dimensionality dependence of multifractality




Analytics $(2+\epsilon$, one-loop) and numerics

$$\tau_q = (q-1)d - q(q-1)\epsilon + O(\epsilon^4)$$

$$f(\alpha) = d - (d+\epsilon-\alpha)^2/4\epsilon + O(\epsilon^4)$$

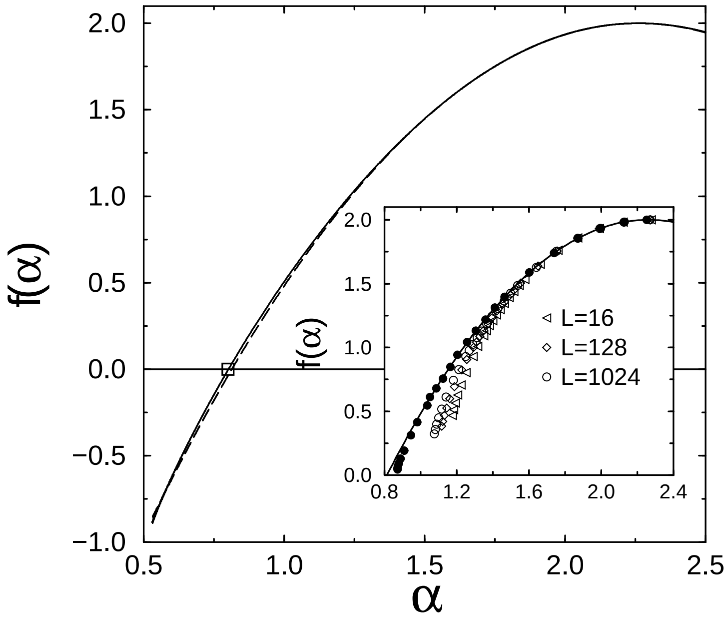
$d = 4$ (full)
$d = 3$ (dashed)
$d = 2+\epsilon, \quad \epsilon = 0.2$ (dotted)
$d = 2+\epsilon, \quad \epsilon = 0.01$ (dot-dashed)

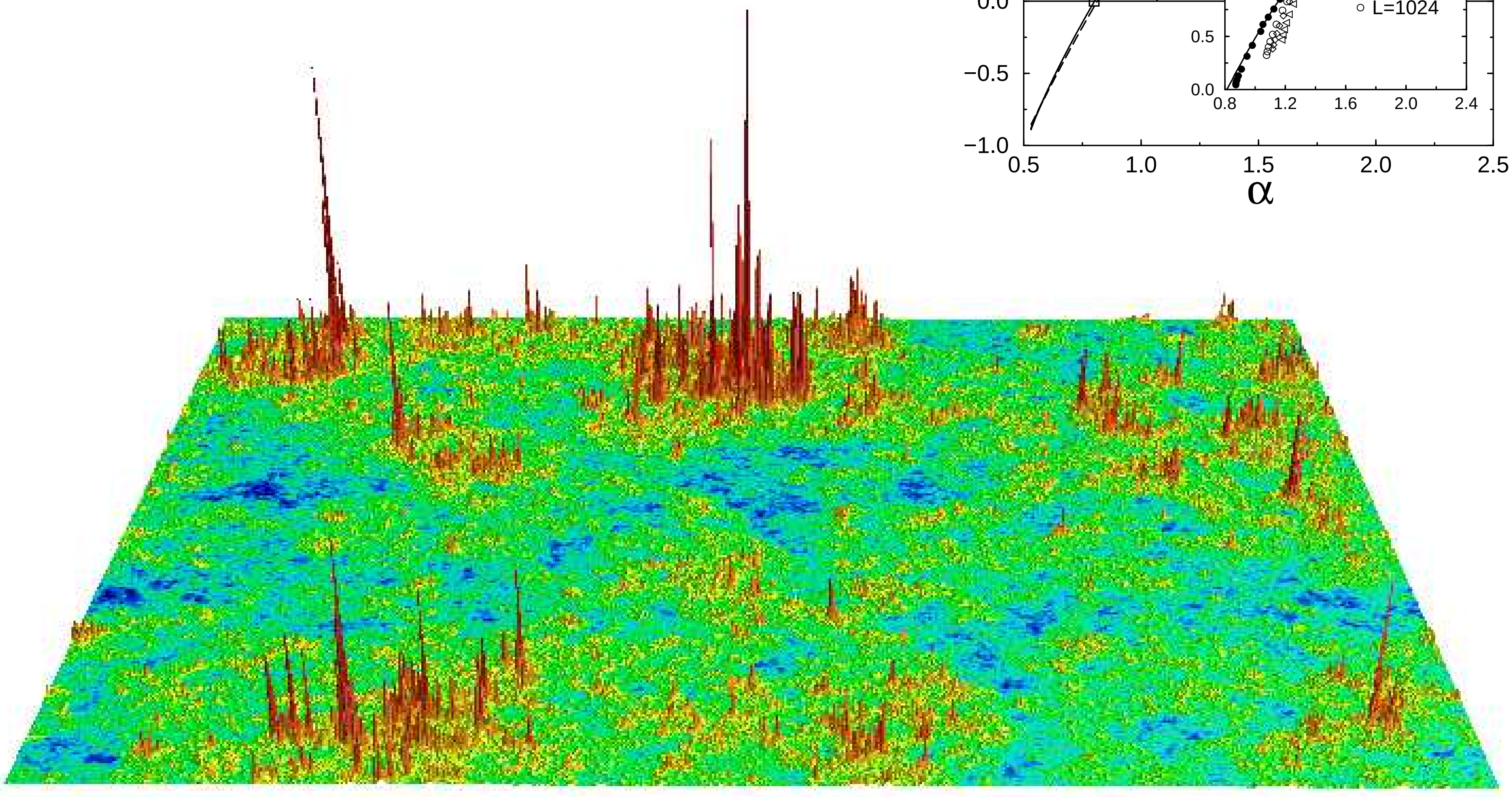
Inset: $d = 3$ (dashed)
vs. $d = 2+\epsilon, \; \epsilon = 1$ (full)

Mildenberger, Evers, ADM '02

# Multifractality at the Quantum Hall transition

**Evers, Mildenberger, ADM '01**



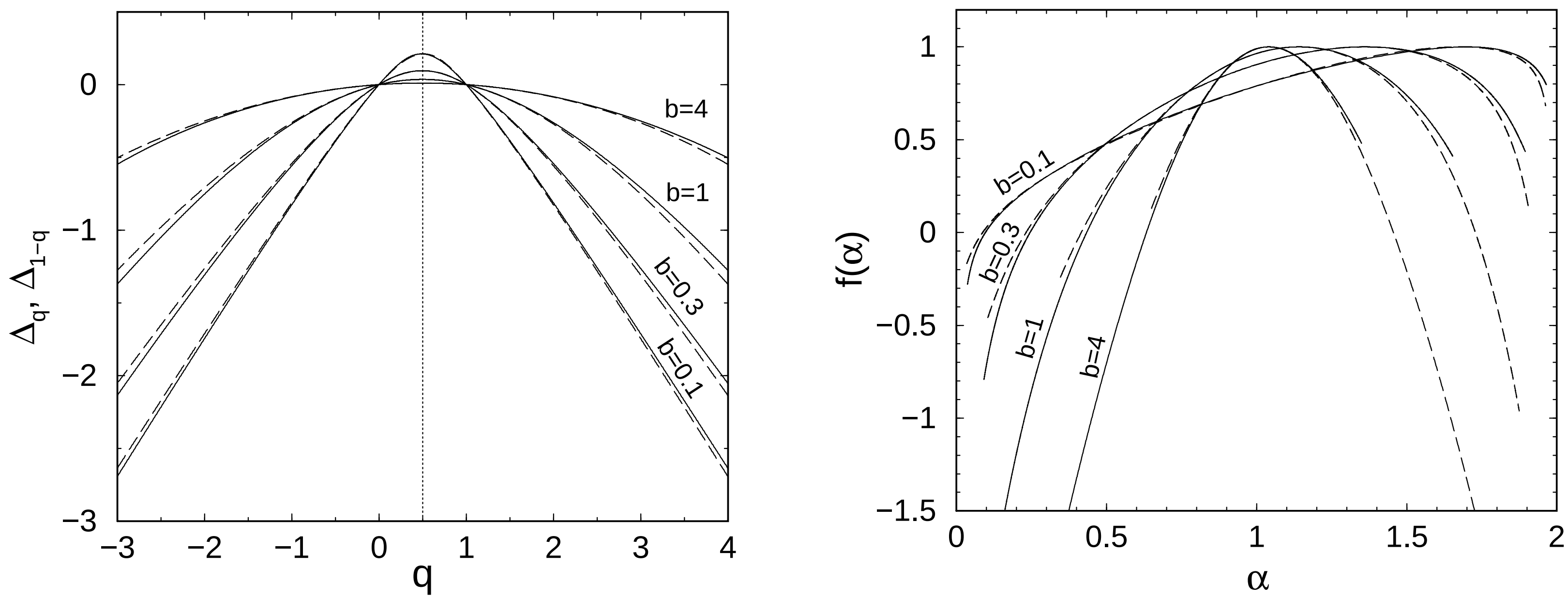
# Symmetry of multifractal spectra

ADM, Fyodorov, Mildenberger, Evers '06

**LDOS distribution in $\sigma$-model + universality**

**$\longrightarrow$ exact symmetry of the multifractal spectrum:**

$$\Delta_q = \Delta_{1-q} \qquad\qquad f(2d-\alpha) = f(\alpha) + d - \alpha$$




**$\longrightarrow$ probabilities of unusually large and unusually small $|\psi^2(r)|$ are related !**

# Multifractality: Generalizations

- **Symmetry of multifractal spectra as a consequence of invariance of the $\sigma$ model correlation functions with respect to Weyl group of the $\sigma$ model target space;**

**generalization to unconventional symmetry classes**

**Gruzberg, Ludwig, ADM, Zirnbauer PRL'11**

- **generalization on full set of composite operators,**

**i.e. also on subleading ones.**

**Gruzberg, ADM, Zirnbauer, PRB'13**

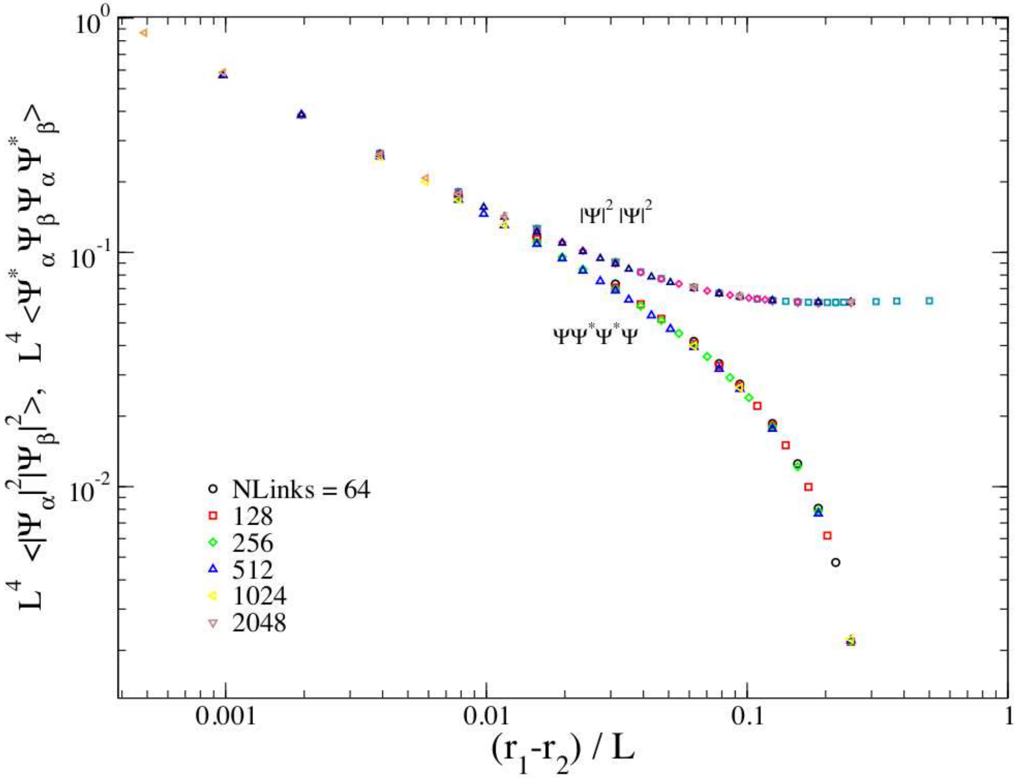
**Important example:**

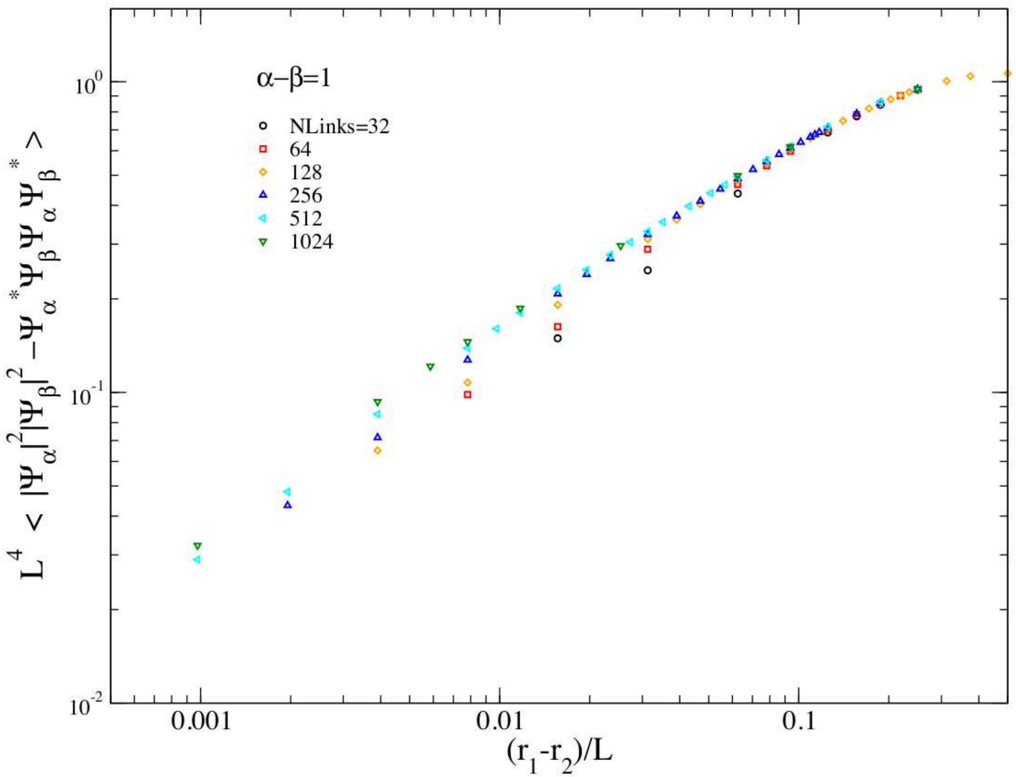
$$A_2 = V^2 |\psi_1(r_1)\psi_2(r_2) - \psi_1(r_2)\psi_2(r_1)|^2$$

$\longleftrightarrow$ **Hartree-Fock matrix element of e-e interaction**

**scaling:** $\langle A_2^q \rangle \propto L^{-\Delta_q^{(2)}}$ **symmetry:** $\Delta_q^{(2)} = \Delta_{2-q}^{(2)}$

# Interaction scaling at criticality




**Hartree, Fock**

**enhanced by multifractality**

**exponent** $\Delta_2 \simeq -0.52 < 0$

**Hartree – Fock**

**suppressed by multifractality**

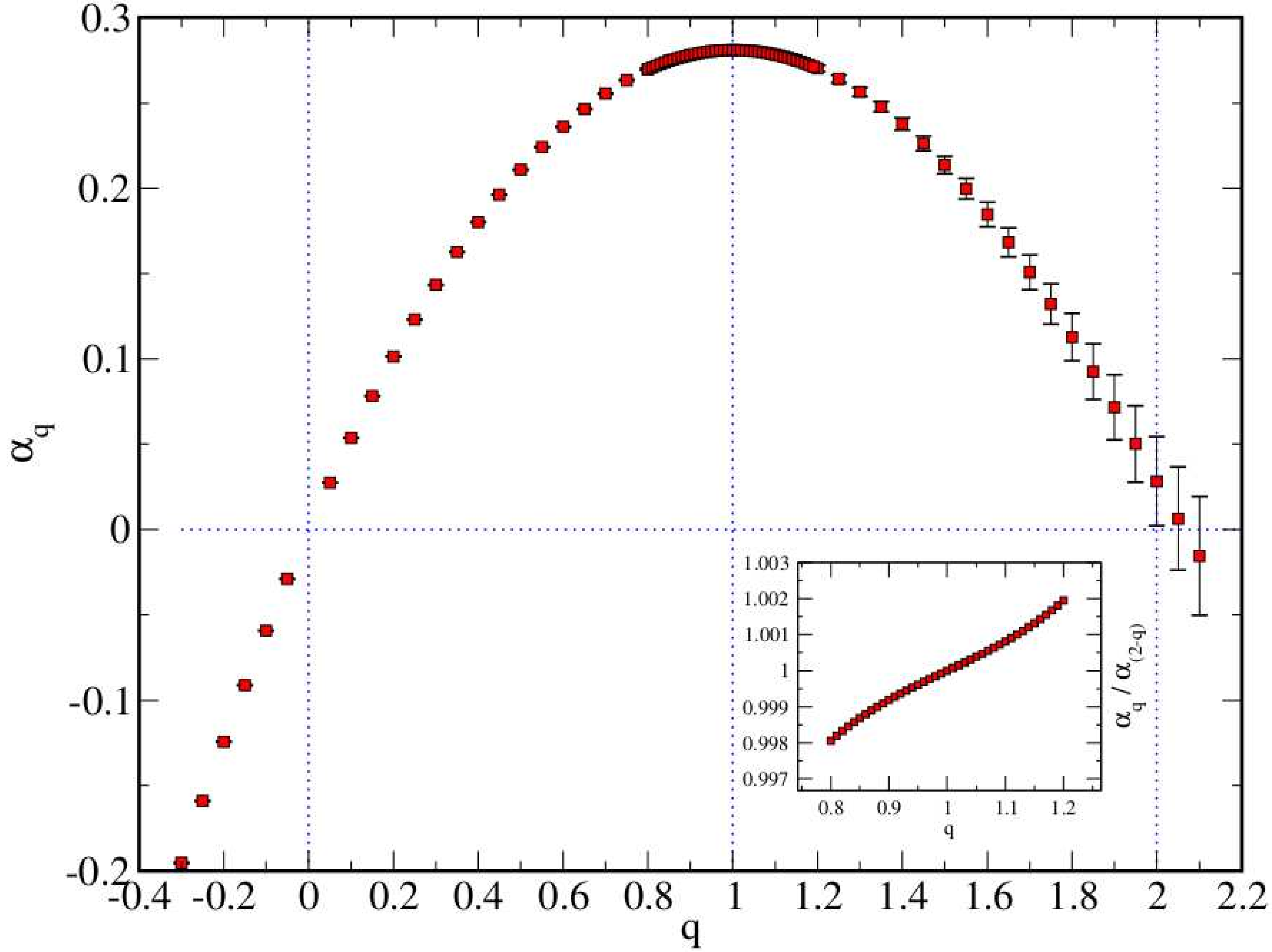
**exponent** $\Delta_1^{(2)} \simeq 0.62 > 0$

**Burmistrov, Bera, Evers, Gornyi, ADM, Annals Phys. 326, 1457 (2011)**

$\longrightarrow$ **Temperature scaling at quantum Hall and metal-insulator transitions with short-range interaction**

# Multifractal spectrum of $A_2$ at quantum Hall transition

**Numerical data: Bera, Evers, unpublished**

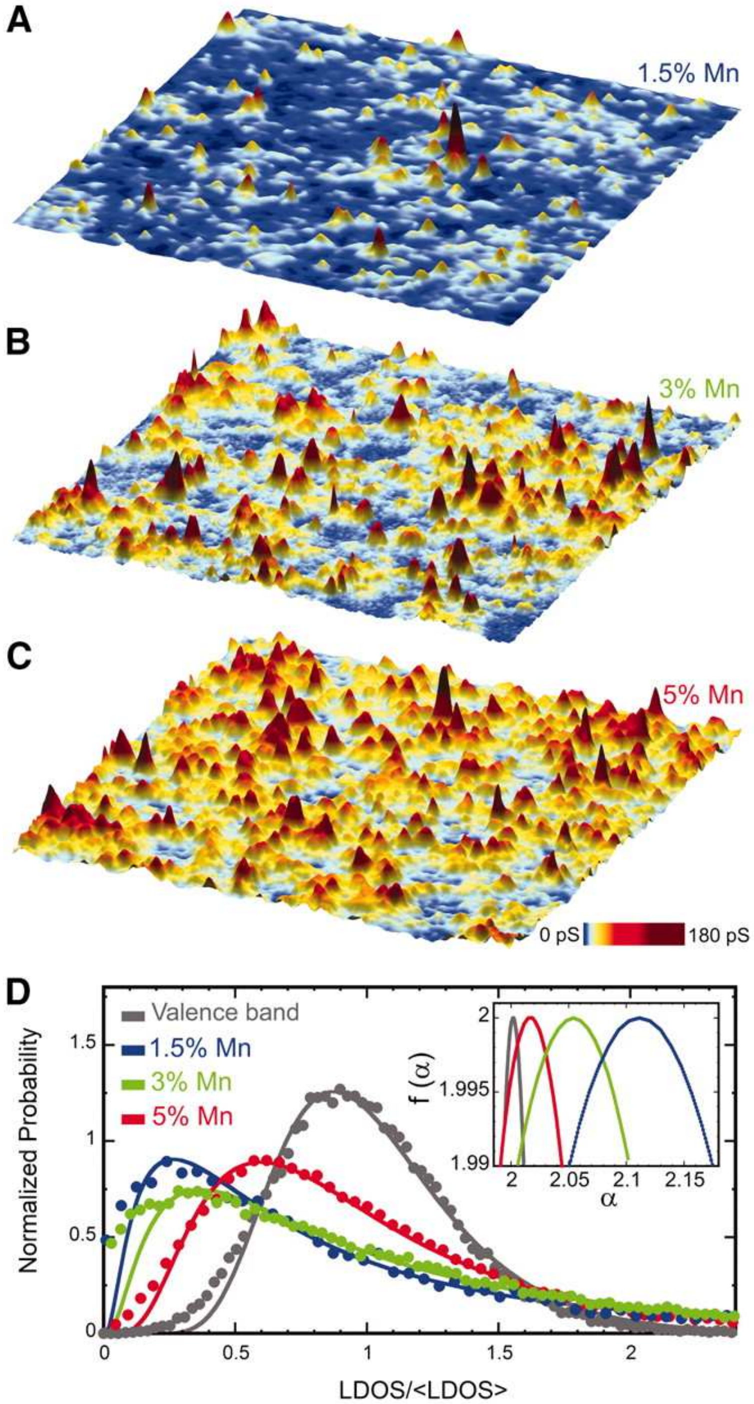



**Confirms the symmetry** $q \longleftrightarrow 2 - q$

# Multifractality: Experiment I

**Local DOS flucutuations near metal-insulator transition in $Ga_{1-x}Mn_xAs$**

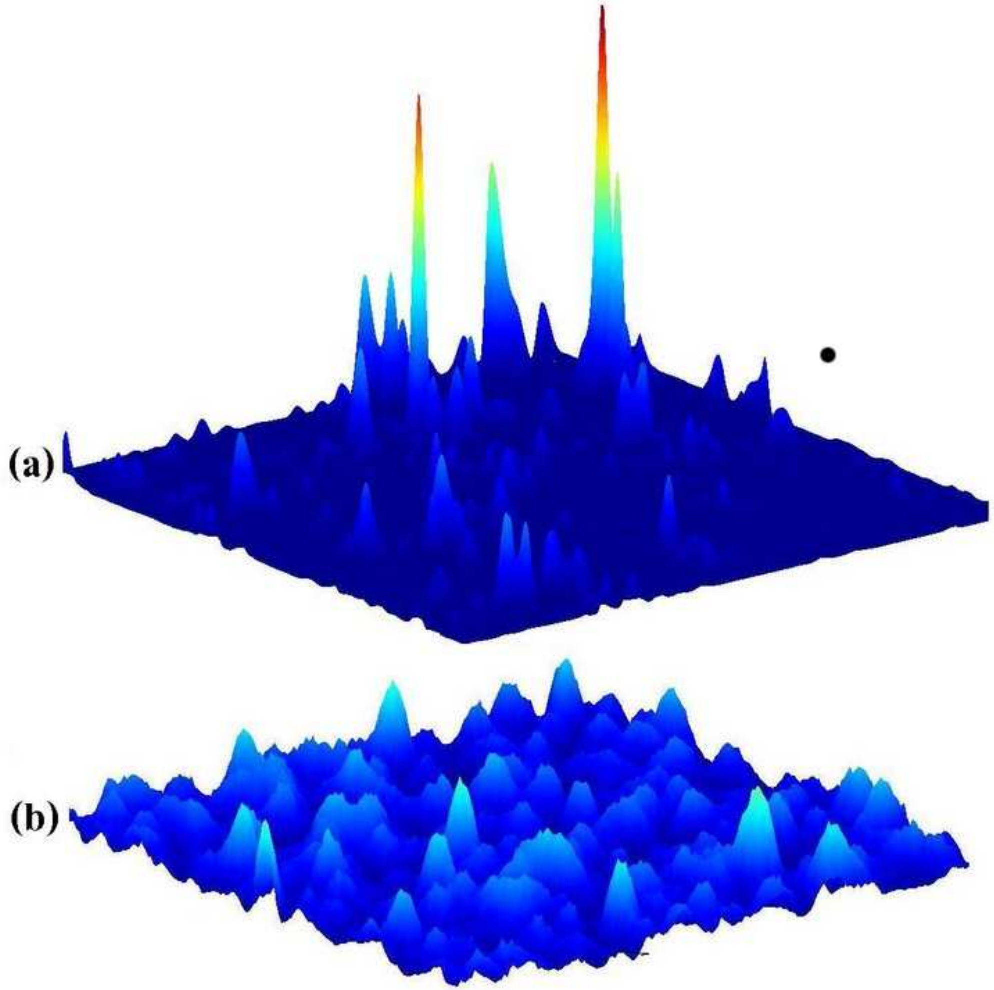
**Richardella,...,Yazdani, Science '10**

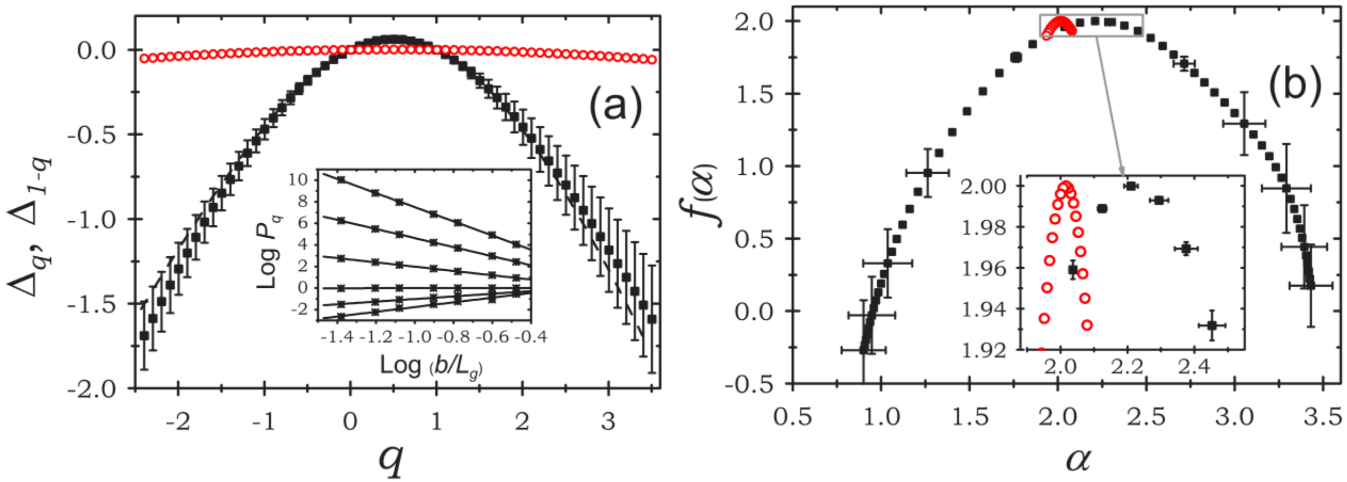


# Multifractality: Experiment II

**Ultrasound speckle in a system of randomly packed Al beads**

Faez, Strybulevich, Page, Lagendijk, van Tiggelen, PRL'09

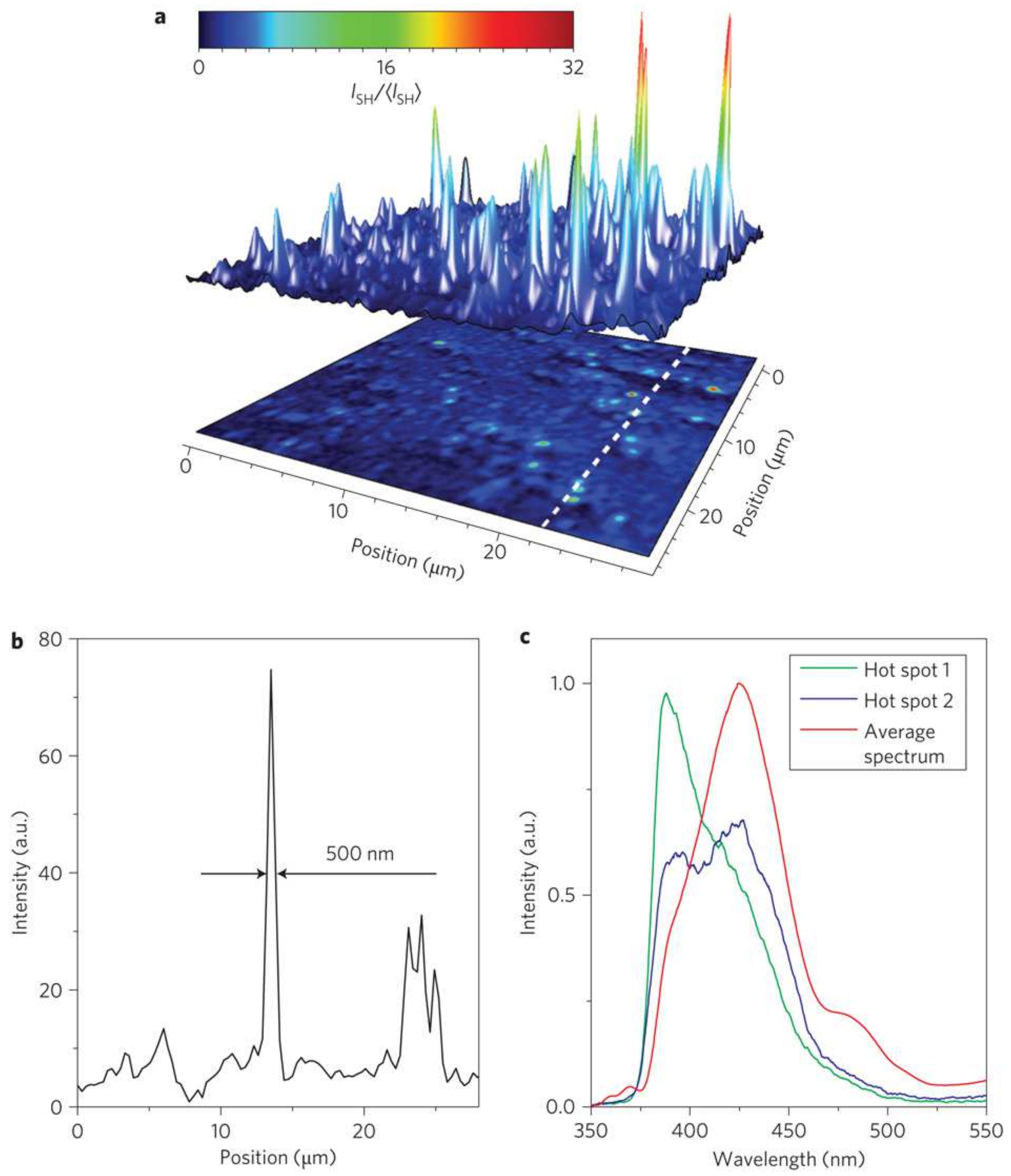


# Multifractality: Experiment III

**Localization of light in an array of dielectric nano-needles**

Mascheck et al, Nature Photonics '12



# Disordered electronic systems: Symmetry classification

Altland, Zirnbauer '97

## Conventional (Wigner-Dyson) classes

| | T | spin rot. | symbol |
|---|---|---|---|
| GOE | + | + | AI |
| GUE | − | +/− | A |
| GSE | + | − | AII |

## Chiral classes

| | T | spin rot. | symbol |
|---|---|---|---|
| ChOE | + | + | BDI |
| ChUE | − | +/− | AIII |
| ChSE | + | − | CII |

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{0} & \mathbf{t} \\ \mathbf{t}^\dagger & \mathbf{0} \end{pmatrix}$$

## Bogoliubov-de Gennes classes

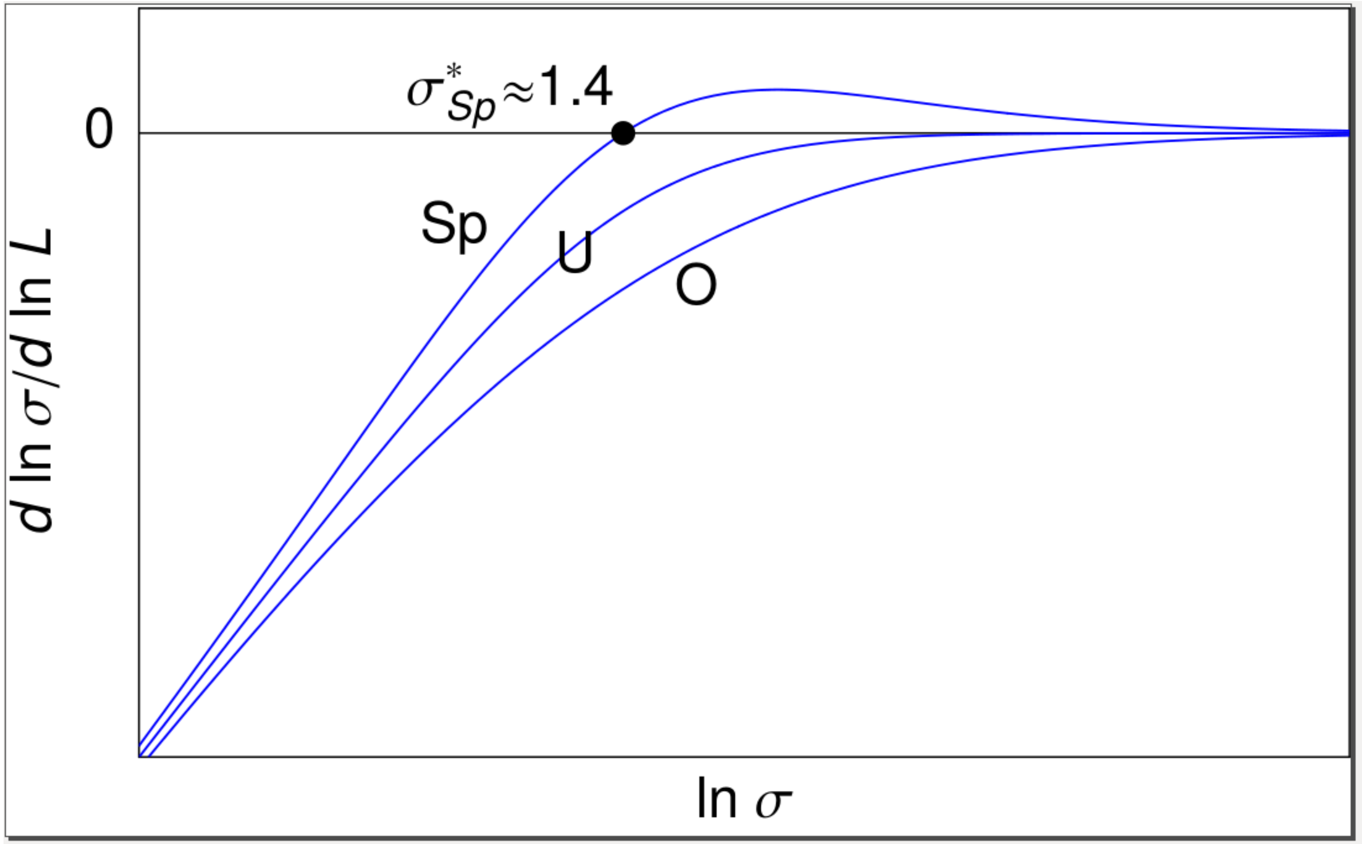
| | T | spin rot. | symbol |
|---|---|---|---|
| | + | + | CI |
| | − | + | C |
| | + | − | DIII |
| | − | − | D |

$$\boldsymbol{H} = \begin{pmatrix} \mathbf{h} & \boldsymbol{\Delta} \\ -\boldsymbol{\Delta}^* & -\mathbf{h}^T \end{pmatrix}$$

# Disordered electronic systems: Symmetry classification

| Ham. class | RMT | T | S | compact symmetric space | non-compact symmetric space | $\sigma$-model B\|F | $\sigma$-model compact sector $\mathcal{M}_F$ |
|---|---|---|---|---|---|---|---|
| **Wigner-Dyson classes** | | | | | | | |
| A | GUE | $-$ | $\pm$ | $\mathrm{U}(N)$ | $\mathrm{GL}(N,\mathbb{C})/\mathrm{U}(N)$ | AIII\|AIII | $\mathrm{U}(2n)/\mathrm{U}(n)\times\mathrm{U}(n)$ |
| AI | GOE | $+$ | $+$ | $\mathrm{U}(N)/\mathrm{O}(N)$ | $\mathrm{GL}(N,\mathbb{R})/\mathrm{O}(N)$ | BDI\|CII | $\mathrm{Sp}(4n)/\mathrm{Sp}(2n)\times\mathrm{Sp}(2n)$ |
| AII | GSE | $+$ | $-$ | $\mathrm{U}(2N)/\mathrm{Sp}(2N)$ | $\mathrm{U}^*(2N)/\mathrm{Sp}(2N)$ | CII\|BDI | $\mathrm{O}(2n)/\mathrm{O}(n)\times\mathrm{O}(n)$ |
| **chiral classes** | | | | | | | |
| AIII | chGUE | $-$ | $\pm$ | $\mathrm{U}(p+q)/\mathrm{U}(p)\times\mathrm{U}(q)$ | $\mathrm{U}(p,q)/\mathrm{U}(p)\times\mathrm{U}(q)$ | A\|A | $\mathrm{U}(n)$ |
| BDI | chGOE | $+$ | $+$ | $\mathrm{SO}(p+q)/\mathrm{SO}(p)\times\mathrm{SO}(q)$ | $\mathrm{SO}(p,q)/\mathrm{SO}(p)\times\mathrm{SO}(q)$ | AI\|AII | $\mathrm{U}(2n)/\mathrm{Sp}(2n)$ |
| CII | chGSE | $+$ | $-$ | $\mathrm{Sp}(2p+2q)/\mathrm{Sp}(2p)\times\mathrm{Sp}(2q)$ | $\mathrm{Sp}(2p,2q)/\mathrm{Sp}(2p)\times\mathrm{Sp}(2q)$ | AII\|AI | $\mathrm{U}(n)/\mathrm{O}(n)$ |
| **Bogoliubov - de Gennes classes** | | | | | | | |
| C | | $-$ | $+$ | $\mathrm{Sp}(2N)$ | $\mathrm{Sp}(2N,\mathbb{C})/\mathrm{Sp}(2N)$ | DIII\|CI | $\mathrm{Sp}(2n)/\mathrm{U}(n)$ |
| CI | | $+$ | $+$ | $\mathrm{Sp}(2N)/\mathrm{U}(N)$ | $\mathrm{Sp}(2N,\mathbb{R})/\mathrm{U}(N)$ | D\|C | $\mathrm{Sp}(2n)$ |
| BD | | $-$ | $-$ | $\mathrm{SO}(N)$ | $\mathrm{SO}(N,\mathbb{C})/\mathrm{SO}(N)$ | CI\|DIII | $\mathrm{O}(2n)/\mathrm{U}(n)$ |
| DIII | | $+$ | $-$ | $\mathrm{SO}(2N)/\mathrm{U}(N)$ | $\mathrm{SO}^*(2N)/\mathrm{U}(N)$ | C\|D | $\mathrm{O}(n)$ |

# Role of symmetry: 2D systems of Wigner-Dyson classes




**Orthogonal and Unitary:** localization;
parametrically different localization length: $\xi_U \gg \xi_O$

**Symplectic:** metal-insulator transition

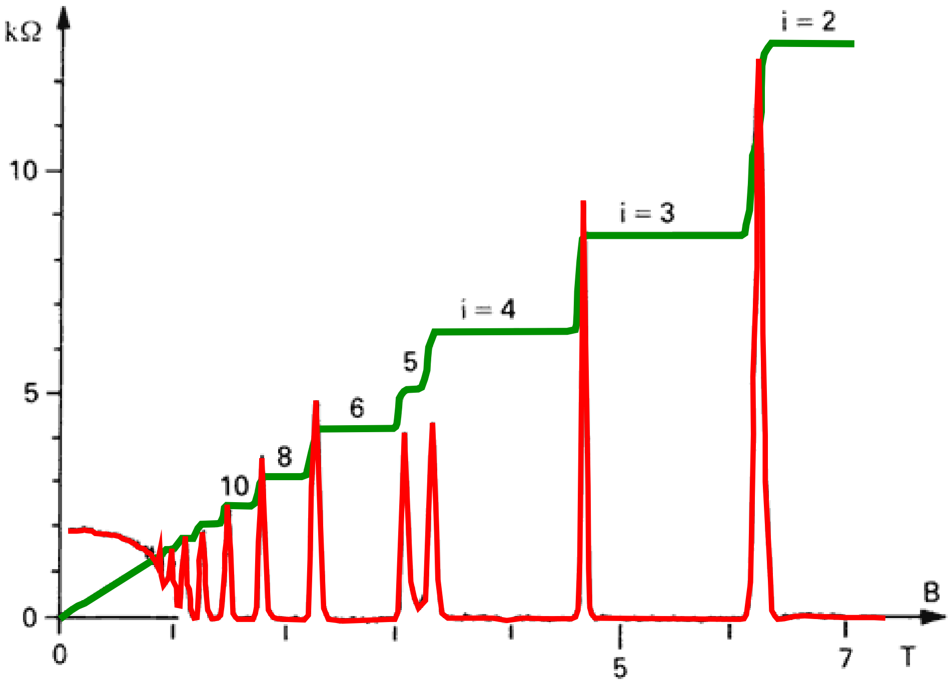
Usual realization of Sp class: spin-orbit interaction

**Symmetry alone is not always sufficient to characterize the system.**

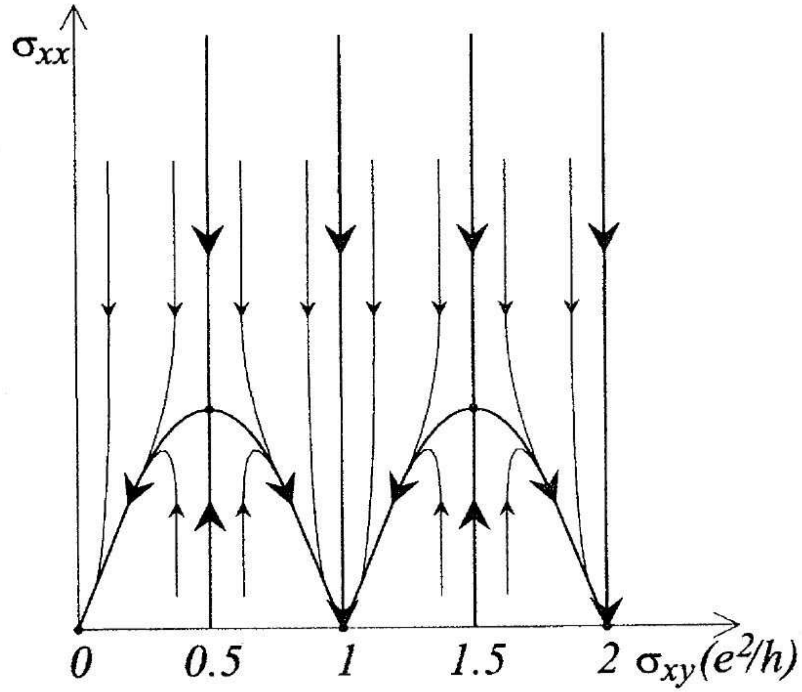
**There may be also a non-trivial topology.**

**It may protect the system from localization.**

# Integer quantum Hall effect

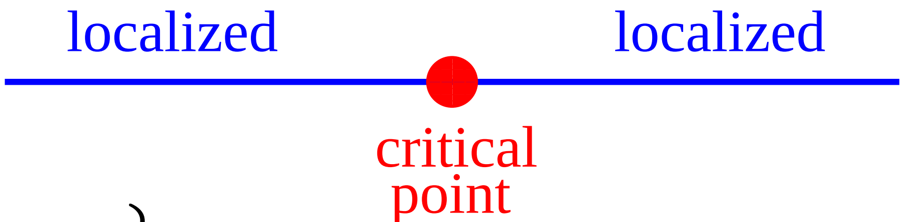



von Klitzing ’80 ; Nobel Prize ’85




**IQHE flow diagram**

**Khmelnitskii’ 83, Pruisken’ 84**




**Field theory (Pruisken):**

## $\sigma$-model with topological term

$$S = \int d^2r \left\{ -\frac{\sigma_{xx}}{8} \mathrm{Tr}(\partial_\mu Q)^2 + \frac{\sigma_{xy}}{8} \mathrm{Tr}\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q \right\}$$

**QH insulators** $\longrightarrow$ $n = \ldots, -2, -1, 0, 1, 2, \ldots$ **protected edge states**

$\longrightarrow$ $\mathbb{Z}$ **topological insulator**

# Periodic table of Topological Insulators

| | Symmetry classes | | | | Topological insulators | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ | $H_p$ | $R_p$ | $S_p$ | $\pi_0(R_p)$ | d=1 | d=2 | d=3 | d=4 |
| 0 | AI | BDI | CII | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ |
| 1 | BDI | BD | AII | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 | 0 |
| 2 | BD | DIII | DIII | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 |
| 3 | DIII | AII | BD | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 |
| 4 | AII | CII | BDI | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ |
| 5 | CII | C | AI | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| 6 | C | CI | CI | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ |
| 7 | CI | AI | C | 0 | 0 | 0 | $\mathbb{Z}$ | 0 |
| $0'$ | A | AIII | AIII | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ |
| $1'$ | AIII | A | A | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 |

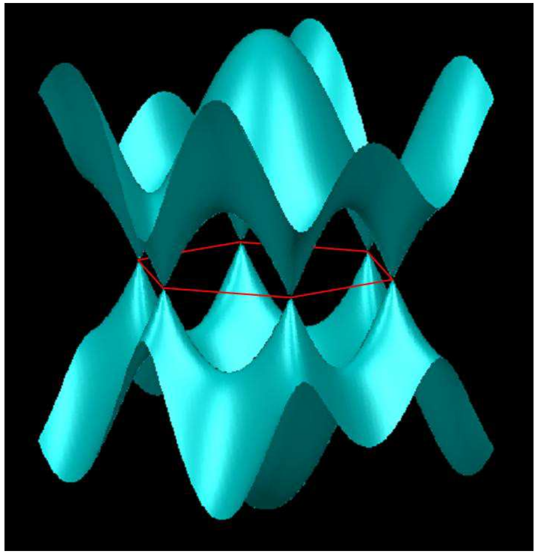
$H_p$ – symmetry class of Hamiltonians

$R_p$ – sym. class of classifying space (of Hamiltonians with eigenvalues $\to \pm 1$)

$S_p$ – symmetry class of compact sector of $\sigma$-model manifold

Kitaev'09; Schnyder, Ryu, Furusaki, Ludwig'09; Ostrovsky, Gornyi, ADM'10

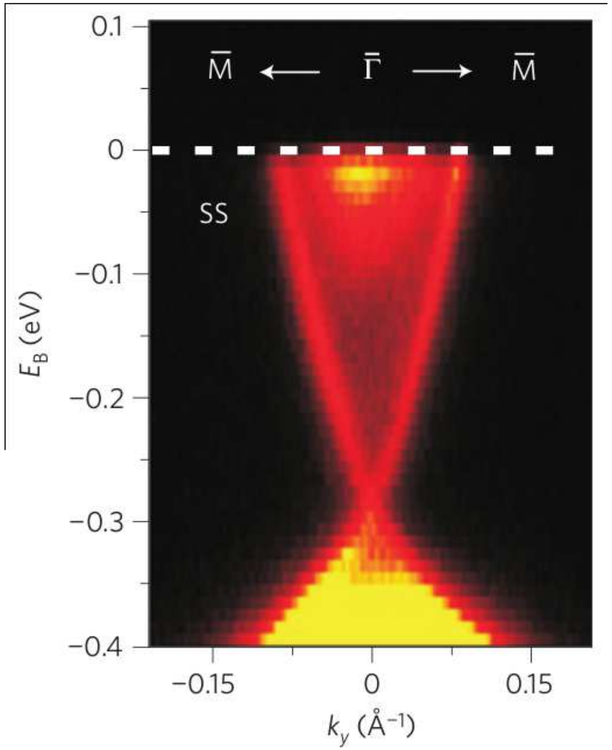
# 2D massless Dirac fermions



Graphene

Geim, Novoselov'04

Nobel Prize'10




Surface of 3D topological insulators

BiSb, BiSe, BiTe Hasan group '08

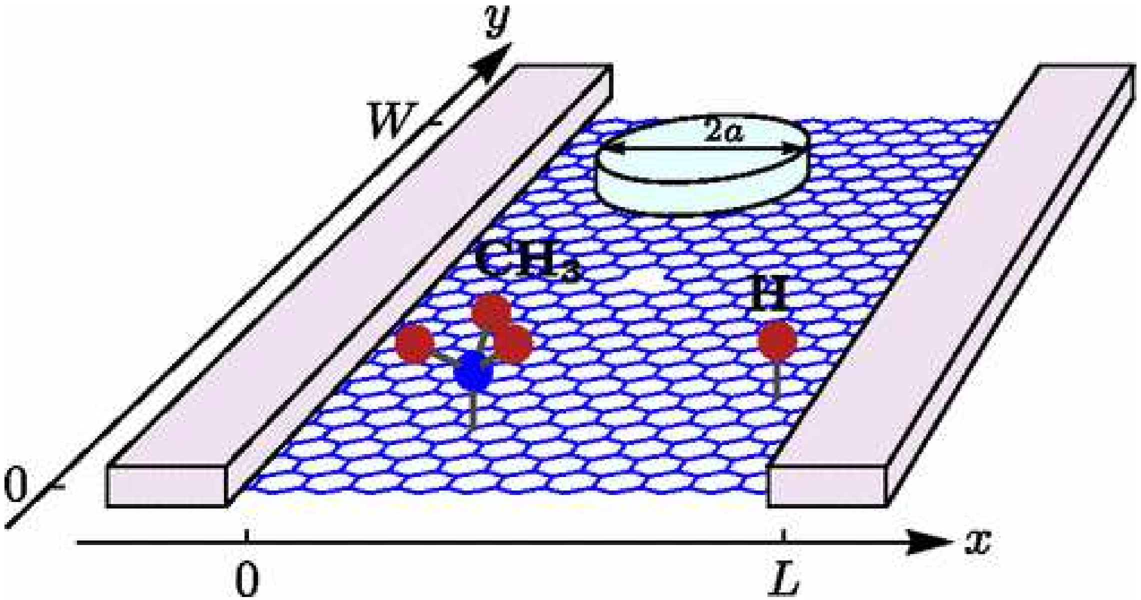
$\sigma$-model field theory with a topological term

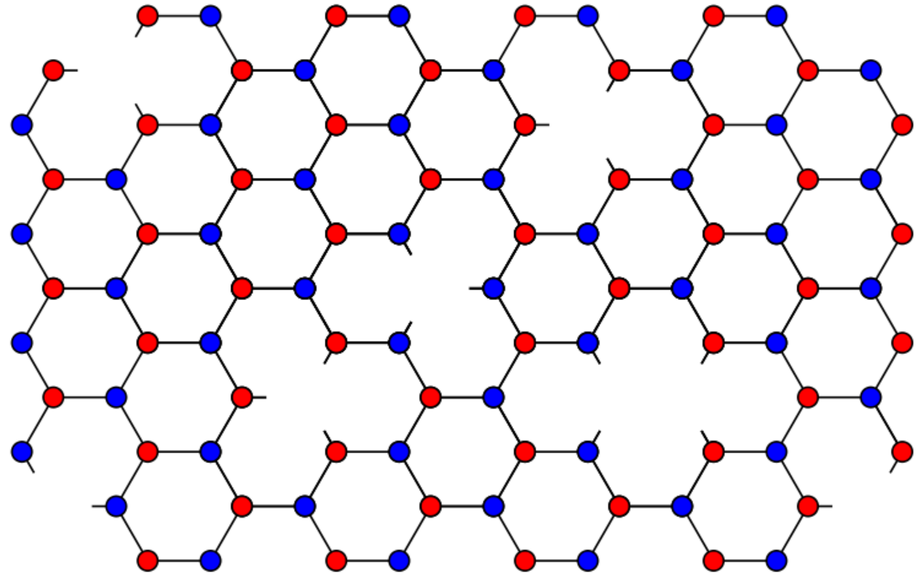
Ostrovsky, Gornyi, ADM '07

- Graphene: long-range disorder (no valley mixing)
- Surface states of 3D TI: no restriction on disorder range

# Role of symmetry and topology: Graphene at the Dirac point

Ostrovsky et al, PRL'10; Gattenlöhner et al, PRL'14



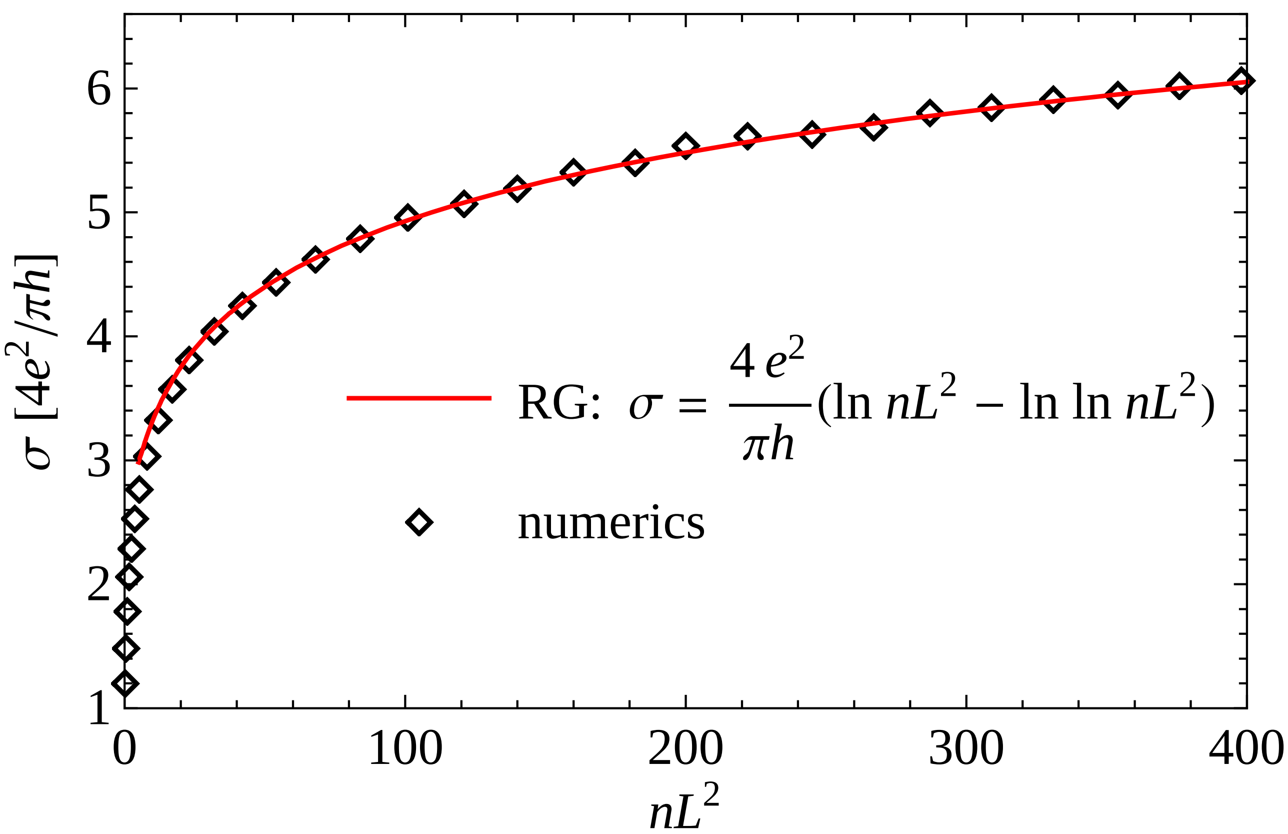



Models of scatterers:

- scalar impurity: smooth on atomic scale (no valley mixing)
- resonant scalar impurity: diverging scattering length, quasi-bound state at the Dirac point
- adatom: on-site potential (valley mixing)
- vacancy: infinitely strong on-site potential

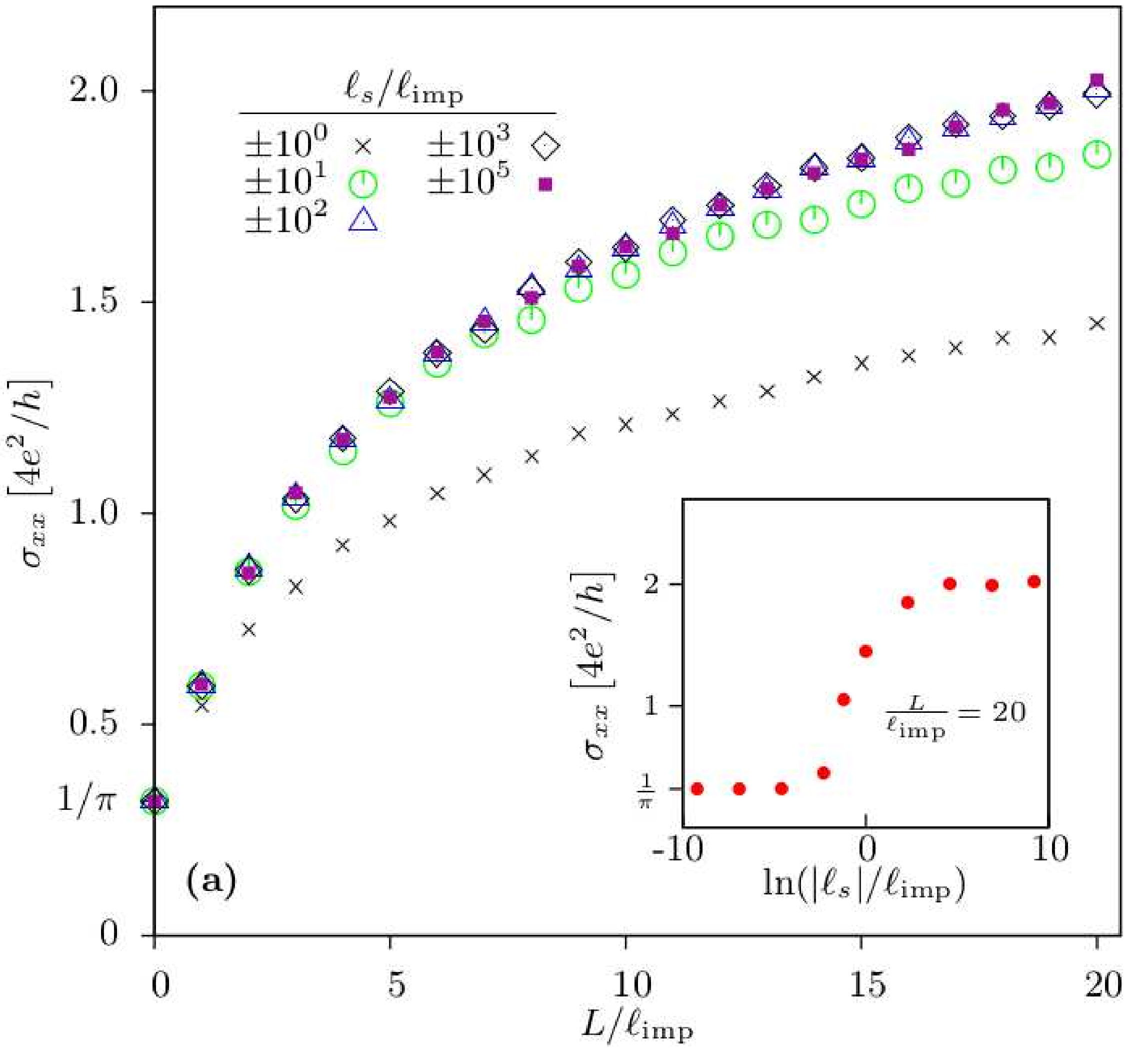
# Resonant scalar impurities ($l_s = \infty$)

**Ostrovsky, Titov, Bera, Gornyi, ADM, PRL (2010)**




- **flow towards supermetal** $\boldsymbol{\sigma \to \infty}$
- **agreement with** $\boldsymbol{\sigma}$ **model RG**

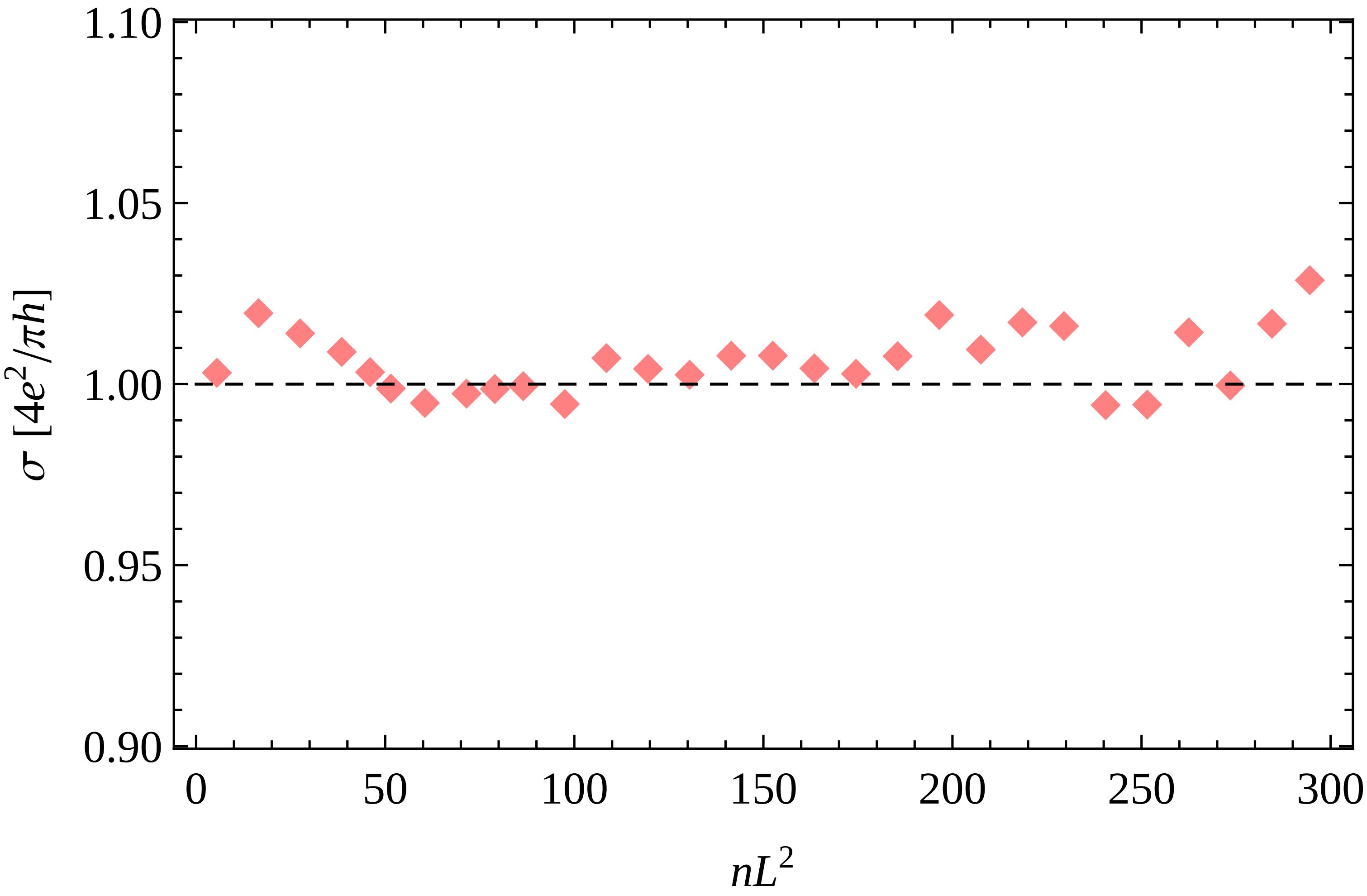
# Scalar impurities (finite $l_s$, random sign)




**Large $l_s$ ⟶ Symmetry breaking pattern:**

**DIII (with WZ term) ⟶ AII (with $\mathbb{Z}_2$ $\theta$-term)**

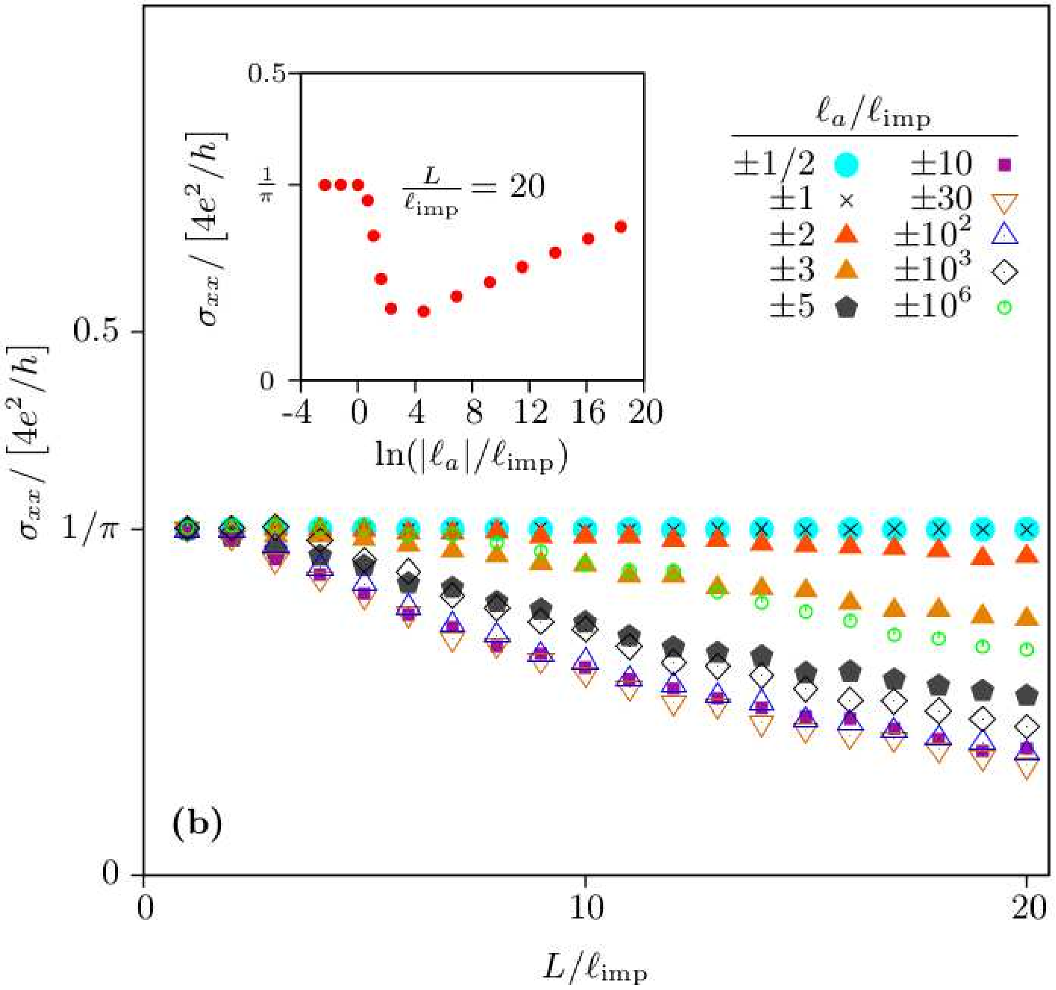
# Vacancies




symmetry class BDI (chiral orthogonal)

No localization, $\boldsymbol{\sigma \to \mathrm{const} \simeq \frac{4}{\pi}\frac{e^2}{h}}$

# Adatoms (finite $l_a$, random sign)

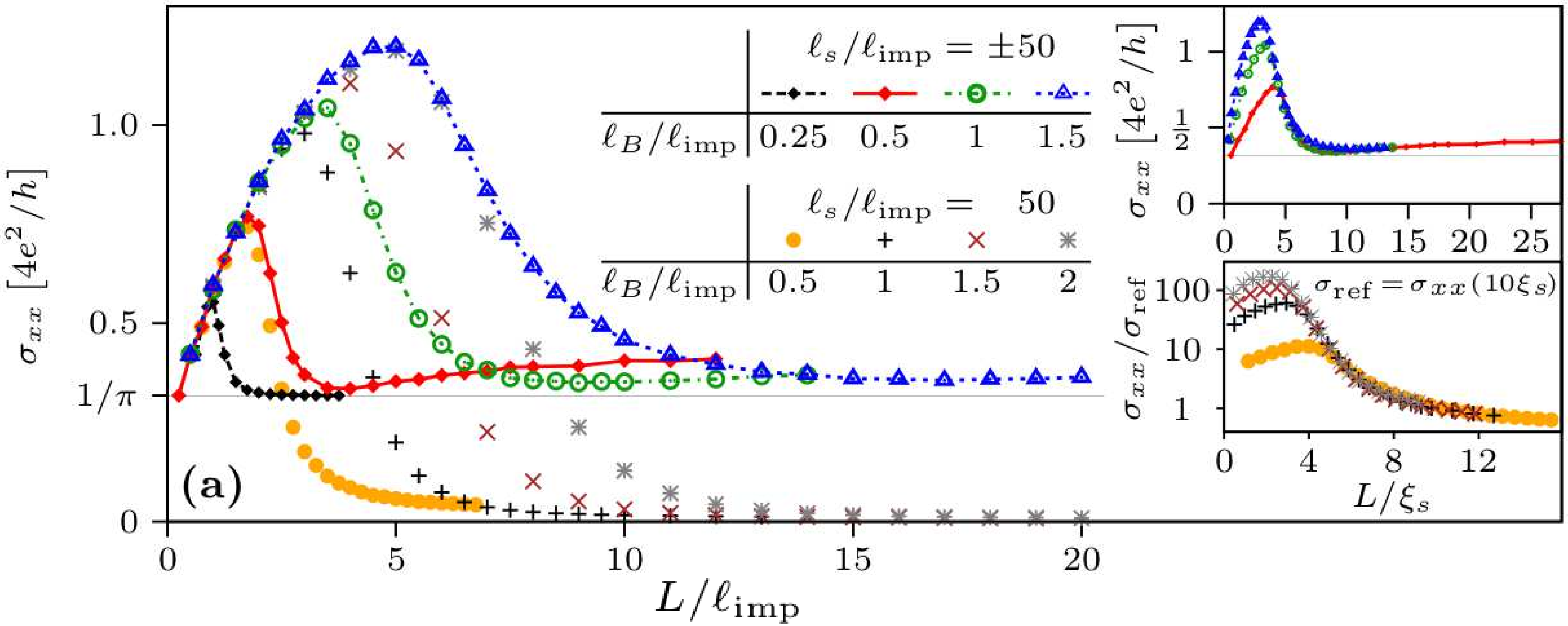



**Large $l_a$ $\longrightarrow$ Symmetry breaking pattern: BDI $\longrightarrow$ AI**

**Vacancies ($l_a \to \infty$): finite conductivity $\sigma \simeq \frac{4}{\pi}\frac{e^2}{h}$ for $L \to \infty$**

**Localization length $\xi$ – non-monotonous function of $l_a$**

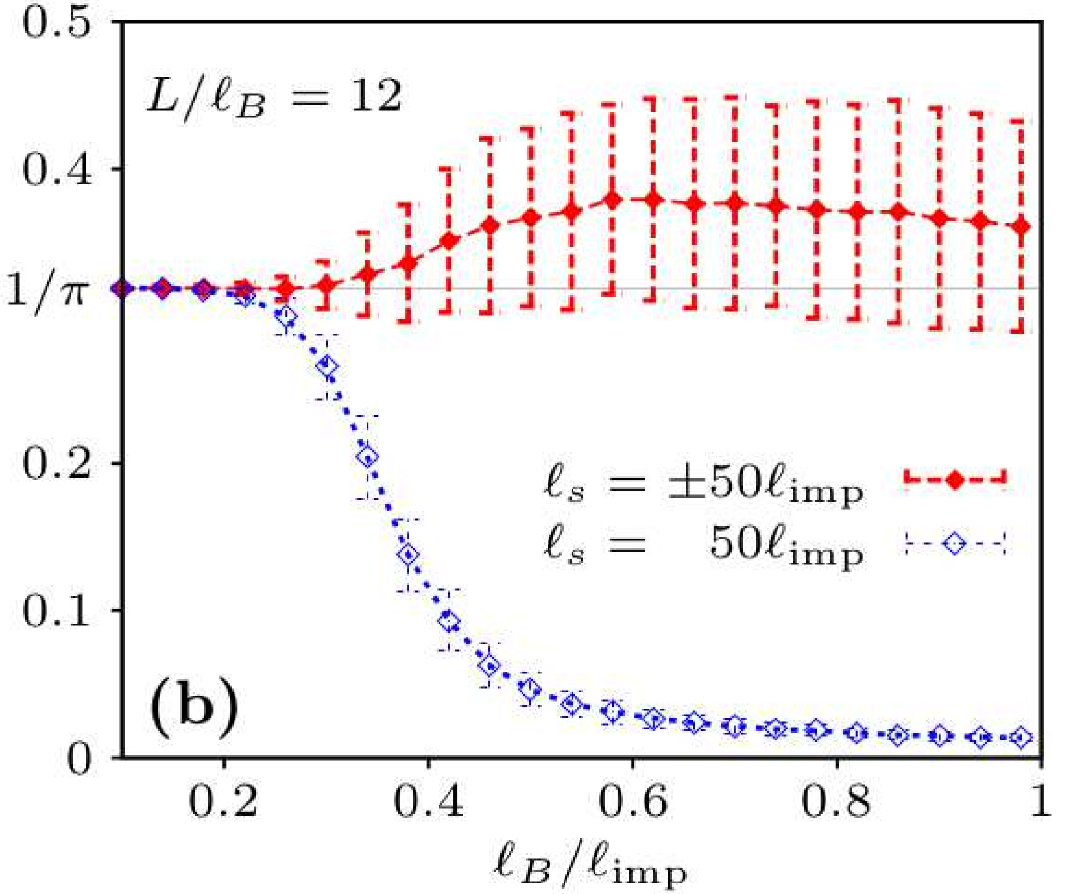
# Scalar impurities in magnetic field $B$

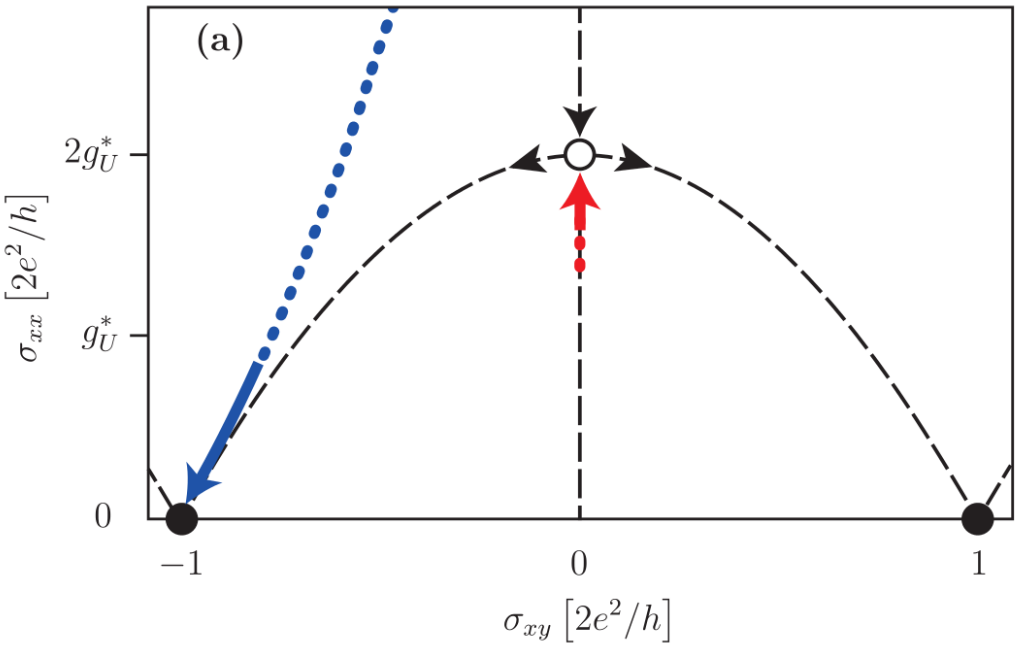



**Symmetry breaking pattern:** DIII → AII → A for weaker B and DIII → AIII → A for stronger B

**Ultimate fixed points:** Quantum Hall criticality (random sign of impurity potentials) and localization (fixed sign)

# Scalar impurities in magnetic field $B$

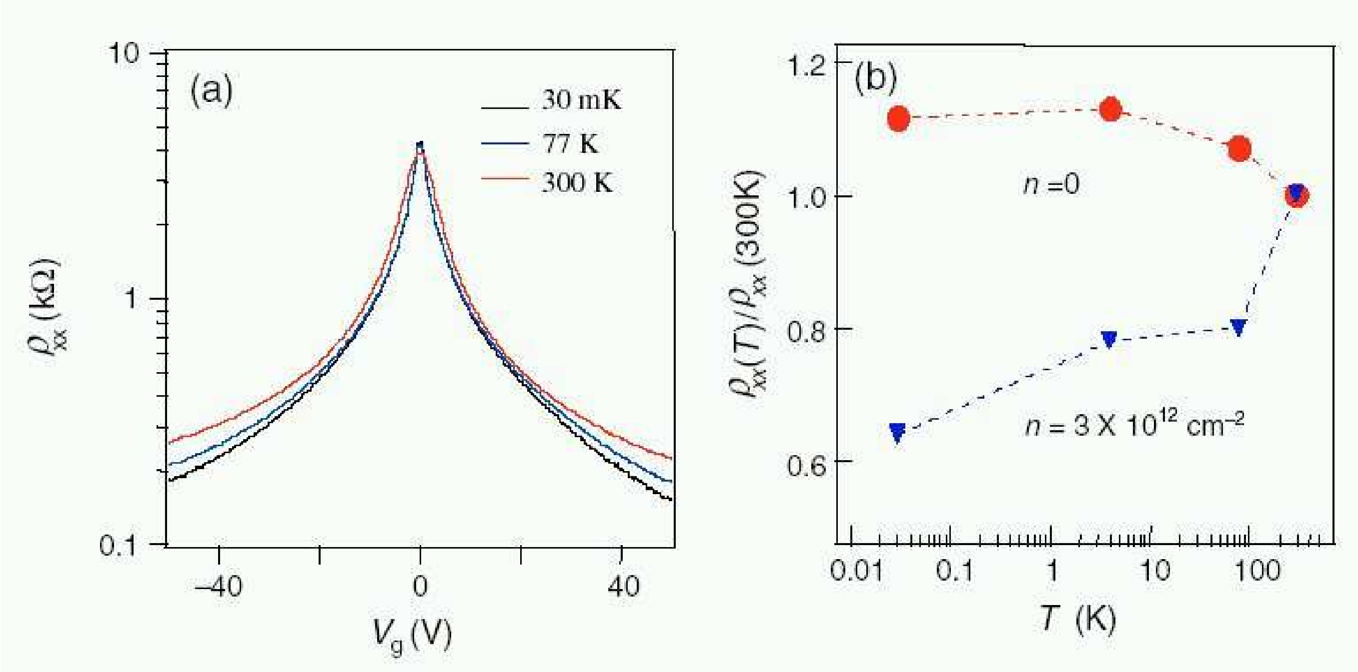


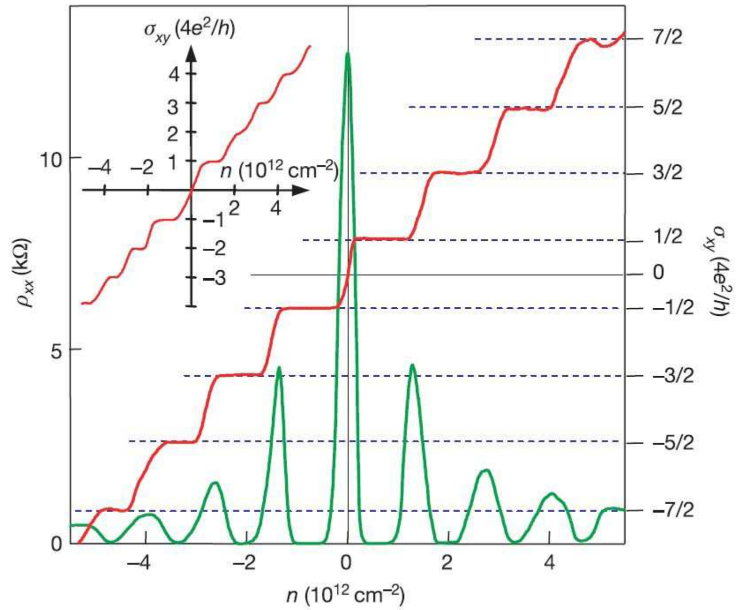




**Ultimate fixed points:** **Quantum Hall criticality** **(random sign of impurity potentials) and** **localization** **(fixed sign)**

**Vertical bars: mesoscopic fluctuations**

# Graphene: Experiments    Geim-Novoselov and Kim groups




Topological terms explain unconventional properties of high-quality graphene samples:

- absence of localization at Dirac point down to very low temperatures (30 mK), although conductivity $\simeq e^2/h$ per spin per valley

- anomalous QHE: $\sigma_{xy} = (2n+1) \times 2e^2/h$;

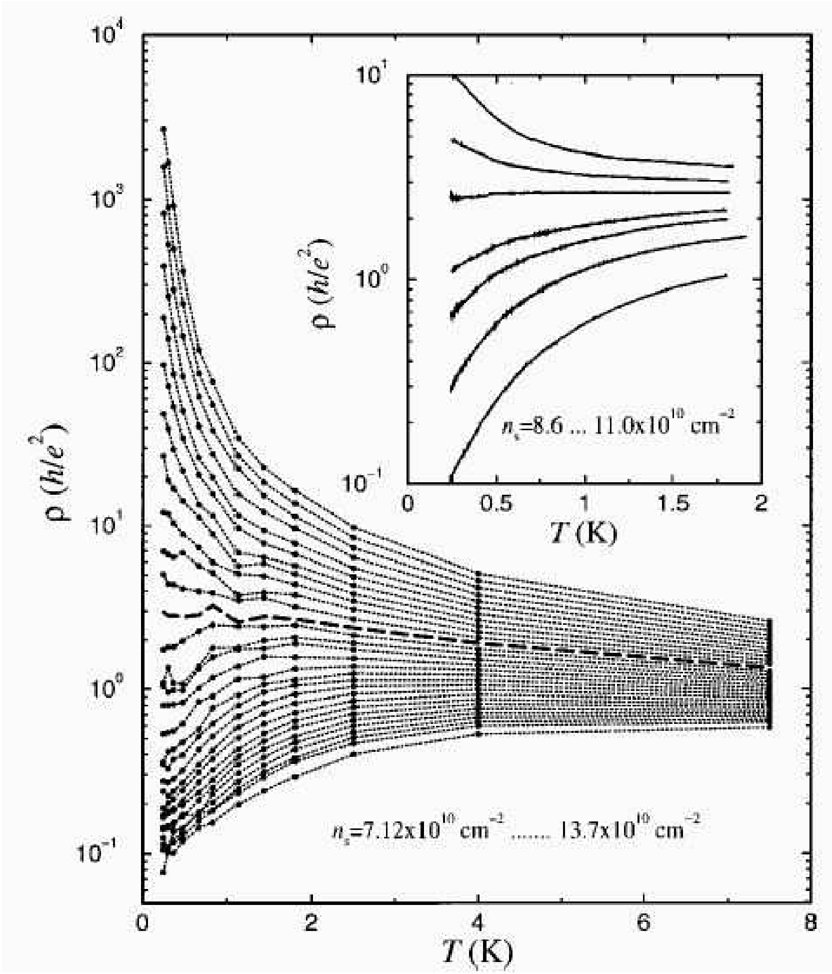
QHE transition at $n = 0$ (Dirac point), i.e. at $\sigma_{xy} = 0$

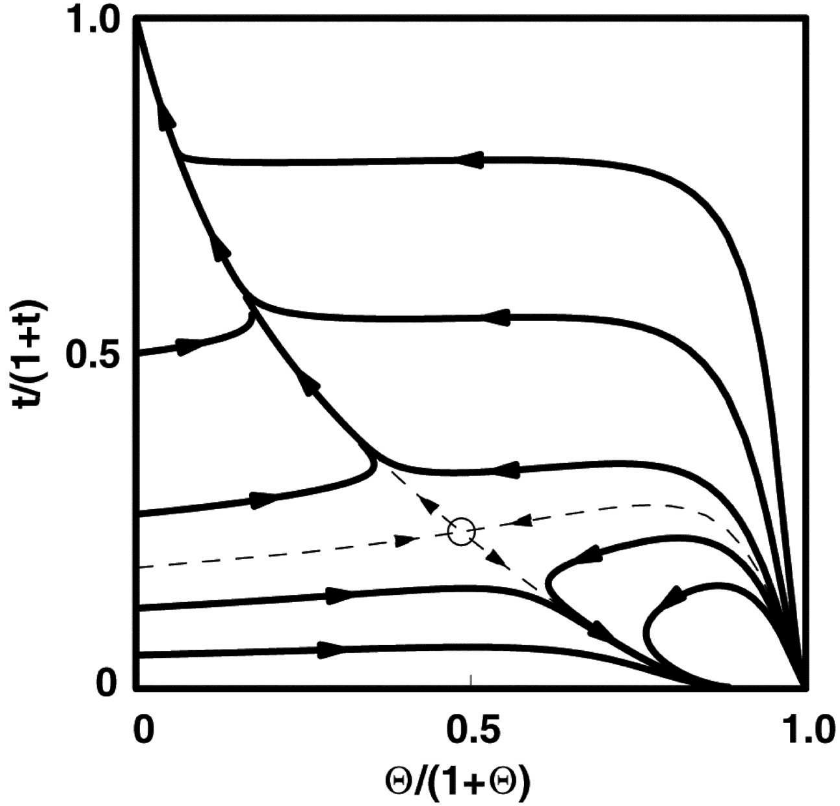
# Electron-electron interaction

**E-e interaction can be incorporated within the same general theoretical framework ($\sigma$ model);**

**in some cases essentially modifies localization properties**

# MIT in a 2D gas with strong interaction




Kravchenko et al '94, . . .

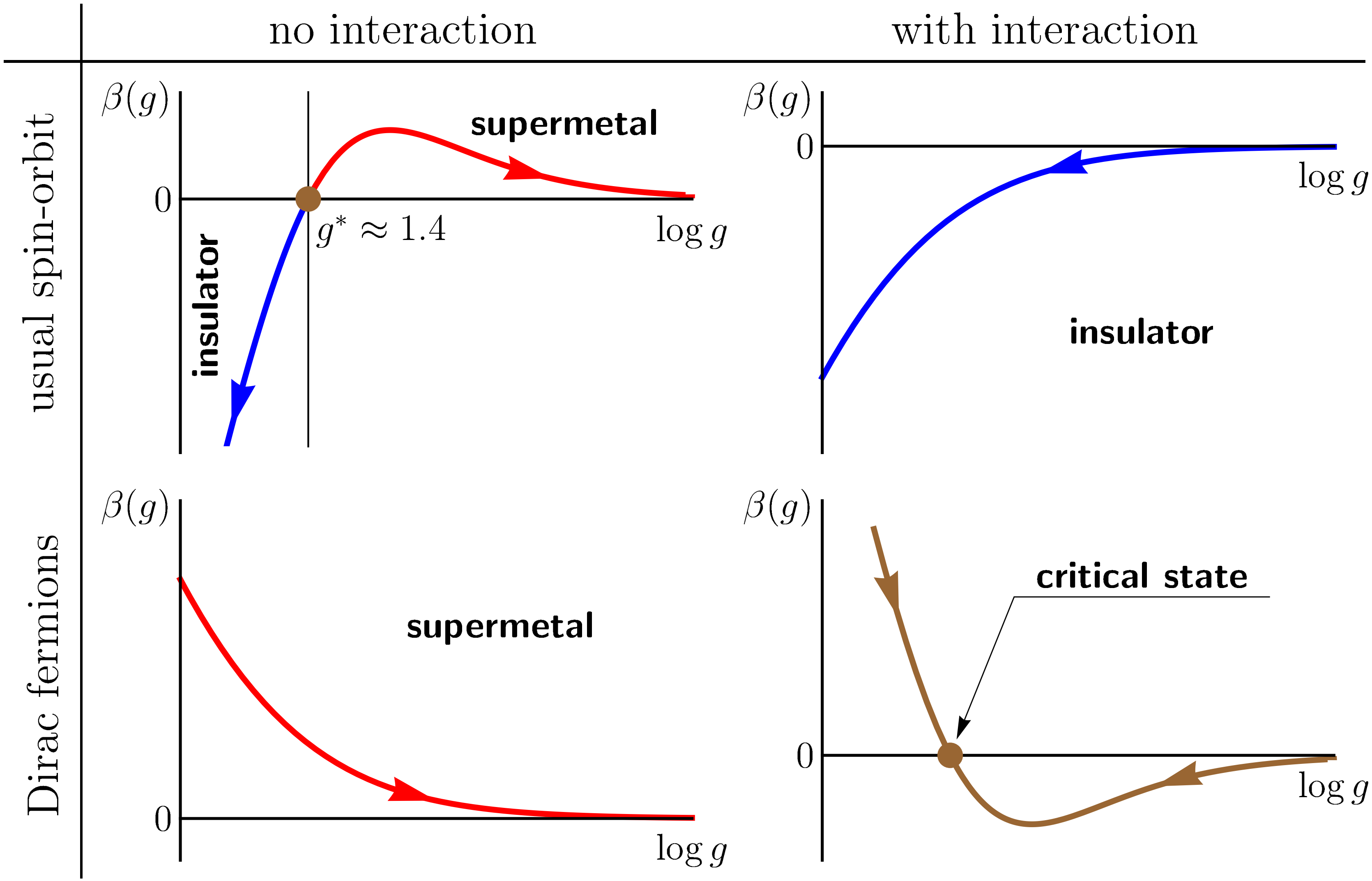
Finkelstein '83;

Punnoose, Finkelstein, '02-05

(number of valleys $N \gg 1$;

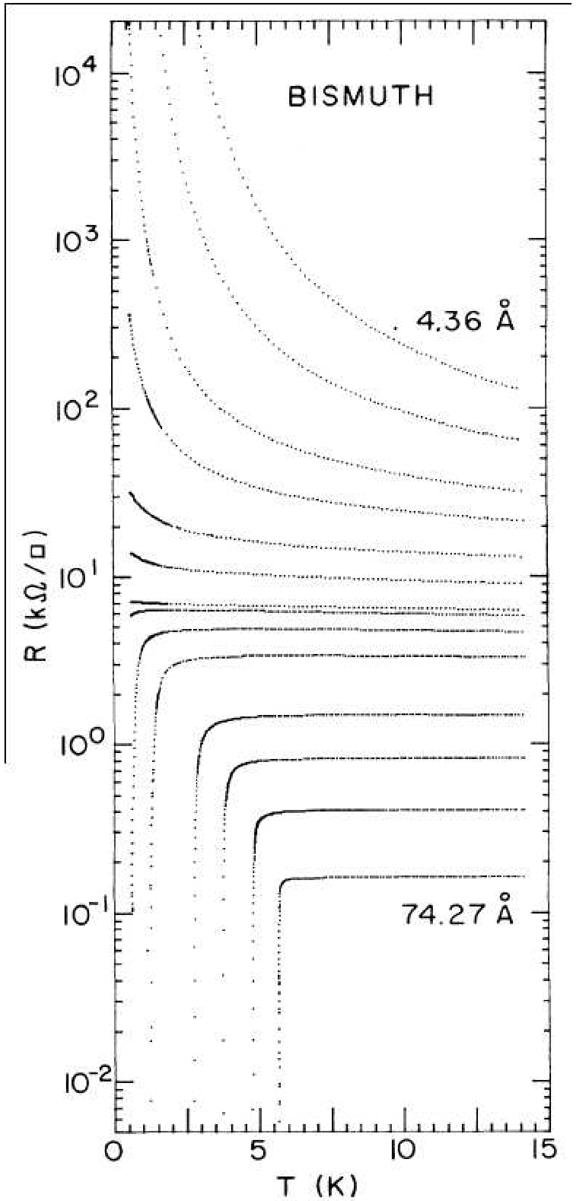
in practice, $N = 2$ sufficient)

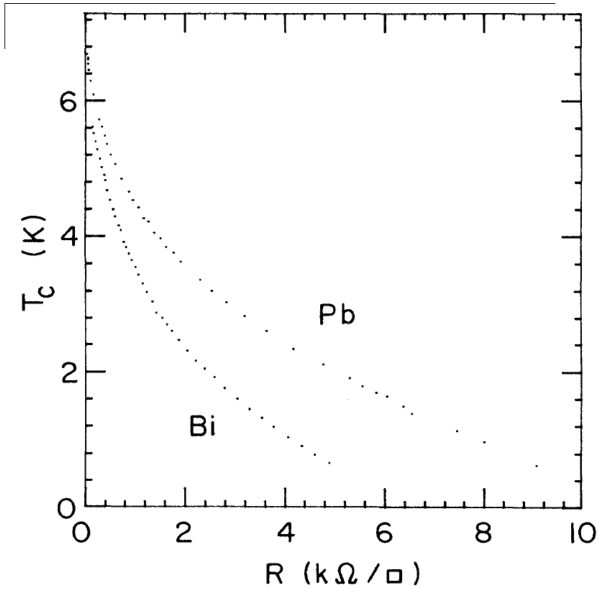
# Localization behavior in 2D Symplectic class: Effects of symmetry, topology, and Coulomb interaction



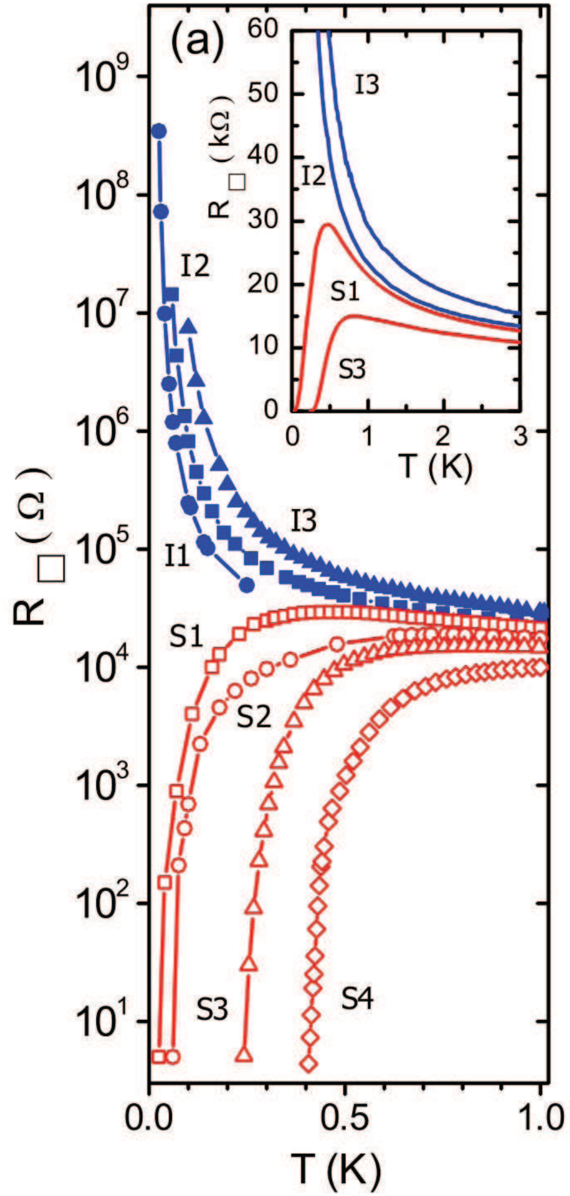

**Ostrovsky, Gornyi, ADM, PRL '10**

# Superconductor-Insulator Transition




**Suppression of $T_c$ by disorder**




Haviland, Liu, Goldman, PRL'89

Bi and Pb films

Baturina et al, PRL'07

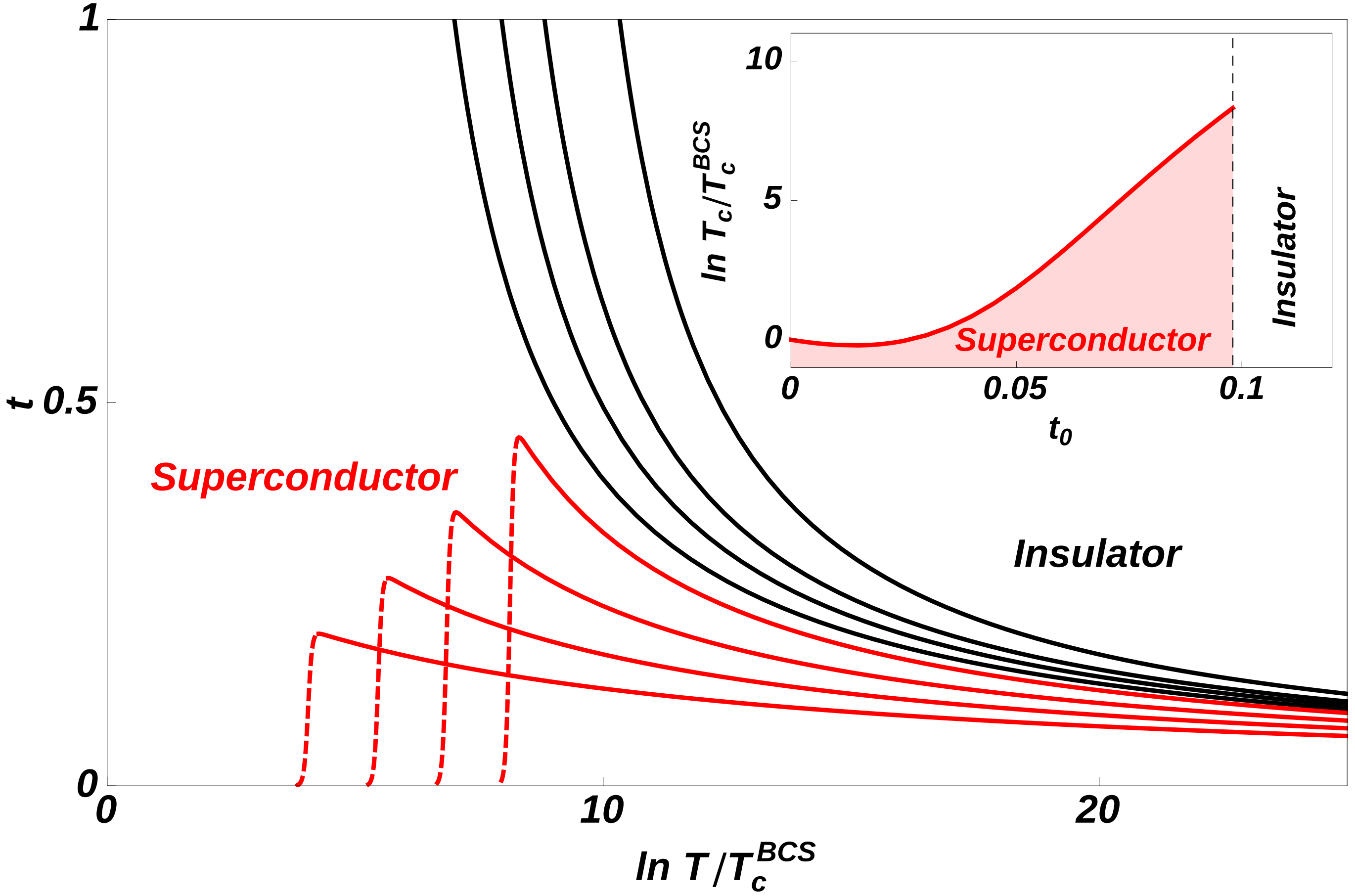
TiN films

# Enhancement of superconductivity by Anderson localization: interplay of interaction and multifractality

## short-range interaction

Feigelman et al, PRL '07, Ann. Phys.'10: **near 3D Anderson transition**

Burmistrov, Gornyi, ADM, PRL '12: **2D films**



**International group on Localization, Interactions, and Superconductivity**

**Landau Institute for Theoretical Physics**

**RSF grant**
**"Superconductor-insulator and metal-insulator transitions in interacting disordered electronic systems"** **2014-2016**

**Workshop 22-25 December 2014, Chernogolovka**

# Summary

- **Anderson localization: basic properties, field theory**
- **Wave function multifractality**
- **Symmetries of disordered systems**
- **Manifestations of topology in localization theory**
- **Influence of electron-electron interaction**

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